Spatial Coherence and Entanglement of Light

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1 Introduction

1.1 Light

It is quite difficult to introduce the notion of *light* avoiding any philosophical detour. Most of the time, physicists don't even bother to say what the light *is*; they are mainly interested in describing how it behaves. Quantum mechanics provides a very good description of the behavior of light (and matter) in most of its details and, in particular, on an atomic scale. However, as one of the fathers of quantum electrodynamics, Richard Feynman, once expressed [1]:

> *At the quantum level things do not behave like waves, they do not behave like particles, they do not behave like clouds, or billiard balls, or weights on springs, or like anything that you have ever seen. Newton thought that light was made up of particles, but then it was discovered that it behaves like a wave. Later, however (in the beginning of the twentieth century), it was found that light did indeed sometimes behave like a particle. Historically, the electron, for example, was thought to behave like a particle, and then it was found that in many respects it behaved like a wave. So it really behaves like neither. Now we have given up. We say : "It is like neither."*

The behavior of the fundamental particles of light*, known as *photons*, can be quite accurately described by a wave function. Although the photon is neither a particle nor a wave, within the paradigm of the wave-particle duality one is tempted to think that the photon will exhibit either a wave-like *or* a particle-like behavior. An evidence of wave-like properties is the observation of interference fringes in a Young double slit experiment. These fringes are, however, a signature of the *coherence* of light. By modifying this property one can also make the fringes disappear. In this case, the result of a Young's experiment would be very similar

* We refer to any electromagnetic radiation, not only visible light.

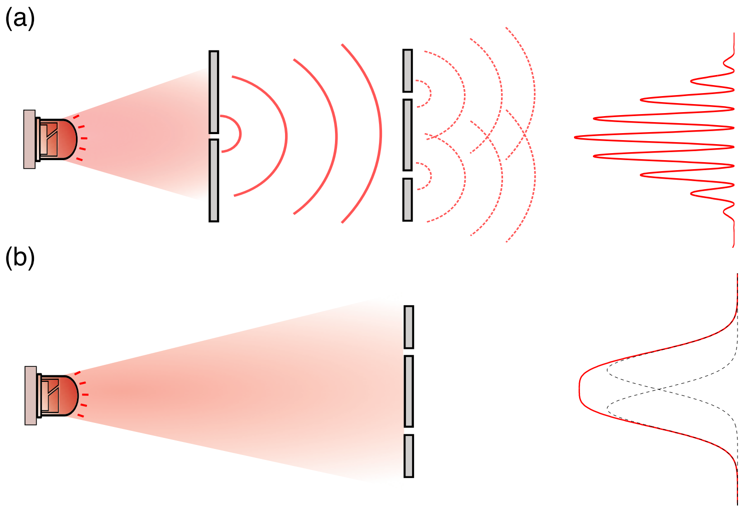

Figure 1.1: (a) Young's double-slit experiment. A monochromatic and incoherent light source is first filtered by a narrow slit and is then transmitted through two slits. An interference pattern is observed on a screen behind the slits; this is a signature of the coherence of the incident field. (b) The first aperture is removed and the interference pattern disappears. Since the incident field is now incoherent, the measured pattern is just the sum of the intensities transmitted by each aperture individually.

to what is expected from a stream of particles. Coherence is thus also one of the most fundamental concepts in both classical and quantum optics. With this notion in mind, let's revisit Young's experiment.

1.2 Coherence in optics

Coherence is the property of waves that allow them to interfere. For an ideal sinusoidal wave, if we know the amplitude and phase at a certain point in space, we know how this wave will be oscillating in the entire space. In other words, the swing of the wave at one point is perfectly correlated with all other points.

Figure 1.1(a) shows a schematic realization of the double-slit experiment. First, a thin slit is used to filter a light beam, producing to a good approximation a coherent *wave**. The light is then transmitted through a plate with two narrow slits and observed at a screen behind the slits. Since the oscillations at the upper and lower slits are perfectly correlated, these two sources will produce a high-contrast interference pattern. By removing the first plate, however, the interference pattern will disappear. As shown in Fig. 1.1(b), the result is now just the sum of the intensities transmitted through each aperture individually. The waves transmit-

* We are describing the experiment in a classical language.

ted through both apertures don't "feel" each other and no interference occurs. In this situation we say that the two transmitted beams are *incoherent* with respect to each other. The relative phase of the oscillations at the two slits is completely random.

