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## **Beyond beauty : reexamining architectural proportion in the Basilicas of San Lorenzo and Santo Spirito in Florence**

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## 2. Sets of Proportions in the San Lorenzo Nave Arcade Bays (with a Santo Spirito Comparison)

In his slim *Pocket Guide to Architecture*, we have seen, Patrick Nuttgens summarizes the significance of the Basilica of San Lorenzo in Florence in one sentence: “S. Lorenzo is notable for the precision of its proportions.”<sup>1</sup> Indeed, the notion that the proportions of this building, guided by a mathematically rational set of proportions embedded in its dimensions, impart to it positive qualities such as precision, beauty, harmony, perspectival rationality and *all’antica* refinement, today stands as a virtual axiom of architectural history.<sup>2</sup> Survey textbooks are extremely important vehicles for summarizing and shaping both popular and scholarly perceptions of architecture, and Nuttgens’ statement is but a brief summation of a more detailed interpretation of the Basilica of San Lorenzo that has been retold, with slight variation, in virtually every art and architectural history textbook published during the past fifty years. Nevertheless, no one has ever determined, based on verifiable measurements, what the proportions of this building are, whether an intentional set of proportions exists in any part of it, and if so, what significance it may have held for early fifteenth-century Florentines. In this study I revisit these questions pertaining to sets of proportions in the Basilica of San Lorenzo—and for comparison in the Basilica of Santo Spirito as well—based on a comprehensive methodology that supports observation with documentary research.

### 2.1 Methodology: San Lorenzo Nave Arcade Bays

Although the comprehensive new surveys of the basilicas of San Lorenzo and Santo Spirito that I have recorded may be considered the foundation of this study, measurements alone can be misleading no matter how accurate and comprehensive they may be. Any large building contains enough complexity to justify virtually any hypothesis regarding architectural proportion. Search hard enough in the measurements and you will find what you seek—a process that scientists unceremoniously refer to as “data dredging.” Indeed, since even consistent proportional patterns can be coincidental, distinguishing intentional proportions from coincidental ones may be considered one of the central challenges of the study of architectural proportion.<sup>3</sup> Conversely, documentary evidence pertaining to architectural proportion (proportion-1) can be equally misleading without corroboration through measurements and other forms of observation, for no building can ever be assumed to correspond to any verbal remark or graphic representation recorded on paper, no matter how authoritative it may seem to be. When preconceptions guide the research process, such as the belief that Renaissance architecture is based on whole number proportions, the risk of misinterpreting historical evidence only increases.

The present study addresses these concerns by using various types of observation-based and documentary evidence in complementary ways. In addition, it forgoes common preconceptions such as the Wittkower Paradigm, in particular the Geometry vs. Number theory contained within it, and the

various beauty-in-proportion belief systems, in favor of a more open approach: Rather than interpret the basilica of San Lorenzo as the first Renaissance building, distinct from the medieval past, or as some expression of transition between the two periods, it examines the basilica as a product of the late medieval civilization from which it arose. By using comprehensive measurements, subjected to rigorous analysis, as a primary source in the study of the basilica of San Lorenzo, this study arrives at novel conclusions pertaining not only to sets of proportions, but to the question of attribution and early site conditions as well. (The survey and survey methodology are described in Appendix 9.1.)

This study of the San Lorenzo nave arcade bays began with two simple measurements taken at the floor level between two adjacent nave columns selected at random. The choice of the specific points of measurement was inadvertently innovative. Scholars have thus far acknowledged two types of intercolumnar measurement, termed “on center” and “in the clear.”<sup>4</sup> There is a third type however, which I will term “plinth-to-plinth,” and it constitutes the key to understanding the proportions of the basilica of San Lorenzo.<sup>5</sup> The distance between the plinths of Columns 9 and 10 (Figure 2-1), for example, is 564.0 cm, and the distance between the farther edges of the same two plinths is 797.5 cm (see Appendix 9.1). Note that when the larger of these measurements is divided by the smaller, the result is the square root of 2, with a negligible error of about 2 mm, or, 0.2–0.3%.<sup>6</sup> The ratio between these two measurements, therefore, is nearly exactly  $1:\sqrt{2}$ . The only geometrical construction that will produce this ratio between two co-linear dimensions is a root-2 rectangle inscribed within a square.<sup>7</sup> Judging from photographs, merely as an initial exploration, this construction appears to correspond to the width-to-height proportions of each nave arcade bay, up to the tops of the column shafts (Figure 2-2).

This procedure of estimating key building heights by extrapolating from plinth-to-plinth measurements has a notable precedent: according to Brunelleschi’s fifteenth century biographer, Antonio di Tuccio Manetti, when Brunelleschi, in his youth, studied the remains of ancient buildings with his friend Donatello “... they drew elevations, roughly in drawings, of almost all the buildings in Rome.... and in this way, when they were able to, they estimated the heights [by measuring] from base to base ....” (*da basa a basa*). Since the bases of classical columns are typically equal in width to the plinths, we may assume that for Manetti “base-to-base” meant plinth-to-plinth.<sup>8</sup>

Even in light of Manetti’s comment, however, we are as yet quite far from being able to hypothesize either that these observations constitute an accurate description of the proportions of the nave arcade bay, or that those proportions result from intention rather than coincidence. In order to propose and test such hypotheses, we need to determine: 1) the exact dimensions of the nave arcade bays, 2) how best to interpret and manage dimensional irregularities in a proportional analysis, and 3) when measurements reveal the presence of a particular proportional relationship ( $1:\sqrt{2}$ , for

example) within an acceptable level of dimensional tolerance, how to establish that it is the result of intention rather than coincidence.

In this study, a proportional relationship found within the measurements will be considered a likely product of the architect's intentions if it meets the following three criteria:

Criterion #1: The proportion must match the building measurements within the range of construction and measurement error established by statistical analysis. If a proportion fails to meet this standard but nevertheless appears, in the opinion of the author, to be intentional, it may remain under consideration if some convincing historical explanation, supported by documentary or observation-based evidence, can be found to account for all dimensional discrepancies.

Criterion #2: The proportion must appear in documentary sources relevant to the early fifteenth century, or closely resemble other proportions that do.

Criterion #3: The proportion must be related to other proportions in the building as part of a logical *set* of proportions that occur *simultaneously* within the measurements.

## 2.2 Refining the Survey Data: Metrical Observations

Seen from the floor with the unaided eye, all the bays of the nave arcades appear identical (Figure 2-3). Sighting along the tops of the columns from scaffolding, however, irregularities in the column heights become clearly visible (Figure 2-4). The bar charts indicate that the most pronounced of these irregularities are concentrated in the five easternmost bays of the nave, or to the outsides of the thin vertical lines in each chart (Figure 2-5).

The single largest height difference between any two adjacent columns in the nave is the 10.8 cm difference between the heights of Columns 12 and 13 (Figure 2-4, second and third columns from the right; see Figure 2-1 for column numbers). That this discrepancy is virtually all constructive, and not due to uneven foundations, is proven by the measurement breakdowns in the survey. The two column capital heights are identical at 96.9 cm each, but the monolithic shaft of Column 13 is taller than that of Column 12 by 6.7 cm, the base is taller by 2.1 cm, and the plinth by 0.7 cm, for a total of 9.5 cm, all contained within the masonry units and their mortar joints. Virtually all of the remaining 1.3 cm of difference ( $10.8 - 9.5 = 1.3$ ) is due to the slightly raised foundation beneath Column 13, which rises 1.2 cm higher than that beneath Column 12 (for measurements, see Appendix 8.1).

While Column 13 consequently stands out as the most pronounced height anomaly of the north arcade, that distinction might have gone to Column 11 had the masons not taken what appears to have been remedial action. The monolithic shaft of Column 11 is taller than that of Column 13, albeit by a mere 0.7 cm, but this excess height was compensated for in the manufacture of the base and plinth for this column. These two elements are together 8.6 cm shorter than those of Column 13. An unusually short entablature block atop Column 11 (Figure 2-5, upper right), furthermore, makes the total order height measured at this column consistent with most others in the nave (Figure 2-5, lower left). No such remedial action was taken at Column 13, however, and so it remains unusually tall in the nave, whether measured to the top of the capital (Figure 2-5, lower right) or the entablature block (Figure 2-5, lower left).

The survey also calls our attention to Column 4 as the shortest column in the basilica. Here the masons appear to have taken a different kind of remedial action, for close inspection reveals that this column contains the only bilithic shaft in the basilica (Figures 2-6 and 2-7). Perhaps the shaft was damaged in transit to the site from the quarry or *bottega*, or perhaps it was originally made grossly too short or too tall. Whatever the problem, evidently the solution, rather than make a whole new shaft, was simply to make a new top, matched so precisely that only a mortar joint and a slight shaft height deficiency betray the repair.

All the significant dimensional irregularities in the nave arcades—and with them, all the remedial actions which, as we have seen, are themselves rather irregularly deployed—are concentrated in the five easternmost bays, or to the outsides of the thin vertical lines in the bar charts in Figure 2-5. If connoisseurship teaches us that judgments of artistic quality can be elevated to the level of empirical historical evidence, then it may now be useful to observe that the capitals and entablature blocks in these areas of greatest dimensional irregularity are also notably cruder in design than their counterparts in the western three bays, and rife with mistakes, corner-cutting, and sloppy workmanship—a sharp contrast to the exceptionally well crafted western three bays, as discussed in greater detail below (Figures 2-8 to 2-21).

Since my survey reveals countless slight dimensional irregularities from one nave arcade bay to the next, even though all bays appear to have been intended to be identical, this study examines the dimensions of *all* the bays, as a group, using statistical analysis. Such analysis accounts for dimensional irregularity by examining proportions in the non-definitive terms of probability. For example, since any given width-to-height proportion, defined between a particular set of points, is slightly different in each of the sixteen nave arcade bays, at best we can make an informed conjecture as to which single width-to-height proportion was intended for that set of points in every bay, identically. To do so, we can use the measurements to calculate a “confidence interval” within which

a particular proportion, such as that of the root-2 rectangle, can be said, with 95% confidence, to be present in the nave arcade bays. The smaller and fewer the irregularities, the smaller the interval, and the more precise our conjecture can be.<sup>9</sup> Unfortunately, the dimensional irregularities in the nave arcades are too large to permit a sufficiently precise conjecture regarding the proportions of the arcade bays. Most of the heights of the nave columns vary by no more than a centimeter or so from one to the next, but a few of them vary by as much as 14 cm—discrepancies large enough not only to muddy proportional calculations, but to be visible from certain vantage points (Figure 2-4). A simple solution to this problem would seem to be to ignore the most pronounced irregularities, which appear as notable spikes and dips in bar charts representing the measurements (Figure 2-5). To remove these measurements from consideration, however, requires historical evidence that the more regular measurements are indeed more likely to represent the original design intentions than the notably irregular ones. This evidence is found in both the physical fabric of the building and surviving documents.

### 2.3 Refining the Survey Data: Sculptural Observations

The column capitals in the western three bays of the nave are sumptuously articulated with acanthus leaves that appear fleshy and alive (Figures 2-8, 2-10, 2-12 and 2-14). Some even bear delicate triplets of sickle-shaped incisions that are only visible at close range (Figure 2-12). The capitals in the eastern four bays, by contrast, appear stiff and stylized, articulated with pointed leaflets of unvarying thicknesses and squared-off edges, as if cut from thick sheets of rolled dough (Figures 2-9, 2-11, 2-13 and 2-15). Throughout the capitals and entablature blocks, highly polished surfaces in the western bays contrast with rough, chiseled surfaces in the eastern bays (Figures 2-12 to 2-21]. Some of the surfaces of the eastern stonework are also pitted, indicating a lower quality of stone than in the western stonework (Figures 2-16 to 2-17). These distinctions in quality are most pronounced in the entablature block friezes. In the western bays the pairs of carved winged angels are expressive and sublime. Some were probably commissioned from the celebrated Rossellino brothers (Figures 2-22 to 2-24).<sup>10</sup> In the eastern bays the carved angels appear clumsy and crude by comparison (Figures 2-25 to 2-26), occasionally distressingly so (Figure 2-27).

The masons responsible for the eastern portions of the nave arcades appear to have availed themselves of every opportunity to spare time and expense. In the western entablature blocks, all the egg-and-dart crown mouldings have nine eggs per side (Figure 2-24). Most of the eastern blocks only have eight, however (Figure 2-28), and some as few as seven (Figure 2-29). In the crown moulding located atop the ill-starred Column 13, the left-most of the eight eggs is much wider than the others, as if the mason had begun carving in haste from right to left, without first laying out equal

subdivisions (Figure 2-28). A more serious mistake is found directly below this imperfect crown moulding. In all the other entablature block friezes in the nave the lamb faces left and looks right (Figures 2-22, 2-24 and 2-25). This one is backwards (Figure 2-28). The compound Corinthian pilaster capitals that terminate the arcades on the interior façade display similarly egregious errors: ill-fitting parts seem carelessly jammed together, the large gaps that remain having been filled with thick dabs of plaster (Figure 2-30). At the western ends of the arcades, by contrast, the corresponding pilaster capitals join up in perfect alignment (Figure 2-31).

