

Cover Page



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Profinite groups with a rational probabilistic zeta function

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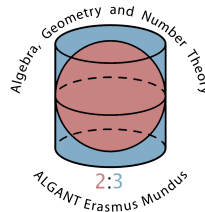
geboren te Dong Nai, Vietnam in 1985

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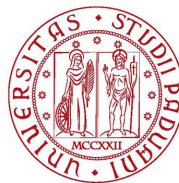
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To my wife

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