We have discussed the two extreme examples of complete coherence or complete incoherence. However, by changing the width of the first slit, one can continuously change from the situation in Fig. 1.1(a) to the situation in Fig. 1.1(b). The intermediate case corresponds to a *partially coherent* field. The field has both wave-like and particle-like behavior at the same time. Curiously, all these remarks remain true even at the single photon level. The patterns shown in Figs. 1.1(a) and (b) are then interpreted as the probability distribution of measuring the photon at a certain position.

The subject of coherence is extremely broad and rich. Entire books have been written on diverse aspects of coherence in different domains, such as space, time, frequency or polarization [2–4]. In this thesis we present an extensive study of spatial coherence of light, in particular for a field containing two photons, in which the quantum features become even more prominent. We begin by introducing some of the basic mathematical tools necessary to understand one-photon and two-photon spatial coherence.

1.2.1 Second-order coherence: classical and quantum description

The ingenuity behind Young's double slit experiment is that it allows the field at two different space-time coordinates to be superposed before measuring the combined intensity. In Fig. 1.1, the field at the position of the upper slit is diffracted and superposed with the field at the position of the lower slit. Other optical experiments can also be designed to allow such superposition, like a Michelson or a Mach-Zehnder interferometer. In general, when the field $E^{(+)}(\mathbf{r}_1 t_1)$ at position $\mathbf{r_1}$ and time t_1 is superposed with $E^{(+)}(\mathbf{r}_2 t_2)$, the resulting intensity will be

$$\begin{aligned} I &= \left\langle \left| E^{(+)}(\mathbf{r}_1 t_1) + E^{(+)}(\mathbf{r}_2 t_2) \right|^2 \right\rangle , \\ &= I_1 + I_2 + 2\mathrm{Re}\left[\left\langle E^{(-)}(\mathbf{r}_1 t_1) E^{(+)}(\mathbf{r}_2 t_2) \right\rangle \right] , \end{aligned} \tag{1.1}$$

with the understanding that $E^{(+)}$ is the positive-frequency component of the complex representation of the electric field. The first two terms are the contributions of the intensities of each field individually, whereas the last term represent the interference effect of the superposed fields*. The brackets $\langle \ldots \rangle$ denote time averaging†, as the measurement time is usually much longer than the period of

* To be more precise, the fields should be physically superposed at the same point $\mathbf{r}t$. The interference term can then be written as $\left\langle E_1^{(-)}(\mathbf{r}t) E_2^{(+)}(\mathbf{r}t) \right\rangle$, where E_i is the transformation or propagation of the field at point $\mathbf{r}_i t_i$ to the observation coordinate $\mathbf{r}t$.

† We assume a stationary stochastic process. In this situation, time averaging equals *ensemble* averaging.

the oscillation.

The coherence properties of the field is thus completely described by the correlation function

$$\Gamma(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = \left\langle E^{(-)}(\mathbf{r}_1 t_1) E^{(+)}(\mathbf{r}_2 t_2) \right\rangle . \tag{1.2}$$

The field is said to be completely coherent if the correlation function factorizes in the form

$$\Gamma(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = \mathcal{E}^*(\mathbf{r}_1 t_1) \mathcal{E}(\mathbf{r}_2 t_2). \tag{1.3}$$

This implies that there is a fixed phase relationship between $\mathcal{E}(\mathbf{r}_1 t_1)$ and $\mathcal{E}(\mathbf{r}_2 t_2)$ and the contrast of the interference fringes will be maximum. This is precisely the definition of coherence that optics has traditionally used.

