Broadband polarimetry of exoplanets : modelling signals of surfaces, hazes and clouds

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Observing the Earth as an exoplanet with LOUPE

Based on:

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Abstract The detections of small, rocky exoplanets have surged in recent years and will likely continue to do so. To know whether a rocky exoplanet is habitable, we have to characterize its atmosphere and surface. A promising characterization method for rocky exoplanets is direct detection using spectropolarimetry. This method will be based on single pixel signals, because spatially resolving exoplanets is impossible with current and near-future instruments. Well-tested retrieval algorithms are essential to interpret these single pixel signals in terms of atmospheric composition, cloud and surface coverage. Observations of Earth itself provide the obvious benchmark data for testing such algorithms. The observations should provide signals that are integrated over the Earth's disk, that capture day and night variations, and all phase angles. The Moon is a unique platform from where the Earth can be observed as an exoplanet, undisturbed, all of the time. Here, we present LOUPE, the Lunar Observatory for Unresolved Polarimetry of Earth, a small and robust spectropolarimeter to observe our Earth as an exoplanet.

6.1 Introduction

Since the discovery of the first exoplanet (Mayor & Queloz 1995), more than 700 exoplanets have been detected as of today. Even though most of these exoplanets are gas giants, in recent years the number of detected smaller mass planets has surged (see e.g. Wordsworth et al. 2011). Indeed, according to (Cassan et al. 2012), about 62% of the Milky Way stars should have an Earth–like planet. A near-future detection of an Earth-sized exoplanet inside its star's habitable zone seems inevitable. Whether or not an Earth-sized planet in a habitable zone is actually habitable, depends strongly on the composition and structure of its atmosphere. As an example, the Venusian surface is about 500° hotter than expected from Venus' orbital distance and albedo, thanks to an extremely strong greenhouse effect in its thick CO_2 atmosphere. Hence, a characterization of the planetary atmosphere will be needed to address a planet's habitability.

Currently, atmospheres of exoplanets are being characterized using the so–called *transit method* (see e.g. Beaulieu et al. 2010, Miller-Ricci & Fortney 2010). This method is based on measurements of the wavelength dependence of starlight that filters through the upper layers of the planetary atmosphere during the primary transit (when the planet passes in front of the star), or of the planetary flux just before or after the secondary eclipse (during which the planet passes behind its star). The transit method is mostly applied to gaseous planets that orbit close to their star. Earth–sized exoplanets in the habitable zone of a solar–type star are too small to yield a strong enough signal for a spectroscopic characterization during transits (Kaltenegger & Traub 2009).

The best way to characterize the atmosphere and surface of an Earth-sized exoplanet, is through *direct detection*, using large ground–based telescopes such as the European Extremely Large Telescope (E-ELT) (Keller et al. 2010) or space telescopes with diameters of a few meters. With direct detection the light of a planet is measured separately from the stellar light (except for some background starlight). But even if we observe an exoplanet with a direct detection, the planet itself will be unresolved, i.e. it will appear as a single pixel. If the planet resembles the Earth, this single pixel holds information on oceans and continents, coverage by vegetation, desert, and, for example, snow and ice, all overlaid by various types of patchy clouds.

Polarimetry promises to play an important role in exoplanet research both for exoplanet detection and characterization. In particular, because the direct starlight is unpolarized (Kemp et al. 1987), while the starlight that is reflected by a planet will usually be polarized, polarimetry can increase the planet–to–star contrast ratio by 3 to 4 orders of magnitude (Keller et al. 2010), thus facilitating the detection of an exoplanet that might otherwise be lost in the glare of its parent star. Additionally, as in the case of Solar System planets (see e.g. Hansen & Hovenier 1974, Mishchenko & Travis 1997), polarimetry will help the characterization of planetary surfaces and atmospheres, because the polarization of the reflected starlight is very sensitive to the physical properties of an atmosphere and surface. In particular, it has been argued (Williams & Gaidos 2008) that polarimetry could help to detect the glint of starlight reflected on liquid surfaces, such as those of oceans, on exoplanets. Such a detection would be major step forward in the search for life. Combining flux with polarimetric observations will also help to break retrieval degeneracies that flux–only measurements have (see e.g. Stam (2008) and Chapters 2 and 3). Finally, while measuring the state of *linear* polarization of reflected starlight helps to characterize a planetary atmosphere and surface, the degree of *circular* polarization of this light appears to be an indicator for the existence of life on a planet, since circular polarization, and in particular its wavelength dependence, is linked to homochirality of the complex molecules that are essential for life (Sparks et al. 2009b, Sterzik et al. 2010).

