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Three-Mode Principal Component Analysis: Theory and Applications

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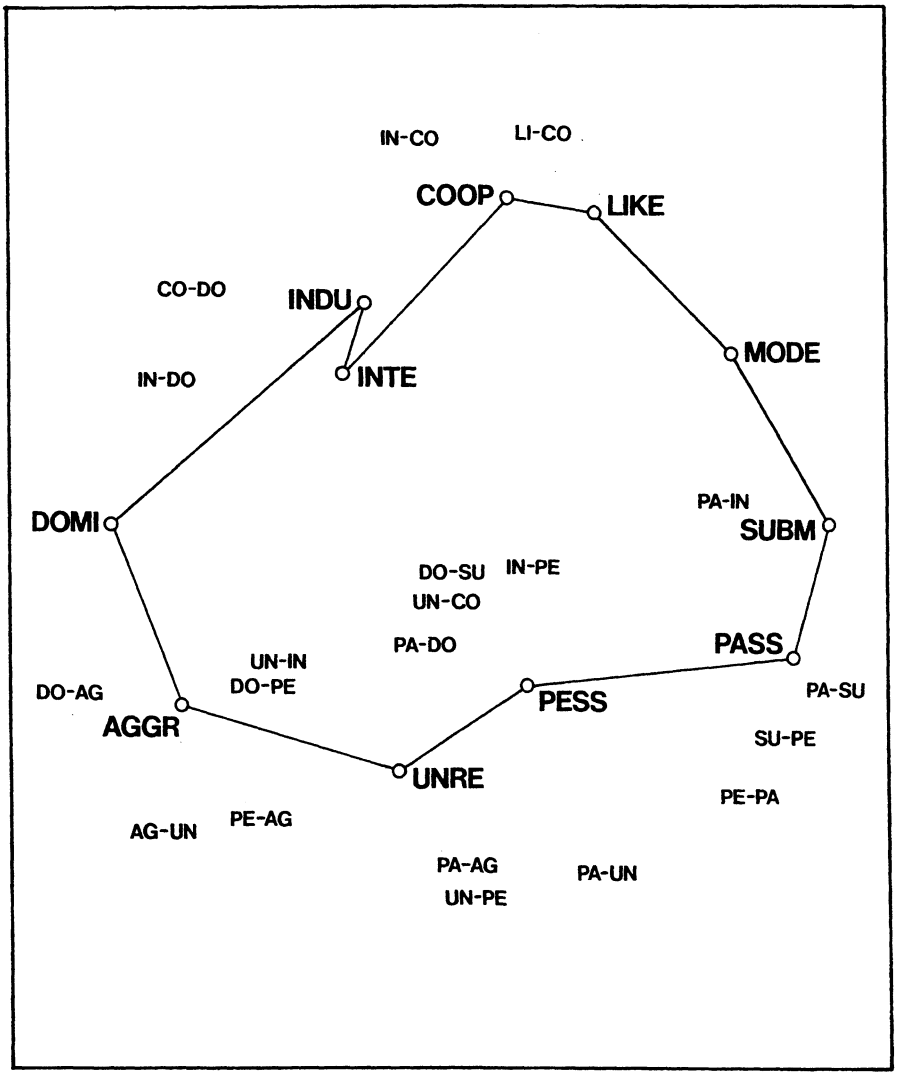
ASYMMETRIC SIMILARITY DATA

10

ITP study



10.1 INTRODUCTION

The main purpose of this section is to show how theoretical subjects may serve to aid the interpretation of subject spaces. A *theoretical subject* is defined as the collection of scores (or response pattern) on the variables in a study which has been derived on the basis of theoretical considerations. In other words, substantive knowledge is used to derive how a subject should score if he conformed to (a particular aspect of) the theory. The particular illustration used here is taken from Van der Kloot & Kroonenberg (1982).

