# Between politics and administration : compliance with EU Law in Central and Eastern Europe

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# APPENDIX II
# FORMALIZING THE THEORETICAL MODEL

## II.1 Deriving the solution of the constrained optimization problem

The objective is to minimize:

$$f(x, y) = \sqrt{(x - x_0)^2 + (y - y_0)^2},$$

given the constraint:

$$y = (a + sx_1) - sx.$$

Define the function:

$$g(x, y) = y - (a + sx_1) + sx.$$

Let

$$\Lambda\,(x, y, \lambda) = f(x, y) + \lambda g(x, y) = \sqrt{(x - x_0)^2 + (y - y_0)^2} + \lambda(y - (a + sx_1) + sx).$$

The critical values of Λ occur when its gradient is zero. The partial derivatives are:

$$\frac{\partial\Lambda}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} + \lambda s = 0,$$
$$\frac{\partial\Lambda}{\partial y} = \frac{y}{\sqrt{x^2 + y^2}} + \lambda = 0,$$
$$\frac{\partial\Lambda}{\partial\lambda} = y - (a + sx_1) + sx = 0.$$

The first equation implies that:

$$\lambda = -\frac{x}{s\sqrt{x^2 + y^2}}.$$

Substituting this in the second equation implies:

$$\frac{y}{\sqrt{x^2 + y^2}} - \frac{x}{s\sqrt{x^2 + y^2}} = 0,$$

which simplifies to:

$$ys = x.$$

Substituting in the third equation:

$$y - (a + sx_1) + ys^2 = 0.$$

Solving for y:

$$y = \frac{a + sx_1}{s^2 + 1},$$

and then:

$$x = \frac{s\,(a + sx_1)}{s^2 + 1}.$$

Since the policy-making constraint changes direction at $x_1$, if

$$x \leq sa,$$

then

$$y = a$$

The second constraint demands that

$$(x_1 - d) \leq x \leq x_1$$

The value of y at $(x_1 - d)$ is $(a + sd)$. Substituting, we get:

$$x_1 > \frac{(a + sd)(s^2 + 1) - a}{s},$$

which simplifies to:

$$x_1 > s(a + sd) + d.$$

Thus, for values of $x_1$ greater than

$$x_1 > s(a + sd) + d,$$

the solution for x is

$$x = x_1 - d.$$

## II.2 Proof of Hypotheses 3a and 3b

Taking the derivative of y with respect to s:

$$\frac{\Delta y}{\Delta s} = \frac{2s(a + sx_1) - (s^2 + 1)x_1}{(s^2 + 1)^2} = \frac{x_1 s^2 + 2as - x_1}{(s^2 + 1)^2}.$$

Taking the derivative of x with respect to s:

$$\frac{\Delta x}{\Delta s} = \frac{2s(sa + s^2 x_1) - (s^2 + 1)(a + 2sx_1)}{(s^2 + 1)^2} = \frac{as^2 - 2x_1 s - a}{(s^2 + 1)^2}.$$

In order to find the local maximum we set the first derivative of y with respect to s to 0:

$$\frac{x_1 s^2 + 2as - x_1}{(s^2 + 1)^2} = 0.$$

Then:

$$s = \frac{-2a \pm \sqrt{(2a)^2 + 4{x_1}^2}}{2x_1} = \frac{-a \pm \sqrt{a^2 + {x_1}^2}}{x_1}.$$

Since we are interested only in the cases in which $s > 0$, the only solution is:

$$s = \frac{-a + \sqrt{a^2 + x_1^2}}{x_1}$$

At this value of *s, y* has a local maximum.

Similarly for x, we set the first derivative of x with respect to s to 0:

$$\frac{as^2 - 2x_1 s - a}{(s^2+1)^2} = 0.$$

Then:

$$s = \frac{2x_1 \pm \sqrt{(-2x_1)^2 + 4a^2}}{2a} = \frac{x_1 \pm \sqrt{x_1^2 + a^2}}{a}.$$

Again, since we are interested only in the cases $s > 0$, the only solution is:

$$s = \frac{x_1 + \sqrt{x_1^2 + a^2}}{a}.$$

At this value of *s, x* has a local maximum.

## II.3 Proof of Hypotheses 4a and 4b

Lets redefine the utility function:

$$u(x, y) = -\sqrt{(x - x_0)^2 + w(y - y_0)^2},$$

and the function to be minimized:

$$f(x, y) = \sqrt{(x - x_0)^2 + w(y - y_0)^2}.$$

It follows that:

$$y = \frac{a + sx_1}{w(s^2+1)}.$$

Solving for x:

$$x = \frac{s\,(a + sx_1)}{s^2+1} * \sqrt{1 + w - \frac{1}{w^2}}.$$

The first derivative of y with respect to w is:

$$\frac{\Delta y}{\Delta w} = -\frac{(s^2+1)sx + as^2 + a}{w^2},$$

which is negative, hence the function is decreasing.

The first derivative of x with respect to w is:

$$\frac{\Delta x}{\Delta w} = \frac{(w^3+2)|w| \;\; \frac{s\,(a + sx_1)}{s^2+1}}{2w^3\sqrt{(w^3 + w^2 - 1)}},$$

which is positive, hence the function is increasing.