Random walks in dynamic random environments

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Preface

In the past forty years, models of Random Walks in Random Environments (RWREs) have been intensively studied by the physics and the mathematics community, giving rise to an important and still lively research area that is part of the field of disordered systems. RWREs on $\mathbb{Z}^d$ are Random Walks (RWs) evolving according to a random transition kernel, i.e., their transition probabilities depend on a random field or a random process ξ on $\mathbb{Z}^d$ called Random Environment (RE). What makes these models interesting is that, depending on the RE, several unusual phenomena arise, such as sub-diffusive behavior, sub-exponential decay of probabilities of large deviations, and trapping effects. The REs can be divided into two main classes: *static* and *dynamic*. We refer to *static* RE if ξ is chosen at random at time zero and is kept fixed throughout the time evolution of the RW, while we refer to *dynamic* RE when ξ changes in time according to some stochastic dynamics. For *static* RE, in one dimension the picture is fairly well understood: recurrence criteria, laws of large numbers, invariance principles and refined large deviation estimates have been obtained in a series of papers. In higher dimensions many results have been obtained as well, but still many questions remain open. In *dynamic* RE the state of the art is poorer, even in one dimension. In this thesis we will focus on a class of RWs in dynamic REs constituted by interacting particle systems. The analysis of these models leads us to derive new results and to formulate challenging questions for the future.

The thesis is organized as follows. In Chapter 1 we review what is known in the literature, both for static and dynamic RE, and we introduce the class of models we are interested in. In Chapter 2 we prove a strong law of large numbers under a certain space-time mixing condition on the RE, both in one and in higher dimensions. Furthermore, by using a perturbation argument, we give a series expansion in the size of the drift for the asymptotic speed of RWs with small drifts in highly disordered REs. Chapter 3 focuses on the scaling limits of such processes. By adapting to our context a proof of Comets and Zeitouni [36] for multi-dimensional RWs in static REs, we show that, under a certain space-time mixing condition, an annealed invariance principle holds in any dimension. We further give an alternative proof of this invariance principle in the context of highly disordered REs under small drift assumptions. Chapter 4 deals with the large deviation analysis for the empirical speed of one-dimensional RWs in dynamic REs. We prove a quenched and an annealed large deviation principle and we exhibit some qualitative properties of the associated rate functions. In particular, we give examples of fast and slow-mixing REs for which, respectively, exponential and sub-exponential decay of large deviation probabilities occur. In Chapter 5 we prove a law of large numbers for transient RWs on top of a simple symmetric exclusion process and we conclude with a brief discussion about possible extensions to more general slow-mixing REs, which are part of an ongoing project.