10.2 THEORY, DESIGN, AND DATA

Theory. The notion that people use naive, common sense, or *implicit theories of personality (ITP's)* when they form impressions of another person's personality was introduced in 1954 when Bruner and Tagiuri proposed a cognitive approach to the study of person perception. In its original meaning, a person's ITP is a set of perceived or expected relations among personality traits; these perceptions and expectations may vary from person to person. Since then, numerous studies have been conducted on various aspects of the concept of the ITP. These studies were reviewed by Schneider (1973).

Van der Kloot & Van den Boogaard (1978) conducted an experiment to gain insight in the way people process information about other persons. Experiments in this field usually follow Asch's (1946) original paradigm in which a stimulus person is described by

a number of personality trait adjectives. The subjects have to express their impressions of a stimulus person by either checking similar adjectives in a checklist, or by giving numerical judgments on one or more rating scales.

Design and data. Van der Kloot and Van den Boogaard used 11 personality trait adjectives: *Likeable, cooperative, intelligent, industrious, dominant, aggressive, unreliable, pessimistic, passive, submissive, and modest.* These traits were selected because earlier research had shown that these stimuli lie on a circle in the order in which they are presented above. These stimuli were used in two experimental tasks.

In the first task, subjects had to rate 11 stimulus persons. Each stimulus person was described by one of the adjectives mentioned above (for instance: somebody is *aggressive*). In the second task the subjects had to rate 20 stimulus persons, each described by combinations of two personality trait adjectives (see Table 10.1). For further details see Van der Kloot & Kroonenberg (1982).

Table 10.1 *Combinations of adjectives in experimental task*

Likeable-cooperative	(LI-CO)	Unreliable-intelligent	(UN-IN)
Cooperative-dominant	(CO-DO)	Unreliable-pessimistic	(UN-PE)
Intelligent-cooperative	(IN-CO)	Pessimistic-aggressive	(PE-AG)
Intelligent-dominant	(IN-DO)	Pessimistic-passive	(PE-PA)
Intelligent-pessimistic	(IN-PE)	Passive-intelligent	(PA-IN)
Dominant-aggressive	(DO-AG)	Passive-dominant	(PA-DO)
Dominant-pessimistic	(DO-PE)	Passive-aggressive	(PA-AG)
Dominant-submissive	(DO-SU)	Passive-unreliable	(PA-UN)
Aggressive-unreliable	(AG-UN)	Passive-submissive	(PA-SU)
Unreliable-cooperative	(UN-CO)	Submissive-pessimistic	(SU-PE)

The descriptions of the stimulus persons were presented in two booklets, each preceded by an instruction page. The descriptions were printed on top of each page, and were followed by 11 ten-point rating scales. These rating scales were labelled with the 11 per-

sonality traits mentioned before, including the adjective (or adjectives) used in the description of the stimulus person. The rating scales ranged from 1 to 10, with end points denoted by "extremely not ..." and "extremely ..." (e.g. "extremely not *cooperative*" and "extremely *cooperative*"). In the two tasks subjects rated a total of 31 stimulus persons on 11 criterion variables.

The data were re-analyzed by Van der Kloot and Kroonenberg using three-mode principal component analysis. The solutions were based on double-centred subject matrices (see section 6.5). Thus the subjects were made identical with respect to scale and stimulus means, leaving the configurational aspects of the ITP's (i.e. the stimulus x scale interactions) as the data to be analysed.

10.3 THEORETICAL SUBJECTS *)

The data set was extended with six *theoretical subjects* in order to improve the interpretability of the solutions. The first, or *average subject* (A1), consists of the mean ratings of the stimuli averaged over the 59 real subjects. The second, or *dominance subject* (A2), has been constructed as if the stimuli were judged only with respect to their apparent dominance. The third, or *evaluation subject* (A3), was constructed as if the subject only judged the evaluative content of the stimuli. The fourth, or *random subject* (A4), consists of uniform random error superimposed on the overall scale means. The data of the fifth, or *uniform scorer* (A5), are equal to the grand mean, i.e. the average over stimuli, scales and subjects. The ratings of the sixth, or *extreme scorer* (A6), consists of either 2 or 9 scores. His scores are equal to 9 when the ratings of the average-subject (after being centred) are larger than 0. His scores are 2 when the average-subjects's double-centred ratings are smaller than 0.

