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Light with a twist : ray aspects in singular wave and quantum optics

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Light with a Twist

Ray Aspects in Singular Wave and Quantum Optics

Steven J. M. Habraken

The pictures on the front cover respectively show the phase and the intensity patterns of one of the long-lived modes of a rotating two-mirror cavity with general astigmatism. The background shows a detail of original handwritten notes.

Light with a Twist

Ray Aspects in Singular Wave and Quantum Optics

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in 1980

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aan mijn ouders
en aan Astrid

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