A fixed point approach towards stability of delay differential equations with applications to neural networks

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Summary

Asymptotic behavior and stability of solutions of delay differential equations play an important role in the qualitative analysis of delay differential equations. This thesis studies asymptotic behavior and stability of determinsitic and stochastic delay differential equations.

The approach used in this thesis is based on fixed point theory, which does not resort to any Liapunov function or Liapunov functional. This approach relies mainly on three principles: an elementary variation of parameters formula, a complete metric space and a constraction mapping principle. The benefit of this approach is that the fixed point arguments can yield existence, uniqueness and stability of a system in one step. The main difficulty of this approach is to define a suitable complete metric space and a suitable mapping. Different choices of norms can be considered on defined spaces. The norms we choose should be such that the space under consideration is complete and the equation yields a contraction with respect to the norm.

The main contribution of this thesis is to study the approach using fixed point theory in a systematic way and to unify recent results in the literature by considering some general classes of equations. The equation we considered is a combination of time dependent delays, distributed delays, impulses and stochastic perturbations. In addition, an application to stochastic delayed neural networks is investigated. The results in this thesis extend and improve some exist results in the literature in some ways. Examples are discussed in each chapter to illustrate our main results. Chapter 2 presents three methods concerning asymptotic behavior of autonomous neutral delay differential equations. More specifically, we address the relations of the spectral method and the ODE method by considering a class of second order delay differential equations. For a case when there are no neutral terms to the considered equations, we illustrate a third method, fixed point method. Chapter 3 focuses on asymptotic behavior of a class of nonautonomous neutral delay differential equations. Chapter 4 addresses a fixed point approach to stability of deterministic delay differential equations and Chapter 5 discusses the stability of two classes of neutral stochastic delay differential equations with impulses. Chapter 6 studies stability properties of a class of stochastic delayed neural networks.