# Dislocations in stripes and lattice Dirac fermions

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# Chapter 8

# Conclusions

In this thesis we have presented theoretical analysis of two types of electronic systems: (i) massless spinors in one and two dimensions, and (ii) collective bosonic liquid crystalline type order parameters. Most of our work is directly related to experimental endeavors, but some of it grazes on interesting general theoretical problems. Although each Chapter contains a section devoted to concluding remarks, we decided to close this thesis with a presentation of additional thoughts, outlooks and some general views on the role of the work presented here.

In Chapter 4 we make direct predictions of the LDOS in graphene grain boundaries, based on the model of free tight binding electrons. However this is only the first necessary step in understanding the real grain boundaries of graphite, and the problem is now back in the court of experiments, we believe. Firstly, we suggest that low angle grain boundaries have zero energy peaks in the LDOS pending the orientation of the Burgers vector of the constituent dislocations with respect to the lattice. It is an ongoing experimental effort to improve the resolution at the defects to allow the identification of the precise core structure. Only after this step one can judge just how useful the free electron model is. Our prediction is based on the well-known zero mode structure of the Dirac equation in an Aharonov-Bohm field, whose role is here played by the dislocations. The zero modes are localized on all dislocation cores, and at least one of them has a LDOS scaling with measurement area that is favorable for detection.

Secondly, the question of experimental testing of predictions is acute in view of the surprising measurement of ferromagnetism in these grain boundaries. The LDOS peak features near the Fermi energy in the free electron model suggest en enhanced role of interactions, which could lead to local magnetic moments.

However, the nature of distribution of these magnetic moments is still not known experimentally. This additionally leaves open the question of sufficiency of the graphene model, as opposed to a 3d graphite model with a 2d grain boundary structure.

We also feel that additional high-resolution measurements on large-angle grain boundaries should be used to reveal the microscopic, more chemical, aspect of disordered carbon structures. Such feedback could be used for revision of the standard molecular dynamics models for equilibriation of 2d carbon, as the one we used in this thesis.

Chapter 3 of this thesis was focused on revealing a puzzling possibility of a non-equilibriated transport situation in graphene, that would result in the violation of a very general Onsager relation on the evenness of magnetoconductance. This proposal should be exciting from several viewpoints. Firstly, a correct modeling of electron dephasing in a mesoscopic model is mainly an *ad hoc* affair, and fraught with caveats. Our predicted effect is a direct possibility to learn more about this subject, and the experiments are near the stage of having all ingredients ready. One can easily find many real life objections to the simple model of graphene device we used in the calculation, and indeed dephasing models explicitly built to respect the Onsager relations exist. However the essence of our arguments are simple lengthscale considerations, and they must extend beyond the level of minimal models like the one we used. The real question becomes what other (non-graphene) physical realizations of the setup could be feasible, especially in regard to a time reversal invariant filter for "valleys" which are connected by time reversal. We imagine an investigation of 3d TIs and their Dirac cones could be fruitful. Another candidate, which shares such valley structure, appears in ultracold atomic gases on a lattice with local rotational driving. This system, including its fermion spectrum, is highly tunable through the external fields, leaving room for constructing elements to our will [257].

The subject of Chapter 2 is the most remote from direct experimental efforts. It however leaves intriguing questions for the future. It is a fundamental question to explore the non-trivial geometric structure carried by the point-like defects. The identification of the electron holonomies we achieved is the first step, and an investigation of the proper extended algebra and braiding rules should be the next. The direct manipulation of topological defects, or their condensation, in a graphene sample does not seem feasible at the moment; but the specific realization of the algebra of defect holonomies might lead to new insights when investigated within the quantum double symmetry group perspective. This theory is tailored for the full consideration of such non-trivial defect group structures. The spinor realization of the defect holonomies adds to their elemental Euclid group based relationship, and we therefore expect novel results.

