Errata

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"Three-mode Principal Component Analysis: Theory and Applications"

(including a very selective list of new three-mode papers)

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Page	Line	Correction
10	12ff	These lines should read: 'but it specifies common angles between the axes of the stimulus space. However, differential weighting of these axes is allowed.'
18	6	Formula should read: $h_{j1}s_{i11} + h_{j2}s_{i21}$.
19		In Figure 2.1 The labels 'Standard PCA' and 'Q-PCA' should be interchanged.
25	8	'SS(Data)' should read 'SS(Total)'.
25	10	'SS(Data$_f$)' should read 'SS(Total$_f$)'.
25	-12	Delete two sentences: 'Similarly [..] information'.
26	15	'j-centring' should read 'j-centring /standardization'.
26	19	'jk-centring' should read 'jk-centring /standardization'.
48	- 1	'183, 186' should read '83,86'.
48ff		Chapter 3. Kiers (in press-a, in press-b) has presented two new taxonomies for three-way models. His treatment includes several models from the French school (see also Carlier, et al., 1989, and Lavit, 1988).
49		Figure 3.1. The statement for going from IDIOSCAL to PARAFAC2 is incorrect. The proper expression can be found on p. 55.
49	-5	model~~s~~
50	2-4	The 'number of parameters' indicated in the table are not corrected for dependence between the parameters. e.g. for the Tucker3 model, the number

		of independnet parameters should be decreased by '$s^2+t^2+u^2$', and the Tucker2 model by 's^2+t^2'.
50	4	In the table stubs 'diff.' and 'red.' should be interchanged.
50	7	Three-mode scaling: '21s' should read '1s'.
51	10-13	The Tucker2 and Tucker3 models are not necessarily orthonormal, in fact Tucker did nor require this. The orthonormality is an expedient constraint for solving the estimation problem, and may be dropped later, as can for instance be seen in Chapter 5.
53	3	'$c_{pp'r} = c_{pp'r}$' should read '$c_{pp'r} = c_{p'pr}$'.
53	-10	'p.90-92, and section 6.2)' should read 'p.90-92), and section 6.2'.
56	18,19	The 'weak conditions' mentioned here are too strongly formulated, see the references mentioned.
57	13	In contrast to what is implied in the text, the expression Z = GC(H'■E') refers to a 'lateral plane' representation, i.e. both Z and C are juxtaposed lateral planes. The proper expression for a 'frontal plane' representation as used verbally in the text and in the next formula is Z = GC(E'■H').
58	4	'W =' should read 'W_k ='.
58	12,13	Such procedures have been recently discussed by Harshman, Kruskal, and Lundy in various permutations (see reference list).
61	3	'for the communalities' should read 'for factor analysis by estimating the communalities'.
62	8	'estimate' should read 'estimating'.
64	- 4	S and s should be underlined.
65	4	S ans s should be underlined.
68		Section 3.7. One of the reviewers missed a section on resampling plans, such as the bootstrap (Efron, 1982). One application using this approach is contained in Kroonenberg & Snyder (1989).
70	-10,-12	The papers referred to in this section have now been published: Van der Kloot & Kroonenberg (1985) and Van der Kloot, Kroonenberg, & Bakker (1985).
80	- 2	'c_{pqr}' should read '$\hat{c}_{pqr}$ '

80	20	'Penrose (1955 -' should read 'Penrose (1956 -', and the reference given on p. 376 is incorrect (see below).
82		It has been shown in Brouwer (1985) that the SS(Fit)/SS(Total) = R^2, given the data have been centred such that the overall mean is zero.
83	5	Formula should start with a minus sign.
83	- 6	Index of last summation sign should be 'r' rather than " q' ".
83	11	The statement 'the Hessian is negative' is not applicable for the present maximization with restrictions. Correct approaches may, for instance, be found in Luenberger (1973, p. 226).
84	15	'only the assess' should read 'only to assess'.
84	- 2	Add " $z_{i'j'k'}$ " to the end of the formula.
86		Two alternatives have been proposed for the TUCKALS algorithm. Weesie & Van Houwelingen (1983) constructed an algorithm based solely on regression techniques, rather than on eigenvalue-eigenvector decompositions, by including the estimation of the core matrix into the iterative process. The advantage of their approach is that missing data can be handled in a natural straightforward way. Murakami (1983) produced an ALS algorithm for the Tucker2 model which uses the multivariable-multioccasion matrix as its starting point. This gives the possibility of analysing published matrices of this kind, and is an alterantive for the Invariant Factors Model of McDonald (1984).
87-89		Recently Kroonenberg, Ten Berge, Brouwer, & Kiers (1989) have shown that the Bauer-Rutishauser step in the TUCKALS algorithms may be replaced by the modified Gram-Schmidt orthogonalization procedure. The latter is slightly faster than the former.
103		The proof assumes orthonormality, while at p.81, it was suggested that the proof should be valid without this assumption. A correct proof is given in Ten Berge, De Leeuw, & Kroonenberg (1987).

