Beyond beauty : reexamining architectural proportion in the Basilicas of San Lorenzo and Santo Spirito in Florence

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6. Alternatives to the Wittkower Paradigm

The preceding chapters constitute a fundamental challenge to the Wittkower Paradigm. The first of the three characteristics of the Wittkower Paradigm, the premise that certain sets of proportions determine the aesthetic character of architecture, including the aesthetic distinctions between architectural styles, would seem to be entirely without foundation in light of the findings of this study. The vertical sets of proportions found in the Old Sacristy, if they can be considered intentional sets of proportions at all, have little geometrical or numerical interest, yet art and architectural historians consider the Old Sacristy to be a masterpiece of Renaissance architecture. According to what principles—aesthetic, philosophical, scientific or otherwise—could these sets of proportions contribute to the aesthetic appeal of that building? The same may be asked in relation to the basilica of Santo Spirito, since the set of proportions found in the nave arcade bays appears cursory and carelessly conceived compared to those of San Lorenzo. Even the sets of proportions of San Lorenzo, we have seen, contain significant errors and irregularities, yet neither critics nor historians have ever questioned the aesthetic quality of the building or its pivotal role in the history of architecture.

Furthermore, we have seen that the Gothic-style basilica of Santa Maria del Fiore and the Renaissance-style basilica of San Lorenzo contain similar sets of proportions, as do the Gothic-style Cathedral of Milan and the Renaissance-style basilica of Santo Spirito, thus indicating, contrary to the Wittkower Paradigm, that sets of proportions are stylistically neutral. That the second premise of the Wittkower Paradigm, suppression of the object in favor of documentary research, is an unnecessarily limiting approach to the study of architectural proportion (proportion-1 and proportion-3) is indicated by the many conclusions presented in this study that are based on evidence that is only accessible through observation.

The third characteristic of the Wittkower Paradigm, the theory of Geometry vs. Number, or, the theory that the transition from the medieval to the Renaissance periods was accompanied by, according to Wittkower, a "[…] transition from a primarily geometrical to an arithmetical approach to proportion," finds no expression in any of the sets of proportions examined in this study.[1] The sets of proportions in the fourteenth-century nave arcade bays of the basilica of Santa Maria del Fiore appear to accord equal importance to both geometry and number, as do those of the nave arcade bays of the basilica of San Lorenzo.

While the latter falls under Wittkower's *quattrocento* exception of the theory of Geometry vs. Number within the Wittkower Paradigm (see Chapter 1), the throughness and sophistication with which geometry and number are blended in the San Lorenzo sets of proportions challenges the validity of both this exception and the theory. The combinations of geometry and number found there appear to be the results of centuries of development rather than a temporary, transitional phase in the history of geometry and mathematics. Furthermore, the Santa Maria del Fiore evidence indicates that the *quattrocento* was not preceded, as Wittkower claims, by a medieval period that emphasized geometry over number. Indeed, Theon of Smyrna's formula for generating accurate numerical approximations of the ratio 1:√2 appeared some 1300 years before Dolfini began his work on that basilica, and such approximations (if not Theon's forumal) were known throughout the medieval period.[2]

The preceding chapters not only challenge the Wittkower Paradigm, but suggest alternatives to it that I will now explore, beginning with the third characteristic of the paradigm. While the evidence presented in this study challenges the main premises of Geometry vs. Number, it is consistent with this theory at least insofar as it seems to confirm that the medieval and Renaissance periods exhibited notable differences in their attitudes toward geometry and number. Those differences do not appear to have been characterized by preferences for one over the other, however, since geometry and number do not ever appear to have been separated in the history of architecture, but rather, by ever-increasing degrees of precision in quantification. This proposal can be elucidated through further analysis of the sets of proportions found in the nave arcade bays of Santa Maria del Fiore, considered in the context of fourteenth-century developments in mathematics.

6.1. The Crosby Thesis Instead of Geometry vs. Number

The mid-fourteenth century was a period of transition in the history of mathematics. Hindu-Arabic numerals began to appear together with the older Roman numerals in treatises on arithmetic in Europe as early as the late 11th and early 12th centuries, but Roman numerals were still preferred in these works for calculation.[3] Roman numerals, however, being non-positional and lacking a symbol for zero, made calculation cumbersome. By the end of the 13th century, Hindu-Arabic numerals had become so common as instruments of calculation in Florence that in 1299 a statute of the *Arte del Cambio* (the guild of the money changers) was enacted to prohibit their use.[4] Perhaps, as Dirk Struik suggests, the supporters of the statute saw the new number system as an asset of the *Arte del Cambio* that could be profitably withheld from others.[5] Similar prohibitions were enacted in Padua in 1348.[6] Ball proposes that "[…] by the year 1400, we may consider that the Arabic symbols were generally known throughout Europe, and were used in most scientific and astronomical works."[7] The archives of the *Opera* of Santa Maria del Fiore seem to support this estimate. The document of 1357 quoted above specifies dimensions in Roman numerals, but a few decades later in these archives Hindu-Arabic numerals appear with increasing frequency.[8] The historical development of fractional, rather than whole number, notation provides a more detailed view into the development of calculation with Hindu-Arabic numerals.

Roman Fractions

The use of Roman numerals combined with fractions composed of Hindu-Arabic numerals appears to have gained wide acceptance slightly before the use of Hindu-Arabic numerals for both whole numbers and fractions, probably because the Roman system of fractions is so unwieldy. It is a system in which all fractions are based on twelfths, and in which a finite number of fractions is recognized. These fractions are represented by obscure, never fully-standardized symbols that are as difficult to remember as to distinguish from one another. Each increment of twelfths, from $\frac{1}{12}$ to $\frac{12}{12}$, has its own unique symbol, and certain fractions smaller than $\frac{1}{12}$ are represented by yet more unique symbols (Figure 6-1).[9]

When a fraction is needed that either is not a factor of $\frac{1}{12}$, or, if smaller than $\frac{1}{12}$, has a numerator greater than one, such a fraction has to be formed by combining available fractions. One such example is $\frac{1}{8}$ which, apparently because of its usefulness, has its own symbol even though its name refers to other fractions. Thus in the Roman system of fractions $\frac{1}{8}$ is simply understood as one-and-a-half twelfths ($\frac{1}{12}+\frac{1}{24} = \frac{1}{8}$), as its name *sescuncia*, or “inch and a half”, implies (Figure 6-1). The arithmetician who needs the fraction $\frac{11}{144}$, however, is not so fortunate. This fraction must be expressed as $\frac{1}{24}$ (*semuncia*) + $\frac{1}{36}$ (*duella*) + $\frac{1}{144}$ (*hemisecla*), and such combinations must be memorized, for Roman fractional notation, like Roman numeration in general, is not conducive to calculation. Rather, according to Gillian Evans, in order to subdivide the quantity of “one” this system requires a “shift in thinking”; a shift that I would describe as more verbal than mathematical.[10]

Evans observes that the arithmetician “ […] begins by renaming his unit an *as* [$\frac{12}{12}$], and this seems to be a signal for him to begin to think of it as a whole, divisible into twelve parts….”[11] Returning to the sets of proportions in the nave arcade bays of the basilica of Santa Maria del Fiore, we now see how simple it would have been for Talenti, who was perhaps trained in the Roman system of fractions in his youth, to determine the mean between 19 $\frac{1}{3}$ br and 19 $\frac{1}{2}$ br. No calculation would have been required, but rather, a “shift in thinking” and a correct selection of words. In the Roman system of fractions, $\frac{1}{3}$, called *triens* (“one third”) is understood to equal $\frac{4}{12}$. The fraction $\frac{1}{2}$, called *semis* (“half”) is understood to equal $\frac{6}{12}$. Talenti merely had to select the fraction that falls between them, called *quincunx* (“five twelfths”) and append it to 19 in order to determine the height of the nave pier shafts, 9 $\frac{5}{12}$ br. Thus, calculation with Roman fractions is essentially a qualitative, verbal procedure. As such, it has a notable parallel with some of the earliest fractional notations recorded in the fourteenth-century Florentine cathedral archives.

Verbal Fractions

The collection of documents pertaining to the Basilica of Santa Maria del Fiore published by Guasti in 1887 provides a convenient case study in the evolution of fractional notation from the late thirteenth to the early fifteenth centuries in Florence. A document dated January 5, 1351 (new style), for example, contains numerous examples of verbal fractions, such as: “in length one *braccio* and one-half of another *braccio*” (*longitudinis unius bracchii et dimidii alterius bracchii*), “in width two-thirds of a *braccio*” (*largitudinis duorum tertiorum bracchii*), and “in width one *braccio* and one-eight part of another *braccio*” (*largitudinis unius bracchii et octave partis alterius bracchii*).[12] Similar to Roman fractions in that they are verbal, but dissimilar in that they use only common Latin terms to denote fractional quantities, these verbal fractions perhaps indicate that the complex system of Roman fractions found only limited use.

Serial Fractions

Seven years later in the Guasti transcriptions, a new system of fractional notation appears. Now Roman numerals are followed by a string of interdependent fractions that I will term “serial fractions”. These fractions contain only Hindu-Arabic numerals, as in the dimension br. xxxiij $\frac{3}{8}\,\frac{1}{1}$, which is the specification for the Santa Maria del Fiore nave width, measured on center, that immediately precedes the 34 br specification in the 1357 document discussed above.[13] This strange and complex system of fractional notation, which according to Louis Charles Karpinski originates in the ancient Egyptian system of “unit fractions”, but which I find also reminiscent of Roman fractions, gained a foothold in the Latin West in part through its use by Leonardo of Pisa (alias

Fibonacci, ca. 1175–ca. 1250), in his *Liber abaci* of 1202.[14] It appears to have gained wide enough acceptance in Florence to have delayed the definitive adoption of common fractions (the system that we use today, which happens to be another system described by Leonardo of Pisa) until the end of the fourteenth century.[15]

In the preceding example, $33\frac{3}{8}\,\frac{1}{1}$, the second fraction denotes a fractional portion of the denominator of the first. Thus, the serial fraction $\frac{3}{8}\,\frac{1}{1}$ must be read: $\frac{3}{8}+\frac{1}{8}=\frac{1}{2}$. This interpretation is consistent with the teaching found in a contemporary schoolbook on arithmetic, discussed in detail below. Thus the total dimension noted above equals $33\frac{1}{2}$ br. The hapless scribe of the aforementioned 1357 document appears to have been sufficiently confused by this system that as a precaution he repeated the dimension, apparently incorrectly, in the old verbal system as: "thirty-three and three-eighths and a half *braccia*" (*br. trentatre e tre ottavi e mezo*); or $33\frac{3}{8}+\frac{1}{2}=33\frac{7}{8}$ in modern notation.[16] My survey supports the first interpretation, rather than the scribe's. The nave width, measured between the plinths of the engaged piers on the interior façade (because this westernmost bay of the nave is the most likely to reflect the dimensions in the 1357 document, as noted above) is 1667.5 cm (Figure 5-18, between Piers 1 and 10), or 28.57 br; or, only 4 cm larger than $28\frac{1}{2}$ br plinth to plinth; or, virtually the same as the specified dimension of $33\frac{1}{2}$ br on center once we add 5 br for half the width of each plinth on either side.

