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Error bounds for discrete tomography

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Wagner Rodrigues Fortes

Error bounds for discrete tomography

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To my parents

Angela and Paulo

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