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Profinite groups with a rational probabilistic zeta function

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Summary

In this thesis, we investigate the connection between finitely generated profinite groups G and the associated Dirichlet series $P_G(s)$ of which the reciprocal is called the probabilistic zeta function of G . In particular, we consider the conjecture of Lucchini saying that given a finitely generated profinite group G , the associated Dirichlet series $P_G(s)$ is rational if and only if the quotient group $G/\text{Frat}(G)$ is finite. Detomi and Lucchini first showed that the conjecture holds when G is prosoluble. For non-prosoluble groups, they later showed that the conjecture also holds when almost every nonabelian composition factor of G is an alternating group. In this thesis, we prove the conjecture in several other cases. We first show that it holds when almost every nonabelian composition factor of G is isomorphic to a simple group of Lie type over a field of characteristic p , where p is a fixed prime. When there are different characteristics, the problem becomes quite difficult and we do not have any answer yet. However, we obtain that the conjecture holds when almost every nonabelian composition factor of G is $\text{PSL}(2, p)$ for some prime $p \geq 5$. This is also the case when we replace $\text{PSL}(2, p)$ by a sporadic simple group. The conjecture is still open in general and it cannot be proved by our techniques. We give some examples supporting this. Nevertheless, we also obtain a partial result by showing that the conjecture holds when almost every nonabelian composition factor is isomorphic to either $\text{PSL}(2, p)$ for some prime $p \geq 5$, or a sporadic simple group, or an alternating group $\text{Alt}(n)$ where n is either a prime or a power of 2.