Common Fractions

Judging from our limited sample in the *Opera* documents transcribed by Guasti, and other evidence discussed below, serial fractions appear to have been short-lived in Florence, having been supplanted by common fractions at about the same time that Roman numerals finally gave way to Hindu-Arabic numerals as the primary form of numeration. Thus, in a document of 1411 we find a *braccio* dimension expressed as an Hindu-Arabic numeral plus a common fraction: "124 and $\frac{23}{24}$ *braccia*" (*brachiis 124 et* $\frac{23}{24}$).[17]

This new system would seem to have appeared just in time from the point-of-view of the cathedral accountants who had to figure out how much, according to the aforementioned document of 1411, Ugho de Alessandris was owed for the $124\frac{23}{24}$ br of wood planks he supplied, at a monetary rate of 8 *soldi* and 4 *denari* per *braccio*, to be used in the centering of the squinches and occuli of the cathedral tambour.[18] The calculation would have been far more laborious had the fraction $\frac{23}{24}$ been expressed as a serial fraction. We can begin to appreciate just how laborious by perusing a Florentine schoolbook of arithmetic, or *trattato d'abbaco*, written by a noted mathematician about forty years earlier.

A more accurate title for the *Trattato d'aritmetica*, or, *Treatise on Arithmetic*, written by Paolo dell'Abbaco probably around 1373, might have been *Treatise on Common Fractions*, for the purpose of the work appears to have been to explore every aspect of the then-new system of common fractional arithmetic that might have had any practical application.[19] Paolo's 197 problems range from simple exercises such as: "divide 12 by $3\frac{1}{4}$ " (Problem 2) and "multiply $5\frac{1}{4}$ by $8\frac{3}{5}$ " (Problem 14), to complex word problems involving areas of cloth (Problem 36), divisions of a testament (Problem 100), the length of a hemp rope strung between two towers to support a lead weight (Problem 158), and many other situations.[20] Paolo apparently wanted to make sure his students understood the older system of serial fractions before moving on to common fractions, and so his first problem is devoted to a serial fraction equivalent to, as it turns out, the common fraction $\frac{23}{24}$.

Paolo explains at length that the serial fraction $\frac{2}{3}\ \frac{1}{2}\ \frac{3}{4}$ must be read, in effect: "two thirds, plus one half of one third, plus three quarters of one half of one third", or, $\frac{2}{3} + \frac{1}{6} + \frac{1}{8}$, or $\frac{16}{24} + \frac{4}{24} + \frac{3}{24}$, or $\frac{23}{24}$. He also describes a shortcut for carrying out this conversion that, if it is indeed a shortcut, leaves us marveling over how such a complicated system of fractional notation could ever have come into being.[21] Paolo himself seems to have shared this view, for he ends this first problem with apparent impatience, declaring: "and this is sufficiently clear and so, enough" (*e questo è assai chiaro e basta*).[22] Never again to return to serial fractions, he proceeds through the remaining 196 problems, virtually every one of which involves computation with common fractions, some with quite large numerators and denominators, and concludes each with a more satisfied refrain such as: "and all is well" (*E sta bene*), "and it is done" (*Ed è fatta*), "and see how it turns out well" (*E echo che ttorna* [sic] *bene*).[23]

Soldi and *denari*

By the end of the fifteenth century Florentine architects had evidently even found common fractions to be so cumbersome when dealing with measurements denoted by all but the most ordinary fractions such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{8}$ that they adopted a new system of subdivision notably similar to serial fractions, but much easier to use.[24] In the new system, which was perhaps derived from the Roman monetary system (see, for example, the debt to Ugho above), one *braccio* is divided into 20 *soldi*, and each *soldo* is divided into 12 *denari*. Thus, the above-noted fraction $\frac{23}{24}$ in the new system is equivalent to 19 *soldi* and 2 *denari*, which by the late fifteenth century would have been written "s 19 d 2."[25] This system is both practical, because it renders any fractional quantity immediately comprehensible as some number of twenty parts plus-or-minus a little bit, and precise, because no fractional quantity is ever left out.[26]

The Venerable Model

The history of fractional arithmetic outlined above evidences a distinct development toward ever-increasing precision in thought and calculation during the late fourteenth and early fifteenth centuries. Thus, a customer who asked a merchant during the earlier part of this period for an eighth of a *braccio* of cloth represented a quite different attitude toward quantification than one who asked during the later part for a length of cloth measuring 2 *soldi* and 6 *denari* of a *braccio*, even though the two lengths are identical. Indeed, the earlier customer is not likely to have ever asked for, say, $\frac{23}{24}$ br of cloth, for he would have lacked both the conceptual tools with which to comprehend such a quantity, and the arithmetical vocabulary with which to request it; and if that were in fact the exact length needed, he is not likely to have been concerned with the difference between $\frac{23}{24}$ br of cloth and a full *braccio*, but more likely would have simply bought the full *braccio* and trimmed it down to suit his purpose. Thus for the earlier customer, $\frac{23}{24}$ br of cloth and a full *braccio* of cloth would have been, for all practical purposes, indistinguishable. More significant in the context of this study than the practical implications of the first attitude are the non-practical implications. The mental process of equating mathematically imperfect conditions, readily found in the real world, with imaginary idealized conditions facilitated a profoundly mystical interpretation of the world that may be considered one of the fundamental characteristics of western civilization before, and to an ever-decreasing extent during, the Renaissance.

Alfred W. Crosby, in his book *The Measure of Reality: Quantification and Western Society, 1250–1600*, elucidates this interpretation of medieval and Renaissance thought as follows:

> "Today we utilize numbers when we want narrow focus on a given subject and maximum precision in our deliberations. The old Europeans preferred broad focus and settled for imprecision in the hope of including as much as possible of what might be important. Often they were reaching not for a handle on material reality, but for a clue as to what lay beyond the scrim of reality. They were as poetic about numbers as about words."[27]

Crosby notes, for example, that Roger Bacon (c. 1221–c. 1292) readily equated a cycle in history of 693 years that he found in the writings of the Arabic astrologer Abu Ma'shar with the number 663, which he believed to be the number of the Antichrist. The correct number of the Beast of Revelation, Crosby notes, is 666 (Rev. 13:18), a discrepancy that perhaps arose because Bacon's copy of Revelation was defective. Crosby notes however, that:

> The other defect is more interesting. Abu Ma'shar's 693 and the Bible's 663 (or 666, if you want) are not the same number. [...] But Bacon believed that the message is more important than the vehicle, numbers. So he fudged the numbers, justifying himself by saying, "Scripture in many places takes something from a complete number, for this is the custom of Scripture" and "Perhaps God willed that this matter should not be explained fully, but should be somewhat veiled, like other matters which are written in the Apocalypse."[28]

Crosby traces the accelerated shift from this early attitude regarding quantification, which he calls the Venerable Model, after about 1250 to " […] the emerging New Model, [...] [which] was distinctive in its growing emphasis on precision, quantification of physical phenomena, and mathematics."[29] He thereby provides a far-reaching framework within which fundamental differences between medieval and later Renaissance thought can be productively characterized. In its emphasis on quantification, this framework is directly applicable to the study of medieval and Renaissance sets of architectural proportions.

Crosby's Venerable Model helps to explain medieval attitudes toward not only number but geometry. In September 1391 the mathematician Gabriele Stornaloco was summoned to the building site of the Cathedral of Milan to help resolve a problem involving both, and his solution is as remarkable for its mathematical precision as for its tolerance for geometrical and mathematical imprecision. The *capomaestro* at that moment, Annas de Firimburg, wanted the cross-section of the rising cathedral to conform to the proportions of an equilateral triangle. Since the foundations had already been laid with a width of 96 *braccia milanesi*, such a triangle would have an incommensurable height of 83.138… br. Annas and the building committee, however, evidently wanted the flexibility of easily divisible whole numbers. In a letter and accompanying diagram sent to the building committee, Stornaloco calculates a very close approximation of the height in question, which in modern notation comes out to 83.2 br, and describes it as "[…] somewhat less than 84 […]" (*aliquid minus de LXXXIIII*).[30] Stornaloco rounds up this height to 84 br and then divides it into six horizontal stages of 14 br each. That he continued to think of this vertically-stretched, formerly equilateral triangle as equivalent to a truly equilateral one is indicated by his diagram (elucidated by his accompanying verbal description), which consists of a single-line diagram of the cathedral cross-section, superimposed with a framework of diminishing equilateral triangles, all inscribed within an outer hexagon and circle.[31]

Thus, to paraphrase Crosby, for Stornaloco the message, equilateral triangle, is more important than the vehicle, a significantly compromised equilateral triangle. As a mathematician summoned to give expert advice on a major building project, Stornaloco may be assumed to represent the highest level of quantitative thinking of his time; and according to his way of thinking, an equilateral triangle 83.138… br tall, and another triangle of the same width but 84 br tall, could under some circumstances be considered identical. Thus, compared to this well-documented example of medieval willingness to ignore nearly 1 br *milanese* of difference between the heights of two triangles related to the Cathedral of Milan cross-section, the difference of just $\frac{1}{6}$ br *fiorentino* that I propose Talenti and his building committee ignored between the heights of two rectangles embedded in the Cathedral of Florence nave arcade bay thirty-four years earlier (Figure 5-19) represents a quite high level of design precision. By 1418, near-perfect precision would be achieved in Dolfini's San Lorenzo nave arcade bay set of proportions, where an overlapping square and root-2 rectangle, and their numerical dimensions $9\frac{2}{3}$ br and $13\frac{2}{3}$ br, all correspond within (in modern metric units) just 1-3 mm (Figure 4-12).[32]

Crosby's Venerable Model is akin to the medieval and Renaissance concept of *ordine*, which may be understood as both the image of God expressed in terms of geometry and number, and the antidote to that most dreaded state, disorder.[33] A building could be imbued with *ordine* through the use of sets of proportions employing culturally-valued numerical and geometrical constructs. Such constructs included, for example, not just any series of numbers or any four-sided figure, but numbers and geometrical figures that possessed certain culturally-recognized attributes such as symmetry, consistency, symbolism, and a name. Thus, an equilateral triangle—or if need be, an approximate one—possessed *ordine* whereas any randomly-selected triangle, even if symmetrical, would not. Similarly, to revisit the Santa Maria del Fiore example, a square-and-a-half and a two-square rectangle each possessed *ordine*, because they could be understood in terms of that perfectly symmetrical and individually named four-sided figure, the square, whereas the rectangle that would constitute the dimensional average of those two figures would not. Once these two figures were constructed to overlap precisely—with a little help from Venerable Model tolerance—they possessed even more *ordine* than they did individually (Figure 5-19).

