Foam Rheology Near the Jamming Transition

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Chapter 9

Appendices

9.1 Z

We have found a clear dependence of the contact number, Z, on both the strain rate and the density; see figure 9.1. In this appendix we will explain the steps we have taken to try to find a functional form for this dependence, as well as the results of those.

In the static, 2-dimensional case the dependence of Z on $\dot{\gamma}$ is given by $Z - Z_c = Z_0 \Delta\phi^\theta$, with $Z_c = 4$, $Z_0 = 3.6$ and $\theta = 0.5$ [5]. We assume for now that this general form will still hold true in the dynamic case, though with constants that are now a function of $\dot{\gamma}$. Furthermore, we assume that we recover the static case in the limit that $\dot{\gamma}$ goes to 0. Blindly fitting to find values for $Z^*(\dot{\gamma})$, $Z_0(\dot{\gamma})$ and $\theta(\dot{\gamma})$ is difficult, as the zero with respect to which everything is logarithmic itself depends on $Z^*(\dot{\gamma})$.

We have done various variants of such fits: forcing Z_0 to be independent of

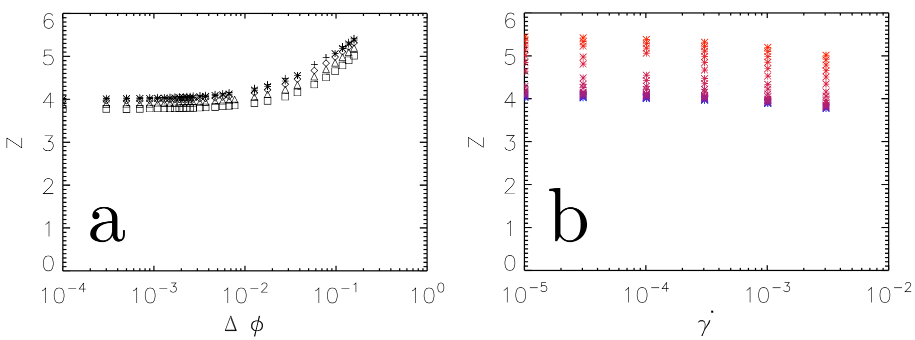

Figure 9.1: Plots of the contact number vs. the density, **a**, and the strain rate, **b**.

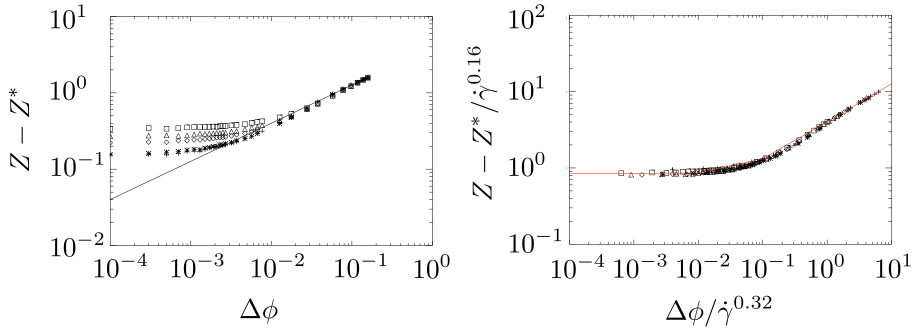

Figure 9.2: **a**: plot of $Z - Z^*(\dot{\gamma})$ vs. $\Delta\phi$. $Z*$ has been obtained by finding the best fit to a 1/2-power law for high density data. Note that all data collapses onto the same line for high density. **b**: the same plot, now rescaled by $\dot{\gamma}$ to achieve collapse, the red line is $y = (0.85^4 + (4x^{0.5})^4)^{0.25}$. Symbols indicate strain rate but are not consistent with table 3.3.

