Dirac and Majorana edge states in graphene and topological superconductors

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Summary

The rising sun creates a narrow strip of light at the horizon, separating the earth from the sky — regardless of the profile of the horizon. Similarly, the electronic states studied in this thesis appear at the edge of a material not because of a particular structure of the edge, but because of a fundamental difference between the material inside and empty space outside.

The first part of the thesis is dedicated to edge states in graphene, a single sheet of graphite with carbon atoms on a honeycomb lattice. It was originally thought that only a particular type of boundary, the zigzag edge, has states confined to the edge. However the analysis of chapter two shows that, instead, edge states appear generically whenever the honeycomb lattice is terminated. The key ingredient of this analysis is the formulation of the most general boundary condition of the Dirac equation (which was unknown, although the Dirac equation entered physics early in the 20th century).

The remainder of the first part addresses observable consequences of edge states. In the third chapter we show how to use a combination of superconductivity and strong magnetic fields to probe the elusive valley polarization of an edge state. (The band structure of graphene has two valleys, related by time reversal symmetry.) Next, in chapter four, we present an analytical theory of the current switching effect (known as the "valley valve"), which had been discovered in computer simulations. This analysis revealed an unexpected difference between zigzag and anti-zigzag nanoribbons, which cannot be described by the Dirac equation. Finally in chapter five, we propose a method for direct detection of edge states in a graphene quantum dot, including also the perturbing effects of disorder and next-nearest-neighbor hopping.

In the second part of the thesis the focus shifts from edge states in graphene to Majorana edge states in superconductors. Both types of edge states appear from fundamental considerations, regardless of the microscopic properties of the edge. The special property of Majorana particles is that they are their own antiparticles, and consequently are chargeless and spinless. Majorana edge states are predicted to appear in superconductors with an unusual "chiral *p*-wave" pairing symmetry. Conventional *s*-wave superconductors can be used as well, in combination with materials having strong spin-orbit coupling (topological insulators). Majorana particles are in demand because they are predicted to have very long coherence times. Our goal in this part of the thesis is to identify experimental signatures of Majorana edge states, as well as to investigate their potential for quantum computation.

In the sixth chapter we analyze the reasons for the protection against decoherence of Majorana particles and show that the key principle is conservation of particle number parity. Contrary to concerns raised in the literature, thermal excitations do not lead to decoherence. This is of crucial importance, since suppression of thermal excitations would require unrealistically low temperatures of 10^{-4} K. In the following two chapters we propose methods for detection of Majorana edge states. Since they are charge neutral, the central problem is how to couple them to an electrical current. In chapter seven we show that a Cooper pair is split into two electrons when it is injected through a pair of Majorana states. In the next chapter we propose a "Dirac-to-Majorana" converter, which reversibly transforms an electrical current carried by ordinary (Dirac) electrons into a neutral current carried by Majorana edge states. (The charge deficit is absorbed by the superconducting condensate.) This idea was independently proposed by Liang Fu and Charles Kane, and appeared in an episode of the American sitcom *The Big Bang Theory*.

In chapter nine we apply these ideas to chiral p-wave superconductors. We predict that if two domains of opposite chirality are brought into close contact, their joint edge starts conducting electrical current even though a single edge carries only a charge-neutral current. In chapter ten we study the topological phase transition into a phase which supports Majorana states. We find that the transition point in a wire geometry is signaled by a peak of quantized thermal conductance and quantized electrical shot noise — without any of the finite size effects that usually accompany a phase transition. In chapters eleven and twelve we use a formal correspondence with the Ising model in a transverse field to analyze the non-Abelian statistics of Majorana particles. (These are the technically most involved chapters of the thesis.) Finally, in chapter thirteen we present a new scheme for quantum computation with Majorana particles, based on the Aharonov-Casher effect (the dual of the more familiar Aharonov-Bohm effect). The advantage of this new scheme over earlier proposals is that it is insensitive to thermal excitations (in accord with the findings of chapter six).