These striking differences in quality between the western and eastern portions of the nave arcades allow us to identify the precise structural boundary between the two construction phases. Proceeding from west to east, the sculptural archivolt that spring from Columns 5 and 10 on opposite sides of the nave each undergo a notable change after the first mortar joint: the carved laurel motifs become more simplified and crudely executed, and the spiraling carved leather bindings become thicker and reverse direction (Figures 2-32 and 2-33).<sup>11</sup>

Documentary evidence confirms what these observations suggest: the nave was built in two distinct phases. The first, from 1446 to 1450, took four years, and included six columns (see Figure 2-1, Cols. 5-10). The second, from 1461 to 1464, took three years, but included eight columns (see Figure 2-1, Cols. 1-4 and 11-14).<sup>12</sup> More work in less time generally leads to lower quality, as Cosimo de' Medici, the patron for both phases, must have been acutely aware. Already ailing by 1461 and anxious to complete this basilica that he had conceived as his own grand mausoleum lest his heirs be unable to do so, Cosimo appears to have ordered the remaining five bays of the nave (encompassing eight columns) rushed to completion (see Chapter 5). This novel historical interpretation of the nave construction provides the necessary justification to drop the measurements of the later portion of the nave from the data used in this analysis.<sup>13</sup> Doing so causes the standard deviations for all dimension sets to drop substantially, and the accuracy of all proportional calculations to improve enough to virtually eliminate ambiguity in the results (Appendix 9.4).

## 2.4 Geometry

The proportional calculations presented earlier in this chapter confirm our initial observation that the distance between the nearer edges of any two adjacent column plinths (first measurement) and the farther edges (second measurement) correspond to the ratio  $1:\sqrt{2}$  with negligible error. They also indicate that an overlapping square and root-2 rectangle, when drawn to touch the edges of the column plinths, does indeed rise to the tops of the column shafts (Figure 2-2), though a seemingly minor discrepancy of 1.5% (about 11-12 cm) in this alignment falls outside the statistically allowable limits established for this study (Figure 2-5).<sup>14</sup> These calculations also indicate that a variant of the

root-2 rectangle known as a “dual diagonal,” when inscribed between the column plinths, corresponds to the heights of the San Lorenzo entablature blocks within just 0.05%, or, about 0.5 cm (Figure 2-34).<sup>15</sup>

The overlapping square and root-2 rectangle shown in Figure 3-2 can be derived through a simple geometrical procedure known as “the rotation of squares technique.” As such, it resembles the technique for determining the ground plan proportions of a Gothic pinnacle described in *Booklet Concerning Pinnacle Correctitude*, the mason’s manual of Mathes Roriczer which was published in 1486, but very likely records a centuries-old oral tradition (Figure 2-35).<sup>16</sup> Roriczer’s manual is a pragmatic work that reveals a method for determining the “correct proportions” (*Gerechtigkeit*) of a Gothic pinnacle through a series of illustrated, step-by-step instructions. In the first three steps Roriczer instructs his readers to draw three diminishing squares, one inscribed within the next, each turned forty-five degrees relative to the previous one (Figure 2-35, top left). In the fourth step Roriczer instructs his readers to “rotate the square e h g f,” meaning the middle square, until its sides are parallel to those of the other two (Figure 2-35, bottom left). In the fifth step the sides of the innermost square are extended to touch the sides of the newly rotated middle square, thus closing off the corners of the middle square to form four much smaller squares (Figure 2-35, top right). In subsequent steps he describes smaller geometrical manipulations (Figure 2-35, bottom right). The completed figure, he tells us, represents the floor plan (in collapsed view) of the shaft of a pinnacle. To determine the height of the shaft, he multiplies the width of the largest square by six, thereby deriving the height from the floor plan. The spire on top of the pinnacle rises another seven of these modules.<sup>17</sup>

The diagram in Roriczer’s fifth step (Figure 2-35, top right), when simplified to exclude the outermost square (Figure 2-36, third iteration), corresponds precisely to the floor plan of each bay of the San Lorenzo side aisles, provided that the side aisle pilaster plinths are imagined to project into the walls as full squares (Figure 2-1). Of the two large squares in our simplified Roriczer diagram (Figure 3-36, third iteration), the width of the smaller of them corresponds to the plinth-to-plinth distance at San Lorenzo, and the width of the larger one, to the distance between the farther edges of the column plinths (Figure 2-2). We know that these two squares bear a  $1:\sqrt{2}$  relationship to one another, just like the two plinth-to-plinth distances of San Lorenzo, because before we rotate the smaller square, we can plainly see that its diagonal is equal in length to the sides of the larger square (Figure 2-36).

If we now take our simplified Roriczer diagram (Figure 2-36, third iteration), stand it up for use in elevation, and remove the two inside horizontal lines (Figure 2-36, fourth iteration), we have

the San Lorenzo nave arcade proportions of overlapping square and root-2 rectangle (Figure 2-2). We have previously seen that the dual diagonal that marks the heights of the San Lorenzo entablature blocks can be derived from a root-2 rectangle inscribed between the column plinths (Figure 2-34). The Roriczer diagram now suggests another way to derive this figure: in the preceding operation we removed two horizontal lines that formed narrow bands along the top and bottom of the figure (Figure 2-36, fourth iteration). If we were now to duplicate these bands instead of removing them, and to add the duplicates to the top and bottom of the geometrical figure, the two innermost vertical lines of the new figure would describe the sides of a dual diagonal. Such a geometrical elaboration would be in the spirit of Roriczer's technique, which consists of incremental permutation of a basic figure of rotated squares.

The overlapping geometrical set of proportions shown in Figure 2 bears even closer similarities to the proportions contained in certain illustrations in Sebastiano Serlio's *Secondo Libro* of 1545 and *Terzo Libro* of 1540.<sup>18</sup> Serlio frequently invites his readers to measure the proportions of his illustrations directly, and I have done just that, working from original editions.<sup>19</sup> Two of Serlio's illustrations bear the same proportions of overlapping square and root-2 rectangle observed at San Lorenzo, with errors of 1–2 millimeters, or, 0.5% (Figures 2-37 and 2-38, and Appendix 8.5).<sup>20</sup> Serlio even includes a pavement pattern of rotated squares in one of them (see Figure 2-37), perhaps as a hint to help his readers reconstruct the illustration's proportions. In this context it may be significant that several of the San Lorenzo nave windows contain wooden muntins, perhaps original, arranged as rotated squares (Figure 2-39). The dual diagonal proportion is also found, with similar accuracy, in certain illustrations of Roman buildings in Serlio's *Terzo Libro* (Figures 2-40 and 2-41, and Appendix 8.5).<sup>21</sup> We know that the latter proportions are of Serlio's invention, and not based on measurements of the buildings themselves, because Serlio explicitly tells us so.<sup>22</sup>

Additional documentary support for the preceding geometrical interpretation of the San Lorenzo nave arcade bay proportions is provided by a variety of sources. In *De architectura*, Vitruvius recommends three rectangles having the proportions 3:5, 2:3 (a square-and-a-half rectangle) and  $1:\sqrt{2}$  (a root-2 rectangle) for the proportions of the atria of Roman houses.<sup>23</sup> This treatise, which had been widely disseminated by the late medieval period—indeed, Nicola Acciaioli willed a copy to the church of San Lorenzo in 1359, and Giovanni Bocaccio willed another to a monk of the church of Santo Spirito in 1374—thus established a canon of rectangular proportions that was elaborated upon in every architectural treatise of the Renaissance.<sup>24</sup> Filarete, in his fifteenth-century *Trattato di architettura*, recommends a slight modification of the Vitruvian canon of rectangular proportions for use in the designs of doorways and arched portals. For such purposes

Filarete variously recommends the ratios 1:2 (a two-square rectangle),  $1:\sqrt{2}$  (a root-2 rectangle) and 1:1 1/2 (a square-and-a-half rectangle).<sup>25</sup> Alberti recommends two of these proportions for doorways, 1:2 and  $1:\sqrt{2}$ ; and a different selection of four rectangular proportions for arcade and colonnade bays.<sup>26</sup> Francesco di Giorgio Martini presents a complex array of ten simple quadrangular proportions for rooms and courtyards.<sup>27</sup> The one rectangular proportion common to all of these authors is  $1:\sqrt{2}$  (a root-2 rectangle).

These observations suggest that architects and masons of both the medieval and Renaissance periods shared a common geometrical tradition incorporating root-2 proportional relationships. Indeed, Serlio even includes the rotation of squares technique on the title page of his *Primo libro* as one of the fundamental tools of the architect, together with the compass, plumb line and set square (Figure 2-42). Similarly, the author of the *Antiquarie prospettiche Romane* (c. 1500), probably Bramante, included a figure of rotated squares in his title page illustration that depicts an architect, perhaps himself, surrounded by ancient Roman monuments and the instruments needed to study them (Figure 2-43).<sup>28</sup> Thus, to describe this technique as a primarily medieval phenomenon, as many scholars do, is to deny its continued importance during both the medieval and Renaissance periods; and to interpret rotation of squares-related proportions at San Lorenzo as medieval characteristics in an otherwise Renaissance building would be inaccurate.

## 2.5 Number

Geometry can be used to establish architectural proportions, but geometry's companion, number, is needed to translate those proportions into built form. Number gives architecture quantity and dimension. The architect had to communicate to the masons quantitative information such as how many columns were needed, how tall they were to be and how far apart they were to be, and number provided the simplest means by which to do so. Number also gives architecture scale. Any geometrical proportion can be executed at the jeweler's scale of a reliquary, the monumental scale of a major cathedral, or any increment in between. At San Lorenzo, number allowed the architect to reconcile abstract geometry with dimensioned proportions at a scale appropriate to the site, budget and civic importance of the basilica. Number, however, has two sides: number can represent magnitude, and number can represent itself. This second side—the vast intellectual landscape of number divorced of magnitude—constituted a significant branch of western thought prior to the Industrial Revolution of the eighteenth and nineteenth centuries. Aristotle (384–322 B.C.) described this dual nature of number in a discussion of time, as follows:

“...Since ‘number’ has two meanings (for we speak of the ‘numbers’ that are counted in the thing in question, and also of the ‘numbers’ by which we count them and in which we calculate), we are to note that time is the countable thing that we are counting, not the numbers we count in—which two things are different.”<sup>29</sup>

Johannes Kepler (1571–1630) reaffirmed this distinction between dimensioned and un-dimensioned numbers, terming the former “counted numbers” (*numeri numerati*), i.e. the quantities that we count; and the latter, “counting numbers” (*numeri numerantes*), i.e., the numerals with which we count.<sup>30</sup> Although we still recognize this distinction today, we rarely assign much importance to un-dimensioned numbers, which we find, for example, on automobile license plates and athletes’ jerseys; or in popular superstitions such as lucky number 7 and unlucky (in America) or lucky (in Italy) number 13. For architects of the medieval and Renaissance periods both sides of number were equally important, and the distinction between them was paramount. Dimensioned numbers were for them utilitarian instruments while un-dimensioned numbers, which could simply be dimensioned numbers interpreted in a different way, represented abstract values that formed hierarchical patterns of infinite beauty and complexity. Thus, dimensioned numbers linked architecture to the mundane, while un-dimensioned numbers linked architecture to an abstract intellectual and perhaps even spiritual realm.