In the centuries before the advent of modern structural engineering, architects relied on *ordine* to help ensure structural stability.[34] Buildings that possessed *ordine* thus became associated with stability and, in turn, with the unified attribute of *fortezza* and *bellezza* (strength and beauty). A letter of 1589 by the architect Francesco Terribilia pertaining to the completion of the late fourteenth-century basilica of San Petronio in Bologna, for example, explicates the structural benefits of an overall cross-section proportion that would make the basilica "[…] as high as it is wide […]" and thus conform to the proportions of a square. Terribilia notes that "[…] from this proportion is born a principal strength of the building […]". The letter goes on to attribute this strength to the properties of an implied circle: "[…] because if one places a circle in the middle of it [the basilica cross-section], drawing a circumference that touches the altar, and the walls at the sides, and the peak[s] of the vaults, one forms there a circle, embracing all three of the vaults together with the buttresses, that confers a very strong binding force throughout the work […]."[35]

Similarly, that medieval and Renaissance architects used the rotation of squares technique in part in the belief that it helped ensure structural stability is indicated in an anonymous passage inserted into the 1599 Venice edition (Italian translation) of *Margarita philosophica*, a popular and widely disseminated encyclopedic digest originally authored by Gregor Reisch.[36] According to both this anonymous passage and the today well-known medieval mason's manual published by Mathes Roriczer in 1486, the rotation of squares technique can be used to determine the proportions of vertical, multi-stage structures (such as a Gothic pinnacle, in Roriczer's example) by drawing three inscribed squares, the middle one rotated forty-five degrees, to determine the floor plan dimensions of each stage relative to the others.[37] The proportional ratio between the width of any two adjacent squares in such a series is $1:\sqrt{2}$. The largest square in the series is then taken as a module and multiplied some specified number of times for the height of the structure (seven in the case of Roriczer's pinnacle). While Roriczer is silent with regard to structural considerations, the anonymous encyclopedist notes, both verbally and with a diagram (Figure 6-2), that a height of six modules would be appropriate for the weight of stone, and that different multiples of the module are required depending on the weights of different materials.[38] Talenti thus perhaps made the Santa Maria del Fiore nave pier plinths 5 br square, and the foundation below them 7 br square on the belief that the ratio 5:7, which closely approximates the ratio 1:√2 (which, of course, can be generated by the rotation of squares technique), would help ensure the structural stability of the nave arcades.[39]

Concluding Thoughts on the Crosby Thesis

The Crosby thesis serves as a valuable reminder that in the study of medieval sets of architectural proportions we must not hold the subjects of our study to New Model standards of quantitative precision if they were built under the influence of the Venerable Model. Thus, in the present study I propose that certain geometrical and numerical relationships in the Santa Maria del Fiore nave arcades constitute an intentional medieval set of proportions even though they contain notably less precision and consistency than do similar relationships in the San Lorenzo nave arcades.[40] Medieval culture changed during the approximately six decades that separate these two nave arcade designs. The quantitative criteria by which these nave arcade bay sets of proportions are judged therefore, must be adjusted accordingly as we search for evidence of geometrical and numerical intentions related to both the concept of *ordine*, and the unified concept of *fortezza* and *bellezza*.

Perhaps additional research into the sets of proportions found in other late medieval buildings will show that the notably greater precision of the San Lorenzo nave arcade bay sets of proportions compared to that of Santa Maria del Fiore represents one incremental step toward ever-increasing geometrical and arithmetical precision in medieval sets of proportions. Conversely, perhaps such research will show that the greater precision observed in the San Lorenzo nave arcades represents an abrupt leap forward such that the basilica deserves recognition as the first architectural expression of the New Model. Regardless of which of these interpretations eventually proves more accurate, the Crosby thesis provides a more promising framework for the study of medieval and Renaissance sets of architectural proportions than the Wittkower Paradigm.

Unlike the Crosby framework, the Wittkower Paradigm encourages unnecessary separation between our understanding of medieval architecture, and Renaissance architecture from the Basilica of San Lorenzo forward, and unecessary linkage between historical and aesthetic considerations in the study of sets of architectural proportions.[41] The eye cannot perceive intellectually satisfying numerical relationships such as $9\frac{2}{3} : 13\frac{2}{3}$, or 19, 29, 39, nor can it meaningfully distinguish between intellectually satisfying rectangles such as those derived from the square, and similar ones that are not. Thus, quite the opposite of the Wittkower Paradigm, historically accurate interpretations of sets of architectural proportions require *separation* of historical and aesthetic considerations, and *unification* of the periods we label medieval and Renaissance, such that these periods can be studied as one continuous, incrementally changing historical phenomenon.

Understood within this proposed new conceptual framework, the sets of proportions in the basilica of San Lorenzo cannot be understood as a "radical departure" from medieval precedent, but rather, as a logical development of it.[42] The set of proportions found in the Santa Maria del Fiore nave arcade bays may have provided Dolfini (or possibly Brunelleschi) with some of the seeds for that development; and the history of quantification now provides a useful framework within which to explore the methods and assumptions that accompanied that development from the mid-fourteenth to the early fifteenth centuries.

6.2 Sets of Proportions as Rhetorical Rather Than Aesthetic Structures

As noted in Chapter 1, an alternative to Wittkower's aesthetic interpretation of architectural proportion (proportion-1 and proportion-3) is to interpret sets of proportions (proportion -3) as forms of communication that are incapable of producing aesthetically-significant visual outcomes. According to this new interpretation, sets of proportions can be understood as narratives that enhance the experience of architecture for those who are both capable of reading them and receptive to their messages. Understood as such, they belong to the rhetorical rather than the aesthetic or physical structures of architecture. While I propose here that historians can productively interpret sets of proportions as rhetorical devices however, I do not propose that medieval and Renaissance architects necessarily interpreted them that way.

Thus, while these architects appear to have recognized sets of proportions as rhetorical devices at least in some cases, such as when they incorporated number symbolism into architectural dimensions and quantities, in other cases they appear to have believed that sets of proportions served not merely to communicate through non-physical means, but to bring about *physical* outcomes that they considered to be both beautiful and structurally stable. Such beliefs held by medieval and Renaissance architects that could not possibly be true—at least, it is a premise of the present study that sets of proportions can bring about neither beauty nor structural stability in architecture—may also be considered rhetorical devices, in that they imbued architecture with meaning for those architects.

A medieval mason, for example, may have *believed* that a particular set of proportions caused a vault to be structurally stable, when in fact the stability of the vault depended most critically on a variety of other variables, such as the quality of the materials and the composition and mode of application of the mortar. The rhetorical interpretation of sets of proportions that I propose, furthermore, accommodates the possibility that at any given moment in history, medieval and Renaissance individuals may have held *diverse* beliefs with regard to sets of architectural proportions. This interpretation assumes, however, that virtually everyone who had some involvement with architecture during the periods in question, whether as designer, builder, patron or informed observer, held *some* beliefs with regard to them.

In Chapter 2, I suggested that Dolfini embraced a recognizably more intricate and theoretical approach to sets of architectural proportions than Brunelleschi. While Dolfini and Brunelleschi may have approached the problem of architectural proportion with different attitudes and intentions, however, both worked from a common foundation of medieval and Renaissance proportional belief that was characterized by a comprehensive notion of architectural correctness for which we have no equivalent today: A building that was di *proportione* (in correct proportion), possessed *ordine* (order), *fortezza* and *bellezza* (strength and beauty), and was right in every way.[43]

The concept of *ordine* was paramount in medieval and Renaissance thought, for the idea of a lack thereof, in a world ever threatened by war, corruption, famine, and disease; and haunted by the ubiquitous ruins of the once eminently orderly and powerful Roman Empire, was so utterly disagreeable.[44] Architecture was for these periods a permanent and reassuring symbol of humankind's perpetual quest for triumph over the forces of disorder. Indeed, Manetti describes the very origin of architecture as the process by which people first learned how to "expurgate disorder" (*purghare de' disordini*), and "discover order" (*scoprire qualche cosa di ragione*).[45] Thus the importance of *ordine* to the work of the architect could not be overstated in the medieval and Renaissance periods. Serlio accordingly excoriates those architects—though he would even deny them the honor of that title—whose buildings, wanting proportional order based on the principles of geometry, are merely "arbitrary and random" (*a ventura & a caso*).[46] Serlio laments the lack of art (*senza arte alcuna*) and order (*con pocca ragione*) in such works, using the words *arte* and *ragione*, respectively, and such words appear to be closely related to, if not synonymous with, the seemingly more common word *ordine*.[47] Even casual architectural criticism from these periods frequently refers to *ordine*, as in a late sixteenth century assessment of the church of San Petronio in Bologna that notes: "…it appears at first sight to be a beautiful work, and with some order…."[48]