*) Van der Kloot & Kroonenberg (1982) used the term *artificial subjects*, but *theoretical subjects* seems to be a better term.

We will use the theoretical subjects as some other authors have used 'conceptual individuals' or 'idealized individuals' (e.g. Tucker & Messick, 1963; Cliff, 1968; Tucker, 1972). The advantage of our theoretical subjects is that they were created on the basis of possible scoring behaviours of individuals from, or in analogy with, the original data. When included in the analysis they therefore provide a priori information about the subject space which is not the case with 'conceptual individuals'. The interpretation on the basis of theoretical subjects thus rests on more solid ground.

10.4 SCALE AND STIMULUS CONFIGURATIONS

The two-dimensional TUCKALS3 configuration of the *scales* explained 51.3% of the total sum of squares. Since the addition of a third dimension reduced the residual sum of squares by only 4.0%, we found the two-dimensional solution quite satisfactory, especially because the first two dimensions of the three-dimensional solution were virtually identical to those of the two-dimensional configuration. The configuration of the rating scales is pictured in Figure 10.1. It should be noted that each dimension explains an almost equal amount of variation: respectively 26.2% and 25.2%. This means that they are of equal importance for the group as a whole. The shape of the T3 configuration is roughly circular, and the horizontal and vertical dimensions can be interpreted as a dominance-submission and an evaluation dimension.

The T3 configuration of the *stimuli* for the 2x2x2-solution is represented in Figure 10.2. The 11 stimuli, consisting of single adjectives, lie on a polygon which is more or less the same as that of the scales in Figure 10.1, with the exception of *intelligent* and *industrious* which have switched places. Notwithstanding this difference, one may conclude that the stimulus space and the scale space are virtually identical; these spaces and their respective dimensions seem to have the same cognitive structure. Moreover, the two dimensions of the stimulus space also account for an almost equal proportion of the sum of squares (resp. 26.1% and 25.2%). Therefore