In the quantum theory, measurable quantities, like the electric field, are no longer associated with a complex number $E^{(+)}$, but with an operator $\hat{E}^{(+)}$. The state of the system is represented by a vector or ket $|\ \rangle$. The electric field operator $\hat{E}^{(+)}$ is an annihilation operator in the sense that it lowers the number of quanta present in the field by one. Likewise, the Hermitian conjugate $\hat{E}^{(-)} = [\hat{E}^{(+)}]^\dagger$ raises the number of quanta by one. If the field is in the pure state $|\psi\rangle$, the second-order coherence* in quantum language is defined by

$$\Gamma(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = \langle \psi | \, \hat{E}^{(-)}(\mathbf{r}_1 t_1) \hat{E}^{(+)}(\mathbf{r}_2 t_2) \, | \psi \rangle . \tag{1.4}$$

However, most light sources don't produce pure states. We should then consider the state $|\psi\rangle$ as depending on some random and uncontrollable parameters of the source, for instance, the fluctuating relative phases between the fields at the two slits in Fig. 1.1(b). Partially-coherent fields in classical optics are represented by mixed states in the quantum language and are described by the density operator

$$\varrho = \{ |\psi\rangle \langle \psi| \}_{\text{av}} , \tag{1.5}$$

where we consider a statistical average over the fluctuating parameters. The most general quantum-theoretical form of the correlation function is

$$\begin{aligned} \Gamma(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) &= \left\{ \langle \psi | \, \hat{E}^{(-)}(\mathbf{r}_1 t_1) \hat{E}^{(+)}(\mathbf{r}_2 t_2) \, | \psi \rangle \right\}_{\text{av}} , \\ &= \text{Trace} \left\{ \varrho \hat{E}^{(-)}(\mathbf{r}_1 t_1) \hat{E}^{(+)}(\mathbf{r}_2 t_2) \right\} . \end{aligned} \tag{1.6}$$

For stationary fields, the correlation function depends per definition only on the time difference $\tau = t_1 - t_2$. For many applications, especially when monochromatic fields are involved, it is advantageous to work in the space-frequency domain. In this domain, one defines the *cross-spectral density* W as the Fourier

* Second order on the electric field operators.

transform of the coherence function

$$W(\mathbf{r}_1, \mathbf{r}_2; \omega) = \int_{-\infty}^{+\infty} \Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) e^{i\omega\tau}. \tag{1.7}$$

Although the quantum treatment of coherence is generally carried out in the space-time domain, recent studies have provided a more exact formulation in the space-frequency domain. This quantum description can be made more comprehensible with the following notation. Let's consider a single photon with frequency ω that can be in one of a set of orthogonal states $\{|\psi_i\rangle\}$ with probabilities $\{P_i\}$. The density operator of this photon is then*

$$\varrho = \sum_i P_i \left|\psi_i\right\rangle \left\langle\psi_i\right|. \tag{1.8}$$

The cross-spectral density is the spatial-coordinate representation of the density operator (Eq. 4.7-61 of Ref. [2]),

$$W(\mathbf{r}_1, \mathbf{r}_2; \omega) = \langle \mathbf{r}_1 | \, \varrho \, | \mathbf{r}_2 \rangle = \sum_i P_i(\omega) \; \psi^*(\mathbf{r}_1, \omega) \psi(\mathbf{r}_2, \omega). \tag{1.9}$$

This decomposition is known classically as the *coherent-mode representation*. It represents the field generated by the source as a linear combination of completely coherent fields; each of them can be found with probability P_i. The degree of coherence of the field is related to the number of terms in this modal decomposition. A completely coherent field has just one mode, whereas a completely incoherent field has infinite terms.

The description presented here is sufficient to understand earlier experiments on spatial coherence, in which single detectors were used to measure optical intensities or counting rates. In the 1950's, however, new experiments were developed that involved intensities or counts correlations between *two* detectors [5–7]. A more general theoretical approach was then necessary to explain, for instance, unexpected results on the correlations in the arrival times of photons. Such a generalization was introduced by Glauber, in his prestigious paper "quantum theory of optical coherence" [8]. In the next section we introduce the next higher-order correlation function.

1.2.2 Fourth-order coherence and the two-photon field

In order to elucidate coherence phenomena when correlations between multiple detectors are involved, Glauber defined higher order correlation functions $\Gamma^{(2N)}$. The previous section discussed the case $N = 1$ of one detector. When two detec-

* By a proper change of basis, the density matrix can always be written in a diagonal form.

tors are involved, the relevant function is

$$\Gamma^{(4)}(\mathbf{r}_1t_1, \mathbf{r}_2t_2, \mathbf{r}_3t_3, \mathbf{r}_4t_4) = \text{Trace}\left\{\varrho\hat{E}^{(-)}(\mathbf{r}_1t_1)\hat{E}^{(-)}(\mathbf{r}_2t_2)\hat{E}^{(+)}(\mathbf{r}_3t_3)\hat{E}^{(+)}(\mathbf{r}_4t_4)\right\}, \tag{1.10}$$

which is known as the fourth-order correlation function.