105		A different more direct proof is contained in Ten Berge, De Leeuw, & Kroonenberg (1987).
110		The discussion on this page should have referred to Green (1952).
111	12	Delete from (5.3): ' and $K'K = I_S$ '
113		The diagonality problem (NS) is essentially equivalent with Harshman's PARAFAC (Harshman, & Lundy, 1984a). A more detailed report on this equivalence is contained in Brouwer (1985), and related material is given by Harshman, Kruskal, and Lundy in various permutations (see reference list).
128		Sections 6.1-6.8. The issue of centring and standardization has recently received a much more detailed and algebraic treatment (Harshman & Lundy, 1984b; Kruskal, 1984). Some additional commentary see Harshman and Lundy (1985).
131		'pca-data' are commonly called 'profile data'; 'mds-data' are commonly called 'proximity data'; 'anova-data' resemble 'conjoint measurement data'. see Shepard (1972).
138ff		Three-mode data. The ANOVA-first approach (see also p.195) has been treated in considerable detail by Kettenring (1983a,b) using the PARAFAC model for the three-way decomposition. An application with the Tucker models can be found in Kroonenberg & Van der Voort (1987).
141	- 5	'$g_{ip}h_{jq}e_{kr}$' should be '$g_{ip}h_{jp}e_{kp}$'.
151	3-9	In contrast to the statement, averages of correlations <u>are</u> correlations.
153	5	Before the third summation sign an '=' should be inserted.
154	14ff	Components as latent elements. Probably all that can be said is that the major components span a space which captures most of the common variability. Whether there are directions in this space which correspond to latent entities or theoretical constructs is a different matter, and thus the statements in this section stand to be corrected.

155	8	'session' should read 'section'.
155ff		Rotation of components. Most reviewers of the book have chided the author for not treating this issue in more depth. (see the reviews for detailed arguments).
159	18,19	The diagonal matrix lambda should be absorbed into H rather than G (see also remark, p. 19).
160	1-4	A diagonal cork matrix as suggested here, can only occur if $\Sigma_{p,q}\ c_{pqr}c_{pqr'} = 0$, as required by the all-orthogonality of the core matrix (see Weesie & Van Houwelingen, 1983). Thus $G_1 = \begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix}$ and $G_2 = \begin{matrix} -1 & 0 \\ 0 & 1 \end{matrix}$ is possible, but $G_1 = \begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix}$ and $G_2 = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$ is not. The cited example indeed shows the required pattern.
160	7-15	The diagonal/antidiagonal phenomenon is also in concordance with the all-orthogonality.
164	- 3	'HV_r' should read 'HV_r'.
165	2,3	'When C_r [..] be used' should read 'When C_r is not square with, say s<t, C_r selects as-dimensional subspace from the t-dimensional space of H'.
167	-4,5	'$h_{.q}$' should read '$\bar{h}_{.q}$'.
180	20	'1.67' should read '1.64'.
202ff		Chapter 8: The final and corrected data have now been reported in Van IJzendoorn, et al. (1985) with, however, very condensed information on the three-mode analysis.
227ff.		Chapter 9. Triple personality data. A reworked version of this chapter was published as Kroonenberg (1985).
257		Section 11.2. From the introduction of this section one might get the impression that unlike methods for multidimensional scaling three-mode principal component analysis needs similarities. The remarks made refer to the fact that the output of the three-mode programs are easiest to read when high values indicate closeness, rather than separateness. Such

an effect can easily be obtained by converting dissimilarities into inner products by the standard Torgerson procedure for classical multidimensional scaling. When the input data themselves are already considered proportional to inner products, then high values should indicate closeness.

274ff Chapter 12: The reference to Levin (1966) was omitted in this chapter. An extensive treatment of the analysis of sets of correlation/covariance matrices is now available in Kroonenberg & Ten Berge (1987, 1989). Furthermore, insufficient justice was done to Procrustes procedures (see Gower, 1975; Ten Berge, 1982).

