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Random walks in dynamic random environments

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Random walks in dynamic random environments

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Random walks in dynamic random environments

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THOMAS STIELTJES INSTITUTE
FOR MATHEMATICS



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Contents

Preface	ix
1 Introduction: Random walks in random environments (RWRE)	1
1.1 Static RE	1
1.1.1 One dimension	2
1.1.1.1 Ergodic behavior	2
1.1.1.2 Scaling limits	3
1.1.1.3 Large deviations	4
1.1.1.4 An example	5
1.1.2 Higher dimensions	7
1.2 Dynamic RE	10
1.2.1 Early work	10
1.2.2 Space-time i.i.d. RE	11
1.2.3 Time-dependent RE	12
1.2.4 Space-time mixing RE	12
1.3 RW on an Interacting Particle System (IPS)	13
1.3.1 IPS	13
1.3.1.1 Definition	13
1.3.1.2 Examples	14

1.3.2	RW on IPS	16
1.4	Related models	17
2	Law of large numbers for a class of RW in dynamic RE	19
2.1	Introduction and main result	20
2.1.1	Model	20
2.1.2	Cone-mixing and law of large numbers	21
2.1.3	Global speed for small local drifts	22
2.1.4	Discussion and outline	24
2.2	Proof of Theorem 2.2	25
2.2.1	Space-time embedding	26
2.2.2	Adding time lapses	27
2.2.3	Regeneration times	28
2.2.4	Gaps between regeneration times	30
2.2.5	A coupling property for random sequences	31
2.2.6	LLN for Y	32
2.2.7	From discrete to continuous time	35
2.2.8	Remarks on the cone-mixing assumption	37
2.3	Series expansion for $M < \epsilon$	38
2.3.1	Definition of the environment process	38
2.3.2	Unique ergodic equilibrium measure for the environment process	39
2.3.2.1	Decomposition of the generator of the environment process	40
2.3.2.2	Expansion of the equilibrium measure of the environment process	44
2.3.3	Expansion of the global speed	46
2.4	Examples of cone-mixing	48
2.4.1	Spin-flip systems in the regime $M < \epsilon$	48

2.4.2	Attractive spin-flip dynamics	49
2.4.3	Space-time Gibbs measures	50
2.5	Independent spin-flips	50
3	Annealed central limit theorem for RW in mixing dynamic RE	53
3.1	Introduction and main result	53
3.2	Proof of Theorem 3.2	55
3.2.1	A chain with complete connections	55
3.2.2	Invariance principle for the chain with complete connections	60
3.2.3	Invariance principle for the random walk	61
3.2.4	Examples of mixing dynamic RE	63
3.3	CLT in the perturbative regime	64
4	Large deviation principle for one-dimensional RW in dynamic RE: attractive spin-flips and simple symmetric exclusion	67
4.1	Introduction and main results	68
4.1.1	Random walk in dynamic random environment: attractive spin-flips	68
4.1.2	Large deviation principles	69
4.1.3	Random walk in dynamic random environment: simple symmetric exclusion	72
4.1.4	Discussion	73
4.2	Proof of Theorem 4.2	76
4.2.1	Three lemmas	76
4.2.2	Annealed LDP	79
4.2.3	Unique zero of I^{ann} when $M < \epsilon$	80
4.3	Proof of Theorem 4.3	82
4.3.1	Three lemmas	82
4.3.2	Quenched LDP	83

4.3.3	A quenched symmetry relation	84
4.4	Proof of Theorem 4.4	89
4.4.1	Traffic jams	89
4.4.2	Slow-down	92
5	Law of large numbers for one-dimensional transient RW on the exclusion process	97
5.1	Introduction and result	97
5.1.1	Slow-mixing REs and the exclusion process	97
5.1.2	Model and main theorem	98
5.2	Proof of Theorem 5.1	98
5.2.1	Coupling and minimal walker	99
5.2.2	Graphical representation: symmetric exclusion as an interchange process	100
5.2.3	Marked agents set	101
5.2.4	Right walker and a sub-additivity argument	103
5.2.5	LLN	105
5.3	Concluding remarks	107
	Bibliography	109
	Samenvatting	117
	Acknowledgements	119
	Curriculum Vitae	120

Preface

In the past forty years, models of Random Walks in Random Environments (RWREs) have been intensively studied by the physics and the mathematics community, giving rise to an important and still lively research area that is part of the field of disordered systems. RWREs on $\mathbb{Z}^d$ are Random Walks (RWs) evolving according to a random transition kernel, i.e., their transition probabilities depend on a random field or a random process ξ on $\mathbb{Z}^d$ called Random Environment (RE). What makes these models interesting is that, depending on the RE, several unusual phenomena arise, such as sub-diffusive behavior, sub-exponential decay of probabilities of large deviations, and trapping effects. The REs can be divided into two main classes: *static* and *dynamic*. We refer to *static* RE if ξ is chosen at random at time zero and is kept fixed throughout the time evolution of the RW, while we refer to *dynamic* RE when ξ changes in time according to some stochastic dynamics. For *static* RE, in one dimension the picture is fairly well understood: recurrence criteria, laws of large numbers, invariance principles and refined large deviation estimates have been obtained in a series of papers. In higher dimensions many results have been obtained as well, but still many questions remain open. In *dynamic* RE the state of the art is poorer, even in one dimension. In this thesis we will focus on a class of RWs in dynamic REs constituted by interacting particle systems. The analysis of these models leads us to derive new results and to formulate challenging questions for the future.

The thesis is organized as follows. In Chapter 1 we review what is known in the literature, both for static and dynamic RE, and we introduce the class of models we are interested in. In Chapter 2 we prove a strong law of large numbers under a certain space-time mixing condition on the RE, both in one and in higher dimensions. Furthermore, by using a perturbation argument, we give a series expansion in the size of the drift for the asymptotic speed of RWs with small drifts in highly disordered REs. Chapter 3 focuses on the scaling limits of such processes. By adapting to our context a proof of Comets and Zeitouni [36] for multi-dimensional RWs in static REs, we show that, under a certain space-time mixing condition, an annealed invariance principle holds in any dimension. We further give an alternative proof of this invariance principle in the context of highly disordered REs under small drift assumptions. Chapter 4 deals with the large deviation analysis for the empirical speed of one-dimensional RWs in dynamic REs. We prove a quenched and an annealed large deviation principle and we exhibit some qualitative properties of the associated rate functions. In particular, we give examples of fast

and slow-mixing REs for which, respectively, exponential and sub-exponential decay of large deviation probabilities occur. In Chapter 5 we prove a law of large numbers for transient RWs on top of a simple symmetric exclusion process and we conclude with a brief discussion about possible extensions to more general slow-mixing REs, which are part of an ongoing project.