To decipher future signals of directly detected Earth-like exoplanets, numerical models that can simulate single pixel signals of exoplanets with inhomogeneous atmospheres and surfaces, are essential. Such models are essential for the design and optimization of telescope instruments and mission profiles (spectral bands, spectral resolution, integration times, revisiting times, etc.), and, once observations are available, they are a necessary tool to interpret the observations. There are a number of numerical models that are used to calculate signals of gaseous and terrestrial exoplanets, for reflected starlight and/or thermally emitted radiation (see e.g. Seager et al. (2000), Ford et al. (2001), Stam (2008), Tinetti et al. (2006a), Bailey (2007), Williams & Gaidos (2008) and Chapter 3). In order to validate the results of such numerical models, it is important to compare them against observations. The obvious test–case for numerical models for Earth–like exoplanets, is Earth itself. To fully validate these models, we need observations of the Earth as if it were an exoplanet, hence single pixel observations that cover the diurnal rotations of the Earth, and all phases of the Earth. And, ideally, the observations should cover different seasons to record the changes in surface albedos and weather patterns.

An excellent location for performing such observations and for building a benchmark dataset is the lunar surface facing the Earth. From there, we can observe the whole disk of the Earth, all of the time, at all phase angles, throughout the year. As we will argue in more detail in Sect. 6.3, such observations cannot be achieved by e.g. combining observations of Low Earth Orbit (LEO) satellites. In this Chapter, we present LOUPE, the Lunar Observatory for Unresolved Polarimetry of Earth. LOUPE is a small and robust spectropolarimeter that measures the flux and state of polarization of sunlight that is reflected by the Earth e.g. from ESA's Lunar Lander (Carpenter et al. 2012).

This Chapter is structured as follows. In Sect. 6.2, we present calculated flux and polarization spectra of a single pixel Earth. In Sect. 6.3, we summarize the advantages of observing the Earth from the moon. In Sect. 6.4, we describe the LOUPE instrument. Section 6.5, finally, contains the summary and our conclusions.

6.2 Flux and polarization spectra of the Earth as an exoplanet

6.2.1 Flux and polarization definitions

Sunlight that is reflected by a planet is described by a flux vector $\pi\vec{F} = \pi[F, Q, U, V]$, with πF the total flux, πQ and πU the linearly polarized fluxes and πV the circularly polarized flux (see e.g. Hansen & Travis 1974, Hovenier et al. 2004, Stam 2008). Each flux parameter depends on the wavelength λ, and has dimensions W m^{-2}m^{-1}. Linearly polarized fluxes πQ and πU are defined with respect to the plane through the center of the star, the planet and the observer (see Chapter 3). The degree of polarization P of the reflected sunlight is defined as the ratio of the polarized flux to the total flux, thus $P = \sqrt{Q^2 + U^2 + V^2})/F$. Specifically, the degree of linear polarization is defined as $P_{\rm L} = \sqrt{Q^2 + U^2}/F$, and the degree of circular polarization as $P_{\rm C} = V/F$.

6.2.2 Sample flux and linear polarization signals of the Earth

Figure 6.1 shows numerically calculated total fluxes πF and degrees of linear polarization $P_{\rm L}$ as functions of the wavelength λ. The Earth's phase angle, α, is 90° (from the moon, one would see a 'half' Earth). The spectra have been calculated using the radiative transfer algorithm described in (Stam 2008), which assumes horizontally homogeneous model planets. We used four model planets, covered by sand, forest, ocean, or ice, combined with a cloud free or a completely cloudy atmosphere (composed of the model B cloud particles of Chapter 2) with an optical thickness of 10 (at 0.55 μm) and located between about 3 and 4 km. The forest and ice surfaces are treated as Lambertian reflectors, with albedos taken from the ASTER library. The ocean surface is completely flat and black with a Fresnel reflecting interface on top. The bi-directional and polarized reflection by the sand surface is modeled using an optically thick ($\tau = 20$ at all λ) layer of dust particles (Laan et al. 2009), with a single scattering albedo chosen such that the albedo agrees with that measured from an airplane above the Sahara (Bierwirth et al. 2009).

