



Universiteit
Leiden
The Netherlands

On the geometry of fracture and frustration

Koning, V.

Citation

Koning, V. (2014, November 26). *On the geometry of fracture and frustration. Casimir PhD Series*. Retrieved from <https://hdl.handle.net/1887/29873>

Version: Not Applicable (or Unknown)

License: [Leiden University Non-exclusive license](#)

Downloaded from: <https://hdl.handle.net/1887/29873>

Note: To cite this publication please use the final published version (if applicable).

Cover Page



Universiteit Leiden



The handle <http://hdl.handle.net/1887/29873> holds various files of this Leiden University dissertation.

Author: Koning, Vinzenz

Title: On the geometry of fracture and frustration

Issue Date: 2014-11-26

ON THE GEOMETRY OF
FRACTURE AND FRUSTRATION

PROEFSCHRIFT

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van Rector Magnificus prof. mr. C.J.J.M. Stolker,
volgens besluit van het College voor Promoties
te verdedigen op woensdag 26 november 2014
klokke 11:15 uur

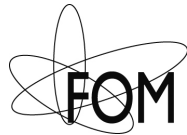
door

VINZENZ KONING

GEBOREN TE HAARLEMMERMEER IN 1988

PROMOTIECOMMISSIE

Promotor: prof. dr. M. L. van Hecke
Copromotor: dr. V. Vitelli
Overige leden: prof. dr. A. Achúcarro
prof. dr. C. W. J. Beenakker
prof. dr. E. R. Eliel
prof. dr. A. Fernandez-Nieves (Georgia Institute
of Technology)
dr. L. Giomi
prof. dr. W. T. M. Irvine (University of Chicago)



This work is part of the research programme of the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organisation for Scientific Research (NWO).

The cover shows a cartoon of twisted director fieldlines in toroidal nematic droplets, originally published in *Soft Matter* 10, 4192-4198 (2014). The bookmark shows crossed-polarised images of nematic double emulsion droplets created by Teresa Lopez-Leon and Alberto Fernandez-Nieves, originally published in *Nature Physics* 7, 391-394 (2011). Cover and bookmark design by Shivendra Shah.

Casimir PhD series, Delft-Leiden 2014-25
ISBN 978-90-8593-198-0

CONTENTS

1	INTRODUCTION	1
1.1	Geometric frustration	1
1.2	Liquid crystals	4
1.3	Differential geometry of surfaces	4
1.3.1	Preliminaries	4
1.3.2	Curvature	6
1.4	Elasticity on curved surfaces and in confined geometries	9
1.4.1	Elasticity of a two-dimensional nematic liquid crystal	9
1.4.2	Elasticity of a two-dimensional solid	11
1.4.3	Elasticity of a three-dimensional nematic liquid crystal	13
1.5	Topological defects	13
1.5.1	Disclinations in a nematic	14
1.5.2	Disclinations in a crystal	15
1.5.3	Dislocations	16
1.6	Interaction between curvature and defects	17
1.6.1	Coupling in liquid crystals	17
1.6.2	Coupling in crystals	19
1.7	Nematic order on the sphere	22
1.8	This thesis	23
1.8.1	Spherical nematic shells (part I)	23
1.8.2	Toroidal nematics (part II)	25
1.8.3	Cracks in curved solids (part III)	27
I	SPHERICAL NEMATICS	29
2	THIN SHELLS	31
2.1	Introduction	31
2.2	Divalent, trivalent and tetravalent shells	32
2.3	Trivalent ground state	36
2.4	Valence transitions	40
2.5	Bond fidelity	42
2.6	Conclusion	44
3	THICK SHELLS	47

3.1	Introduction	47
3.2	Director fields in divalent nematic shells	49
3.2.1	The inverse stereographic projection and the Ansatz for the homogeneous shell	50
3.2.2	An electrostatic analogy and the <i>Ansatz</i> for the inhomogeneous shell	52
3.3	Energetics of homogeneous shells	57
3.4	Energetics of inhomogeneous shells	59
3.4.1	Buoyancy versus elastic forces	59
3.4.2	Confined and deconfined defect configurations	61
3.4.3	Phase diagram	61
3.4.4	Comparison with experiment	63
3.5	Conclusion	66
II	TOROIDAL NEMATICS	69
4	CHIRAL SYMMETRY BREAKING	71
4.1	Introduction	71
4.2	Toroidal director fields	73
4.2.1	Free energy of a nematic toroid	73
4.2.2	Double twist	76
4.3	Chiral symmetry breaking	77
4.3.1	Results for divergence-free field	77
4.3.2	Effects of external fields and cholesteric pitch	79
4.3.3	Results for the two-parameter <i>Ansatz</i>	82
4.4	Comparison with experiment	85
4.5	Conclusions	89
III	FRACTURE OF CURVED SOLIDS	91
5	ONSET OF CRACK GROWTH	93
5.1	Linear elastic fracture mechanics	93
5.2	Cracks in a Gaussian bump	96
5.2.1	Problem formulation	96
5.2.2	The Gaussian bump	97
5.2.3	Decomposing the problem	98
5.2.4	Stress fields in the absence of cracks	99
5.2.5	Results for small center cracks	102
5.3	Conclusions	110
6	CURVED CRACK PATHS	111
6.1	Cotterell and Rice theory	111

6.2 Crack paths on a Gaussian bump	112
6.3 Conclusions	114

APPENDIX 117

A ANSATZ FOR THE HOMOGENEOUS SHELL	119
B CONFORMAL MAPPINGS AND THE CIRCLES OF APOLLONIUS	121
C EXPERIMENTAL PATH THROUGH PHASE SPACE	125
D CRACK INCREMENT (IN)DEPENDENCE	127

BIBLIOGRAPHY	131
SAMENVATTING	143
CURRICULUM VITAE	147
PUBLICATIONS	149