$\dot{\gamma}$, forcing θ to be independent of $\dot{\gamma}$ etc. When doing one such a fit, enforcing $\theta = 0.5$ and, erroneously, dramatically overvaluing the goodness of fit at high density, we got results for which Z_0 was also independent of $\dot{\gamma}$ and moreover was very close to the epitome value: 4, see fig. 9.2 **a**; in this case, $Z^*(\dot{\gamma})$ seems to be reasonably approximated by $4 - 2.6\dot{\gamma}^{0.28}$. We then tried if we could get collapse for low densities by rescaling the axes with $\dot{\gamma}$: plotting $\frac{Z-Z^*}{\dot{\gamma}^{\alpha}}$ vs. $\frac{\Delta\phi}{\dot{\gamma}^{\beta}}$. Because all data falls onto a 1/2 power law, we know that we should have $\beta = 2\alpha$. Trial and error finds good collapse for $\alpha = \frac{1}{6}$ and $\beta = \frac{1}{3}$, see figure 9.2 **b**. The data then seems to fall onto

$$\frac{Z - 4 + 2.6\dot{\gamma}^{0.28}}{\dot{\gamma}^{\frac{1}{6}}} = (0.85^4 + (4\frac{\Delta\phi}{\dot{\gamma}^{\frac{1}{3}}}^{0.5})^4)^{0.25} \tag{9.1}$$

While fit and collapse are pretty good, this has 5 fit parameters; a little much. Additionally, for low $\Delta\phi$ the dependence is given by $Z = 4 - 2.6\dot{\gamma}^{0.28} + 0.85\dot{\gamma}^{0.17}$, which contains two terms with similar dependence on $\dot{\gamma}$.

A third problem of this result is that it contains a rescaling of $\Delta\phi$ with $\dot{\gamma}$ that is different from any such rescaling we have already found in our scaling model. We therefore wonder if we can also get good collapse and fit if we start from a rescaling of $\frac{\Delta\phi}{\dot{\gamma}^{0.5}}$; if we still want to end up with a 1/2 power law, this means we need to rescale $Z - Z^*$ with $\dot{\gamma}^{0.25}$. In figure 9.3 **a** we show such a graph, in which we have adjusted Z^* by hand to achieve collapse. The collapse is decent but not phenomenal. If we now keep the same values for Z^*, but revert the rescaling to $\frac{Z-Z^*}{\dot{\gamma}^{0.17}}$ and $\frac{\Delta\phi}{\dot{\gamma}^{0.33}}$, the collapse is much better, as can be seen in figure 9.3 **b**. A power law fit to $Z^*(\dot{\gamma})$ gives $Z^* = 4 + 0.03 - 7.8 * \dot{\gamma}^{0.58}$.

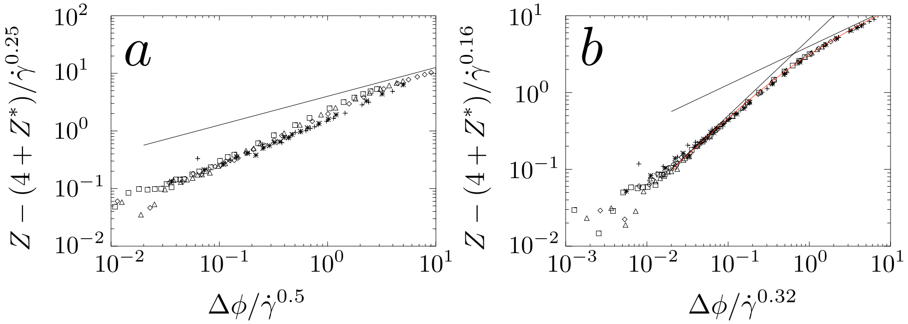

Figure 9.3: **a** plot of $\frac{Z-Z^*(\dot{\gamma})}{\dot{\gamma}^{0.25}}$ vs. $\frac{\Delta\phi}{\dot{\gamma}^{0.5}}$. $Z^*(\dot{\gamma})$ has been selected by hand to result in good collapse. **b** plot of $\frac{Z-Z^*(\dot{\gamma})}{\dot{\gamma}^{0.17}}$ vs. $\frac{\Delta\phi}{\dot{\gamma}^{0.33}}$ using the same values for $Z^*(\dot{\gamma})$. The red line is $y = \frac{1}{((5x)^{-2}+(4\sqrt{x})^{-2})^{0.5}}$. Symbols indicate strain rate but are not consistent with table 3.3.