To model the spectra of the horizontally inhomogeneous Earth, we apply the weighted averages method (Stam 2008) using the total and polarized flux spectra of the horizontally homogeneous model planets. In Fig. 6.1, we have chosen the weighting factors such that they represent a case in which Africa and Eurasia are on the centre of the planetary disk and a case in which the Pacific ocean is on the centre. For comparison, the latter case is also shown with a cloud layer.

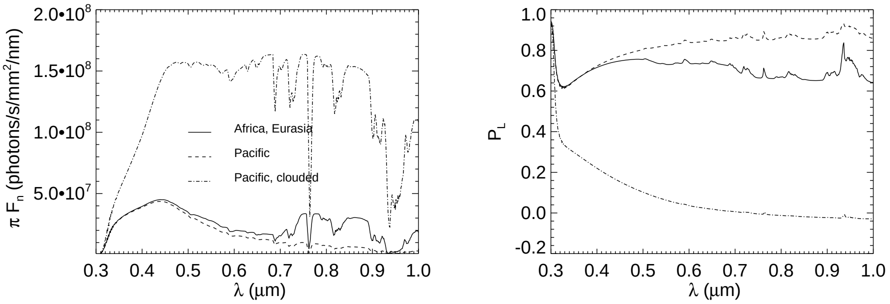

Figure 6.1: Calculated flux πF (left) and degree of linear polarization $P_{\rm L}$ (right) of sunlight reflected by the Earth as functions of λ, for $\alpha = 90^\circ$: with Africa and Eurasia in view and no clouds (solid lines), with the Pacific ocean in view and no clouds (dashed lines) and when completely cloudy (dashed-dotted lines).

For the solar flux that is incident on the Earth, we adopted the solar flux (in photons s^{-1} mm^{-2} nm^{-1}) as measured on February 25th, 2008, by the GOME-2 spectrometer (Callies et al. 2000) on the Earth-observing MetOp satellite. The measured spectrum runs from 0.3 μm to 0.8 μm, and we extrapolated it smoothly towards 1.0 μm.

The flux and polarization spectra of the cloud-free planets in Fig. 6.1 clearly show the traces of the Earth's surface and atmosphere. For a detailed explanation of the spectral features due to gaseous absorption by O_3, O_2, and H_2O, see (see the explanation below (box A) or (Stam 2008)). Longwards of 0.7 μm, the characteristic red–edge albedo feature of the vegetation (Seager et al. 2005) clearly shows up in πF when the continents are in full view: πF is higher by almost a factor of 5 (at 0.85 μm) than when the Pacific is in view (a small fraction of this increase will be due to the sand surface). The red–edge feature shows up as a decrease of $P_{\rm L}$ (of about 20% in absolute value) because an increase in surface albedo increases the amount of unpolarized light that is reflected towards the observer. Adding clouds to the model atmosphere increases πF strongly (except in the deepest gaseous absorption bands): πF is ~12 times (~ 23 times) higher at λ =0.65 μm (0.85 μm). At the same time, the clouds significantly decrease $P_{\rm L}$ at this phase angle ($\alpha = 90^\circ$): ~80% at λ =0.65μm. The model planets used in Fig. 6.1 are either cloudfree of completely cloudy. In reality, the Earth is only partly covered by clouds (with a range of optical thicknesses) and the real flux and polarization spectra will be mixtures of the spectra that are shown here.

In Fig.6.2 we show πF and $P_{\rm L}$ at λ=550 nm, as functions of phase angle α for the model Earth that has a cloud coverage of about 42 % (the cloud properties are the same as in Fig. 6.1). The narrow features on top of the curves are due to the daily rotation of the planet as it orbits its star, showing ocean and/or continents through the holes in the clouds. The "bump" in the curves around $\alpha = 38^\circ$ are due to the primary rainbow: sunlight that has been scattered by the cloud droplets once. Clearly, the rainbow is much more pronounced in $P_{\rm L}$ than in πF. Finding a rainbow in exoplanetary polarization signals will be a direct indication for the presence of liquid water droplets in the planetary atmosphere (for a more detailed description of rainbows on exoplanets, see Bailey 2007). The angular separation of the colors in exoplanetary rainbow could help to determine the cloud particle sizes (see Chapter 2).