We can find the dimensioned numbers within the San Lorenzo nave arcade sets of proportions by converting key measurements in the survey from meters and centimeters to the early fifteenth-century Florentine *braccio*. This conversion is a two-step process: first the measurements are divided by the correct centimeter equivalent of one *braccio*, which appears to be the widely accepted standard of 58.36 cm, derived from an eighteenth century *campione* conserved in the Florentine state archives (Figure 2-44).<sup>31</sup> Second, any decimal remainders are converted to the correct system of *braccio* subdivision. That system consisted of simple fractions and not, as many scholars believe, the more complex system of *soldi* and *denari*, which does not appear to have come into use until the sixteenth century.<sup>32</sup>

Converted to *braccia*, these dimensioned numbers immediately begin to communicate in the terms of their un-dimensioned alternate identities. Note that most of the nave arcade dimensions end in the fraction  $\frac{2}{3}$  (Figure 2-45): the capital heights measure  $1\frac{2}{3}$  br (approximately 1.659 br); the plinth-to-plinth distances,  $9\frac{2}{3}$  (approximately 9.662); the distances between the farther edges of the

column plinths,  $13\frac{2}{3}$  (approximately 13.669); and the minor order height to the tops of the entablature blocks,  $17\frac{2}{3}$  (approximately 17.637).<sup>33</sup> Furthermore, if we assume that the column shaft heights were intended to correspond to the overlapping square and root-2 rectangle shown in Figure 2-2, the aforementioned 11-12 cm discrepancy for now notwithstanding, then the heights of the column shafts would also measure precisely  $13\frac{2}{3}$  br. When all these dimensions ending in  $\frac{2}{3}$  are separated out, arranged in order of magnitude (understood as abstract magnitude rather than measurable dimensions), and the duplicates removed, the following number progression results:

$$1\frac{2}{3}, 9\frac{2}{3}, 13\frac{2}{3}, 17\frac{2}{3}$$

The fractions, having served their graphic functions as flags calling out these dimensions as a group, may now be removed:

$$1, 9, 13, 17$$

Note that the intervals between these numbers are 8, 4, and 4,

$$\begin{array}{cccc} 1 & 9 & 13 & 17 \\ \vee & \vee & \vee & \\ 8 & 4 & 4 & \end{array}$$

which imply that a 5 is missing from the progression. When it is included, the intervals become a series of repeating 4s:

$$\begin{array}{cccc} 1 & (5) & 9 & 13 & 17 \\ \vee & \vee & \vee & \vee & \\ 4 & 4 & 4 & 4 & \end{array}$$

Why this 5—or more pertinently, the dimension  $5\frac{2}{3}$  br—might be missing from the nave arcade bay dimensions will be discussed below. Even without it though, any educated person of the early fifteenth century would have recognized the progression 1, 9, 13, 17 as a small, if incomplete, piece of a vast network of similar progressions, all interrelated according to the principles of Boethian number theory. Once a cornerstone of western learning and virtually synonymous with the very concept of number during both the medieval and Renaissance periods, Boethian number theory has been all but forgotten today.<sup>34</sup> Thus, a brief review is in order.

In about 503 A. D., Anicius Manlius Severinus Boethius (c. 480–c. 525 A. D.)—a Roman aristocrat, philosopher, and advisor to the Ostrogoth King Theodoric<sup>35</sup>; dubbed “the first scholastic and last Roman” by historian E. K. Rand<sup>36</sup>; an early Christian who was tortured to death, placed in *Paradiso* by Dante<sup>37</sup> and revered as a saint for centuries before his cult was confirmed in 1883<sup>38</sup>; and who is today best known as the author of *The Consolation of Philosophy* (*De consolazione philosophiae*)—gave to the Latin West a translation and slight elaboration of a Greek treatise on arithmetic, written by Nicomachus of Gerasa in about the first century A. D., that would serve as the basis of mathematical knowledge in Europe for over a millennium.<sup>39</sup> The fundamental importance of Boethius's *De institutione arithmetica*, which was completed in about 503 A. D., to the history of mathematics is reflected in an illustration in the *Margarita philosophica*, a popular and widely disseminated encyclopedic digest first published a millennium later, in 1503. In the illustration, Boethius is depicted as equivalent in importance to Pythagoras, the man considered since antiquity to be the father of all number theory (Figure 2-46).<sup>40</sup> By the early fifteenth century copies of *De institutione arithmetica* could be found in all the major libraries of Europe.<sup>41</sup>

According to Boethian number theory, which is rooted in Neo-Pythagorean doctrines, numbers are philosophical rather than utilitarian instruments, and occur in progressions rather than individually.<sup>42</sup> Each progression is identified by the intervals between the consecutive numbers of which it is composed, and these intervals in turn form new progressions. Boethius describes many different classes of numbers, but devotes particular attention to those that “deal with geometric figures,”<sup>43</sup> and these numbers—let us call them geometrical numbers—may be considered the foundations of Boethian number theory.<sup>44</sup> There are three principal types of geometrical numbers: linear (let us call them one-dimensional), polygonal<sup>45</sup> (two-dimensional), and pyramidal (three-dimensional). These numbers are written as sets of dots or pen strokes<sup>46</sup> arranged as lines, polygons; and, in certain Renaissance and later sources, axonometric pyramids, respectively. In Boethian number theory every progression of geometrical numbers begins with, 1, or “unity,” an entity that is considered to be the basis of all numbers but is not itself considered to be a number, just as in

classical geometry the point is considered to be the basis of all geometry but is not itself considered to be geometrical because it has no dimension.<sup>47</sup>

The *Liber mathematicalis bernwardi*, c. 1000, conserved in the archives of St. Michael's Church in Hildesheim, includes a manuscript of *De institutione arithmetica* that contains one of the earliest surviving illustrations of the so-called "polygonal numbers" (Figure 2-47).<sup>48</sup> In it, the "triangular numbers," 1, 3, 6, 10, 15, 21, 28..., are shown as pen strokes arranged as triangles; the "square numbers"—a term we still use today—1, 4, 9, 16, 25, 36, 49..., as pen strokes arranged as squares; and so on through the polygons. The intervals between the polygonal numbers produce new progressions of numbers variously called "root numbers" or "gnomons," and the root numbers produce intervals—for clarity let us call them "root intervals"—that consist of repeated integers.<sup>49</sup> The polygonal numbers themselves constitute the intervals between the so-called pyramidal numbers, which may be visualized as stacks of cannonballs. The polygonal numbers illustrated in the Hildesheim manuscript are broken down into these various intervallic progressions in Figure 3-48. Note that the root hexagonal numbers begin with the now familiar progression 1, 5, 9, 13, 17, the same one found at San Lorenzo.<sup>50</sup>

Boethian number theory may be understood as a fundamental part of the intellectual framework underlying the medieval world view. Its influence can be found, for example, in the many *trattati d'abbaco* (schoolbooks of arithmetic) that appeared in Florence in the fourteenth and fifteenth centuries, in mathematician Gabriele Stornaloco's famous letter of 1391 regarding the proportions of the Cathedral of Milan, in Alberti's remarks about number, and even in the rules underlying the medieval number game *rithmomachia* which prominently incorporates the squares of the progression 5, 9, 13, 17, among many other Boethian number relationships.<sup>51</sup>

## 2.6 Arithmetic

Geometry and number give architecture buildable form, but sometimes the two simply refuse to work together. When Vitruvius attempts to calculate the length of the diagonal of a 10 foot x 10 foot square he concludes: "nobody can find this by means of arithmetic."<sup>52</sup> Serlio warns of the similar difficulty posed by the root-2 rectangle, noting: "...[it] is irrational; one finds no whole number proportion at this increment."<sup>53</sup> Many scholars incorrectly assume that architects and masons of the medieval and Renaissance periods lacked the mathematical knowledge necessary to calculate the length of the diagonal of a square of known numerical width and that, when using such a diagonal to determine the heights of columns, vaults or other tall features (see Figure 2-2), they laid out the square full-scale on the ground and measured the diagonal with a piece of chain or string in order to avoid using numbers.<sup>54</sup> Wittkower, for his part, claims that the side/diagonal and other

irrational ratios ran contrary to “the new organic approach to nature” that characterized the “Renaissance attitude to proportion” and that consequently, whole number ratios constitute “the nodal point of Renaissance aesthetics.”<sup>55</sup> Such speculations would seem to have little relevance to the San Lorenzo proportions, however, for my measurements indicate that while the architect who designed them indeed regarded the irrationality of the side/diagonal ratio with special interest, far from avoiding it in favor of number, he engaged it numerically with great bravura.

At San Lorenzo, we have seen, he expressed the irrational ratio  $1:\sqrt{2}$ —that is, the proportions of the root-2 rectangle—with the numerical ratio  $9\frac{2}{3} : 13\frac{2}{3}$ , with no apparent dimensional compromise.<sup>56</sup> How did he do it? The ratio  $9\frac{2}{3} : 13\frac{2}{3}$  is not a true numerical expression of the side/diagonal ratio (no such expression is possible), but an extremely accurate approximation thereof. The error, four one-hundredths of one percent (0.04%), translates into about 1-3 mm at the scale of the San Lorenzo nave arcades. Since even in the extremely accurately constructed western portion of the nave the degree of construction error is about ten times larger, or, 1-2 cm, for the purposes of architecture the ratio  $9\frac{2}{3} : 13\frac{2}{3}$  may be considered numerically *equivalent* to the ratio  $1:\sqrt{2}$ .<sup>57</sup>

The use of precise numerical approximations of the side/diagonal ratio was far from new in the early fifteenth century. The Old Babylonians knew how to calculate them, Plato refers to them in the *Republic*, and in the first century, A.D., the mathematician Theon of Smyrna wrote a treatise that presents a simple formula for calculating an infinite progression of them.<sup>58</sup> Since a copy of Theon’s treatise reached Florence by the mid-fifteenth century (it was translated by Marsilio Ficino for Cosimo de’ Medici) and since Theon’s formula helps to illuminate a particular characteristic of the San Lorenzo sets of proportions that appears to have been progressive for its day, the formula deserves our close attention.<sup>59</sup> Theon’s formula begins with a theoretical square, the dimensions of the side and diagonal of which are 1, or unity. It then generates successive theoretical squares according to the following relationships: the sum of the side and diagonal (s+d) of the first square generates the side of the next square, and twice the side plus the diagonal (2s+d) of the first square generates the diagonal of the next square. The second square in the progression thus has a side/diagonal number pair of 2:3, which is a poor approximation of the ratio  $1:\sqrt{2}$  (error: 6.07%). Successive generations—5:7 (error: 1.01%), 12:17 (error: 0.17%), 29:41 (error: 0.04%)—approximate this ratio with a roughly six-fold increase in accuracy with each generation, and continue to improve as such *ad infinitum* (Figure 2-49).<sup>60</sup> Theon’s formula, however, would not have been very useful to architects working much before the years in which the San Lorenzo sets of

proportions were designed. As the numerical approximations that it generates increase in accuracy, so too do they increase in magnitude, and all but a few of them are simply too large relative to common units of measure such as the Florentine *braccio* to have had much practical application. The San Lorenzo architect, however, possessed a new tool, in widespread use perhaps only since the mid-fourteenth century, that allowed him to overcome this problem.

Common fractional arithmetic was the new math of the late fourteenth and early fifteenth centuries, the most significant development in the use of number in the West since the displacement of Roman in favor of Hindu-Arabic numerals in the thirteenth and fourteenth centuries.<sup>61</sup> The *Trattato d'aritmetica* by Paolo dell'Abbaco (c. 1281–1374) is a particularly useful source of insight into the mathematical knowledge of the period because its author was both an esteemed mathematician in Florence, and master of his own well-regarded school of arithmetic (*scuola dell'abbaco*) in which this *trattato* very likely served as a required text.<sup>62</sup> The *Trattato* consists of 197 word problems, nearly every one of which involves fractional calculations difficult enough to tax the mathematical skills of any junior high school student—not to mention many adults—today.<sup>63</sup>

Also central to the curricula of the *scuole del'abbaco* in fourteenth and fifteenth century Florence was the concept of equivalent ratios. Pier Maria Calandri notes in his *Trattato d'abbaco*, for example, that the progression 8, 12, 18, 27 is composed of “proportional quantities” (*quantità proporzionali*) because the ratio 8:12 (denoted verbally as *8 a 12*, or, “8 to 12”), represents the same ratio as 12:18 and 18:27—i.e., they all represent the irreducible whole number ratio of 2:3.<sup>64</sup> Combining his knowledge of fractional arithmetic and equivalent ratios, the architect of the San Lorenzo sets of proportions understood that any numerical ratio that included fractions could be converted to an equivalent whole number ratio, and vice versa, through simple arithmetic. He thus realized that he could capture the impressive accuracy of the ratio 29:41 as a side/diagonal approximation, perhaps derived from Theon's formula (see Figure 2-49), while reducing that ratio to a smaller, more practical scale relative to the Florentine *braccio*, as follows:

$$29:41 \Rightarrow \frac{29:41}{3} = \frac{29}{3} : \frac{41}{3} = 9\frac{2}{3} : 13\frac{2}{3}$$

These fractional calculations bear a particular resemblance to the solutions to three word problems explicated in Paolo dell'Abbaco's *Trattato d'aritmetica*, and shed light on the new opportunities that common fractional arithmetic opened up to architects who were interested in reconciling two of the principal sciences of the medieval and Renaissance periods—geometry and arithmetic—in the one area in which they seemed diametrically opposed: geometrically-derived

magnitudes that in theory cannot be expressed in terms of number.<sup>65</sup> Indeed, such reconciliation of apparent opposites in the San Lorenzo sets of proportions could possibly have taken on philosophical or even religious significance for those early fifteenth-century intellectuals who were aware of them.<sup>66</sup>

### 2.7 The Column Shaft and Entablature Block Height Discrepancies

Our analysis of the overlapping square and root-2 rectangle shown in Figure 2-2 has proceeded even though the survey indicates that when this geometrical construction is drawn to touch the edges of the column plinths, the top line lands along a smooth, architecturally undistinguished area 11-12 cm below the tops of the column shafts, rather than aligning precisely with the tops of those shafts as I propose the architect intended. A convincing historical explanation for this discrepancy must now be provided. I propose that the masons, perhaps working from incorrect or ambiguous information, mistakenly made the column shafts 11-12 cm taller than the architect wanted them. I furthermore propose, since the present minor order height of  $17\frac{2}{3}$  br and the present capital height of  $1\frac{2}{3}$  br both appear, in light of the preceding analysis, to correspond precisely to the architect's intentions, that the excess 11-12 cm of shaft height came at the expense of the originally intended entablature block height. The entablature blocks presently measure about 2.13 br in height, which is 11-12 cm short of  $2\frac{1}{3}$  br (2.33 br).<sup>67</sup> An entablature block height of  $2\frac{1}{3}$  br plus the present capital height of  $1\frac{2}{3}$  br would equal 4 br, or, twice the plinth width of 2 br.<sup>68</sup> The dimension  $2\frac{1}{3}$  br thus has a ring of intention to it, even in the absence of the additional supportive evidence now to be examined.