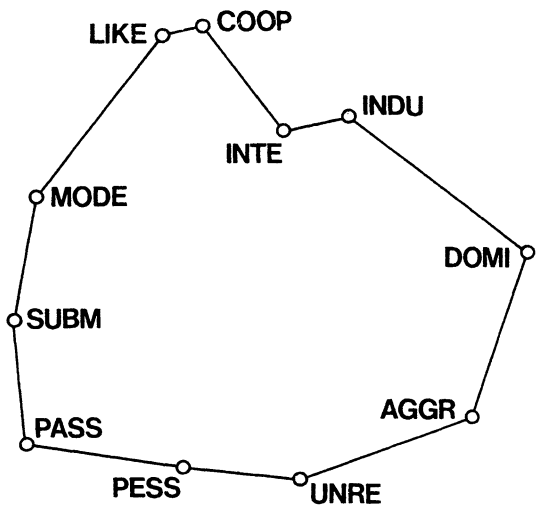


Fig. 10.1 *ITP study: Scale configuration*

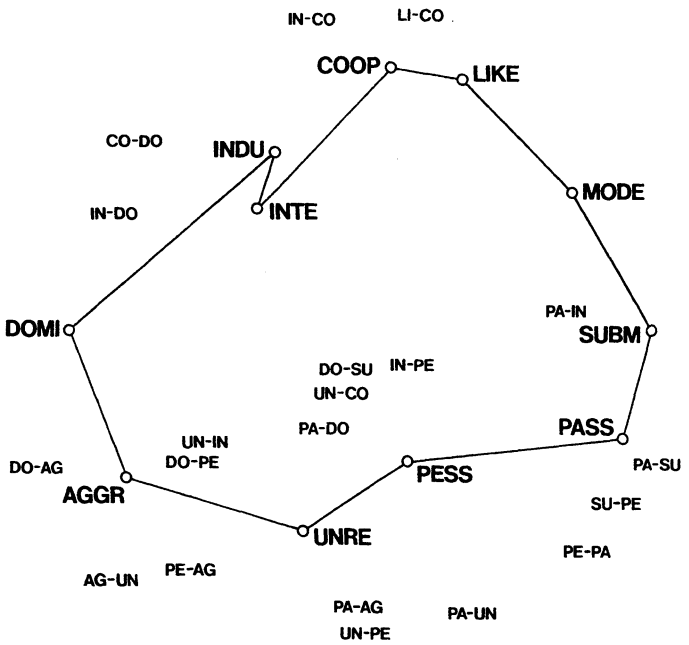


Fig. 10.2 *ITP study: Stimulus configuration*

Figure 10.1 and Figure 10.2 may directly be superimposed (after reflection) without further standardization of the projections, and we will refer to both the scale and the stimulus space as *the (personality) trait space.*

10.5 SUBJECT SPACES

The eigenvalues of the two components of the subject space from the T3 analysis (based on 65 real and theoretical subjects) were .498 and .015 respectively. Since the first component, which reflects the covariance of the individuals, is much larger than the second, it may be concluded that the subject space is largely one-dimensional (Fig. 10.3).

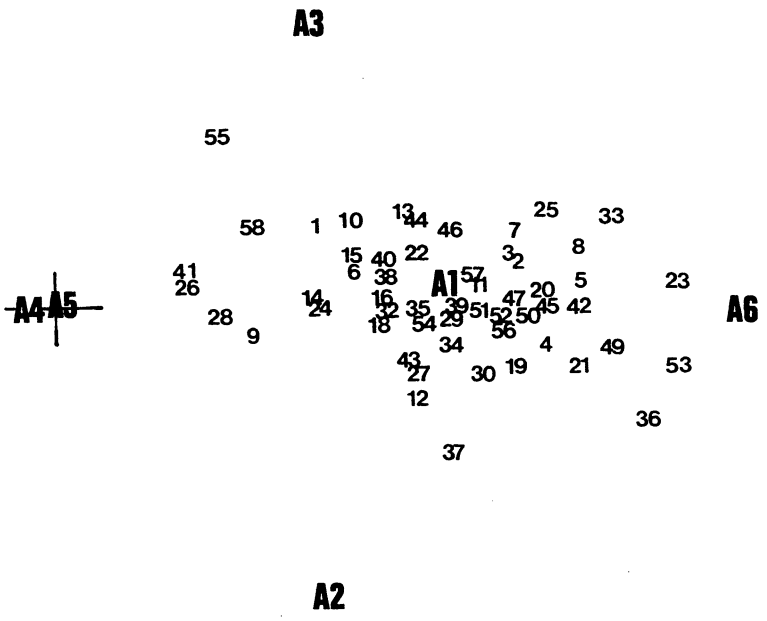


Fig. 10.3 *ITP study: Subject space* (+ = origin; unscaled)

The usefulness of introducing theoretical subjects now becomes clear, as they mark the end-points of the axes. The *uniform scorer* (A5) and the *extreme scorer* (A6) demarcate the first axis, and the *dominance subject* (A2) and the *evaluation subject* (A3) take on the extremes of the second axis. The *average subject* (A1) is located in the middle of the configuration. The *random subject* (A4) differs only marginally from the *uniform scorer* (A5). The conclusions (drawn on the basis of the theoretical subjects alone) are that subjects along the first axis of the subject space emphasize the dominance and evaluation axes of the personality trait space equally strongly with increasing emphasis going from left to right, and that subjects along the second axis of the subject space emphasize dominance at the cost of evaluation or vice versa.

