Random-matrix theory and stroboscopic models of topological insulators and superconductors

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Date: 2012-11-21

Summary

Topological phases of matter are exceptional because they do not arise due to a symmetry breaking mechanism. Instead they are characterized by topological invariants – integer numbers that are insensitive to small perturbations of the Hamiltonian. As a consequence they support conducting surface states that are protected against disorder and other imperfections. Furthermore, a variety of unusual transport properties arise due to the presence of topology. In this work the interplay between topology and sample imperfections is investigated with a focus on transport phenomena.

The first part of the thesis treats superconducting systems, where topology leads to boundary states that are protected by a superconducting gap. Among the topological superconductors, one-dimensional systems are of particular interest. They can be realized in semiconductor - superconductor heterostructures, and support Majorana bound states at their ends – zero energy particles that are their own anti-particles. These Majorana states are regarded as promising candidates for the realization of a topological quantum memory.

The generic transport properties of superconducting systems and their dependence on symmetries can be studied analytically using random scattering matrices. To this end, the superconducting circular ensembles of random-matrix theory are studied in chapter two. In the third chapter we generalize these known ensembles by distinguishing between phases of different topology. This enables us to determine the influence of topology on the superconducting transport properties. In particular the signatures of Majorana fermions in the Andreev conductance of disordered normal-superconductor (NS) junctions are analyzed.

The fourth chapter proposes a setup for the unambiguous detection of topological superconductivity: a quantum point contact attached to

a superconducting wire. The signature of the topological phase is a quantized conductance in the single-channel limit. In contrast, the conductance is forced to be zero for a trivial superconductor. The advantage of this setup over the usual tunnel contact setup is a strongly reduced sensitivity to finite voltage or temperature.

In the fifth chapter we move over to nodal superconductors, introducing a scattering formulation for their topology. Although they are gapless, a variety of lower dimensional topological invariants can be defined that, as we show, have strong impact on the transport properties of these systems.

In the second half of the thesis the focus shifts from topological superconductors to topological insulators. The first two chapters of this part are concerned with the localization properties of electrons close to topological phase transitions. In two-dimensional systems without time reversal symmetry, disorder forces the electronic wave functions to localize – a phenomenon called Anderson localization. At the topological phase transition the localization length of the wave function diverges with a universal critical exponent.

In chapter six we introduce a new approach to investigate this delocalization behavior in the quantum Hall universality class, using a stroboscopic model. This method is computationally efficient and makes it possible to study higher-dimensional systems in one spatial dimension. In chapter seven we extend our description to the quantum spin Hall universality class (two-dimensional systems with time-reversal symmetry). Its phase diagram is different from that of the quantum Hall effect, because the phase transition happens via a metallic phase. For both universality classes we calculate the critical exponent numerically.

In the final chapter of the thesis we go over to three-dimensional topological insulators, which are characterized by a topologically protected metallic surface. Since electrons moving on this surface are constrained to follow its geometry, they effectively live in a curved space and are thus subject to geodesic scattering. This leads to a novel contribution to the resistance of the surface, caused by surface roughness.