The construction error hypothesis stated above is supported by three categories of evidence. The first is the subtle and complex San Lorenzo nave arcade sets of proportions themselves, and their thorough consistency with late medieval learning. That these sets of proportions, with their cunning integration of geometry, number and arithmetic, might have come about by coincidence seems highly unlikely. These sets of proportions only come to light, however, when we correct the masons' apparent 11-12 cm mistake by imagining that the column shafts measure  $13\frac{2}{3}$  br high (and not 11-12 cm higher, as at present), and the entablature blocks,  $2\frac{1}{3}$  br (and not 11-12 cm lower, as at present).

The *present* column shaft and entablature block heights, by contrast, lack any apparent logical basis whatsoever.

The second category of evidence in support of my construction error hypothesis is provided by the basilica of Santo Spirito, which I also measured comprehensively. In that basilica, which Brunelleschi designed in the 1430s with striking stylistic and compositional resemblances to the basilica of San Lorenzo, the top of a root-2 rectangle inscribed between the plinths of any two adjacent columns aligns with the tops of the column shafts, not including the astragals, with absolute statistical precision, exactly as I have proposed this proportion was intended to align at San Lorenzo (Figure 2-50 and Appendix 8.3).<sup>69</sup> Furthermore, the Santo Spirito capitals measure  $1\frac{2}{3}$  br high (or, the same as those of San Lorenzo), and the entablature blocks measure  $2\frac{1}{3}$  br high—exactly the dimension I have proposed was intended at San Lorenzo. Thus, Brunelleschi appears to have used the San Lorenzo nave arcade sets of proportions (whether or not he was their author), absent the future construction errors, as the basis for his design of the Santo Spirito arcade sets of proportions (to be considered in greater detail below).

The third category of evidence in support of my construction error hypothesis consists of early documentary accounts that describe numerous divergences from Brunelleschi's intentions in the construction of all the works traditionally attributed to him. Whether or not Brunelleschi designed the San Lorenzo nave arcade sets of proportions, he clearly played a significant role in shaping the present above-ground form of the basilica during his brief term as *capomaestro* early in the construction process. Manetti notes that Brunelleschi continually grappled with the problem of construction error, and while repeatedly lambasting those who, in his estimation, bore responsibility for these divergences, does not hesitate to lay some of the blame in the master's own lap.<sup>70</sup> According to the biographer, Brunelleschi, ever fearful that others would "discover his every secret" (*intendessi ogni suo segreto*), was notoriously ambiguous when documenting his designs in drawings and models, preferring instead to give verbal instructions directly to the masons as they worked (*a bocca di mano in mano*). Indeed, Manetti specifically notes that "...he worked in this way at San Lorenzo..."<sup>71</sup> This method of communication, according to Manetti, led to significant errors in several buildings, particularly pertaining to capitals, entablatures and other ornamental articulations of structure (*capitelli o d'architravi, fregi, e cornici ecc.*). These errors, he notes, caused Brunelleschi "much annoyance and sorrow" (*molte noie e rincrescimenti*) when he invariably discovered them too late to correct them.<sup>72</sup>

Manetti singles out the Ospedale degli Innocenti as a building that suffered particularly egregious departures from Brunelleschi's intentions during one of the architect's frequent absences from Florence, a claim consistent with that of the early sixteenth century biographer Antonio Billi. Manetti notes that during one of these absences the architect left just one drawing (*el disegno solo*), measured to scale (*misurato a braccia picchhole*), and "...gave oral instructions (*e a bocca mostrò*) to the master builders, the stonecutters, certain citizens, the leaders of the Guild, and the workers assigned to the undertaking."<sup>73</sup> These instructions, however, appear to have been insufficient to avert, among other errors "...an architrave that turns down and continues to the water table of the building."<sup>74</sup> This down-turning architrave is indeed one of the oddest details in the Brunelleschi *oeuvre*—an apparent relapse to Romanesque improvisation in an otherwise rigorously classical, Early Renaissance façade—and Billi sees fit to elaborate upon the circumstances that brought it about. According to him, upon returning from a trip to Milan, Brunelleschi confronted Francesco della Luna, the *capomaestro* in charge of the work during his absence, and asked "why he made such a thing." Francesco replied that he had "taken it from the church of St. John" (i.e., the Baptistery of Florence), which Brunelleschi had apparently instructed him to follow in certain details. In one of the wittiest repartees in the history of architecture, Brunelleschi replied: "There is but one error in that building, and you have chosen to copy it."<sup>75</sup>

The errant nave column and entablature block heights of San Lorenzo appear to have originated with a problem involving applied architectural ornament very similar to the problem Brunelleschi encountered at the Ospedale. The errant dimensions in question must have been established as soon as the first minor order pilaster was installed in the transept, for the heights of its various component parts would have automatically established permanent horizontal benchmarks across the entire site of the future basilica, including both the transept and nave (Figure 2-51). That first San Lorenzo pilaster was installed in the first private chapel built in the basilica, the Medici double chapel (see Figure 2-1, SP 17-23), which was completed, together with the adjacent Old Sacristy, by 1428.<sup>76</sup> Though documentary evidence gives no reason to doubt that Brunelleschi supervised the construction of both the chapel and sacristy, the apparent dimensional errors in the pilasters suggest that his supervision either lapsed or ended soon after the chapel walls reached the most critical *structural* benchmark in the basilica—the height of the springing line of the minor order arches and vaults. That benchmark occurs precisely  $17\frac{2}{3}$  br above the floor of the future basilica nave, and corresponds to the tops of the entablature blocks of the nave arcades and the corresponding entablature that circumscribes the entire basilica (see Figures 2-3 and 2-51). During this early period of construction Brunelleschi also appears to have supervised the manufacture, but not the

installation, of the pilaster capitals that embellish the Medici Chapel in question, for the exceptionally high sculptural quality of these capitals (Figure 2-52), unmatched anywhere else in the basilica proper (Figure 2-53), seem to bear the imprint of his personal attention. These capitals display exceptionally smooth finish, and fine vertical ridges along the leaf stems that resemble metalwork (see Figure 2-52), perhaps a reflection of Brunelleschi's training as a goldsmith.

Brunelleschi appears to have missed his opportunity to establish correctly the next most important structural benchmark of the minor order—the height of the transept pilaster shafts, and by extension, the height of the future nave column shafts. He appears to have been absent when the pilaster capitals were mounted on the walls of the Medici Chapel, and to have neglected to leave any completed pilaster plinth + base assemblies, shafts, or entablature segments.<sup>77</sup> Imagine the hapless mason standing in the then-recently vaulted Medici double chapel, looking up at the horizontal springing line of the chapel vaults and arches, and wondering how far below it to mount Brunelleschi's pilaster capitals (see Figure 2-51, right). We now see that the mason simply needed to place the bottoms of those capitals (marked by the bottoms of the astragals) exactly 4 br below the springing line to accommodate the  $2\frac{1}{3}$  br entablature height plus the  $1\frac{2}{3}$  br capital height (see Figures 2-45 and 2-51), but for some reason he apparently never received that crucial piece of information. Perhaps Brunelleschi, with characteristic ambiguity, failed to transmit it to his successor as *capomaestro*, or perhaps his successor failed to pass it along to the mason. We may never know exactly what went wrong the day the first San Lorenzo pilaster took its place on one of the walls of the Medici Chapel. The apparent 11-12 cm errors in question, however, serve as valuable reminders that irregularity is the normal condition of architecture, and that no building can ever be assumed fully to reflect the architect's intentions.

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<sup>1</sup> Patrick Nuttgens, *The Pocket Guide to Architecture* (New York, 1980), 115.

<sup>2</sup> A typical and widely disseminated example of this view, including an admission of its speculative nature, is that of Janson: “What makes the interior of San Lorenzo seem so beautifully integrated? There is indeed a controlling principle that accounts for the harmonious, balanced character of his design: the secret of good architecture, Brunelleschi was convinced, lay in giving the ‘right’ proportions—that is, proportional ratios expressed in simple whole numbers—to all the significant measurements of a building. The ancients had possessed this secret, he believed, and he tried to rediscover it by painstakingly surveying the remains of their monuments. What he found, and how he applied his theory to his own designs, we do not know for sure.” H. W. Janson, *History of Art*, 3<sup>rd</sup> ed. (New York, 1986), 410.

<sup>3</sup> Coincidental occurrences of highly ordered structures must be expected in architecture, as in geometry and mathematics. This phenomenon is aptly illuminated by Arnheim, who notes: “Only in a world based exclusively on the chance combination of independent elements is an orderly pattern a most improbable thing to turn up; in a world replete with systems of structural organization, orderliness is a state universally aspired to and often brought about.” Rudolf Arnheim, *Entropy and Art: An Essay on Disorder and Order* (Berkeley, Los Angeles and London: 1971), 37. For a mathematical analysis of this phenomenon, see Roger Fischler, “How to Find the ‘Golden Number’ Without Really Trying,” *Fibonacci Quarterly* 19, no. 5 (Dec. 1981), 406-410.

<sup>4</sup> According to Saalman, “[t]wo things required proportioning in Early Renaissance architecture: the building members (mass) and the spaces between them (void)... All geometrical relationships are determined ‘in the clear,’ that is, from edge to edge of the building members, *not* from axis to axis.” Saalman, *Brunelleschi: The Buildings*, 361; and Howard Saalman, “Early Renaissance Architectural Theory and Practice in Antonio Filarete's Trattato di Architettura,” *Art Bulletin* 41, No. 1 (Mar., 1959), 94. Several references to “on center” dimensions are found in fourteenth century documents pertaining to the Cathedrals of Florence and Milan. A document of 1357 repeatedly refers to the dimensions between the nave piers in the church of Santa Maria del Fiore in Florence as “...from center of column to center of column...” (“...da meza cholonna a meza cholonna...”), as transcribed in: Cesare Guasti, *Santa Maria del Fiore: La Costruzione della Chiesa e del Campanile*, Firenze, 1887, p. 94. Similarly, in a letter of 1392, the mathematician Gabriele Stornaloco twice refers to the on center distances between the nave columns (piers) of the Cathedral of Milan: “The appropriate height is in accordance with the distance from center to center of the columns...” (“Competens altitudo est secundum distantiam centri ad centrum colonarum”), and: “I have taken all the widths

from center to center; therefore I have not bothered to enter into the drawing the thickness of the columns because it is sufficiently clear to the master architects how much they occupy within the bodies [scil., central nave and aisles] of the church.” (“Omnes lactitudines acepi mensuratione centri ad centrum icdeo non curavi in designamento ponere spissitudinem colonarum quia satis est manifestum Magistris Inzigneriis quantum occupant in corporibus ecclesie...”), as quoted and translated in: Paul Frankl, “The Secret of the Medieval Masons”, *Art Bulletin*, XXVII, 1, 1945, pp. 54 and 55. I have substituted Frankl’s word “pier” for “column,” which I believe more closely follows Stornaloco’s text. In another example, Antonio di Vincenzo, architect of the church of San Petronio in Bologna, annotates his drawings of the Cathedral of Milan with the following reference to on center dimensions: “...from the center of the pilaster [i.e., pier] to the other center...” (“...da mezo del pilastro al altro mezo...”), as quoted in: James S. Ackerman, ““Ars Sine Scientia Nihil Est:’ Gothic Theory of Architecture at the Cathedral of Milan,” *Art Bulletin*, XXVI, 2, 1949, p. 88, n. 14). For references to in the clear measurement see: Vitruvius, *De Architectura*, III, iii, 7; and Leone Battista Alberti, *De re aedificatoria*, VII, v and VII, xv.

