Random-matrix theory and stroboscopic models of topological insulators and superconductors

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Cover: *Topological invariant of a nodal Rashba superconductor in contact with a metal, as a function of interface orientation and momentum. Compare with the left panel of Fig. 5.4.*

To Nina and my parents.

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