The spectra in Figs. 6.1 and 6.2 do not include the contribution of sunlight that is scattered by the zodiacal dust surrounding the Earth. Sunlight scattered by this dust will usually be linearly polarized (see Renard et al. 1995), with a degree and direction of polarization that will depend on the phase angle, the wavelength, and on local variations in the dust particle properties. Since the dust is optically thin and dark (Renard et al. 1995), its contribution to the measured signal will depend strongly on the spatial resolution of the observations. When the instrument design allows spatially resolved observations (see Sect. 4), the signal of the zodiacal light could be measured separately from the Earth's signal, and provide valuable benchmark data for (exo)zodiacal dust disk models. In the case of spatially unresolved observations (see Sect. 4), the contribution of the zodiacal dust to the total flux and polarization signal is expected to be negligible, because of the brightness of the Earth's disk. Modelling of this contribution will be part of the future instrument studies.

6.2.3 Circular polarization

All known living material on Earth exhibits homo–chirality: sugars and nucleic acids occur exclusively in the right–handed form, and amino-acids and proteins in the left-handed form. Homo–chirality makes light scattered by organic material partially circularly polarized, and circular polarimetric spectra of various samples of biological material have been published (Wolstencroft et al. 2004, Sparks et al. 2009b, Sterzik et al. 2010). The reasons for homo–chirality are unknown, but if similar evolutionary scenarios naturally occur elsewhere in the universe, measuring $P_{\rm C}$ could be a unique tool for the detection of life on exoplanets. Since the Earth is the only planet we know that has life on it, Earth observations are the only way to empirically test this remote-sensing method. Some abiotic scattering

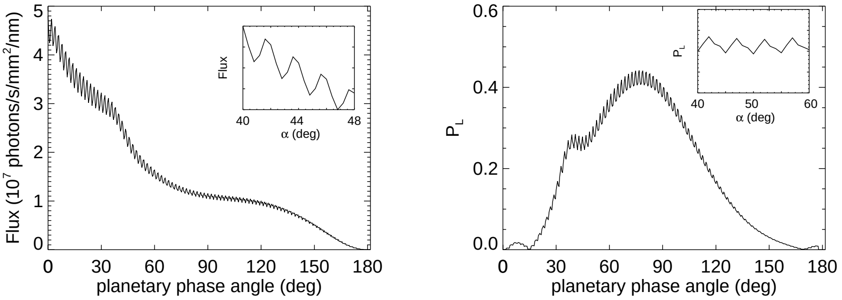

Figure 6.2: Calculated πF (left) and $P_{\rm L}$ (right) of sunlight reflected by a rotating model Earth with 42% cloud coverage as functions of α, at 550 nm.

processes (e.g. by optically active atmospheric aerosols or minerals) may also give a measurable $P_{\rm C}$, but as shown in (Sparks et al. 2009b) the wavelength dependence of these signals is very different from that of the circular polarization of biological material (Sparks et al. 2009a).

The degree of circular polarization produced by biological material is generally weak, on the order of $10^{-3} - 10^{-4}$ (see Sparks et al. 2009b). Therefore, it would be very interesting to perform spatially resolved observations of the Earth, as this would allow us to study whether such signals can be measured locally (e.g. over the Amazonian rainforest) and whether they could be measurable in disk integrated signals.

6.3 The advantages of using the Moon as observation platform

In order to build a comprehensive database of benchmark data of the Earth as an exoplanet and to be able to fully test numerical algorithms for signal simulation and planet characterization, the requirements on the flux and polarization observations are as follows:

1) Each observation of the Earth should be (nearly) instantaneous, to observe different regions on the illuminated and visible part of the Earth simultaneously and hence to capture the effects of the differences in local solar zenith angles and viewing angles.