<sup>5</sup> When references to plinth-to-plinth measurement appear in the fifteenth century literature, scholars do not recognize them. Saalman, for example, accepts Enggass’s translation of Manetti’s words *da basa a basa*, quoted above, as “from base to base,” but nevertheless interprets the passage to be a reference to the significantly larger “intercolumnal dimension measured from shaft to shaft.” Antonio di Tuccio Manetti, *The Life of Brunelleschi*, Howard Saalman, ed., Catherine Enggass, transl., University Park and London, 1970, 34, 52 and 132 n. 33. In their translation of *De re aedificatoria*, furthermore, Rykwert et al. omit Alberti’s word “base” (*basis*) from his description of the proportions of a portal flanked by two Corinthian columns, and with it, his reference to plinth-to-plinth (i.e., base-to-base) measurement. Alberti writes (author’s translation): “The height of the columns, including the capitals, is equal to the distance between the farther edge of the right base to the farther edge of the left.” (“Longitudo columnarum cum capitulis tanta est: quanta sit ab angulo extremo basis dextræ ad angulum extremum sinistrae.”) Leonis Baptiste Alberti, *De re aedificatoria* (Florence, 1485), VII, xii, fol. riiii. By contrast, Rykwert et al.’s translation reads (brackets are the translators’): “The length of the columns, complete with capitals, should equal that of the diagonal [of the void] from bottom right to top left.” Leon Battista Alberti, *On the Art of Building in Ten Books*, Joseph Rykwert, Neil Leach and Robert Tavernor, transl. (Cambridge, Mass. and London, 1988), 226. In their study of the basilica of Santo Spirito, Benevolo, Chieffi, Mezetti, et al. provide a floor plan annotated with every unobstructed plinth-to-plinth dimension in the church but never use any of them in their numerous proportional calculations. Rather, they use exclusively on center

dimensions. When considering elevation proportions, furthermore, these authors deny any architectural significance to the column plinths whatsoever. Stating that Brunelleschi considered column plinths to be merely portions of “exposed foundation” [*fondazione allo scoperto*], they set the base line for their proportional calculations at the tops of the column plinths, rather than at floor level. Leonardo Benevolo, Stefano Chieffi, Giulio Mezzetti, “Indagine sul S. Spirito di Brunelleschi,” *Quaderni dell’istituto di storia dell’architettura*, XV, 85-90, 1968, p. 12, and Dis. XXVI. Scarchilli follows suit in his proportional studies of the basilica of San Lorenzo. Renzo Scarchilli, “Il complesso laurenziano: La chiesa (prima parte),” *Controspazio*, IX, 1, 1977, p. 45, Fig. 3. A second part appears never to have been published. For a Vitruvian plinth-to-plinth reference, see *De architectura*, III, iii, 2.

<sup>6</sup> Thus  $797.5 \div 564.0 = 1.4140$ , which is nearly identical to  $\sqrt{2}$ , or, 1.4142....

<sup>7</sup> A root-2 rectangle is formed by extending two parallel sides of a square to equal the length of its diagonal (see Figure 2-2, center). The resultant side/diagonal ratio is  $1:\sqrt{2}$ .

<sup>8</sup> “...quantunque insieme e’ levassono grossamente in disegno quasi tutti gli edifici di Roma, ed in molti luoghi circustanti di fuori, colle misure delle larghezze ed altezze, secondo che potevano, arbitrando, certificarsi, e longitudini, ecc. E in molti luoghi facevano cavare per vedere e riscontri de’ membri degli edifice e le loro qualità, s’egli erano quadri o di quanti anguli, o tondi perfetti o ovati o di che condizione, e così, dove e’ potevano congetturare, l’altezze, così da basa a basa per altezza, come da’ fondamenti e riseghe e tetti degli edifici....” Antonio Manetti, *Vita di Filippo Brunelleschi*, Giuliano Tanturli, ed. (Milan, 1976), 67. Saalman nevertheless interprets Manetti’s phrase “from base to base” as a reference to the significantly larger “... intercolumnal dimension measured from shaft to shaft” (i.e., in the clear). Antonio di Tuccio Manetti, *The Life of Brunelleschi*, Howard Saalman, ed. (University Park and London, 1970), 34, 52, and 132 note 33. Cf. a record of payment from 1443 for construction of a pier “from the base to the capitals,” a specification that could not possibly include the base but not the plinth of the pier. Isabelle Hyman, “Fifteenth Century Florentine Studies: The Palazzo Medici; and a Ledger for the Church of San Lorenzo.” PhD diss., New York University, 1968 (Ann Arbor, Michigan, 1970), 332, 464. See also *ibid.*, 337, 516, 539.

<sup>9</sup> For a more detailed discussion of the statistical analysis used in this study, see Appendix 9.4.

<sup>10</sup> Isabelle Hyman, ‘Fifteenth Century Florentine Studies: The Palazzo Medici; and a Ledger for the Church of San Lorenzo.’ PhD diss., New York University, 1968 (Ann Arbor, Michigan: University Microfilms, Inc., 1970), pp. 349, 400-403, 496. Although these winged beings with young childrens’ faces resemble cherubim, contemporary construction documents refer to them as ‘seraphim’ (*serafini*). *Ibid.*, pp. 349, 496, 511. I refer to them as angels, however, in keeping with the

iconography of the entablature block friezes that depict the Lamb of God and the seven-sealed Book of the Apocalypse—apparent references to Revelation, which refers frequently to angels, but neither cherubim nor seraphim. Furthermore, the correspondence between the number of columns in each nave arcade, seven, and the recurrent theme of seven in the Book of Revelation, is perhaps not coincidental. See Revelation 5-10.

<sup>11</sup> For previous, less detailed San Lorenzo nave capital observations, see Martin Gosebruch, “Florentinische Kapitelle von Brunelleschi bis zum Tempio Malatestiano und der Eigenstil der Frührenaissance,” *Römisches Jahrbuch für Kunstgeschichte* 8 (1958), 84-91; Howard Saalman, “Filippo Brunelleschi: Capital Studies,” *Art Bulletin* 15, no. 2 (Ju. 1958), 123-127; and Gabriele Morolli, “L’ordine brunelleschiano: morfologia e proporzioni,” in *San Lorenzo 393-1993*, 81-94. For other observations of discontinuities in parts of the nave other than the arcades, see: Piero Roselli, ‘Brunelleschi in San Lorenzo, Contributi alla cronologia dell’edificazione,’ *Antichità viva* 2, 1979, p. 38, Fig. 2; Piero Roselli and Orietta Superchi, *L’edificazione della basilica di San Lorenzo* (Florence: Cooperativa Editrice Universitaria, 1980), pp. 71-72, Figs. 10-11; and Gabriele Morolli, “San Lorenzo da Piero a Lorenzo (1465-1480 circa),” in *San Lorenzo 393-1993: L’architettura, Le vicende della fabbrica*, ed. by Gabriele Morolli and Pietro Ruschi (Florence: Alinea Editrice, 1993), p. 76.

<sup>12</sup> The old basilica occupied the site of the eastern four bays of the nave until about 1465 (see Chapter 5).

<sup>13</sup> Morolli assumes that unspecified eastern nave columns were constructed after 1465 under the patronage of Piero dei’ Medici. Gabriele Morolli, “L’ordine brunelleschiano: morfologia e proporzioni,” in *San Lorenzo 393-1993*, 84ff. Other scholars discuss the construction of the eastern nave chapels without mentioning the nearby columns, which need not be assumed contemporaneous with the chapels. Volker Herzner, “Zur Baugeschichte von San Lorenzo in Florenz,” *Zeitschrift für Kunstgeschichte* 37, 1974, 90 ff; Piero Rosselli, “Brunelleschi in San Lorenzo, Contributi alla cronologia dell’edificazione,” *Antichità viva* 2 (1979), 41; repeated in Piero Roselli and Orietta Superchi, *L’edificazione della basilica di San Lorenzo....* (Florence, 1980), 24; Miranda Ferrara and Francesco Quinterio, *Michelozzo di Bartolomeo* (Florence, 1984), 204-206, 285-287; and Howard Saalman, “Capital Studies,” 124 Plate III, and *Brunelleschi: The Buildings*, 188, 439 Docs. 13.1, 13.2, 14.1.

<sup>14</sup> In this study column shaft heights are measured from the bottoms of the plinths to the mortar joints below the astragals, which are integral with the capitals. Measuring to the tops of the astragals would make the discrepancy in question worse.

<sup>15</sup> A dual diagon is a rectangle formed by extending two parallel sides of a square to equal the length of its diagonal (thus forming a root-2 rectangle) and then extending those two sides again by the same amount. The resultant width-to-height ratio is  $1:2\sqrt{2}-1$ . I borrow the term “dual diagon” from Kenneth J. Conant, “The After-life of Vitruvius in the Middle Ages,” *Journal of the Society of Architectural Historians* 27 (1968), 34. Conant in turn attributes the term, without references, to “investigators attached to the National Museum at Ljubljana, Yugoslavia, including Tine Kurent.” See Tine Kurent, *Kosmogram romanske bazilike v stični: Cosmogram of the Romanesque Basilica at Stična, Yugoslavia*, Univerza v Ljubljani, Fakulteta za Arhitekturo, Gradbeništvo in Geodezijo, Ljubljana, no. 2 (1977/78), 31. Neither Conant nor Kurent, however, provide verifiable measurements demonstrating the use of this figure in medieval architecture.

<sup>16</sup> Mathes Roriczer, *Büchlein von der Fialen Gerechtigkeit*, Regensburg, 1486. For an English translation, see: Lon R. Shelby, *Gothic Design Techniques: The Fifteenth-Century Design Booklets of Mathes Roriczer and Hanns Schmuttermayer* (Carbondale, Ill., 1977), 1, 2, 32.

<sup>17</sup> *Ibid.*, pp. 84–98. Frankl quotes a document of 1459 that mentions rules that prohibit members of mason’s lodges across central Europe from revealing “...how to take the elevation from the ground plan,” and from this evidence assumes that the rotation of squares technique was a “secret of the mediaeval masons” (Frankl, Paul, “The Secret of the Mediaeval Masons,” *Art Bulletin*, XXVII, 1, 1945, p. 46), an interpretation that Shelby refutes (Lon R. Shelby, “The ‘Secret’ of the Medieval Masons,” in Bert S. Hall and Delno C. West, *On Pre-Modern Technology and Science Studies in Honor of Lynn White, Jr.*, Malibu, 1976, pp. 201-219).

<sup>18</sup> Serlio, *Il Secondo libro d’Architettura* (Paris, 1545); and Serlio, *Il Terzo Libro di Sabastian Serlio Bolognese....* (Venice, 1540).

<sup>19</sup> See for example, Serlio, *Regole generali di Architettura....* (Venice, 1537), Xir, IXv, Xv, XXVIIIv, Lv, LIr.

<sup>20</sup> Note that in Figure 2-37, the root-2 rectangle aligns with the columns in the clear rather than plinth-to-plinth, perhaps to make the illustration easier to draw, since it forms part of an explication of one-point perspective drawing. Serlio, *Il Secondo libro*, 45r. Serlio also provides a preparatory wire-frame drawing of the portal show in Figure 2-37 that contains these same proportions. For a quite different interpretation of the proportions of this illustration in Figure 2-37, which is supported neither by diagrams nor measurements, see Saalman, “Early Renaissance Architectural Theory,” 98. For Figure 2-38, see Sebastiano Serlio, *Il Terzo Libro*, XLIX.

<sup>21</sup> Serlio, *Il terzo libro*, LVI, LXXI.

<sup>22</sup> Regarding the portal in his Spoleto illustration (see Figure 2-40), Serlio concedes, "... I did not measure it, but from horseback I designed its conception and form." ("... non la misurai, ma così a cavallo disegnai la invenzione, e la forma"). Serlio, *Il Terzo Libro*, LVI. For his drawing of the Spello city gate (see Figure 2-41), Serlio provides his ground plan measurements but then similarly notes: "the heights I did not measure: rather, I invented them completely in the drawing, because I liked them" ("le altezze io non le misurai, ma tolsi la inventione solamente in disegno, perche mi piacque."). Serlio, *Il Terzo Libro*, LXXI.