The concept of *ordine* had more than merely symbolic import for the notion of architectural fitness in the medieval and Renaissance periods. Prior to the advent of modern structural analysis in the mid-eighteenth century, sets of proportions encoded a vast body of proto-engineering wisdom, hard-won through centuries of trial and error, that helped architects impose *certain kinds* of order in the hope of ensuring the structural stability of every part of a building, from foundations to vaults.[49] The repeated collapse of the immense dome of Hagia Sophia in the sixth, tenth, and fourteenth centuries, and of the soaring vaults of Beauvais Cathedral in 1284, are particularly spectacular examples of what must have been a continuous succession of structural failures in these periods.[50] Lorenz Lechler, in his booklet on Gothic design principles and techniques written for his son in 1516, acknowledges the inherent precariousness of both architecture and the architect's lot when he advises: "… if you give proper attention to my teaching, you can meet the needs of your building patron and yourself, and not be despised as the ignorant are, for an honorable work glorifies its master, if it stands up."[51]

In this climate of structural uncertainty, the proportions of buildings that stood up, and stayed up, came to represent strength (*fortezza*), and strength simultaneously came to embody beauty (*bellezza*), in a sense very different from our aesthetic understanding of the term today.[52] A building not only had to *be* strong but *look* strong to earn the confidence of those whose reputations depended on its structural stability, and perhaps of the general public as well. Thus, if the term *fortezza* came to mean "is strong," *bellezza* came to signify in part its essential complement, "looks strong." Thus, furthermore, did *bellezza* come to influence *fortezza*, as much as the inverse, for many a medieval and Renaissance building is structurally over-built for purely visual reasons, most notably when classical columns are involved.[53]

These two terms, which are subsidiary to the overarching concept of *ordine*, are often linked in the documentary record, sometimes accompanied by any of a host of other laudatory terms. Serlio's note that a particular set of antique pilasters "...uphold the corner by strength and with beauty of work" is typical, as is the late sixteenth century observation of architect Francesco Terribilia that the basilica of San Petronio in Bologna suffers "…some defects in its parts with regard to both strength and beauty."[54] Numerous variations on this theme can also be found in the documentary record. In the archives of the Cathedral of Florence, a newly proposed design for a column is described in 1357 as "more strong and beautiful and praiseworthy" than a previous one.[55] Another entry dated 1366 notes that various designs were evaluated to determine which "... is most beautiful and most useful and most secure.…"[56] In his late fifteenth century biography, Manetti notes that Brunelleschi completed the cupola of the Cathedral of Florence "... with very great beauty and strength and usefulness ...," and in 1587 a group of experts, including Terribilia, submitted a similarly worded opinion that the vault over the San Petronio nave "…must be made [in a way that provides] strength, beauty, and usefulness."[57]

The concept of *fortezza* and *bellezza* appears to be related to the canonical Vitruvian triad of architectural fitness—"strength, convenience, beauty"—which resembles the last three examples above.[58] It is also fundamental to Vitruvius's understanding of columnar proportions. Of the history of such proportions Vitruvius writes:

> "Wishing to set up columns in that temple, but not having rules for their symmetry, and being in search of some way by which they could render them fit to bear a load and also of a satisfactory beauty of appearance, they measured the imprint of a man's foot and compared this with his height…. Thus the Doric column, as used in buildings, began to exhibit the proportions, strength, and beauty of the body of a man."[59]

What, exactly, were the proportions that would imbue a work of architecture with *fortezza* and *bellezza*, and perhaps other attendant positive qualities? Extensive documentary evidence indicates that in the medieval and Renaissance periods, no one was ever quite sure. To determine the proportions of important structural members, Lechler simply advises his son: "Give attention to the divisions of the buttress; for that which is above the springer or capital you may take whatever you think will stand up well."[60] The equally candid remarks of the sixteenth century Spanish architect Rodrigo Gil de Hontañon, after his presentation of a rule for the estimation of rib vault thrusts, further convey a sense of how precarious it all was. Hontañon writes: "I have often attempted to rationalize the buttress needed for any bay, and have never found a rule adequate for me. I have also pursued the inquiry among Spanish and foreign architects, and none appears to have established a rule verified by other than his own judgment. Upon asking how we shall know whether such and such a buttress is enough, we are told that it is needed, but not for what reason. Some take the fourth [of the span], and others arrive [at an estimate] by certain orthogonals, and dare to have confidence…."[61]

The Geometrical Model

As the preceding evidence indicates, while medieval and Renaissance architects typically seem to have employed a great variety of numerical rules of thumb and geometrical techniques to determine the proportions of individual structural members, just as often they relied on simple guesswork.[62] To ensure overall stability, however, these architects appear to have believed that the presence of basic geometrical figures in the overall building proportions was necessary. Among the deliberations of the *Opera* of the Cathedral of Milan from the year 1400, for example, we find a proposal "...to integrate the aforesaid church and transept so that they correspond to a rectangle according to the demands of geometry, but beyond this, for the strength and beauty of the crossingtower."[63] Similarly, a letter of 1589 by Terribilia pertaining to the completion of the basilica of San Petronio explicates the structural benefits of an overall cross-section proportion that would make the basilica "…as high as it is wide…," and thus conform to the proportions of a square, noting that "…from this proportion is born a principal strength of the building…." The letter goes on to attribute this strength to the properties of an implied circle: "…because if one places a circle in the middle of it [the body of the basilica], drawing a circumference that touches the altar, and the walls at the sides, and the peak[s] of the vaults, one forms there a circle, embracing all three of the vaults together with the buttresses, that confers a very strong binding force throughout the work…."[64]

In the absence of a scientific model for structural stability, the geometrical model described here must have seemed perfectly reasonable to medieval and Renaissance architects based on the evidence available to them, which would seem to have been everywhere apparent. A projectile launched upward at a forty-five degree angle, for example, flies farther than one launched with equal force at any other angle. Might not a vaulted bay proportioned according to the diagonal of a square (which has an angle of forty-five degrees), therefore, be stronger and last longer than one built to a different proportion? And if a bay proportioned as such indeed proved to be strong, would not imitating that proportion be a logical strategy for ensuring that another vaulted bay built elsewhere would also be strong? However logical the geometrical model might have seemed to the pre-modern mind however, when it led to successful results, which it must have done at least occasionally, it surely did so most often by accident. Other factors in addition to proportion, after all, such as types of foundations, strength of materials, extent and type of buttressing, and severity of wind loads are equally important in establishing structural stability in large buildings.

When a certain proportion appeared to result in structural stability, it earned the respect of architects and builders and remained in use until a better one was found. Indeed, just as pre-Copernican models of an earth-centered universe still find use among mariners today, so too does the geometrical model retain some limited relevance for architects and builders today.[65] For example, when a lintel above a window or door in a brick wall fails, modern science cannot predict the shape of the void that will result after the bricks above the opening have fallen out—a regular triangle of equilateral or somewhat lower proportions remains the best estimate.[66] Similarly, the root-2 rectangle happens to be a very efficient cross-section for a wood joist, ensuring near-maximum strength per unit of material for species of wood commonly used in construction.[67]

The necessity for structural stability in architecture, and the deeply-rooted belief in the efficacy of the geometrical model during the medieval and Renaissance periods, together help to explain the most notable similarity between the Dolfini- and Brunelleschi-designed proportions in the basilicas of San Lorenzo and Santo Spirito, that being the use of rectangular proportions based on the diagonal of the square for the proportions of large arched openings, or major portions thereof. The preceding evidence suggests that both architects may have understood such proportions to be useful conventions that would help to establish *ordine*, a condition that would also help to maintain architectural dimensions within the structural limits of masonry construction. Thus their works would not only remain standing, but would embody ample *fortezza* and *bellezza*. These geometrical proportions, perhaps in combination with various numerical and arithmetical relationships, could perhaps have carried other layers of significance for Dolfini and Brunelleschi involving number symbolism, and philosophical or religious beliefs, though additional research is needed to establish any such significance (and any differences between these two architects' belief systems) with certainty.[68] Without structural stability, however, there would be no architecture with which to associate such beliefs, and documentary evidence leaves little doubt that in the absence of modern structural engineering methods, geometrical sets of proportions constituted one important strategy that architects of Dolfini and Brunelleschi's day used with the intention of ensuring that their buildings would stand up and stay up.

The preceding analysis highlights the main purposes of sets of architectural proportions for people of the medieval and Renaissance periods: to provide *ordine* and the closely-related quality of structural stability. Architects and builders of these periods used sets of proportions of for a variety of other reasons as well, however, such as to ensure stylistic consistency, in particular where the classical orders were concerned, and to provide diagrammatic clarity, as in the logical subdivisions of floor plans (Figures 3-7 to 3-10). All of these purposes were significant for their narrative qualities, because they had no necessary, functional qualities. Sets of proportions, for example, did not ensure structural stability, and *ordine*, being an effect intended, but not always achieved, may have seemed necessary to some, but could not, in fact, have been necessary in any tangible way. While the columns of both the basilicas San Lorenzo and Santo Spirito, furthermore, appear to be equally correct examples of the Corinthian order, they have quite different proportions, those of Santo Spirito being about one capital height shorter than those of San Lorenzo, though the shaft diameters are equal. Thus, the columns of these two basilicas demonstrate that while sets of proportions can be used to create Corinthian columns that look correctly Corinthian, such sets are not necessary to produce such an outcome. Similarly, diagramatic clarity is not necessary for the function of a building, but has narrative value in leading the informed observer to understand a design as having originated from an earlier, more basic form, or at least to believe that it did (Figures 3-7 to 3-10).

Why medieval and Renaissance architects, including Dolfini and Brunelleschi, incorporated often-elaborate sets of proportions into the designs of their buildings when these sets do not appear to have been functionally necessary is a profound question that highlights the identity of architecture as an art, both in the fifteenth-century sense of *ars*, as the result of methodical, orderly reasoning, and in the modern sense, as a form of human expression.[69] This identity ultimately explains why the basilica of San Lorenzo has such a serene, orderly appearance that tends to make one think of geometry and mathematics. The basilica looks the way it does because Filippo Brunelleschi, the architect who is most responsible for its present appearance, wanted it to look that way, and had the skills as a designer and craftsman to bring his creative vision to fruition. Although the carefully-crafted sets of proportions that are incorporated into the dimensions of the basilica make no contribution to this aesthetic impression, those proportions communicates a wealth of information about early fifteenth-century knowledge pertaining to geometry, number, arithmetic, and perhaps more that future research will illuminate.