A. The spectrum of Earth as an exoplanet

In Fig. 6.1 we plot the flux and polarization spectrum of a simplified Earth as if it were an exoplanet. The high–spectral resolution features that we see in both flux and polarization are due to gaseous absorption bands.
At the shortest wavelengths light is absorbed by the atmospheric O_3, causing a dip in the reflected flux and a corresponding increase in the degree of polarization around 0.3 μm. Between $\sim$0.5 μm and $\sim$0.7μm we notice, especially in the case we observe areas with high surface albedo (here, the case of Africa and Eurasia), a dip in the reflected flux, which is the so–called Chappuis absorption band of O_3. Around 0.76 μm we notice a deep absorption band which is the famous oxygen A–band, the easiest identifiable O_2 band in our atmosphere. The oxygen A–band is useful for the characterization of planetary atmospheres, since its depth compared to the continuum, can help us determine the cloud top heights in a planetary atmosphere (see e.g. Wu 1985, and references therein).
In polarization, we notice that the spectrum of our planet looks similar to that in flux, only now the absorption lines have transformed into "emission" lines, i.e. there is a local increase in the degree of polarization where an absorption line lies. The reason for this is that the absorption of light in a band decreases the amount of multiple scattered light and we see mostly light that is singly scattered ((normally) with a higher degree of polarization) in the atmosphere.
The general increase in polarization that we observe with wavelength in the non–cloudy cases is due to the decrease in the optical thickness of the atmosphere with wavelength, which leads to a small(er) amount of multiple scattering. In this way, in the case we observe the Pacific region, since the ocean surface is almost black the polarization reaches almost as high as the single scattering value of the atmosphere. In the case we observe the Africa–Eurasia area on the other hand, a number of photons will have managed to reach as low as the surface and get reflected. At the longer wavelengths, where the atmosphere is less opaque the number of photons that have managed to penetrate the atmosphere and reflect on the surface is (getting) comparable to the number of singly scattered photons on the upper atmosphere and the degree of polarization decreases.

2) Observations should cover the Earth's diurnal cycle, to capture the effects of different regions of the Earth emerging from the night, and disappearing over the limb (or the other way around), with the corresponding local changes in solar zenith and viewing angles.
3) The Earth should be observed at phase angles from $\sim$0° ('full Earth') to $\sim$180°

('new Earth'), with steps small enough to capture characteristic angular features in the reflected πF and P, such as the glint of sunlight reflected by surface water and the rainbow of sunlight scattered in clouds.
4) The observations should ideally cover all seasons to capture the effects of changes in local solar zenith angles, polar nights, weather and cloud patterns, and surface albedos.

Thanks to the monthly orbit of the Moon around the Earth and the tidal locking of the Moon with respect to the Earth, a spectropolarimeter on the lunar surface could observe the whole Earth, during each day, at all phase angles (depending on the power source), and, in principle, throughout the seasons. Such whole Earth observations cannot be obtained from (existing) artificial satellites, such as Low Earth Orbit (LEO) remote-sensing satellites or geostationary satellites. LEO satellites observe local regions on the Earth, and would require several days to achieve global coverage. In addition, a certain location on Earth will always be observed under similar illumination and viewing geometries (apart for seasonal variations of the local solar zenith angle). Currently, only the POLDER Earth-observing satellite instrument has polarimetric capabilities (broadband, no spectropolarimetry). Geostationary satellites observe the same hemisphere of the Earth all of the time. While these satellites do capture the effects of the diurnal rotation and at the same time the phase angle changes of the Earth, they cannot observe different regions of the Earth, and their observations cannot teach us how to derive a global distribution of oceans and continents from single pixel measurements. There are currently no polarimeters onboard any geostationary satellite. A network of geostationary satellites could be used to capture the whole Earth. However, with such a network, it would not be possible to measure the effects of e.g. continents emerging from the night into the daylight or disappearing over the limb. In addition, to be able to compare total flux measurements from different satellites, every spectropolarimeter should be carefully internally calibrated.

