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Light with a twist : ray aspects in singular wave and quantum optics

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Light with a Twist

Ray Aspects in Singular Wave and Quantum Optics

Steven J. M. Habraken

The pictures on the front cover respectively show the phase and the intensity patterns of one of the long-lived modes of a rotating two-mirror cavity with general astigmatism. The background shows a detail of original handwritten notes.

Light with a Twist

Ray Aspects in Singular Wave and Quantum Optics

PROEFSCHRIFT

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van Rector Magnificus prof. mr. P. F. van der Heijden,
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Steven Johannes Martinus Habraken

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in 1980

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The poem “Love Itself” by Leonard Cohen is reprinted with kind permission of HarperCollins Publishers, New York (USA).

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*aan mijn ouders
en aan Astrid*

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