When only one detector is used, the measured light intensity is $I(\mathbf{r}t) \propto \Gamma^{(2)}(\mathbf{r}t, \mathbf{r}t)$. Similarly, when two detectors are used, the coincidences rate is given by $R_c(\mathbf{r}_1t_1, \mathbf{r}_2t_2) \propto \Gamma^{(4)}(\mathbf{r}_1t_1, \mathbf{r}_2t_2, \mathbf{r}_2t_2, \mathbf{r}_1t_1)$. Let's now focus our attention on a light field containing exactly two photons that are in a pure state $|\psi\rangle$. Since each of the electric field operators $\hat{E}^{(+)}$ "annihilates" one photon from the field, the coincidence rate is [9–11]

$$\begin{aligned} R_c(\mathbf{r}_1t_1, \mathbf{r}_2t_2) &= \langle\psi|\,\hat{E}^{(-)}(\mathbf{r}_1t_1)\hat{E}^{(-)}(\mathbf{r}_2t_2)\hat{E}^{(+)}(\mathbf{r}_2t_2)\hat{E}^{(+)}(\mathbf{r}_1t_1)\,|\psi\rangle\,, \\ &= \left|\langle 0|\,\hat{E}^{(+)}(\mathbf{r}_2t_2)\hat{E}^{(+)}(\mathbf{r}_1t_1)\,|\psi\rangle\right|^2, \\ &= |\mathcal{A}(\mathbf{r}_1t_1, \mathbf{r}_2t_2)|^2\,. \end{aligned} \tag{1.11}$$

The coincidence rate can thus be written as the modulus squared of a field $\mathcal{A}(\mathbf{r}_1t_1, \mathbf{r}_2t_2)$, which is known as the two-photon field. It gives the probability amplitude of detecting one photon at position $\mathbf{r}_1$ and time t_1 and the other photon at $\mathbf{r}_2t_2$. Similarly to the cross-spectral density, one can also write the two-photon field in the frequency domain, $A(\mathbf{r}_1\omega_1, \mathbf{r}_2\omega_2)$, as the double Fourier transform of $\mathcal{A}(\mathbf{r}_1t_1, \mathbf{r}_2t_2)$. Much of this thesis is dedicated to investigate many of the very intriguing properties of the two-photon field, especially when both photons have the same frequency ω, i.e., when they are frequency degenerate. This can be achieved experimentally by using narrow-band frequency filters. Under these conditions, the frequency dependence of the field is trivial, being determined by the filters only and will be omitted from the description from now on*.

The function $A(\mathbf{r}_1, \mathbf{r}_2)$ is the spatial-coordinate representation of the state $|\psi\rangle$ and, like W, it can also be represented in a natural set of biorthogonal mode pairs as [12,13]

$$A(\mathbf{r}_1, \mathbf{r}_2) = \sum_i \sqrt{\lambda_i} f_i(\mathbf{r}_1) g_i(\mathbf{r}_2), \tag{1.12}$$

where f_i and g_i are the eigenstates and λ_i the respective eigenvalues. This representation is known as the Schmidt decomposition and it is closely related to the concept of *entanglement*.

Entanglement is an extraordinary quantum property that allows two or more particles (or degrees of freedom) to be strongly correlated. These correlations cannot be explained by any classical (local) model. Consider Eq. (1.12), for instance, which describes spatial entanglement between two photons. If we determine that one photon is in the state $|f_i\rangle$, we know for sure that the other photon will be in

* The coincidences rate is given by a convolution of the two-photon field with the transmission functions of the filters, $R_c = \int |A(\mathbf{r}_1\omega_1, \mathbf{r}_2\omega_2)|^2 T_1(\omega_1)T_2(\omega_2)\mathrm{d}\omega_1\mathrm{d}\omega_2$. We can only write $R_c \propto |A|^2$ when the filters are sufficiently narrow-banded.

the state $|g_i\rangle$, even if these photons are separated by great distances. This property alone does not yet characterize spatial entanglement, as such correlations could in principle be classical*. However, when measurements are made in different bases, the strong correlations persist. This persistence of correlations has no classical analogous. Entanglement is an old concept in quantum mechanics that has long challenged our understanding of nature, as it violates the philosophical principles of realism and locality. More recently, however, physicists have recognized that entanglement is also an important *resource*, with various applications in the now established field of quantum information. The amount of this resource present in the state (1.12) is related to the number of terms in the decomposition. A very common measurement of the effective number of entangled modes is the Schmidt number, defined by

$$K = \frac{1}{\sum_i \lambda_i^2}. \tag{1.13}$$

In the next chapters we will see how this number can be measured and manipulated.