<sup>23</sup> "In width and length, atriums are designed according to three classes. The first is laid out by dividing the length into five parts and giving three parts to the width; the second, by dividing it into three parts and assigning two parts to the width; the third, by using the width to describe a square figure with equal sides, drawing a diagonal line in this square, and giving the atrium the length of this diagonal line." Vitruvius, *The Ten Books on Architecture*, transl. Morris Hicky Morgan, New York, 1960 (rpt. Cambridge, Massachusetts, 1914), VI, iii, 3; p. 177; "Atriorum vero latitudines ac longitudines tribus generibus formantur. Et primum genus distribuitur, uti, longitudo cum in quinque partes divisa fuerit, tres partes latitudini dentur; alterum, cum in tres partes dividatur, duae partes latitudini tribuantur; tertium, uti latitudo in quadrato paribus lateribus describatur inque eo quadrato diagonius linea ducatur, et quantum spatium habuerit ea linea diagonii, tanta longitudo atrio detur." Vitruvius, *On Architecture*, The Loeb Classical Library, E. H. Warmington (ed.), Cambridge, Massachusetts and London, 1985, VI, iii, 3; p. 26.

<sup>24</sup> Carol Herselle Krinsky, "Seventy-Eight Vitruvius Manuscripts," *Journal of the Warburg and Courtauld Institutes*, 30, 1967, p. 38, notes 33 and 36. Umberto Eco notes that "from the ninth century onwards, Vitruvius was constantly cited in philosophical and technical manuals alike." Umberto Eco, *Art and Beauty in the Middle Ages*, trans. Hugh Bredin (New Haven: Yale University Press, 1986), 29.

<sup>25</sup> "Doors can be rectangular and they can also be half-round [i.e., semi-circular arched]. But the ancients used mainly rectangular, and in private buildings one never sees any but rectangular. It is true that in city gates such as those that are in Rome they are all round [i.e., semi-circular arched]. The measure of the portals are of three orders [*ragioni*]...of the three orders that pertain to the portals, that is, the width in relation to the height, I will tell you the form; as I said they can be of three orders of measure, as are also the columns or other members aforementioned, and these depend on the place where they are made, so the measures they require depend on the place; and they are made according to two squares, according to one and a half, and according to one and the diameter [i.e., diagonal]; and these are the three orders of measure. Like this the arched ones [i.e., semi-

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circular arched portals] also have these same three orders of measure, that is, Doric, Ionic, and Corinthian, that is, according to one square and a half, and according to one square and the diameter, and according to two squares, the diameter you having already seen how to take from the square...” (author’s translation); “Le porti possono ess(er)e quadre & anche possono ess(er)e mezze tonde Ma pure gliantichi lusauano lamaggiore parte quadre & nelli hedificij priuati no(n)ne uidi mai senon quadre. Vero e che importe dicitta come chesono a Roma sono tonde tutte. Lamisura delle porti sono ditre ragioni....alle ragioni cheuogliono ess(er)e leporti cioe lalarghezza alla altezza uidiro laforma come o detto possono ess(er)e ditre ragioni dimisure come sono ancora lecolonne o altri membri antedetti & queste ancora seco(n)do eluoghi doue sifanno chesecondo illuogho cosi richieghono lamisura & fannosi adue quadri – auno & mezzo – auono diamitro & cosi sono ditre ragioni dimisure cosi gliarchi ancora anno queste medesime ragioni dimisure cioe /dorico/ ionico & corintho cioe auno quadro & mezzo & auno quadro diamitro & adue quadri il diamitro che auete inteso dinanzi come sipiglia dalquadro...”, transcription from Biblioteca Nazionale, Florence, MS II, I, 140 (formerly Cod. Magliab. XVII, I, 30) by Howard Saalman in: “Early Renaissance Architectural Theory and Practice in Antonio Filarete’s *Trattato di Architettura*,” *Art Bulletin*, XLI, 1, 1959, p. 91, n. 1 and 5.

Saalman has, I believe, misinterpreted the last portion of this passage when he claims it to be a description of a rectangle derived from the diagonal of two squares, or, a root-5 rectangle, which has the proportions  $1:\sqrt{5}$ . According to Saalman, “*A due quadri il diamitro* is  $\sqrt{5}:1$ , the diagonal of a 2:1 rectangle to the shorter side.” (Saalman, 1959, p. 92). Filarete clearly states that the arched portals he describes have the “same orders of measure” (“medesime ragioni dimisure”) as the rectangular doorways that he describes earlier in the passage, and he names these “orders of measure” after the three orders of columns that he recognized: Doric (square and a half), Ionic (root-2 rectangle), and Corinthian (double square). Filarete thus makes clear his intention that the same three “orders of measure” be used both for rectangular doorways and arched portals. Saalman’s introduction of a new, fourth “order”—a root-5 rectangle—into Filarete’s arched portal proportions would therefore seem to be mistaken. Saalman’s apparent error perhaps derives from his reading of the words “adue quadri il diamitro” as a verbal unit (Saalman, 1959, p. 92), and from his translation of this unit as: “the diameter of two squares” (Saalman, 1959, p. 91). Note that in the last portion of my translation above: “...according to one square and a half, and according to one square and the diameter, and according to two squares, the diameter [noted here] you having already seen how to take from the square...” I have used a comma to separate the words “two squares” from “the diameter [i.e., diagonal],” so that “two squares” and “diameter” will *not* be read as a verbal unit.

Thus the word “diameter” can only refer to Filarete’s last usage of it, which is in the phrase “one square and the diameter.” My insertion of a comma in this location is consistent with the Finoli and Grassi edition (Antonio Averlino detto il Filarete, *Trattato di Architettura*, Anna Maria Finoli and Liliana Grassi, eds., Milan, 1972, p. 233).

<sup>26</sup> For doorway proportions see Orlandi, I, xii, pp. 84-84; and for arcade and colonnade bay proportions of 1 : 3 1/2, 1 : 1 1/3, 1:2 and 1: 1/3 see *ibid*, VII, xv p. 643.

<sup>27</sup> These proportions may be summarized as follows: for floor plans, 1:1, 3:4, 2:3, 3:5, 1:2; and for room heights: 2:5, 1:3, 1:4, 1:5, and 1:√2. The full array of Martini’s room proportions breaks down as follows: *Atrio* and *sale* (floor plans: 3:5, 2:3, circle; heights: 1:3, 1:4, 1:5, 1:√2, 1:2, 2:5), *cortili* (1:1, 1:1 1/3, 1:1 1/2, 1:1 2/3), *camere* (floor plans: 1:1, 1:1 1/3, 1:1 1/2; heights: 1:√2), *salotti* (floor plans: 1:2, 1:1 2/3, 1:1 1/2; heights: 1:√2 or “*del solaro*”), *altri salotti o teclini* (floor plans: 1:1, 1:1 1/3, 1:1 1/2, 1:1 2/3; heights: 1:√2), *cucina* (2:3, 1:1 2/3). Francesco di Giorgio Martini, *Trattati di architettura, ingegneria e arte militare*, Corrado Maltese and Livia Maltese Degrassi, eds., 2 vols., Milan, 1967 (hereafter Maltese), vol. 2, pp. 345–347, and tav. 194. Francesco describes the proportion 1:√2 as follows: “...one makes a square from the width [of the room], and divides it with a diagonal line from corner to corner, and this diagonal is the height.” (author’s translation); “...si facci uno quadrato della larghezza overo latitudine, e dividisi per linea diagonia da angulo ad angulo, e quello diametro sia l’altezza.” (Maltese, p. 345). I have omitted from consideration a few of Francesco’s more complex constructions, such as a rectangle composed of two side-by-side root-5 rectangles (see Maltese, pp. 347, 349 and tav. 195, 203, 204).

<sup>28</sup> Carlo Pedretti, “Newly Discovered Evidence of Leonardo's Association with Bramante,” *Journal of the Society of Architectural Historians*, XXXI, 3, 1973, pp. 223–227.

<sup>29</sup> Aristotle, *The Physics*, Philip H. Wicksteed and Francis M. Cornford, transl., Cambridge, Massachusetts and London, 1996, IV, xi, 219b3–9; cited in Judith V. Field, “Kepler’s Rejection of Numerology,” *Occult and Scientific Mentalities in the Renaissance*, Ed. Brian Vickers, Cambridge, 1984, pp. 274, and 293 n. 7.

<sup>30</sup> Johannes Kepler, *Narratio de observatis a se quatuor Iovis satellitibus erronibus* (Frankfurt, 1611) in Kepler, *Gesammelte Werke*, ed. W. von Dyck, M. Caspar, F. Hammer, et. al. (Munich, 1938?), IV, 370, II, 19–25, cited in Field, *ibid.*, pp. 274, 293 n. 5.

<sup>31</sup> The *campione* is a brass bar made in 1782 on the orders of Pietro Leopoldo to serve as the standard measure for all of Tuscany. Its length was based on the Florentine *braccio* in use at the time, which was believed to be the same as that used “since ancient times” (*dall’antica*). *Tavole di ragguaglio per la riduzione dei pesi e misure....* (Florence, 1782), xvi. In 1808 this *campione* was precisely

measured to establish the official metric length of the *braccio* at 0.583625839 m. *Tavole di riduzione delle misure e pesi....* (Florence, 1809), 11-12. This measurement became the basis for the metric value of the Florentine *braccio* listed in numerous late nineteenth-century metric conversion tables, most notably Angelo Martini, *Manuale di metrologia* (Turin, 1883), 206. It is consistent with my measurement of this *campione*, 116.73 cm (or, 1 *braccio* = 0.58365 m); that of a drawn *campione*, c. 1500, measured by Burns; and the “average” *braccio* calculated by Zervas based on a variety of sources, though the only reliable source contained within that average is the 1782 *campione*. Howard Burns, “San Lorenzo in Florence Before the Building of the New Sacristy: An Early Plan,” *Mitteilungen des Kunsthistorischen Institutes in Florenz* 23, no. ½ (1979), 145; and Diane Finiello Zervas, “The Florentine Braccio da Panna,” *Architectura*, no. 1 (1979), 6-10. Most significantly, the length of the 1782 *passetto* corresponds with great precision to the widths of the San Lorenzo column plinths (Figure 2-44).

<sup>32</sup> According to the Florentine mathematician Pier Maria Calandri (1419-1467): “Linear measure is used for cloth, drapes and other similar things; this measure is only by length, and the main instrument used in Florence is the *canna* which is divided into four parts, each called a *braccio*; but the *braccio* has no parts other than those taken of itself, such as  $\frac{1}{2}$  *braccio* and  $\frac{1}{3}$  *braccio*, an eighth of a *braccio*, and so on” (“... ma il braccio non à divisione alcuna se none le parti che se ne piglia, come a dire 1/2 braccio et 1/3 di braccio et ottavo di braccio et simili parti.”) Pier Maria Calandri, *Tractato d'Abbacho, dal Codice Acq. e doni 154 [sec. XV] della Biblioteca Medicea Laurenziana di Firenze*, Gino Arrighi, ed. (Pisa, 1974), 32-33. For a nearly identical statement in a similar *trattato* of 1463 by Lionardo Pisano, see Gino Arrighi, “Il codice L.IV.21 della biblioteca degl'intronati di Siena e la ‘bottega dell’abaco a Santa Trinita’ in Firenze,” *Physis* 7, fasc. 4 (1965), 386. Note that in the San Lorenzo “Venice plan,” c. 1500, annotated *braccio* dimensions include only fractional subdivisions while the “Chatworth plan”, c. 1550, contains *braccio* annotations with subdivisions in *soldi* and *denari*. Burns, “San Lorenzo in Florence,” Figures 1, 2, 4. See also the San Lorenzo construction ledger (1441-1453), in which *braccio* subdivisions are expressed only as fractions. Hyman, *Fifteenth Century Florentine Studies*, Part II.

<sup>33</sup> The centimeter measurements noted here are approximate because they reflect bay-by-bay variations. Other notable *braccio* dimensions include the column and pilaster plinth widths, 2 br (approximately 2.001 cm); the column and pilaster shaft diameters, 1 ½ br (approximately 1.494 cm); and the archivolt widths in elevation, also 1 ½ br (approximately 1.513 cm).