7.3 A Disciplinary Triad

The second characteristic of the Wittkower Paradigm that I identified in Chapter 1, suppression of the physical object in favor of documentary sources, is the product of a long philological tradition that Wittkower brought to architectural history from art history as a natural consequence of his training. Since I do not think that sets of proportion can be studied in the absence of the buildings in which they were embedded, I have made the object the focus of my study, not as an image to look at with the unaided eye and assess aesthetically, but as an object to measure and inspect at close range. In planning a study that uses the architectural object itself as a primary source of historical evidence, I found that architectural history offers a rich array of precedents, particularly in the areas of ancient and medieval studies.[70] I found few useful precedents, however, for the use of measurements in the study of architectural proportion.

The major document-based studies of proportion, though valuable, do not seek confirmation from measurements, and virtually all the measurement-based studies contain sufficient methodological shortcomings to have instilled deep skepticism in many architectural historians about the very viability of measurements as a source of historical evidence in the study of architectural proportion.[71] Contributing to this problem is the current lack of rigorous standards for observation-based research, such as those already in place for documentary research. For example, measurements are often cited without specifying the exact locations of all end points, the methods by which they were recorded, or by whom—essential information to enable the reader to understand and verify the data. I have developed my methodology with an eye toward addressing these challenges, aided by a rethinking of the nature of architectural history as a scholarly discipline.

Many architectural historians, particularly in North America, seem to view their discipline as a branch of art history, and perhaps as a consequence, much architectural history reads like art history—it tends to focus on those issues of greatest interest to art historians, such as style, ornament, iconography, aesthetics, patronage, and socio-cultural context; and to rely most often on the methods favored by them, namely documentary research, and to a lesser extent in contemporary scholarship, connoisseurship.[72] Yet I have found that viewing architectural history as a branch of *architecture*—as, let us say, architecture's alter-ego; the part that interprets rather than creates—has tended to focus my attention on other issues, of particular interest to architects, such as *parti* (diagrammatic intention), function, structure, dimension, spatial experience and transformation over time. This perspective has encouraged me to use in my own research the methods and techniques favored by architects today, such as measuring (Figures 6-3 and 6-4), drawing, model building, and structural analysis; for architecture, in its most fundamental operation of conceiving and placing a useful architectural object in the world, has not substantially changed since the time of Vitruvius.[73]

Such a reframing of architectural history as a branch of architecture places art history, with its relevant interests and methods, in a supportive role on one side and, by emphasizing the importance of the physical reality of building, places archaeology in a corresponding role on the other. With its unflinching devotion to the object, and its scientifically rigorous methods of observation, data collection, and analysis, archaeology has already contributed much to architectural history and could yet contribute more.[74] Thus, art history and archaeology as typcially practiced today have complementary strengths: art history favors interpretation through documentary research and formal analysis, keeping the object at a distance, if it is present at all; while archaeology favors quantitative analysis through direct and intensive observation of the object.

I like to think of architectural history, together with its companion architecture, as occupying a Lagrange Point equidistant between art history and archaeology, held there by the equal gravitational pulls of both. This triangular analogy has served as a useful reminder to integrate a variety of observation-based and documentary approaches into my San Lorenzo research. Thus, I have thought like an architect, in striving to reconcile theory with the practical realities of building; like an art historian, in comparing sculptural features throughout the building, and scouring the documentary evidence for insights into building history and intended proportional order; and like an archaeologist, in recording comprehensive measurements and other observations, and subjecting them to rigorous inductive analysis. In the end, however, I have crafted a unique approach to the study of architectural history unlike those of any of the above three disciplines.

Since the purpose of my research is the critical study of architectural proportion as historical evidence, and not the pure documentation of architectural form as an end in itself, nor the examination of human perception of architecture, the approach described here emulates scientific models of research, for hypotheses are rigorously formulated and tested, and empirical data is carefully controlled, but acknowledges the unpredictability of human nature that makes architecture one of the humanities. This integrated, observation-based approach to the study of architectural history has the potential to bring to light new knowledge pertaining not only to architectural proportion, but to many other areas of architectural theory and practice as well.[75]

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[1] Wittkower, *Architectural Principles* cit., p. 161.

[2] Paul Tannery, *Memoires Scientifiques* 5 (Paris and Toulouse: E. Privat, 1922), 236.

[3] G.R. Evans, *From Abacus to Algorism: Theory and Practice in Medieval Arithmetic*, “The British Journal for the History of Science”, X, 35, 1977, pp. 114-115.

[4] D.J. Struik, *The Prohibition of the use of Arabic numerals in Florence*, “Archives internationales d’histoire des sciences” XXI, 84-85, 1968, pp. 291-294. Note that ‘Hindu-Arabic numeral’ is simply a more precise term for ‘Arabic numeral’. Selection of one or the other appears to be a matter of personal preference among scholars.

[5] *Ibidem*.

[6] W.W.R. Ball, *A Short Account of the History of Mathematics*, 3rd. ed., London, MacMillan and Co., 1901, p. 192.

[7] *Ibidem*, p. 193.

[8] See, for example, *Santa Maria del Fiore*, ed. by Guasti cit., p. 310, Doc. 462, dated February 9, 1411 (1412 new style), for Hindu-Arabic numerals; and *ibidem*, p. 310, Doc. 464, dated March 29, 1412, for Roman numerals. Cfr. mathematician Gabriele Stornaloco’s letter of 1391 that describes complicated calculations regarding the proportions of the Cathedral of Milan using Roman numerals. P. Frankl, *The Secret of the Mediaeval Masons*, “The Art Bulletin” XXVII, 1, 1945, p. 53. The most significant vehicles for the introduction of Hindu-Arabic numerals to the West appear to have been the Latin translation of Al-Khowarizmi’s *Arithmetic* in the twelfth century, Leonardo of Pisa’s *Liber abaci* of 1202 (revised 1228), and the numerous almanacs and calendars whose makers were early converts to the new system. G.R. Evans, *From Abacus to Algorism: Theory and Practice in Medieval Arithmetic*, “The British Journal for the History of Science” X, 35, 1977, p. 115; *A Source Book in Mathematics, 1200-1800*, ed. by D.J. Struik, Cambridge, Massachusetts, Havard University Press, 1969, p. 1; and W.W.R. Ball, *History of Mathematics* cit., pp. 192-193.

[9] F. Cajori, *A History of Mathematics*, New York and London, 1909, pp. 122-124; G.R. Evans, “From Abacus to Algorism: Theory and Practice in Medieval Arithmetic,” *The British Journal for the History of Science* 10, no. 35, 1977, p. 122.; L.C. Karpinski, *The History of Arithmetic*, New York, 1965, pp. 124-125; and K. Menninger, Zahlwort und Ziffer, Göttingen, Vandenhoeck and Ruprecht, 1958, Engl. transl. *Number Words and Number Symbols: A Cultural History of Numbers*, Cambridge, Massachusetts, The MIT Press, 1969, pp. 158-162. For the soureces of the symbols shown in Figure 7-1 see Gerbert, *Oeuvres de Gerbert, Pape sous le nom de Sylvestre II*, ed. by Alexandre Olleris, Paris, 1867, pp. 343-348 and 385-389; and *Due trattati inediti d’abaco, contenuti*

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in due codici vaticani del secolo XII, ed. by Enrico Narducci, Roma, 1882, pp. 41 and 50 (republished extract from: “Bolletino di bibliografia e di storia delle scienze mathematiche e fisiche”, XV, 1882, pp. 41-50);

[10] Evans, *From Abacus to Algorism* cit., p. 127.

[11] *Ibidem*, p. 122.

[12] *Santa Maria del Fiore*, ed. by Guasti cit., p. 66. Since the new year in the calendar used in Florence and many other regions of medieval and Renaissance Europe began on 15 March, any date notation between 1 January and 15 March must specify old style (new year on 15 March) or new style (new year on 1 January).

[13] From a document dated June 19, 1357. *Ibidem*, p. 94.

[14] A unit fraction is any fraction having a numerator of 1, or, ‘unity’. In Egyptian mathematics, strings of unit fractions were added together to represent non-unit fractional quantities, since the only non-unit fraction that the Egyptians possessed was $\frac{2}{3}$. Thus, according to Karpinski, “seven-eighths was written as $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ or as $\frac{2}{3}, \frac{1}{8}, \frac{1}{12}$ “. Karpinski, *History of Arithmetic* cit., p. 121. Cf. D.E. Smith, *History of Mathematics*, II, New York, 1953, p. 209 ff. Serial fractions (my term), the various components of which are related to each other by more complex calculations than the simple addition in the Egyptian example above, have to my knowledge received scant attention in the history of mathematics literature. Karpinski simply refers to serial fractions as “[…] an Arabic device […]” that augmented “[…] the complications of unit fractions, common fractions, and sexagesimal fractions […]”. He then proceeds to provide an example, marred by a typographical error, that should read as follows: “This Arabic device consisted in writing a fractional form $\frac{1}{3}\ \frac{1}{5}$ to mean $\frac{1}{3}+\frac{1}{5}$ of $\frac{1}{3}$ or $\frac{4}{13}\ \frac{3}{11}$ to mean $\frac{4}{13}+\frac{3}{11}$ of $\frac{1}{13}$ “. (The actual passage, before my correction, reads: “[…] fractional form $\frac{1}{3}\ \frac{1}{5}$ to mean $\frac{1}{3}+\frac{1}{3}$ of $\frac{1}{5}$ or […]”). Karpinski, *History of Arithmetic* cit., p. 126; and C.B. Boyer, *A History of Mathematics*, II ed., revised by U.C. Merzbach, New York, John Wiley & Sons, 1991, p. 255.

[15] The terms ‘common’, ‘general’, and ‘vulgar’ fraction all appear to be interchangeable references to any fraction with a numerator greater than 1, or ‘unity’. Smith, *History of Mathematics*, II cit., pp. 213-219; and Karpinski, *History of Arithmetic* cit., p. 127.