Chapter 12: The analysis is in this chapter is performed on a set of correlation matrices. There ae several reasons why such an analysis is less than desirable (Harshman and Lundy, 1984, p.141). Especially within the context of structural equation modeling, there is strong opposition to analysing correlation rather than covariance matrices (see e.g. Jöreskog, 1971; McDonald, 1984, 3p.292; Meredith, 1984). The main argument centres around the different size of one standard deviation unit for the same variable across different samples.

275 7ff The second paragraph on p. 275 contains somewhat confusing statements about eigenvalues. The following remarks taken from Harshman's review clarify the issue: "When performing a two-way analysis using a singular value decomposition (SVD) of raw data, the eigenvalues and related sums of squares are obtained from the **squared** singular values, whereas when doing an SVD of a covariance or correlation matrix, one examines the **unsquared** singular values to obtain the eigenvalues and related sums of squares of the original data from which the covariances were computed. Likewise, when TMFA is applied to raw data, one looks at squared core elements, but when it is applied to

covariances, one should examine the unsquared core elements. If this is done, "eigenvalues" obtained from two-way and three-way methods are directly comparable." (p.331).

285ff.		Chapter 13. For a different analysis of the hospital data see Kroonenberg, Lammers, & Stoop (1985).
313ff.		Chapter 14. These data were also analysed by curve fitting with logistic regression, see Jansen & Bus (1984).
322		The column means in Table 14.4 should read: '12 13 17 27' giving column effects '-5 -4 0 10', and residuals for Q of '-4 -2 1 0 2 0'. Eliminating the 'maybe too many positive scores'of Q (1. -4,-5), and invalidating the remark about Q.
326ff		Chapter 15. Strictly speaking, it is not correct to use the name 'correspondence analysis' in this chapter, as the basic properties of correspondence analysis do not hold. For a further investigation into a proper three-mode correspondence analysis, see Kroonenberg (1989).
327	5	Delete '+ log r_{ij}'
339	- 1	'-18' should read '-8'.
360	19	'Nesselroode' should read 'Nesselroade'.
362	16	'model of application' should read 'model by application'.
364		Insert after Einhorn: Fienberg, S.E. **The analysis of cross-classified categorical data** (2nd edition). Cambridge, MA: MIT Press, 1980.
371	- 0	'1987' should read '1978'.
376	6	'791' should read '591'.
376	16-18	The correct reference is: Penrose, R. (1956). On the best approximate solutions of linear matrix equations. **Proceedings of the Cambridge Philosophical Society, 52, 17-19.**
377	- 9	'alternative' should read 'alternating'.
393		Entry Joint plots: '163-165' should read '164-166'.

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Summary of the book

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Three-mode principal component analysis
Theory and applications
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In multivariate analysis the data are usually contained in a single matrix with n rows and m columns, corresponding with n individuals and m variables. Or, to put it differently, the data have two modes: individuals and variables. It has already been known for a long time that this particular two-mode representation of data is too restrictive in a number of very important cases. Often there are three modes, the additional mode being replications, occasions, conditions, points-of-view, and so on. The data must be collected in a three-mode matrix, which has n rows, m columns, and k slices. Of course this 'data-box' can be flattened into an ordinary two-way matrix in various ways, but often there is no unique obvious way in which this should be done. Moreover most data analysis techniques that can be applied to the two-way matrix obtained by summation or concatenation over one of the modes, simply ignore the fact that the data were originally three mode.

In the early sixties Tucker introduced a form of principal component analysis which works directly on the three-mode matrix, and has parameters for all three modes. This was an enormous step ahead, because there was no need to flatten databoxes any more. Somewhat later individual differences models were introduced in multidimensional scaling. They are also based on three-mode data, and pretty soon the two developments were integrated by Tucker and Carroll. For the individual differences models specialized algorithms are available, but for Tuckers's three-mode component model the available algorithms were somewhat ad hoc and suboptimal.

In **Three-mode principal component analysis** perhaps the main emphasis is on the development and study of a satisfactory algorithm for Tuckers's technique, together with a friendly computer program. But this is not all. Models for three-mode data are also discussed in considerable detail. Important data analytic decisions, which must be taken before the programs can be applied, are spelled out. A chapter on the analysis of residuals shows that the job is not done if the program has run. About half of the book is used to analyze meticulously a number of examples, which are chosen in such a way that each of them is representative for a large class of data structures. There are semantic differentials, three-way contingency tables, replicated correlation matrices, three-way similarity data, and multivariate time-series data. All examples are used to show which plots can be made, how the residuals should be analyzed, how the key parameters should be interpreted, and so on.

Three-mode principal component analysis says about everything there is to say about this class of techniques. The book does this on a mathematical and computational level, but more importantly it illustrates everything that is said by using a number of very real and very interesting examples.

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