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Similarity Coefficients for Binary Data

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Multi-way Metrics and Multivariate Coefficients*

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To Sascha

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