<sup>34</sup> Aspects of Boethian number theory survive in the obscure field of modern number theory. See, for example: Leonardo Eugene Dickson, *History of the Theory of Numbers*, II, Washington, 1920. Historians of mathematics tend to emphasize the mathematically rudimentary level of Boethius's writings on arithmetic, and to downplay the importance of these writings in the development of medieval thought. See for example, Boyer and Merzbach's comment: "...[Boethius] was concerned primarily with two aspects of mathematics: its relationship to philosophy and its applicability to simple problems of mensuration. Of mathematics as a logical structure there is little trace"; and later, their reference to "the jejune Latin texts of Boethius." Carl B. Boyer, *A History of Mathematics*, 2nd ed., revised by Uta C. Merzbach, New York, 1991, pp. 191f, 193-194. See also Ifrah's comment: "'Theoretical' arithmetic in the High Middle Ages was drawn from a work attributed to the Latin mathematician Boethius...who had himself drawn handsomely on a second-rate work by the Greek Nicomachus of Gerasa.... As for 'practical' arithmetic, it consisted mainly in the use of Roman numerals, and in operations with counters on the old abacus of the Romans; it also included the techniques of finger-counting transmitted by Isidore of Seville...." Georges Ifrah, *The Universal History of Numbers*, New York, 2000, p. 578. Beseler and Roggenkamp's efforts to link measurements of St. Michael's Church in Hildesheim to Boethian number theory has several shortcomings, including unrigorous use of unverifiable measurements (which may in fact be estimated measurements), and a limited reading of Boethius's *De arithmetica*. Hartwig Beseler and Hans Roggenkamp, *Die Michaeliskirche in Hildesheim*, Berlin, 1954, pp. 134ff.

<sup>35</sup> For general works on Boethius, see: Pierre Courcelle, *Late Latin Writers and Their Greek Sources*, Harry E. Wedeck, transl., Cambridge, Massachusetts, 1969, pp. 273-330; Lorenzo Minio-Paluello, "Anicius Manlius Severinus Boethius," *Dictionary of Scientific Biography*, II, New York, 1970, pp. 228-236; Howard Rollin Patch, *The Tradition of Boethius: A Study of His Importance in Medieval Culture*, New York, 1935; Edward Kennard Rand, *Founders of the Middle Ages*, Cambridge, Massachusetts, 1928, pp. 135-180; and William H. Stahl, *Roman Science: Origins, Development, and Influence to the Later Middle Ages*, Madison, 1962, pp. 193-211.

<sup>36</sup> Edward Kennard Rand, as paraphrased in: André Barbera, "Interpreting an Arithmetical Error in Boethius's *De Institutione musica* (iii. 14-16)", *Archives internationales d'Histoire des Sciences*, Vol. 31, No. 109, 1981, p. 26.

<sup>37</sup> *Paradiso*, X, 125.

<sup>38</sup> *The Book of Saints*, 4<sup>th</sup> ed., London, 1947, p. 536; According to Turner: "The local cult of Boethius at Pavia was sanctioned when, in 1883, the Sacred Congregation of Rites confirmed the custom prevailing in that diocese of honouring St. Severinus Boethius, on the 23rd of October."

William Turner, "Anicius Manlius, Severinus Boethius," *The Catholic Encyclopedia*, II, New York, 1907, Online Edition, 2002, <<http://www.newadvent.org/cathen/02610b.htm>> (accessed February 16, 2003).

<sup>39</sup> Nicomachus of Gerasa, *Introduction to Arithmetic*, Martin Luther D'Ooge, transl. (New York and London, 1926); Anicii Manlii Torquati Severini Boetii, *De institutione arithmetica, libri duo, de institutione musica, libri quinque*, Godofredus Friedlein, ed. (Leipzig, 1867); and Michael Masi, *Boethian Number Theory: A Translation of the De Institutione Arithmetica* (Amsterdam, 1983).

<sup>40</sup> For the estimated date of completion of *De institutione arithmetica*, see C. J. De Vogel, "Boethiana I," *Vivarium* 9, no. 1 (May 1971), 65; and Edmund Reiss, *Boethius* (Boston, 1982), 11. Gregor Reisch, *Margarita philosophica*, (Freiburg, 1503), Book IV, title page. The illustration reflects an historical misconception common during the Renaissance and later periods that itself attests to the high regard in which Boethius was held. Pullan notes: "It has been said that Boethius first showed the Arabic system [of numeration] to the western world, just as Pythagoras is said to have introduced the abacus into Greece nearly 1000 years before." But, he continues, "Boethius's part is dubious," since Arabic figures first appear in European manuscripts in the 10<sup>th</sup> century, and were probably not used for calculation until much later. J. M. Pullan, *The History of the Abacus*, (New York and Washington, D.C., 1968), 36. On this illustration see also Karl Menninger, *Number Words and Number Symbols* (Cambridge, Mass., 1969), 350, 431; and Masi, *Boethian Number Theory*, 19ff. On the original date of publication of *Margarita Philosophica*, see John J. Bateman, "The Art of Rhetoric in Gregor Reisch's *Margarita Philosophica* and Conrad Celtes' *Epitome of the Two Rhetorics of Cicero*," *Illinois Classical Studies* 8, no. 1, 1983, pp. 137 notes 1-2.

<sup>41</sup> For example, the Lombard duke Filippo Maria Visconti (r. 1412–47) had two copies in his palace library in Pavia and Coluccio Salutati owned one in Florence. *Indagini storiche, artistiche e bibliografiche sulla libreria visconteo-sforzesca del castello di Pavia* (Milan, 1875), 50, 95; Marina Passalacqua and Lesley Smith, eds., *Codices Boethiani: A Conspectus of Manuscripts of the Works of Boethius*, 3 vols. (London and Turin, 2001), III: 85. On "the truly fundamental nature of Boethian mathematics in medieval number theory," see Michael Masi, "The Influence of Boethius' *De arithmetica* on Late Medieval Mathematics" in *Boethius and the Liberal Arts: A Collection of Essays*, ed. Masi (Bern, 1981), 95 and references therein.

<sup>42</sup> For Boethius, the highest intellectual pursuit is philosophy, which must be approached through sequential study of arithmetic, music, geometry, and astronomy, or, the *quadrivium*, a term he coins here. *De institutione arithmetica*, I, 1; and Masi, *Boethian Number Theory*, 71-74. See also Pearl Kilbre, "The Boethian *De Institutione Arithmetica* and the Quadrivium in the Thirteenth Century

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University Milieu at Paris,” in *Boethius and the Liberal Arts: A Collection of Essays*, Michael Masi, ed. (Bern, Frankfurt, and Las Vegas, 1981), 67.

<sup>43</sup> *De Arithmetica*, II, 4; Masi, *Boethian Number Theory*, p. 128.

<sup>44</sup> Such numbers are distinct from others because, Boethius tells us, they concern “...that quantity which consists of itself; it is not referred to anything else...” Boethius, II, 4 (Masi, *ibid.*, p. 128). Thus, they may be considered the building blocks of more complex construction in Boethian number theory. Nicomachus calls the geometrical numbers “absolute” numbers. Nicomachus, VI, I (Nicomachus of Gerasa, *Introduction to Arithmetic*, Martin Luther D’Ooge, transl., with *Studies in Greek Arithmetic* by Frank Eggleston Robbins and Louis Charles Karpinski, New York and London, 1926, p. 236).

<sup>45</sup> Boethius calls them “figured numbers” (*figuratis numeris*). *De Arithmetica*, II, 18; Masi, *Boethian Number Theory*, p. 141; and Nicomachus calls them “polyagonals,” the more descriptive term that I will adopt. Nicomachus, II, XII, 4; D’Ooge, *ibid.* p. 248.

<sup>46</sup> Boethius notes that conventional numerals, by which he means Roman numerals, are not “formed by natural institution,” but rather, “by custom,” and so he describes numbers as sets of “strokes” (*virgulas*) so that the magnitudes they denote are manifest. *De Arithmetica*, II, 4; Masi, *Boethian Number Theory*, pp. 128-129; cf. Nicomachus, II, vi, 2; D’Ooge, *ibid.*, p. 237.

<sup>47</sup> Regarding unity Boethius notes: “...unity has the potential of a point, the beginning of interval and longitude; it is not itself capable of interval or longitude, just as the point is the beginning of the line and the interval, although it is itself neither interval nor line. Nor does a point put upon a point bring about an interval, any more than if you joined nothing to nothing. It is nothing and nothing comes from nothing. The same proportionality exists between equalities. Now if there were equal terms, so much it is from the first to the second as from the second to the third, and between first and second, or second and third, there is no quantity of interval or space.” Masi, *Boethian Number Theory*, p. 129; *De Arithmetica*, II, 4, p. 87. On the possible symbolic significance of unity, see the discussion of symbolic numbers, below.

<sup>48</sup> *Liber Mathematicalis Bernwardi*, Domschatz Nr. 31, Hildesheim. For catalogue information see Victor H. Elbern and Hans Reuther, *Der Hildesheimer Domschatz*, (Hildesheim, 1969), 43-44; and Helmar Härtel and Marlis Stähli, *Die Handschriften im Domschatz zu Hildesheim* (Wiesbaden, 1984), 71-73.

<sup>49</sup> Boethius, *De institutione arithmetica*, II, 4, 14-16; Masi, *Boethian Number Theory*, 129, 137-139. Nicomachus of Gerasa, *Introduction*, 245, 246; Sir Thomas Heath, *A History of Greek Mathematics*,

2 vols. (Oxford, 1921), 1:77-79; and Leonardo Eugene Dickson, *History of the Theory of Numbers*, 3 vols. (Washington, D. C., 1910-1923), 2:1.

<sup>50</sup> Boethius writes of this progression: “Now in a hexagon we join [numbers] together with three between [each], and they surpass each other by four, and these will be the roots and foundations from which, when they are joined together, all hexagons are born: 1 5 9 13 17 21 and so on, according to that order.” *De institutione arithmetica*, II, 15; and Masi, *Boethian Number Theory*, 138.

<sup>51</sup> The 1463 *trattato* of Lionardo Pisano repeatedly refers to Boethius and quotes extensively from *De institutione arithmetica*. Arrighi, “Il codice L.IV.21,” 379-381, 382, 385. Stornaloco provides a sketch showing a series of triangles superimposed over a cross-section cathedral diagram, with the following explanation: “The triangles begin with one *unitas* [unity] according to the nature of triangles. For, once the *unitas* has been established, we have a potential triangle, and when we add a second one above it we shall have the first actual triangle.” Stornaloco, trans. in Paul Frankl, “The Secret of the Mediaeval Masons,” *Art Bulletin* 27, no. 1 (Mar. 1945), 53. Boethius’s description of triangular numbers is strikingly similar: “The first triangle which is born from unity, as here, is a triangle in power, but not in act and operation. This power is, as it were, the mother of all numbers; whatever else occurs in the following numbers comes from that unity and it is found to be of those numbers. It is necessary that unity contains those numbers by a certain natural potency. The number three, which is the first triangle in operation and act, has the binary number [i.e., two] as a side, in growing by a unity.” *De institutione aithmetica*, II, 8; and Masi, *Boethian Number Theory*, 134. Cf. Alberti’s description of cubic numbers. *De re aedificatoria*, IX, vi. On the chess-like rithmomachia board, black pieces in the back row are numbered 25, 81, 169, 189. To derive them, according to a sixteenth-century rule book, one takes the number 5, and every fourth number thereafter “... which numbers will be these: 5, 9, 13, 17, and multiply said numbers by themselves....” Francesco Barozzi, *Il nobilissimo et antiquissimo giuoco pythagoreo nominato rythmomachia, cioe: battaglia de consonantie de numeri* (Venice, 1572), 6v and 7r. In the late fifteenth and sixteenth centuries, editions of Boethius's *De institutione arithmetica* were often published in the same bound volumes as *rithmomachia* rule books. For examples see David Eugene Smith, *Rara Arithmetica* (Boston and London, 1908), 62ff. See also David Eugene Smith and Clara C. Eaton, “Rithmomachia, the Great Medieval Number Game,” *American Mathematical Monthly* 18, no. 4 (April 1911), 73-80; and Gillian R. Evans, “The Rithmomachia: A Mediaeval Mathematical Teaching Aid?,” *Janus* 63, no. 4 (1976), 257-273.

<sup>52</sup> Vitruvius, *De architectura*, IX, Preface, 4.

<sup>53</sup> "... la quale e inrationabile, ne si trova proportione alcuna dal quadro perfetto a questo cressimento." Sebastiano Serlio, *Il primo libro d'architettura*, (Paris, 1545), 21r.

