A fixed point approach towards stability of delay differential equations with applications to neural networks

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Propositions belonging to the thesis

A fixed point approach towards stability of delay differential equations with applications to neural networks

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1. Consider a delay differential equation

$$x''(t) + ax'(t) + bx(t-r) + cx(t) = 0,$$

where $a > 0$, $b > 0$, $b + c > 0$ and $r > 0$. If

$$br\left(1 + \int_0^t |Ae^{A(t-s)}|\, ds\right) < 1,$$

where $A = \begin{pmatrix} 0 & 1 \\ -b & -a \end{pmatrix}$, then every solution of the delay differential equation and its derivative tend to 0 as $t \to \infty$.

2. Consider constants $c > 0$, $\lambda_1 > 0, \lambda_2 > 0, \tau_0 > 0, y_0 \in \mathbb{R}$ and continuous functions $y : [-\tau_0, \infty) \to [0, \infty)$ and $\tau : [0, \infty) \to [0, \infty)$ with $s - \tau(s) \geq -\tau_0$ for all s. If $\lambda_1 + \lambda_2 < c$ and the following inequality holds,

$$y(t) \leq \begin{cases} y_0 e^{-ct} + \lambda_1 \int_0^t e^{-c(t-s)} y(s)\, ds + \lambda_2 \int_0^t e^{-c(t-s)} y(s - \tau(s))\, ds, & t \geq 0, \\ y_0 e^{-ct}, & t \in [-\tau_0, 0], \end{cases}$$

then we have $y(t) \leq y_0 e^{-\gamma t}$ $(t \geq -\tau_0)$, where γ is a positive root of the transcendental equation $\frac{1}{c-\gamma}\left(\lambda_1 + e^{\gamma\tau_0}\lambda_2\right) = 1$.

3. Consider a system of delayed cellular neural networks given by

$$\frac{dx_i(t)}{dt} = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t - \tau(t)))$$

for $i = 1, 2, 3, \cdots, n$, where $\tau(t)$ is a bounded continuous function, $c_i > 0$, $a_{ij}, b_{ij} \in \mathbb{R}$. Suppose that the functions f_j and g_j satisfy Lipschitz conditions with Lipschitz constants α_j and β_j, respectively. Further, assume that $f_j(0) = g_j(0) = 0$ for $j = 1, 2, \cdots, n$. If

$$\sum_{i=1}^{n} \frac{1}{c_i} \max_{j=1,2,\cdots,n} |a_{ij}\alpha_j| + \sum_{i=1}^{n} \frac{1}{c_i} \max_{j=1,2,\cdots,n} |b_{ij}\beta_j| < 1,$$

then the trivial solution of the system is exponentially stable.

4. Let X be a complex Banach space. A closed densely defined unbounded operator A in X is *discrete* if there is a complex number z in its resolvent set $\rho(A)$ for which the resolvent $R(z, A)$ is compact in X. Let A be a discrete operator with spectrum $\sigma(A) = \{\lambda_n\}_{n=1}^{\infty}$ in X such that the range of A is dense in X, and $z \mapsto R(z, A)$ is a meromorphic function of finite order $\rho > 0$. Assume that there exist k angles $\theta_1, \theta_2, \cdots, \theta_k$ and $\alpha \geq 1$ such that $(\theta_{j+1} - \theta_j) \ \ mod \ \ 2\pi \leq \frac{\pi}{\alpha}$ for all j, and there exists an integer m and constant M such that $\|R(z, A)\| \leq M|z|^m$ for $z \in ray(\theta_j)$, $j = 1, 2, \cdots, k$. If $\alpha > \rho$, then the span of eigenvectors and generalized eigenvectors of A is complete.

5. The concept of *generalized characteristic equation* for nonautonomous delay differential equations can not be applied as generally as the concept of *characteristic equation* for autonomous delay differential equations.

6. The fixed point method is effective in establishing stability results for delay differential equations, stochastic delay differential equations, and stochastic delayed neural networks. The scope of this method is far from exhausted.

7. Different choices of norms may yield different stability results for the same delay differential equation. This is an interesting feature of the fixed point method, especially for stochastic delay differential equations.

8. Research is like life: the more you try, the more you will receive.

9. Studying abroad is a wonderfully enriching experience of different culture.

10. Mathematics is like love that is worth of your hard striving.