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[16] Note that elsewhere in the documents of the *Opera* of Santa Maria del Fiore, the fraction $\frac{1}{1}$, when used individually, appears to signify $\frac{1}{2}$ (see for example, *Santa Maria del Fiore*, ed. by Guasti cit., pp. 88, 96, 98, and 100). Such usage perhaps explains the above-noted scribe's confused verbalization of the number *xxxiij* $\frac{3}{8}\ \frac{1}{1}$. Occasionally it is used to signify "mid-" (*metà*), as in "mid-July" (*a* $\frac{1}{1}$ *lulglio* [*sic*]), and "mid-September" (*a* $\frac{1}{1}$ *settembre*) *Santa Maria del Fiore*, ed. by Guasti cit., pp. 93 and 102. In light of the evidence presented in this study, Saalman's note regarding the Santa Maria fel Fiore documents that "the fraction 1/1 throughout these documents stands for one-half" appears to be only partially correct. Saalman, *Santa Maria del Fiore* cit., p. 478.

[17] The document is dated February 9, 1411 (old style). *Santa Maria del Fiore*, ed. by Guasti cit., p. 310, Doc. 462. A subsequent document dated March 29, 1412 contains a Roman numeral, demonstrating the resilience of the old system. *Santa Maria del Fiore*, ed. by Guasti cit., p. 311 Doc. 464.

[18] *Santa Maria del Fiore*, ed. by Guasti cit., p. 310, Doc. 462.

[19] Paolo dell' Abbaco, *Trattato d'aritmetica, Secondo la lezione del Codice Magliabechiano XI, 86 della Biblioteca Nazionale di Firenze*, ed. by Gino Arrighi, Pisa, Domus Galilæana, 1964. Cfr. Cohen, *How Much Brunelleschi?* cit., p. 53 note 45.

[20] *Ibidem*.

[21] The shortcut is as follows: To derive a common fraction equivalent of a given serial fraction, first derive the denominator of the common fraction by multiplying together all the serial fraction denominators. Next, derive the numerator of the common fraction by multiplying the first serial fraction numerator by the second denominator, add the second numerator, and repeat to the end of the serial fraction. Thus, in Paolo's example the serial fraction $\frac{2}{3}\ \frac{1}{2}\ \frac{3}{4}$ is equivalent to the common fraction $\frac{23}{24}$. Paolo dell' Abbaco, *Trattato d'aritmetica* cit., p. 23, Problem 1. Note that many different combinations of serial fraction components can usually be found to denote one and the same common fraction.

[22] *Ibidem*.

[23] Paolo dell' Abbaco, *Trattato d'aritmetica* cit., pp. 25-26. Although serial fractions appear to have fallen out of common use after the fourteenth century, they appear to have survived in limited applications for centuries thereafter. In editions of Giovanni Branca's *Manuale d'architettura*

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appearing as late as 1789, for example, a simplified, two-unit form of serial fraction is used in a chart comparing measurements from Rome and Modena. The Roman measurements are listed in whole numbers, while the Modena equivalents are listed as whole numbers plus fractions. Some of the latter fractions are common fractions with denominators of 16, such as $3\,\frac{8}{16}$ and $7\,\frac{15}{16}$. Others are serial fractions composed of a fraction with denominator 16, followed by $\frac{1}{2}$, for example, $3\,\frac{15}{16}\,\frac{1}{2}$ (equivalent to $3\,\frac{15}{16}+\frac{1}{32}$, or, $3\,\frac{31}{32}$), and $8\,\frac{13}{16}\,\frac{1}{2}$ (equivalent to $8\,\frac{13}{16}+\frac{1}{32}$, or, $8\,\frac{27}{32}$). In this form, unit fractions serve as a simple device for maintaining a maximum denominator of 16, perhaps because $\frac{1}{16}$ was a convenient minimum fractional unit for builders to be concerned with. Thus, in the first example, the builder would have known to measure $3\,\frac{15}{16}$, plus an additional $\frac{1}{32}$ of length, if he did not choose to simply drop that last $\frac{1}{32}$. Giovanni Branca, *Manuale d'architettura* III, iv, Modena, 1789, p. 64.

[24] Cfr. Cohen, "How Much Brunelleschi?" cit., p. 51, note 25.

[25] *Ibidem*, and Angelo Martini, *Manuale di metrologia*, Turin, 1883, p. 206.

[26] The fraction $\frac{1}{3}$ cannot be expressed in the modern metric system, for example, because 10 is not divisible by 3, and an endless decimal results; thus $\frac{1}{3}\,\text{m} = 0.333\ldots\,\text{m}$. One-third of an English foot, however, equals 4 inches (because the 12 inches in a foot are divisible by 3). Conversely, the fraction $\frac{1}{5}$, which equals 0.2 m in the metric system, cannot be expressed in terms of English inches because 12 is not divisible by 5. Since the *soldi* and *denari* system consists of two levels of subdivision, the first based on 20 units and the second on 12, all possible divisors can always be accommodated.

[27] A.W. Crosby, *The Measure of Reality: Quantification and Western Society, 1250–1600*, Cambridge, Cambridge University Press, 1997, pp. 46-47.

[28] Crosby, *The Measure of Reality* cit., pp. 121-122.

[29] *Ibidem*, 49, 58.

[30] J.S. Ackerman, "*Ars Sine Scientia Nihil Est*" *Gothic Theory of Architecture at the Cathedral of Milan*, "The Art Bulletin", XXXI, 2, 1949, pp. 89-90; and P. Frankl, *The Secret of the Medieval Masons*, "The Art Bulletin", XXVII, 1, 1945, 5. Guy Beaujouan's interpretation of Stornaloco's

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calculations supersedes Panofsky's, though Beaujouan mistakenly indicates the height of the equilateral triangle in question as 83.136… instead of the correct height of 83.138…. *Ibidem*, pp. 61-64; and Guy Beaujouan, "Réflexions sur les rapports entre théorie et pratique au moyen âge," in *The Cultural Context of Medieval Learning*, ed. by J.E. Murdoch and E.D. Sylla, Dordrecht, Holland and Boston, D. Reidel Publishing Company, 1975, pp. 444-445.

[31] Ackerman, *Gothic Theory of Architecture* cit., pp. 89-90; and Shelby, *Secret*, 53-60.

[32] On the year 1418 as the *terminus ante quem* for the completion of the design of the San Lorenzo sets of proportions, see Cohen, *How Much Brunelleschi?* cit., pp. 41-42. The 1-3 mm discrepancy noted here is the difference between the height of an approximate root-2 rectangle that measures $9\frac{2}{3}$ br wide by $13\frac{2}{3}$ br high, and the height of a true root-2 rectangle that measures $9\frac{2}{3}$ br wide and ($\sqrt{2}$ x $9\frac{2}{3}$) br high.

[33] See Chapter 7.2.

[34] *Ibidem*.

[35] "Questa medesima altezza è proportionata col corpo principale della chiesa, perchè ella viene ad esser tant'alta quanto larga, dico lasciando le capelle che sono parte del corpo: et da questa proportione nasce una fortezza principale del edificio, perchè posto un centro nel meggio di essa, et tirata una circonferenza che tocchi l'ara et le mura dei lati et la cima delle volte, se ne forma un circolo, il quale abbracciando tutte tre quelle volte con li contraforti insieme viene a farsi una ligatura fortissima di tutta la fabrica. […] " G. Gaye, *Carteggio inedito d'artisti* III, Florence, Giuseppe Molini, 1840, p. 492.

[36] The passage appears in a chapter on architecture that does not appear in earlier editions. "Amaestramenti dell'architettura positive" in G. Reisch, *Margarita Philosophica*, Heidelberg, 1496, trad. it. *Margarita filosofica* […], Venice, Iacomo Antonio Somascho 1599, p. 999.

[37] See Figure 3-35.

[38] L.R. Shelby, *Gothic Design Techniques: The Fifteenth-Century Design Booklets of Mathes Roriczer and Hanns* Schmuttermayer, Carbondale, Illinois, Southern Illinois University Press, 1977, pp. 3–5, 32, and 84–87. The anonymous encyclopedist notes: "[…] various are the measures according to the material weighed. Thus different norms are used with stone, wood, and metal masses, one being heavier than the other. In stone, one square supports six of its equals in height, and these support six more smaller squares of a rotated arrangement, which result from the larger square, and proceeding thus, a pyramidal union is formed. " ("[…] diverse sono le misure secondo la

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diversità del corpo ponderoso. Percioche altra norma si serva nei corpi di pietra, altra in quelli di legno, altra in quelli di metallo, essendo l'uno più grave dell'altro. In quelli di pietra un quadrato ne sostenta sei altri uguali a se in alto, & con questi altri sei minori quadrati di figura rivoltata, che risultano dal maggior quadrato, & cosi procedendo si fa una unione piramidale.") "Amaestramenti dell'architettura positive" in Reisch, *Margarita filosofica*, 1599 cit., p. 999. The appearance of this explication of the rotation of squares technique in a book published in 1599 should help lay to rest the widespread misconception among scholars that the rotation of squares technique was a medieval phenomenon that Renaissance architects did not use to any significant extent. Cfr. Cohen, "How Much Brunelleschi?" cit., pp. 24-27

[39] Likewise, Talenti may have believed that the 29 br by 41 br root-2 rectangle that he incorporated into the Santa Maria del Fiore nave arcade bay sets of proportions (Figure 6-20) contributed to the structural stability of the bay because, as a very close approximation of a root-2 rectangle, it too can be derived from the rotation of squares technique.

[40] Note, however, that the San Lorenzo overall basilica set of proportions, by contrast with the nave arcade bay set of proportions, embodies significant dimensional compromises. See Chapter 4.

[41] For passages in which Wittkower links sets of architectural proportions with aesthetics, see Chapter 1 and: *ibidem*, pp. ii, 116, 158-160; Idem, "Systems of Proportion," *Architects' Yearbook* 5, 1953, pp. 9, 16; and Idem, "Brunelleschi and 'Proportion in Perspective'," *Journal of the Warburg and Courtauld Institutes* 16, 3-4, 1953, pp. 132-134.

[42] Regarding the overall proportions of the basilica of San Lorenzo, Janson notes that Brunelleschi's "…clearly defined, separate space compartments represent a radical departure from the Gothic architect's way of thinking." H.W. Janson, *History of Art: a Survey of the Major Visual Arts from the Dawn of History to the Present Day*, II ed., Englewood Cliffs, New Jersey, 1977, p. 389; III ed., 1986, p. 409; and («radical change») VI ed., 2001, p. 397. In the current edition "radical" has been removed but the intent remains essentially unchanged: "[…] a new emphasis on symmetry and regularity distinguishes [Brunelleschi's] design for San Lorenzo [from Gothic precedents] […]." *Janson's History of Art: The Western Tradition*, VII ed. by Penelope J. E. Davies et al., Upper Saddle River, New Jersey, 2007, p. 513.