<sup>54</sup> For example, describing a figure of two rotated squares, Frankl notes: "The proportion of the side of the small square to that of the bigger one is  $1:\sqrt{2}$ , an irrational proportion which could not be calculated by arithmetic in mediaeval times and had to be constructed by geometry." Frankl, "The Secret of the Mediaeval Masons," 51. Cf. Saalman, "Early Renaissance Architectural Theory," 93, 95. Other evidence, however, suggests that both square root calculation and the Pythagorean theorem were widely taught from at least the thirteenth century in Europe. See E.G.R. Waters, "A Thirteenth Century Algorism in French Verse," *Isis* 11, no. 35 (Sept. 1928), 45-84; and Paolo dell' Abbaco, *Trattato d'aritmetica, Secondo la lezione del Codice Magliabechiano XI, 86 della Biblioteca Nazionale di Firenze*, Gino Arrighi, ed. (Pisa, 1964), especially problems 148-150.

<sup>55</sup> Wittkower, "Systems of Proportion," *Architect's Yearbook* 5 (1953), 16; repeated in: Wittkower, *Architectural Principles* (1971), 158; and Wittkower, "The Changing Concept of Proportion," in *Idea and Image: Studies in the Italian Renaissance* (London, 1978), 116.

<sup>56</sup> This statement is not contingent upon the final resolution of the problem posed by the 11-12 cm column shaft height discrepancy noted above, because in any case the ratio  $9\frac{2}{3} : 13\frac{2}{3}$  very accurately describes the *actual* distances between the nearer edges of any two adjacent column plinths and the farther edges of the same plinths.

<sup>57</sup> Equally remarkable is the numerical ratio  $9\frac{2}{3} : 17\frac{2}{3}$ , which accurately approximates the irrational proportions of the dual diagon. See note 16.

<sup>58</sup> O. Neugebauer, *The Exact Sciences in Antiquity* (New York, 1969), 35; Leonardo Dickson, *History of the Theory of Numbers*, 2:341; Plato, *Republic*, Book 8, 546C; Heath, *A History of Greek Mathematics*, 1: 93; James Gow, *A Short History of Greek Mathematics* (New York, 1923), 96 note 3; and Théon de Smyrne, *Philosophe platonicien exposition des connaissances mathématiques utiles pour la lecture de Platon*, J. Dupuis, transl. (Paris, 1892), 71-75. On Theon's formula see Gow, *ibid.*, 95-96; Heath, *ibid.*, 92; and Peter Kidson, "A Metrological Investigation," *Journal of the Warburg and Courtauld Institutes* 53 (1990), 71-97.

<sup>59</sup> Paul Oskar Kristeller, *Iter Italicum: A Finding List of Uncatalogued or Incompletely Catalogued Humanistic Manuscripts of the Renaissance in Italian and Other Libraries* (London, 1967), 2:368. I thank Howard Burns for calling this source to my attention (Kristeller). Theon's treatise, which was written in Greek, could have arrived in Florence at the end of the fourteenth century, when the

Ottoman Turks encroached into Byzantine territory, and Greek-speaking men of learning took refuge in Italy. Among them was Manuel Chrysoloras, who was invited to Florence in 1396 to give public instruction in Greek. Ferdinand Schevill, *History of Florence, from the Founding of the City through the Renaissance* (New York, 1936), 320; and Paul Lawrence Rose, "Humanist Culture and Renaissance Mathematics," *Studies in the Renaissance* 20 (1973), 53-54. One of Chrysoloras's students, Ambrogio Traversari (1386-1439), a promoter of Greek learning in his Camaldolese monastery in Florence, very likely served, in about 1434, as Brunelleschi's patron for the Scolari Oratory of Santa Maria degli Angeli. For many years prior to 1431 Florentine intellectuals, including two who would develop close ties to Brunelleschi, Cosimo de' Medici and Paolo dal Pozzo Toscanelli, regularly gathered in Traversari's cell as an informal academy devoted to the study of ancient Greek and Roman texts. Charles Stinger, "Ambrogio Traversari and the 'Tempio degli Scolari' at S. Maria degli Angeli in Florence," in *Essays Presented to Myron P. Gilmore*, 2 vols., S. Bertelli and G. Ramakus, eds. (Florence, 1978), 1:280-281; and Rose, "Humanist Culture and Renaissance Mathematics," 48 ff. On Chrysoloras see also Giuseppe Cammelli, *Manuele Crisolora* (Florence: Vallecchi, 1941). Thus Brunelleschi, whether or not he designed the San Lorenzo set of proportions, could have had access to the mathematical knowledge needed to understand and appreciate the side/diagonal approximations embedded in it.

<sup>60</sup> Theon's formula may be described algebraically as:  $d = 2s^2 \pm 1$ . Gow, *A Short History of Greek Mathematics*, 95-96. The ratio 5:7 occurs as a side/diagonal approximation in the late Gothic mason's manual of Lorenz Lechler. Lon R. Shelby and Robert Mark, "Late Gothic Structural Design in the 'Instructions' of Lorenz Lechler," *Architectura* 9, no. 2 (1979), 127-128 and Figure 4 therein. Cesariano notes the side/diagonal approximations of 5:7 and 12:17. Cesare Cesariano, *Di Lucio Vitruvio Pollione de architectura libri decem traducti de latino in vulgare* (Como, 1521; New York, 1968), 98r. The latter approximation also appears to have circulated among mathematicians of 11th century France. Paul Tannery, *Memoires Scientifiques*, V, Paris and Toulouse, 1922, p. 236.

<sup>61</sup> On the introduction of Hindu-Arabic numerals see Walter William Rouse Ball, *A Short Account of the History of Mathematics*, 3<sup>rd</sup> ed. (London, 1901), 192-193; D. J. Struik, "The Prohibition of the use of Arabic numerals in Florence," *Archives internationales d'histoire des sciences* 21, no. 84-85 (1968), 291-294; Gillian R. Evans, "From Abacus to Algorism: Theory and Practice in Medieval Arithmetic," *The British Journal for the History of Science* 10, no. 35 (1977), 115; and D. J. Struik, ed., *A Source Book in Mathematics, 1200-1800* (Cambridge, Mass., 1969), 1.

<sup>62</sup> Paolo dell' Abbaco, *Trattato d'aritmetica*. Paolo was born into the prominent Dagomari family of Prato, served as one of the *priori* in his adoptive Florence in 1363, was quoted by Boccaccio, lived

past ninety, and became one of the most esteemed mathematicians of his time. Arrighi, Introduction to Paolo dell'Abbaco, *Trattato d'aritmetica*, 7ff; which supersedes Gustavo Uzielli, *La vita e i tempi di Paolo dal Pozzo Toscanelli* (Rome, 1894), 19 ff. According to Rose, "Paolo is quoted in Boccaccio's humanist work *De genealogia deorum*, and was well known in humanist circles at Florence." Paul Lawrence Rose, "Humanist Culture and Renaissance Mathematics," *Studies in the Renaissance* 20 (1973), 58. He probably completed his *Trattato* near his death in 1374, judging from internal evidence in word problems that involve hypothetical situations placed in the years 1372 to 1374. Paolo dell'Abbaco, *Trattato*, Problems 112 and 113, pp. 93-95. On the date of Paolo's death, see Arrighi, *ibid.*, 7. The manuscript published by Arrighi dates from the fifteenth century, thus evidencing the continued topicality of the work in the decades following Paolo's death. Among those who attended Paolo's school was the doctor, mathematician, and younger friend of Brunelleschi, Paolo dal Pozzo Toscanelli. Arrighi, Introduction to Paolo dell'Abbaco, *Trattato d'aritmetica* 7-13.

<sup>63</sup> The extensive use of fractions in Paolo dell'Abbaco's *Trattato d'aritmetica*, many of which contain large numerators and denominators, calls into question Carpo's claims that "the unavailability of general fractions confronted classical [i.e., Renaissance] architectural writers with many dumfounding problems," and that "...the only fractions available at the time were unit fractions: fractions in the format '1/n,' all numerators being equal to one." Mario Carpo, "Drawing with Numbers: Geometry and Numeracy in Early Modern Architectural Design," *Journal of the Society of Architectural Historians* 62, no. 4 (Dec. 2003), 451, 456-457.

<sup>64</sup> Calandri, *Tractato d'Abbacho*, 61.

<sup>65</sup> The last step in Paolo's solutions to his word problems 148-150 require the following calculations, respectively:  $49:196 \Rightarrow \frac{49:196}{5} = 9\frac{4}{5} : 39\frac{1}{5}$ ;  $49:441 \Rightarrow \frac{49:441}{10} = 4\frac{9}{10} : 44\frac{1}{10}$ ; and  $49:784 \Rightarrow \frac{49:784}{17} = 2\frac{5}{17} : 46\frac{2}{17}$ . Paolo dell'Abbaco, *Trattato*, 120-122.

<sup>66</sup> The interest in reconciling mathematically irrational, geometrically-derived proportions with numbers, evident in the San Lorenzo set of proportions, would seem consistent with certain late medieval intellectual constructs such as the *coincidentia oppositorum*, formulated by the philosopher Nicholas of Cusa (c. 1401–1464), and presented in his influential treatise *De docta ignorantia*, completed in 1440. For Nicholas, opposites in nature or philosophy, such as the infinitely large and the infinitely small, are united because both represent "absolute maximum," a concept that for him can be embodied only by God. Joseph E. Hofmann, "Nicholas Cusa," *Dictionary of Scientific Biography* 3 (New York, 1971), 514. Nicholas was once a classmate in Padua of Brunelleschi's long-

time acquaintance, Paolo dal Pozzo Toscanelli. Uzielli, *La vita e I tempi*, 67; and Manetti, *Vita*, Tanturli, ed., 70 note 3, and 93 note 23.

<sup>67</sup> In this study, when no simple fractional equivalent is implied, remainders are expressed in modern English decimal notation, a form that first appeared in 1616. Carl B. Boyer, *A History of Mathematics*, 2<sup>nd</sup> ed., revised by Uta C. Merzbach (New York, 1991), 317.

<sup>68</sup> Also note that the ratio between these two heights,  $1\frac{2}{3} : 2\frac{1}{3}$ , is equivalent to 5:7, which is one of the side/diagonal approximation pairs generated by Theon of Smyrna's formula (see Figure 2-49).

<sup>69</sup> Note that this finding contradicts the assumption and findings of Benevolo *et al.* (see note 4).

<sup>70</sup> Manetti accuses those who built at variance with Brunelleschi's intentions of having, variously (page numbers refer to Manetti, *Vita*, Tanturli, ed.), "ignorance," 99, 114, 115, 117; "presumption," 99, 110; "arrogance," 100; and "malice," 117; of "believing himself to be an expert," 105; and of "having a sly temperament that does everything wickedly," 112.

<sup>71</sup> Manetti, *Vita*, Tanturli, ed., 117.

<sup>72</sup> *Ibid.*, 116–117.

<sup>73</sup> *Ibid.*, 99–100.

<sup>74</sup> *Ibid.*, 101.

<sup>75</sup> Antonio Billi, *Il libro di Antonio Billi*, Fabio Benedettucci, ed. (Rome, 1991), 33–34. On Della Luna's role at the Ospedale, see Saalman, *Filippo Brunelleschi: The Buildings*, 49ff. Since Billi claims Brunelleschi's trip to Milan took place during the construction of the portico of the Ospedale degli Innocenti, it can be placed in or around 1420, the year in which Filippo Maria Visconti offered a peace treaty to Florence. Hans Baron, *The Crisis of the Early Renaissance* (Princeton, New Jersey, 1966), 371. Manetti simply notes that during construction of the Ospedale Brunelleschi had to be "elsewhere for a time." Manetti, *Vita*, Tanturli, ed., 100.

<sup>76</sup> On 8 November 1428, Giovanni de' Medici endowed two new canonries in the church of San Lorenzo, one dedicated to Saints Cosmas and Damiano to which the chapel is dedicated, and the other to Saint John the Evangelist, to which the Old Sacristy is dedicated. Moreni, *Continuazione*, 2:361–368. The endowment was finalized in a meeting held in the Old Sacristy, which the document describes as "sumptuous and newly built." A similar *ricordo* is quoted in *Donatello e la Sagrestia Vecchia*, 102; and Ruschi, "Una collaborazione interrotta," 85 note 7. These documents are consistent with the date "1428" carved into the original lantern cap of the Old Sacristy, today displayed on the cloister balcony. Additional evidence that the double chapel and Old Sacristy were completed as a unit is the exterior terra cotta frieze that circumscribes these two, but no other, parts

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of the basilica. Carlo Sisi, “Due interventi di restauro sulle decorazioni in terracotta della Sagrestia Vecchia,” in *Donatello e la Sagrestia Vecchia*, 86-99.

<sup>77</sup> Of the three components that make up the entablature, the architrave and cornice are carved from *pietra serena*, while the frieze is simply a blank area of stuccoed wall. Thus, even if Brunelleschi had left correctly dimensioned samples of the architrave and cornice, the *capomaestro* would have had to have known the correct frieze height.