There is certainly much more to be told about coherence, but the concepts introduced so far should be sufficient and indispensable in order to follow the remaining of this thesis. I wish you a pleasant reading!

1.3 Research topics in this thesis

In this thesis we investigate diverse aspects of spatial coherence of light. Nonclassical fields containing two photons can be generated by a nonlinear optical process known as spontaneous parametric down conversion (SPDC), which will be described in more details in the coming chapters. Among the questions we consider are: What is so special about spatial entanglement? How is it revealed in the fourth-order correlations? What are the differences between a highly entangled and a classically correlated state? How can the number of modes be manipulated and measured? For a two-photon system, we measure both intensities and two-photon correlations. Therefore both second-order and fourth-order coherence are relevant. To get deeper insights into how coherence affects interference, we also investigate completely classical sources. The chapters are organized as follows:

- **Chapters 2 and 3** investigate the spatial properties of the two-photon field. Contrary to the far-field (i.e. momentum) properties, which are widely known, the near-field correlations in the two-photon field have hardly been studied. We find extremely rich structures and many interesting parallels with other fields of optics. Chapter 2 presents a short overview of the experiment and the most important results, while Chapter 3 provides a more

* i.e., the mixed state $\varrho = \sum_i \lambda_i^2 \, |f_i\rangle \langle f_i| \otimes |g_i\rangle \langle g_i|$, which is completely classical, also exhibits the correlations just described.

complete theoretical description and extensive discussions on the consequences of the near-field structures.

- **Chapter 4** describes how the two-photon field is affected when the pump laser that generates entangled photons is strongly focused. We generate a state that is almost separable (i.e., non entangled) and investigate intensities and two-photon correlations. We also propose a semi-classical model of SPDC that explains the classical measurements made with a CCD.

- **Chapter 5** shows how the full dimensionality of the spatial entanglement can be manipulated and measured. We exploit a very interesting connection between second-order and fourth-order coherence in order to provide the first operational definition of the Schmidt number.

- **Chapter 6** investigates entanglement in orbital angular momentum (OAM) of light. Similar to the Schmidt decomposition introduced in Sec. 1.2.2, entanglement in OAM implies, in our geometry, that if one photon has an orbital angular momentum $\ell\hbar$, the other photon will have $-\ell\hbar$. We implement an interferometric method that allows the full probability distribution P_ℓ of finding $(\ell, -\ell)$ pairs to be measured.

- **Chapter 7** studies the orbital angular momentum spectrum of partially coherent light. Although most partially coherent fields do not carry an overall OAM, the statistical nature of the field implies that there is still a probability that the photon will have an angular momentum $\ell\hbar$. We show how the interferometric method and the theoretical framework of Chapter 6 can be used to investigate this OAM modal decomposition of partially coherent light. Contrary to the previous chapters, which focus on the quantum properties, this Chapter investigates completely classical beams.

- **Chapter 8** studies partially coherent classical light *with* an overall OAM. In particular, we address the question: "How does a spiral phase plate affect a partially coherent field?" Spiral phase plates are usually employed to transform a completely coherent beam into an approximate Laguerre-Gauss mode carrying OAM. When a partially coherent beam is used instead, the effect of the phase plate is shown to be much less visible in the intensity and much more dramatic in the coherence function, which now acquires a ring singularity.

- **Chapter 9** investigates the statistical properties of non-local speckle patterns that are obtained when entangled light is scattered through a random medium. In this Chapter, the connection between second-order coherence and fourth-order coherence is very prominent, as the statistics measured by a single detector and by two detectors are deeply linked. The differences between an entangled state and a separable state are very appealing. Finally, we use the statistics of the observed speckles as an alternative method to measure the Schmidt number. In this Chapter, many of the concepts discussed in the previous chapters are put together in a single context.