[43] See, for example, Giovanni di Domenico da Gaiole's letter of 1457 to Giovanni de' Medici, which contains the following description of Brunelleschi's unexecuted design for the crossing dome (*tribuna*) of San Lorenzo: "...it would have been less costly to dismantle and rebuild that *tribuna* in the manner of Filippo, which is light, strong, illuminated, and correctly proportioned, than to follow the unsatisfactory plan..." ("...gli era meno ispesa a disfare e rifare quella tribuna nel modo di filippo,

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chè legiera, forte, alluminata e di proporzione, che seguire lo inconveniente...") Giovanni Gaye, *Carteggio inedito d'artisti dei secoli XIV, XV, XVI*, 3 vols. (Florence, 1839), 1:168. Sometimes this concept is expressed with the words *di ragione*, or *in sua ragione*, as in the following excerpt from Bernardo da Venezia's letter to the *opera* of the Cathedral of Milan, which employs an alternate spelling of this word: "…the body of the church would look more beautiful, and more in proportion…" ("…el corpo dela giesia parerave più bello, e più con sova rexone…"). James Ackerman, "'Ars Sine Scientia Nihil Est': Gothic Theory of Architecture at the Cathedral of Milan," *Art Bulletin* 31, no. 2 (June 1949), 95, 111. The preceding terms are comparable to the medieval German term *Gerechtigkeit* (or *gerechtikait*), which, as used by Mathes Roriczer, Shelby translates as "correctitude," or "correct design." Lon R. Shelby, *Gothic Design Techniques: The Fifteenth-Century Design Booklets of Mathes Roriczer and Hanns Schmuttermayer* (Carbondale and Edwardsville, Ill., 1977), 32-33, 106. A similar fifteenth century reference to this concept that is directly associated with Brunelleschi is found in the records of the Cathedral of Florence (the Basilica of Santa Maria del Fiore). In 1404 Brunelleschi sat on a board of advisors that oversaw the demolition and reconstruction of a then-newly completed portion of the tribune of Santa Maria del Fiore because the board believed it was built "at variance with the required and true measures." Frank D. Prager and Gustina Scaglia, *Brunelleschi: Studies of his Technology and Inventions* (Cambridge, Massachusetts: MIT Press, 1970) 16.

[44] For a fourteenth century expression of the fear of societal disorder, see Ambrogio Lorenzetti's fresco cycle, "The Allegories of Good Government and Bad Government," 1338-1339, Sala della Pace, Palazzo Pubblico, Siena. The association of order with goodness and virtue may be considered a fundamental concept of Western culture. See, for example, the Old Testament description of the Creation, and later representations thereof. Genesis, 1:2. For a typical example of a painted depiction of God the Geometer, c. 1250, see Ehrenfried Kluckert, "Romanesque Painting," in *Romanesque: Architecture, Sculpture, Painting*, ed. Rolf Toman (Cologne, 1997), 448.

[45] Antonio di Tuccio Manetti, *Vita di Filippo Brunelleschi*, ed. Giuliano Tanturli (Milan, 1976), 74-75.

[46] "The extent to which this most precise art of geometry is necessary to everyone can be testified by all those who at one time worked without it, but then later came to some understanding of the art; they will honestly confess that all the things that they thought and made without geometry were lacking in any art whatsoever, but were arbitrary and random. Since this most profound art of architecture is the embracer of many noble arts, firstly it is important that the architect be, if not learned, at least sufficiently emersed such that he has some understanding of it, particularly the

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principles, and also more, and not like many consumers of stone, plaster and even marble, who today bear the name architect, but who do not even know what is a point, a line, a surface, or a body, or what correspondence and harmony are. But guided by their own opinions and what pleases their eyes, following the traces of others, which were made with little order (*ragione*), they go on working; and from this comes the disproportion and poor correspondence that one sees in many buildings…." (Quanto sia necessaria a qualunque persona la certissima arte della Geometria ne possono rendere testimonio tutti colloro che hanno un tempo operato senza quella, & dipoi son venuti in qualche cognition di tal'arte li quali veramente confessaranno, che tutte le cose da loro pensate & fatte senza Geometria, furono senza arte alcuna, ma a ventura & a caso. Per il che essendo la profundissima arte dell'Architettura abbracciatrice di molte arti nobili, primieramente fa di mistiero, che l'Architetto ne sia, se non dottato, almen tinto di forte ch'egli n'habbia qualche cognition, & massimamente de i principij, & anco piu avanti, & non come molti consumatori di pietre, & di calcine, imo de marmi, che al di d'hoggi tengono il nome di Architetti, liquali non sanno pur render conto che cosa sia punto, linea, superficie: o corpo, ne che sia corrispondentia, o harmonia. Ma guidati da un suo proprio parere, & complacentia d'occhio, seguitando le vestigie de glialtri, che con pocca ragione han fatto, vano operando, & di qui viene la disproportione e mala corrispondentia che in molti edificij si vede….) Sebastiano Serlio, *Il primo libro d'architettura*, (Paris, 1545), p. iii*v*.

[47] *Ibid.*

[48] "...si mostra in primo aspetto opera bella et con qualche ordine..." Francesco Terribilia, as quoted in Gaye, *Carteggio inedito d'artisti* (Florence, 1840), 3:491.

[49] According to Mainstone, "… the first recorded structural analysis of a building which is recognizably modern" dates to 1742. Rowland J. Mainstone, "Structural Theory and Design Before 1742," *Architectural Review* 142, no. 854 (1968), 303.

[50] Although the great dome of Hagia Sophia collapsed repeatedly due to earthquakes, pier distortions had appeared soon after construction of the first dome in the sixth century. Robert Mark, *Light, Wind, and Structure: The Mystery of the Master Builders* (Cambridge, Mass., 1990), 77. On Beauvais Cathedral see: Mark, *Experiments in Gothic Structure* (Cambridge, Massachusetts: MIT Press, 1982), 58-77. See also Luca Pacioli's lament that buildings often fall down because their builders use geometry without realizing that "everything consists of number, weight and measure." Pacioli, *De divina proportione* (Venice, 1509), I:54, f. 16r, as quoted in Marcus Frings, "The Golden Section in Architectural Theory," *Nexus Network Journal* 4, no. 1 (2002), 13.

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[51] Lon R. Shelby and Robert Mark, “Late Gothic Structural Design in the ‘Instructions’ of Lorenz Lechler,” *Architectura* 9.2 (1979), 115. Cf. the similar remark by Daniele Barbaro: “…one can well praise the effect of proportion, in which is placed the glory of the architect, and the strength of the work…” (“…ne si può lodare abastanza l’effetto della proportione, nella quale è posta la gloria dell’Architetto, la fermezza dell’opera…”), Barbaro, *I dieci libri dell architettura di M. Vitruvio…* (Venice, 1556), 24, as quoted in: Howard Saalman, “Early Renaissance Architectural Theory and Practice in Antonio Filarete’s *Trattato di Architettura*,” *Art Bulletin* 41, no. 1 (March 1959), 98 n. 24.

[52] For a general discussion of the development of modern conceptions of architectural beauty in the eighteenth century, see M.H. Abrams, "Art-as-Such: The Sociology of Modern Aesthetics."*Bulletin of the American Academy of Arts and Sciences* 38 (1985): 8-33. I thank K. Michael Hays for introducing me to this source. Wittkower’s claim that “Italian architects strove for an easily perceptible ratio between length, height and depth of a building” reflects a post-eighteenth century interest in the purported perceptual effects of sets of architectural proportions that is inconsistent with the evidence presented in this study. Wittkower’s statement is also ambiguous, for it does not explain what an “easily perceptible ratio” might be, compared to one that is not easily perceptible. Rudolf Wittkower, *Architectural Principles in the Age of Humanism* (New York, 1971), 74.

[53] Saalman refers to the concept of “*bellezza* and *fortezza*” as “the close theoretical interconnection of structural form and structural stability in mediaeval architecture…,” and a “…nexus of form and statics, basic to an understanding of medieval architectural theory….” Saalman, “Early Renaissance Architectural Theory and Practice,” 97. I prefer to refer to this concept as *fortezza* and *bellezza* in recognition of the order in which these related terms typically appear in relation to one another the primary sources.

[54] “... Fu fatto con buonissimo giudicio, perche et toglie ben su tutto quell’ angolo e con fortezza, e con bellezza di opera...”. Serlio, *Il Terzo Libro di Sabastiano Serlio Bolognese....* (Venice, 1540), LVIr. “... Patisca alcuni diffetti così nelle parti della fortezza come della bellezza...” Gaye, *Carteggio inedito d'artisti* (Florence, 1840), 3:491. In the documentary record this fundamental concept in the history of western architecture occasionally resurfaces as late as the nineteenth century, as in the following passage from Tredgold’s builder’s manual: “... when beauty and solidity are to be combined, the study of the higher branch of Architecture, which consists in the production of visible beauty, must, necessarily, be joined with the study of construction.” Thomas Tredgold, *Elementary Principles of Carpentry* (London, 1828), vii–viii.

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[55] "…la detta nuova colonna fatta per Franciescho essere più forte e bella e laudabile." Cesare Guasti, *Santa Maria del Fiore: La construzione della chiesa e del campanile* (Firenze, 1887), 103.

[56] "...gli pare più bello o più utile e più sichuro...." Guasti, *Santa Maria del Fiore*, 174. Cf. "...is most beautiful and most useful and strong..." ("...è più bello e più utile e forte..."), and a slight variation thereof, "...è più bello utile e più forte...," *ibid.*

[57] "...con grandissima bellezza e fortezza e comodi..." Manetti, *Vita*, 98; "...si debba fare per fortezza, bellezza e commodità...." Gaye, *Carteggio inedito* III (1840), 482. For additional Early Renaissance examples of the terms *fortezza* and *bellezza*, see Saalman, "Early Renaissance Architectural Theory and Practice," 97-98.