The data then falls onto

$$\frac{Z-4.03+7.8\dot{\gamma}^{0.58}}{\dot{\gamma}^{0.17}} = \frac{1}{\left(\left(5\frac{\Delta\phi}{\dot{\gamma}^{0.33}}\right)^{-2}+\left(4\sqrt{\frac{\Delta\phi}{\dot{\gamma}^{0.33}}}\right)^{-2}\right)^{0.5}} \tag{9.2}$$

Depending on how you count, this dependence has about 6 fit parameters, so it's even worse than before. There are two recurring features. The first recurring features is the $\Delta\phi^{1/2}$ scaling of Z for high $\Delta\phi$. This is the same scaling as was found in static jamming [5]. The second recurring feature is that we find collapse when rescaling $\Delta\phi$ with $\dot{\gamma}^{0.33}$ and and rescaling - some function of - Z with $\dot{\gamma}^{0.17}$. The first one was expected, and at some points enforced, but certainly works very well. The second one was not expected; perhaps it is the only new thing we may learn from this (though it's still unclear what it means exactly).

9.2 Appendix: First Moment

We investigate the first moments of the pdf of Δv by considering the following two components of the relative velocity:

$$\Delta v_{\text{par}} = \Delta v_x \frac{r_x}{r} + \Delta v_y \frac{r_y}{r} \tag{9.3}$$

$$\Delta v_{\text{perp}} = \Delta v_y \frac{r_x}{r} - \Delta v_x \frac{r_y}{r}, \tag{9.4}$$

where Δv_{par} signifies the component of the relative velocity *parallel* to the vector r connecting the centres of the two bubbles and Δv_{perp} signifies the

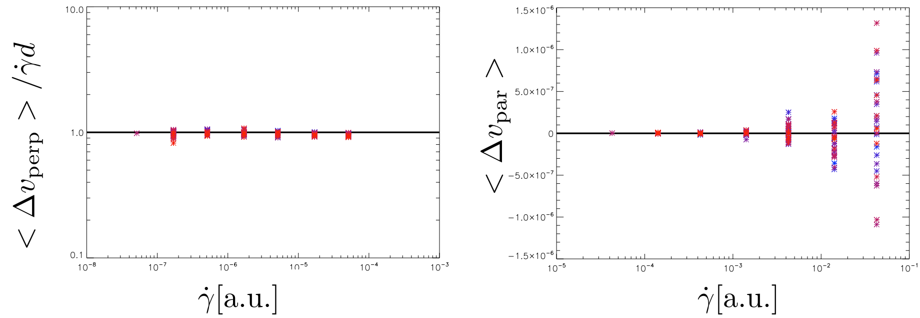

Figure 9.4: Left panel: the rotational component of the relative velocity, $\Delta v_{\rm perp}$, as a function of the strain rate with the expected proportional behavior divided out. Right panel, the compressive component of the relative velocity, $\Delta v_{\rm par}$. In both cases the black line indicates expected behavior. Color denotes density as in table 3.3.

component *perpendicular* to the connecting vector. We have chosen sign conventions such that bubbles moving apart will have a positive $\Delta v_{\rm par}$ and bubbles moving around each other in a clockwise direction will have a positive $\Delta v_{\rm perp}$.

We expect $\langle \Delta v_{\rm par} \rangle = 0$ on physical grounds: there is no net expansion or contraction and therefore every particle movement that brings particles closer together must be balanced by particle movement that brings particles further apart somewhere else. We do not expect $\langle \Delta v_{\rm perp} \rangle = 0$, however, because the applied strain enforces an overall rotation proportional to the applied strain rate $\dot{\gamma}$ and the bubble size d. These relations are tested in figure 9.4. As can be seen, both components of Δv behave as expected, although there are obvious deviations from 0 for $\Delta v_{\rm par}$, they are symmetric and a simple consequence of the fact that the pdfs widen for higher strain rates, as discussed above.

While on the one hand it is good that $\langle \Delta v_{\rm par} \rangle$ and $\langle \Delta v_{\rm perp} \rangle$ behave as we expect, this also means they don't tell us anything new. In the end this is because any component of $\langle \Delta v \rangle$ will just measure some component of the overall velocity, which is prescribed, in the end, by the applied strain rate. Therefore it is only the higher order moments, which tell us something about the way the velocities are distributed around this average, that give us new information about the behavior of the system.