Concerning the most general problem of spinors and topological defects, some of which we approached in Chapters 2, 4, we feel there is exciting motivation for future research. For instance, the standard high-energy approach to gravitating systems favors simpler models where torsion is an afterthought. The condensed

matter perspective rather emphasizes the equal symmetry roles of translations and rotations and corresponding defects. In view of the deep role that single particle spinor formalism plays in recent adS/CFT developments, we wonder about the possible role of the bulk space full of torsion.

The results of Chapter 5 suggest the way to discern the characteristic features of the elusive Majorana bound states appearing on a 2d TI edge interface. A successful experimental characterization of such interfaces is of course the crucial step towards bringing the Majorana states into the realm of elements in a quantum computer. We are however even more excited about the identified TI edge spin currents sourced by crystal dislocations. Due to the deep bulk—edge correspondence of the topologically ordered insulators, this suggests a more fundamental role of dislocations for the topological order. Earlier considerations of 3d dislocation lines revealed the appearance of zero modes, again suggesting a strong influence against the bulk gap. We envisage a future thorough and complete understanding of the interrelation of the defect and bulk topologies, and speculate that the defect melting of TI crystals might also be the disorder operation of destroying the TI bulk gap.

The second type of systems we investigated in Chapters 6, 7 focused on local electronic ordering in underdoped cuprates. There is an important remark to be made about the relationship of physics considered in these two chapters. The subject of Chapter 6 was the phonon anomaly caused by the electron-phonon coupling, and we made several explanations, and direct experimental predictions, based on a simple but efficient model. The results on the incommensurate nature of the stripe CDW (Chapter 7) are however in some tension with respect to the assumption of commensurate static stripes of cuprate families considered in Chapter 6. The experimentalists should in general be strongly motivated to enhance the static stripes in the bulk of cuprates, to make a connection to the stripes found in surface STM measurements. We believe that the framework of on-stripe electron fluctuations we considered in our electron-phonon coupling model, in addition to existing models of transversal stripe fluctuations, enables one to identify the nature of stripes in the bulk by looking at the fate of the phonon anomaly. This would provide a direct comparison between the nature of the bulk and surface stripe orders. At the same time of course, the measurement of the anomaly position in momentum space can reveal just how longitudinal or transversal the fluctuations of stripes are.

Our results on the fluctuating stripes of STM have left us unexpected directions for further research. In Chapter 7 we have seen that the nematic order seems in first instance unrelated to the smectic (stripe) disorder. This observation is crucial for deciphering the liquid crystalline picture of the strongly correlated electron system of cuprates. We however believe that the intra-unit-cell nematic could still represent the liquid crystalline nematic phase. The argument is that generally a nematic phase must be a homogeneous liquid breaking the orientational symmetry; the intra-unit-cell order parameter, defined via the lattice Bragg peaks, is translationally periodic in the square lattice, which can be

regarded as a version of liquid constrained within the network of Cu—O—Cu bonds. The orientational symmetry breaking then comes from within the unit-cell. One could make the observation that a Pomeranchuk instability in a Fermi liquid is also a homogeneous nematic phase, although it does not have an intra-unit-cell structure. Identifying the latter type of ordering in the STM data has not been considered yet.

The completely new open questions in our opinion just add to the thrill of the subject. In the broad perspective, our results should both encourage further experiments on a wider set of compound families, and flare up the prospects of an overall liquid crystalline description of cuprates. The presented framework suggests and allows for a unified analysis of different cuprate families, as they cover various regimes of short/long range ordering in the two order parameters.

Another relevant chain of thought about Chapter 7 concerns the merit of the Fourier analysis of the STM data. Full credit for the introduction of this technique goes to the work in [106]. One can worry about the strength of such a linear filtering approach in the context of strongly disordered, lattice spacing sized, features. For instance, it is impossible to extract the $1/4a$ Fourier peak information from the background noise. The $3/4a$ peak extraction used here therefore leads to identification of just some specific stripe dislocations, where a $4a$ wide stripe CDW looses one of its three crests. We expect that the future application of developed direct local stripe order parameters might lead to some enrichment of the analysis, in terms of potentially detecting additional defect types.