

Cover Page



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Author: Fortes, Wagner Rodrigues

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Propositions

accompanying the thesis

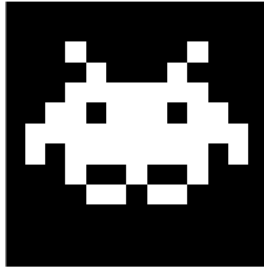
Error Bounds for Discrete Tomography

by Wagner Rodrigues Fortes

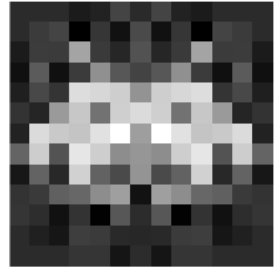
Consider the space invader image of size 13×13 pixels shown in Fig. 1(a). For $1 \leq i, j \leq 13$, the pixel (i, j) of this image is in the i -th column (left to right) and j -th row (bottom to top) and has value either 0 (black) or 1 (white). Let $(a, b) \in \mathbb{Z}^2 \setminus \{(0, 0)\}$, with $a \geq 0$ and a, b coprime. A *projection* of the image in the direction (a, b) is formed by considering the set of parallel lines of the form $ax - by = t$ ($t \in \mathbb{Z}$) through the center of one or more pixels (i, j) , and summing the values of the points on each line. Define the *invader reconstruction problem* from 6 and 8 projections as the problem of finding a binary image that has the same projections as the image in Fig. 1(a) for the sets of directions $D_6 = \{(1, 0), (0, 1), (1, 1), (1, -1), (1, 2) \text{ and } (1, -2)\}$ and $D_8 = D_6 \cup \{(2, 1), (2, -1)\}$, respectively. Both problems can be modeled as a system of linear equations, which we denote by $W^{(6)}x = p^{(6)}$ and $W^{(8)}x = p^{(8)}$, respectively, where $x \in \{0, 1\}^{169}$ is the unknown binary image.



(a) Ground-truth image



(b) A binary reconstruction



(c) Minimum norm solution

Figure 1: Images for propositions 1, 2, 3 and 4.

1. Let $\bar{x}, \bar{y} \in \{x \in \{0, 1\}^{169} : W^{(6)}x = p^{(6)}\}$. Then $\|\bar{x} - \bar{y}\|_1 \leq 6$. (Chapter 2)
2. Let $\bar{x} \in \{x \in \{0, 1\}^{169} : W^{(6)}x = p^{(6)}\}$. Define $\bar{r} \in \{0, 1\}^{169}$ (shown in Fig. 1(b)) by rounding the minimum norm solution of the system $W^{(6)}x = p^{(6)}$ over the real values (shown in Fig. 1(c)) to binary values. Then $\|\bar{x} - \bar{r}\|_1 \leq 3$. (Chapter 2)
3. In the image of Fig. 1(a), the two "eyes" of the *space invader* are the black pixels located at the points (5,8) and (9,8). The reconstruction problem $W^{(6)}x = p^{(6)}$ has no binary solution with two "eyes" in the positions (6,8) and (8,8). (Chapter 8)

4. The reconstruction problem $W^{(8)}x = p^{(8)}$ has a unique binary solution, which is displayed in Fig. 1(a). (*Chapter 2*)
5. Consider a projection matrix W that models a tomographic imaging setup, i.e., the projection data p of an image x is given by $p = Wx$. A tomographic reconstruction algorithm is called *linear* if the computed reconstruction v for any vector p of projection data is of the form $v = Rp$, where R is a fixed matrix. There exists an efficient algorithm for computing the image x with $\|x\|_2 = 1$ such that $\|x - RWx\|_2$ is maximal. In other words, for any given linear reconstruction method (FBP, SIRT, ...) the worst-case ground-truth image can be computed efficiently.
6. Nowadays, several companies provide tomographic image reconstruction software as a black box. The fact that the user of the software does not know the underlying algorithm can lead to wrong interpretation of the results.
7. The DART algorithm may benefit from the feature detection methods of this thesis in the selection of fixed and free pixels.
8. Consider a consistent tomographic reconstruction problem modeled by a system of linear equations $Wx = p$ and suppose that the unknown original image is binary. Denote the real-valued solution of minimal norm of this system by x^* . The pixels in x^* that are close to a boundary of the original image tend to have values close to the average of the grey values of neighboring pixels in the original image.
9. By providing the foundation underneath applied mathematics, pure mathematics is highly useful in practice.
10. Globalization has a major role in the development of mathematics and science. Still, spatial distance is a barrier in sharing knowledge. Therefore, grants for performing research in different countries efficiently spread not only science but also multicultural experiences, benefiting two spatially distant societies.