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## On the geometry of fracture and frustration

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ON THE GEOMETRY OF  
FRACTURE AND FRUSTRATION

PROEFSCHRIFT

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op gezag van Rector Magnificus prof. mr. C.J.J.M. Stolker,  
volgens besluit van het College voor Promoties  
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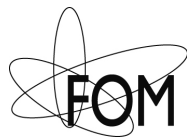
door

VINZENZ KONING

GEBOREN TE HAARLEMMERMEER IN 1988

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The cover shows a cartoon of twisted director fieldlines in toroidal nematic droplets, originally published in *Soft Matter* 10, 4192-4198 (2014). The bookmark shows crossed-polarised images of nematic double emulsion droplets created by Teresa Lopez-Leon and Alberto Fernandez-Nieves, originally published in *Nature Physics* 7, 391-394 (2011). Cover and bookmark design by Shivendra Shah.

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