9.3 Appendix: Correlation Strain

In section 4.5.2 we introduced the correlation strain $\gamma_{\rm corr}$ as the strain over which the autocorrelation of the stress signal has decayed to 0.5. In this appendix we will show how we have determined that this strain is a linear combination of the yield and dynamic strains with a different balance of terms

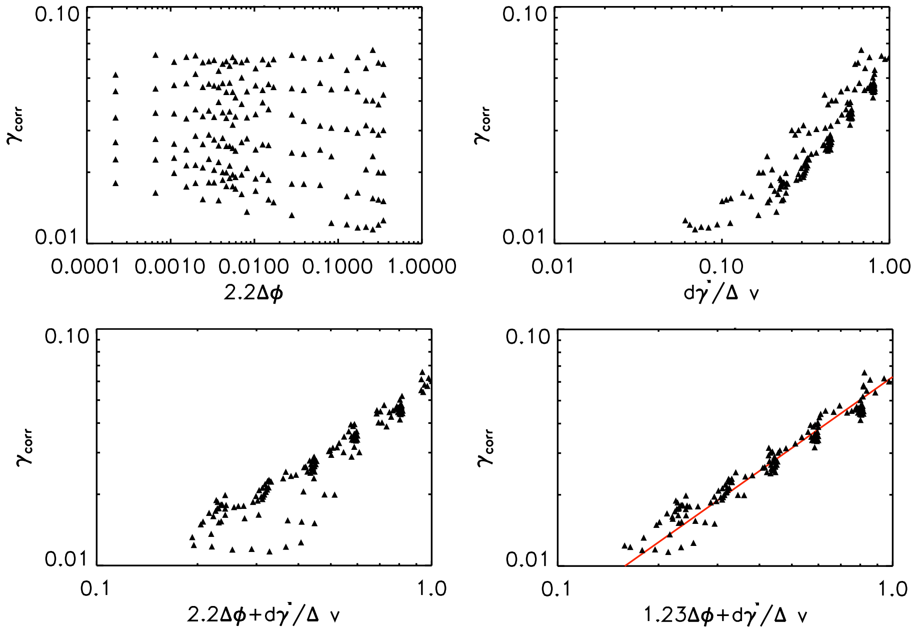

Figure 9.5: Plots of the correlation strain vs. various strains from our model. Top left: the yield strain, γ_y. Top right: the dynamic strain, γ_{dyn}. Bottom left: the effective strain, γ_{eff}. Bottom right: the best fir linear combination of γ_y and γ_{dyn}

from the effective strain.

First, though, we note that γ_{corr} is not a complete characterization of the decay of the autocorrelation, as can be seen in figure 4.13, as the autocorrelation functions do not collapse completely when rescaled with this strain. Moreover, there is a systematic trend: data from high-density simulations tends to have a longer plateau before it commences on a steeper drop. Still, γ_{corr} seems to capture the behavior of the autocorrelation reasonably well.

The next step is to test if γ_{corr} corresponds to the yield strain, γ_y, the dynamic strain, γ_{dyn}, the effective strain, $\gamma_{eff} = B_{eff}\gamma_y + \gamma_{dyn}$, or a more general linear combination of:

$$\gamma_{corr} = B_{corr}\gamma_y + \gamma_{dyn} \tag{9.5}$$

Note that this formulation allows the weight of γ_y but not γ_{dyn} to be zero. This is intentional as we expect that the correlation strain may be dominantly determined by γ_{dyn} as γ_{dyn} is the strain over which particles rearrange and rearrangements play an important role in the fluctuations in the stress.

In figure 9.5 we can clearly see that neither of the three ‘simple’ strains from our Q3E model, the yield, dynamic or effective strains, describe the correlation strain well. A general linear combination of γ_{y} and γ_{dyn} does work. First of all, we see that the data collapses reasonably well, clearly scattering about a line. This scatter is not surprising given that the γ_{corr} that we extracted clearly does not fully capture the behavior of the correlation function as shown. Second, we note that the line around which the data points scatter is actually a linear relation as we hypothesised. In conclusion: the correlation strain is clearly related to the effective strain but is not equal to either the yield, the dynamic or the effective strain.