The T3 core matrix tells the same tale (see Table 10.2). The diagonal elements of the core plane belonging to the first subject component have equal sizes and the same sign, and indicate therefore that both dominance and evaluation are weighted equally. The diagonal elements of the second subject component also have equal

Table 10.2 *ITP study: T3 frontal planes (two subject components and average frontal plane)*

71.89	- 2.18	- 9.20	- 2.14	31.34	- 2.16
2.19	70.39	- 1.78	9.39	.20	39.89
component 1		component 2		average plane	

sizes, but opposite signs, indicating that either dominance or evaluation is emphasized. The larger size of the elements of the first core plane is a direct reflection of the larger eigenvalue of the first subject component.

Although the T3 core matrix supplies information how dominance and evaluation are weighted in relation to each other, it does not tell the size of such weightings for individual subjects. Such information is present in the loadings of the subjects on the two subject components. The extended core matrix from a TUCKALS2 (T2) solution supplies additional and more detailed information: the

diagonal elements of each T2 core matrix indicate the amount of stretching and shrinking each subject applies to the axes of the common personality space, and the off-diagonal elements indicate the angle under which these axes are 'seen' (see section 6.9). It appears that all subjects see these axes as more or less orthogonal because the off-diagonal elements are never really large. Subjects with small and equal diagonal elements in their T2 frontal plane lie on the left hand side of the first axis of the subject space. Subjects with large and equal diagonal elements lie on the right hand side of the first axis, etc. Of the subjects who score most extremely (23, 37, 41 and 55) the core planes are shown in Table 10.3, along with an average subject (47), and the average core plane.

Table 10.3 *ITP study: T2 frontal planes (five subjects and average frontal plane)*

14.83	-.61	6.45	-2.79	3.30	.57
.03	13.34	-1.78	11.23	.04	2.14
subject 23		subject 37		subject 41	
10.13	.36	7.79	.13	9.10	.02
-.58	9.99	-.36	-.57	.00	8.67
subject 47		subject 55		average plane	

The most important feature of the T3 subject space is thus that most individuals emphasize the dominance and evaluation axes equally but with varying values of the weights. This implies that for most subjects the recovered personality trait configuration (or ITP) is circular, and that some have larger circles than others. The subjects with large weights (wider circles) have large sums of squares and thus use most of the ten-point scales. Of secondary importance is that some subjects emphasize either dominance or evaluation. Extreme examples are 55 and 37, who seem to use either the dominance or the evaluation axis as is confirmed by their T2 core plane in Table 10.3.

10.6 RESIDUAL/FIT RATIOS

Theoretical subjects can also assist in checking the assumption that all subjects only applied the transformations allowed by the model to the personality trait space. Subjects who do not conform to the model, or at least less so than other subjects, should have a smaller residual/fit ratio than the average individual. In the sums-of-squares plot for subjects (Figure 10.4) the contributions to the fit are plotted against the residual sums of squares for all real and theoretical subjects. The heavy line in this figure connects points with the overall residual/fit ratio (.49/.51). The other two lines connect points with ratios .39/.61 and .59/.41 respectively. These lines serve as a kind of confidence bands for the overall residual/fit ratio (see section 7.6).