[58] "firmitatis, utilitatis, venustatis." Vitruvius, *De architectura*, I, iii, 2.

[59] "In ea aede cum voluissent columnas conlocare, non habentes symmetrias earum et quarentes quibus rationibus efficere possent, uti et ad onus ferendum essent idoneae et in aspectu probatam haberent venustatem, dimensi sunt virilis pedis vestigium et id retulerunt in altitudinem…. Ita dorica columna virilis corporis proportionem et firmitatem et venustatem in aedificiis praestare coepit." Vitruvius, *De architectura*, IV, I, 6.

[60] Shelby and Mark, "Late Gothic Structural Design," 120.

[61] "Probado he muchas veces a sacar razon del estribo que habra menester cualquiera forma y nunca hallo regla que me sea suficiente, y tambien lo he probado entre los arquitectos españoles y extranjeros, y ninguno paresce alcanzar verificada regla, mas de su solo albedrio; y preguntando por que sabremos ser aquello bastante estribo, se responde porque lo ha menester, mas no por que razon. Unos le dan el 1/4 y otros, por ciertas lineas ortogonales lo hacen y se osan encomendar a ello…," as quoted in George Kubler, "A Late Gothic Computation of Rib Vault Thrusts," *Gazette des Beaux-Arts* 26, series 6 (July-Dec. 1944), 146.

[62] Hontañon's reference to numerical rules of thumb in the determination of buttress proportions recalls similar rules of thumb mentioned in discussions recorded in the archives of the Cathedral of Milan. Jean Mignot, the French architect summoned by the cathedral building committee to help resolve certain important design issues, recommended a 1:3 ratio between the thickness of piers and buttresses, while the building committee countered with their own rule of 1:1 ½. *Ibid.*; and Ackerman, "'Ars Sine Scientia Nihil Est,'" 99. See Mainstone, "Structural Theory and Design Before 1742," p. 303, "geometrical theory."

[63] "...pro retificando praedictam ecclesiam et croxeriam quod respondent ad quatrangulum secundum ordinem geometriae; alia vero pro fortitudine et pulchritudine tiborii..." Ackerman, "'Ars Sine Scientia Nihil Est,'" 100, 109. Cf. the subsequent comment in the same passage: "…the weight on

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these three (*sic*) towers falls evenly on their square, and they will be built properly and strong, and what is vertical cannot fall; therefore they say that they are strong in themselves….” (“…et quod pondus dictis tribus turribus ponderat ubique super suum quadrum, et erunt aedificata recte et fortiter, sed rectum non potest cadere; unde dicunt quod sunt fortes per se…”). *Ibid*., 100, 110.

[64] “Questa medesima altezza è proportionata col corpo principale della chiesa, perchè ella viene ad esser tant’alta quanto larga, dico lasciando le capelle che sono parte del corpo: et da questa proportione nasce una fortezza principale del edificio, perchè posto un centro nel meggio di essa, et tirata una circonferenza che tocchi l’ara et le mura dei lati et la cima delle volte, se ne forma un circolo, il quale abbracciando tutte tre quelle volte con li contraforti insieme viene a farsi una ligatura fortissima di tutta la fabrica.…” Gaye, *Carteggio inedito d’artisti* (Florence, 1840), 3:492. For sixteenth century drawings of the basilica of San Petronio cross-section with superimposed overlays of geometrical proportions, see Guido Zucchini, “Disegni inediti per S. Petronio di Bologna,” *Palladio* 6, no. 5-6 (1942), 153-166.

[65] For examples of earth-centric models, see Thomas Kuhn, *The Copernican Revolution* (Cambridge, Mass., 1976), 10-59.

[66] Even in the early twentieth century, rules of thumb based on idealized triangular shapes were still in circulation to cope with this problem, which has too many unpredictable variables for modern structural analysis to resolve. For determining the necessary strength of a lintel, for example, the author of a popular builder’s manual of 1921writes, alongside an illustration of a roughly triangular void caused by fallen brick above a rectangular opening: “Some authorities recommend considering as the proper load, for brick work, a TRIANGULAR PART [*sic*] of the wall the sides of which triangle have an inclination to the horizontal of 45°; others assume an inclination of 60°. The exact determination of this load by mechanical laws is difficult if not impossible. It is better to consider each case separately….” Frank E. Kidder, *The Architects’ and Builders’ Handbook*, 17th ed. (New York, 1921), 318.

[67] As confirmed by the author’s calculations under the supervision of Daniel Schodek. The root-2 rectangle, and its numerical approximations, is recommended as a useful rule-of-thumb for determining the strongest cross-section for a wood beam in builders’ manuals throughout the nineteenth and early twentieth centuries. For example, according to Kidder: “The strongest [sic] beam cut from a cylindrical log is one in which the breadth is to the depth as 5 is to 7, very nearly….” *Ibid*., 634. According to Gwilt, who cites Tredgold as his source, “…the strongest [sic] beam which can be cut out of a round tree is that of which the depth is to the breadth as √2 is to 1, or nearly 1.4142136 to 1; or as 7 to 5.” Joseph Gwilt, *The Encyclopedia of Architecture* (London,

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1867), 433-434. Gwilt's own source, Tredgold's *Elementary Principles of Carpentry*, notes: "The strongest beam that can be cut out of a round tree is that of which the depth is to the breadth as the square root of 2 is to 1; or nearly as 7 is to 5." In a remarkable footnote to this passage, Tredgold notes: "This was first demonstrated by M. Parent in the 'Mémoires de l'Académie,' Paris, for 1708," thus linking this rule-of-thumb to a pre-1742 source. See note 6 and Thomas Tredgold, *Elementary Principles of Carpentry*, 8th ed. (1828; London, 1892), 74.

[68] See Cohen, "How Much Brunelleschi?," 43.

[69] I thank Caroline van Eck for calling my attention to the importance of the distinctions between *ars* and art. For further discussion of quattrocento ideas pertaining to science, *ars* and method see: Caroline van Eck, "The Structure of *De re aedificatoria* Reconsidered," Journal of the Society of Architectural Historians 57 (1998), 280-297.

[70] A few representative examples exhibiting various approaches include: Francis Cranmer Penrose, *An Investigation of the Principles of Athenian Architecture….* (London, 1851); Walter Horn, "Romanesque Churches in Florence: A Study in their Chronology and Stylistic Development, *Art Bulletin* 25, no. 2 (Ju. 1943), 112-131; and Rowland Mainstone, *Hagia Sophia: Architecture, Structure and Liturgy of Justinian's Great Church* (New York, 1988).

[71] The exemplary document-based study of architectural proportion remains James S. Ackerman, "'Ars Sine Scientia Nihil Est': Gothic Theory of Architecture at the Cathedral of Milan," *Art Bulletin* 31, no. 2 (Ju. 1949), 84-111. In the same year appeared Wittkower's more ideological document-based study, *Architectural Principles in the Age of Humanism* (London, 1949). Note that Wittkower later adds a significant caveat to his assertion: "…Palladio's conception of architecture, as indeed that of all Renaissance architects, is based on commensurability of ratios" in the footnote: "The time for a reliable survey of Renaissance buildings has not yet come, but I feel confident that it would confirm my assumption." Rudolf Wittkower, *Architectural Principles in the Age of Humanism* (London, 1962), 108 and 108 n. 8 (and later editions). Regarding measurement-based studies of architectural proportion Ackerman has noted: "There exists among historians a conviction that it is dangerous to make conclusions from measurements that have no confirmation from the texts because of the unrigorous and/or inconsistent way in which virtually all of those who have published about proportions based on observation have proceeded." James S. Ackerman, in a personal letter to the author, 22 December 1991.

[72] For a useful definition of connoisseurship, see Eric Fernie, *Art History and Its Methods* (New York and London, 1995), 330-331. For an overview of the German contributions to the multiplicity of

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viewpoints that have been accomodated under the umbrella of art history, see Michael Podro, *The Critical Historians of Art* (New Haven and London: Yale University Press, 1982).

[73] See the pertinent comments regarding the "architectural historian-architect" in Arnaldo Bruschi, "Problemi e metodi di ricerca storico-critica sulla architettura" in *Storia e restauro dell'architettura: proposte di metodo*, Gianfranco Spagnesi, ed. (Roma, 1984), 15-34. I thank Francesco Benelli for calling my attention to this source.

[74] Disciplinary relationships between architectural history and the related fields of architecture, art history and archaeology are complex and interwoven, and journals focused on the last three often carry articles pertaining to the first. Since 1996 the journal *Archeologia Medievale* has even produced an annual supplement called *Archeologia dell'Architettura*. Attitudes pertaining to these relationships, furthermore, vary from country to country. Undertaking archaeological surveys, or measured drawings (*rilievi*), for example, is a standard requirement in Italian architecture schools, but rare in American schools. See for example the series *Quaderni d'Architettura*, published by the *Dipartimento di Storia dell'Architettura, Restauro e Conservazione dei Beni Architettonici dell'Università "La Sapienza" di Roma*. On the history of Italian architecture schools see Maristela Casciato, "The Italian Mosaic: The Architect as Historian," *Journal of the Society of Architectural Historians* 62, no. 1 (Mar. 2003), 92-101. For recent discussions of interdisciplinarity in architectural history that tend to focus on disciplines other than those discussed here, see Nancy Steiber, "Learning from Interdisciplinarity," *Journal of the Society of Architectural Historians* 64 (2005), 417-418, and associated essays and references.

[75] One way in which this approach can lead to new insights in architectural history is through the intimate familiarity with the architectural built fabric that it enforces. Recording his surveys of the Parthenon, for example, Penrose observed not only optical refinements—his goal—but traces of an exuberant color scheme. Penrose, *Principles of Athenian Architecture*, 55. Similarly, while recording my surveys for the present study, I made numerous observations not directly related to architectural proportion, some of which are reported in Matthew Cohen, "The Bird Capitals of the Basilica of Santo Spirito in Florence: Some Observations, and a Proposed Iconographical Interpretation," *Quaderni del Dipartimento di Storia dell'Architettura e Restauro…di Firenze* 13-14 (Jan.–Dec. 1995), 48–58, but note that due to publication errors some photographs are misnumbered. A corrected version can be found at: http://www.spokane.wsu.edu/Academics/Design/CohenMatthew.