Recent spectropolarimetric Earthshine observations (Sterzik et al. 2012), in which sunlight that has been reflected first by the Earth and then by the moon is measured with Earth-based instruments (see e.g. Qiu et al. 2003, Sterzik & Bagnulo 2009) confirm that disk-integrated polarimetric observations are extremely sensitive to the visible surface and atmosphere of the Earth. At the same time, discrepancies between theoretical predictions and observations demonstrate that multi-epoch observations of the Earth are needed to constrain the models. The major drawback of Earthshine observations is that the properties of the lunar surface are not known well, especially when polarization is involved. This makes the modelling enormously more difficult than in the case of observations from space (including the Moon). Ground-based Earthshine observations are also hampered by background contamination from the sunlit fraction of the Moon, and do not allow the same phase angle coverage (both in range and in angular resolution) and are unable to capture the full diurnal rotation.

Finally, a number of non–dedicated missions (e.g. Voyager 1, and more recently Deep Impact) have taken snapshots of the Earth.[1] These observations, while often providing interesting data, do not cover the diurnal rotation nor the phase angle range nor the seasonal effects. There have been no polarimetric observations performed by such missions.

A spectropolarimeter could be put onboard a specially designed satellite in an orbit that allows performing the required observations. That orbit would, however, probably closely resemble the orbit of the moon. Including the instrument on a Lunar Lander thus seems a straightforward and economical choice.

6.4 The LOUPE instrument

LOUPE, the Lunar Observatory for Unresolved Polarimetry of Earth shall fulfill the following requirements:

- It performs spectropolarimetric observations of the light from the Earth's disk (at least) at visible wavelengths (400–800 nm).
- The spectral resolution for the polarimetry shall be ∼20 nm, while the O_2A band (∼0.76 μm) is resolved in the flux spectrum. Limited spectropolarimetry can be performed within this and other bands.
- Data is collected on an hourly basis to resolve the Earth's rotation, and span at least a month to cover a full range of phase angles.
- The instrument is small and robust.

For the polarimetry, we explore two different scenarios:
1) *Linear spectropolarimetry only*. For this we adopt the spectral modulation approach (Snik et al. 2009). Using a combination of standard solid-state polarization optics (see Fig. 6.3), the total flux spectrum is multiplied by a sinusoidal modulation for which the amplitude scales with $P_{\rm L}$, and the phase is determined by the angle of polarization. This novel polarimetric concept is being applied in the SPEX instruments. The SPEX prototype exhibits excellent polarimetric performance (van Harten et al. 2011).

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[1] for a nice overview see: http://planetary.org/explore/topics/earth/spacecraft.html

2) *Linear* and *circular spectropolarimetry*. This implementation is more challenging as the data dimensionality is larger, and, moreover, P_{C} ($\sim 10^{-4}$) is much smaller than the average P_{L} ($\sim 10^{-4}$ versus ~ 0.1). The spectral modulation approach in (Nordsieck 1974, Oka & Kato 1999) yields three modulation periods that contain information on the complete flux vector. The modulation approach introduced by (Sparks et al. 2012) yields similar modulations, but along the slit direction.

Various options can be identified for spatial resolution and pointing:
A) The instrument itself averages the light from the Earth's disk ($\sim 2^\circ$ diameter). Because the disk is surrounded by black space, this requires only course pointing. The acceptance angle of the instrument should be wide enough to take lunar libration ($\pm 8^\circ$) into account.
B) The instrument spatially resolves the Earth's disk to obtain data of e.g. just the Amazonian rainforest to maximize the circular polarization signal. Such spatial information can be attained by using a scanning slit or an integral field unit. In any case, accurate pointing and potentially scanning should be implemented. Averaging over the Earth's disk is then performed in the data pipeline. Spatially resolved measurements allow to measure the relatively weak signal of the zodiacal dust.

A sketch of the most basic implementation (1A: only linear spectropolarimetry and no spatial resolution) of LOUPE is presented in Fig. 6.3.

6.5 Summary and conclusions

We present LOUPE, the Lunar Observatory for Unresolved Polarimetry of Earth. LOUPE is a small and robust spectropolarimeter that can observe the Earth as if it were an exoplanet from a vantage point on the lunar surface. The Moon has a unique position with respect to Earth and can provide us with a unique platform from where we can observe the Earth as an exoplanet. From the Moon, LOUPE will be able to observe the whole disk of the Earth, all of the time, at most phase angles and throughout the year.