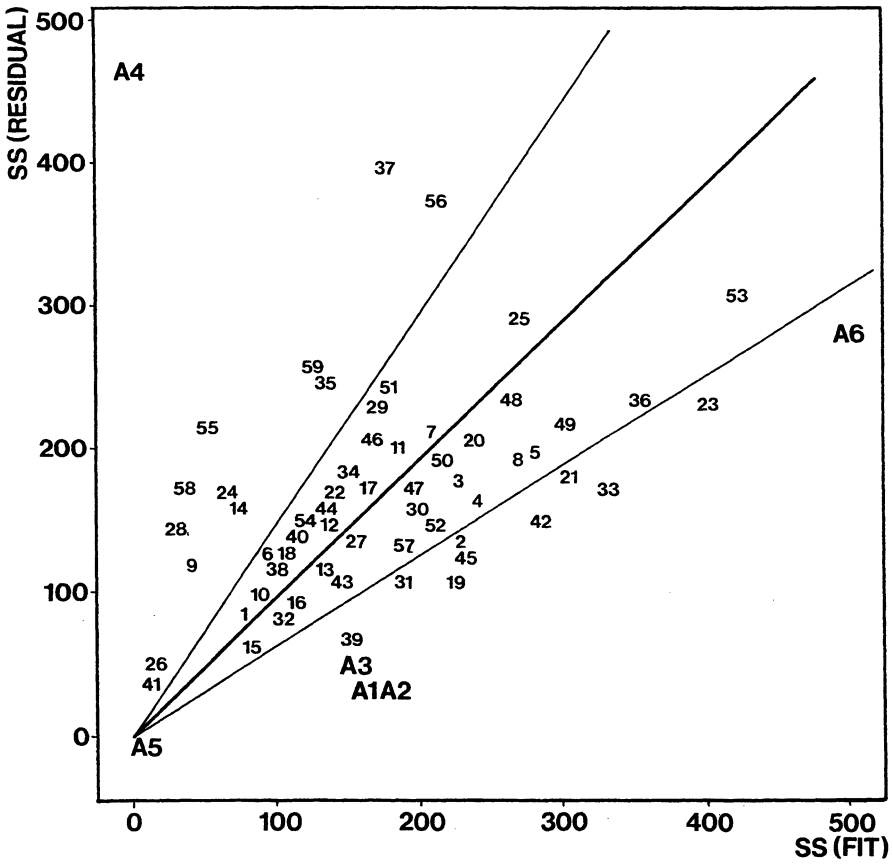


Fig. 10.4 *ITP study: Sums-of-squares plot for subjects*

Twelve real subjects (9, 14, 24, 26, 28, 35, 37, 41, 55, 56, 58, and 59) have rather large residual/fit ratios, and probably do not meet the assumptions of the model. Inspection of their scores on the subject components showed that: (a) the two subjects who have the most extreme scores on the second component space are ill-fitting points; (b) the majority of the 'bad' points have small values for the diagonal elements of their T2 core planes (for some examples see Table 10.3); (c) there are points which do not conform to this pattern, notable 35, 56, and 59, the 'best' of 'bad' points.

The 'good' points (2, 19, 21, 23, 31, 33, 36, 39, 42, and 45) generally have large scores on the first component of the subject space, and thus large overall sums of squares. It is, of course, no surprise that subjects with large sums of squares fit better than subjects with small sums of squares.

In Figure 10.3 the theoretical subjects also lie on the boundary of the configuration, and can thus be used to evaluate the real subjects. The *random subject* (A4) has practically no fit, as it should be. The *average subject* (A1) has roughly the same fit as a real subject (i.e. 17 in the centre of the subject space), but due to the averaging procedure, a smaller residual than such an individual. The *dominance subject* (A2) and the *evaluation subject* (A3) were created from the *average subject* (A1) with comparable sums of squares, which explains their position in Figure 10.3. The *uniform scorer* (A5) has fit nor error as his sum of squares is necessarily zero. The *extreme scorer* (A6) has understandably a very large sum of squares, and also a smaller residual/fit ratio (.36/.64) than the overall one (.49/.51), which indicates his scoring pattern is admissable in terms of the model. In fact, his private trait space is almost a perfect circle.

10.7 CONCLUSIONS

In conclusion one may say that it is especially useful to specify 'ideal' response patterns of theoretical subjects on the basis of a substantive theory, instead of the results of the analysis. By assessing the difference between the real and 'idealized'

subjects, it is possible to accept or reject the models underlying the construction of the latter, and simplify the interpretations of axes.