LOUPE measures the total flux and state (degree and direction) of polarization of sunlight that is reflected by the Earth. Polarimetry appears to be a strong tool for the characterization of exoplanets, allowing the retrieval of the composition and structure of a planet's atmosphere and surface (if present). In particular, the degree of linear polarization can give us information on the presence of liquid water clouds and the degree of circular polarization on the presence of life. LOUPE measurements would be used as a benchmark for future Earth-like exoplanet observations and to test numerical algorithms for the retrieval of planet properties from such observations.

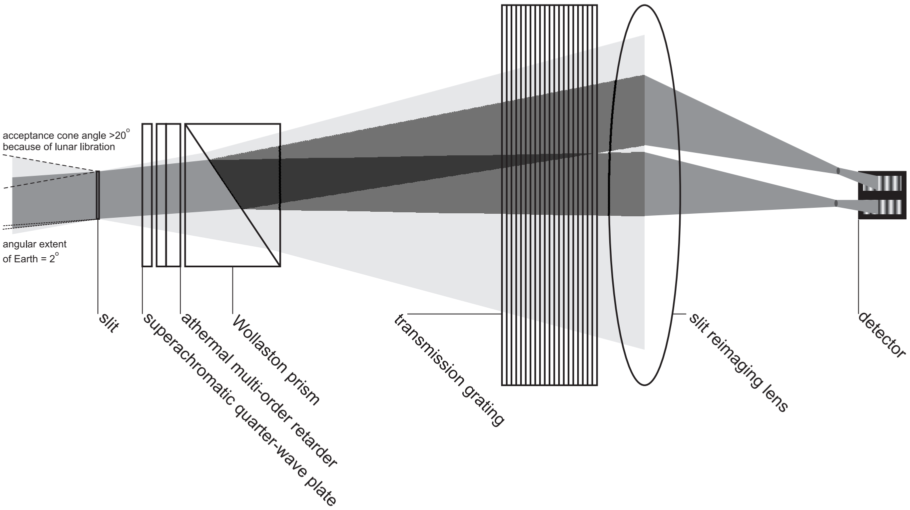

Figure 6.3: Schematic depiction of a potential implementation for LOUPE option 1A (only linear spectropolarimetry, no spatial resolution). The scale is approximately 1:1. The (wide-field) spectral polarization modulation optics are located behind the entrance slit. A Wollaston prism serves as a polarizing beam-splitter. A transmission grating or grism disperses the light, and a reimaging lens focuses the two spectra on the detector. An image of the Earth appears at the focal length of that lens. This instrument only needs to be roughly pointed towards the Earth as it accepts light from all angles within the range determined by the lunar libration.

B. SPEX, the working force behind LOUPE

LOUPE, in its simplest configuration, when we are only interested in the linear polarization of Earth–as–an–exoplanet is working based on the concept of spectral polarization modulation of SPEX (Snik et al. 2009, van Harten et al. 2011).
Classical methods apply temporal or spatial modulation (or both), which demand moving parts (and thus a lot of energy, and risk of failure) and a lot of space (and high energy consumption in order to perform calibration of the system) respectively, while with SPEX we opt for low energy consumption and small volume. For this reason, for the polarization modulation of SPEX a new method was proposed, which takes into advantage the existence of a spectrograph in the system, and maps the polarization properties of the incoming signal onto the spectral dimension. The idea is to apply a "spectral modulation", which is ideally achieved when a sinusoidal modulation of known periodicity is superimposed on top of the incoming spectrum with the amplitude and the phase of the modulation depending on the polarization characteristics of the incoming signal (in particular the amplitude depends on P and the phase on χ). SPEX encodes the degree and angle of linear polarization of the incoming (observed) signal in the sinusoidal form: $I_{out}(\lambda) = (\frac{1}{2}+\frac{1}{2}\mathrm{P}(\lambda)\cos(\frac{C(\lambda)}{\lambda}+2\chi(\lambda,T)))I_{\mathrm{in}}(\lambda)$, where I_{in} is the incoming intensity and $C(\lambda)$ depends on the retardance of the (multiple–order retarder of the) system (see Snik et al. 2009).