Imperfect Fabry-Perot resonators
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Introduction

In 1899, the first Fabry-Perot interferometer (or resonator) was built by Fabry and Perot [1] by placing two planar mirrors parallel to each other. Be it more than 100 years old, it nevertheless presents a challenging topic in the optics course, comprising a number of interesting facets, like the theory of multiple interferences (first analyzed by Airy in 1831) and the presence of circular fringes (first observed by Haidinger in 1855). The high spectral resolution that can be achieved with a Fabry-Perot makes it essential for many (modern) applications; like lasers, laser gyroscopes (more than two mirrors needed), and cavity ring-down spectroscopy [2, 3]. The Fabry-Perot also forms the heart of many state-of-the-art experiments; in cavity QED [4], in experiments with micro-resonators [5, 6], in gravitational wave detectors [7–9], and in even more exotic experiments aimed at superimposing two quantum states of a macroscopic mirror [10]. The first Fabry-Perot interferometers were composed of two planar mirrors; later designs often use two spherical mirrors.

A Fabry-Perot interferometer can be operated in both the angular and spectral domain. In the angular domain, a pattern of "fringes of equal inclination", or so-called Haidinger-fringes, is observed behind a planar Fabry-Perot that is illuminated with a wide-angle beam at a fixed wavelength; fringes formed by illumination with a slightly different wavelength are observed under a slightly different angle. In the spectral domain, different wavelengths show resonances in the spectrum at different cavity lengths, while scanning the cavity length over at least half a wavelength.

The width of a fringe dictates the resolution of a Fabry-Perot and is determined by the cavity finesse $F = \Delta\nu_{\text{FSR}}/\Delta\nu$, where $\Delta\nu_{\text{FSR}}$ is the free spectral range or separation of adjacent maxima and $\Delta\nu$ the width (FWHM) of the individual fringes. Ignoring diffraction, the finesse of a resonator, comprising ideal and lossless mirrors is determined by the reflectivity of the mirrors only. The finesse of practical planar Fabry-Perot resonators, however, is often limited by the losses introduced by both diffraction [11, 12] and mirror imperfections, *e.g.*, surface

roughness and aberrations. Diffraction losses in a planar Fabry-Perot can be neglected for short cavities or wide-beam illumination. In planar cavities with wide-beam illumination, imperfections that introduce a height variation of λ/m over the full mirror aperture, limit the finesse to $F \sim m/2$ [1, 11, 13, 14]. In practice, this means that even for state-of-the-art substrates with a 0.1 nm (RMS) roughness, the finesse of a planar Fabry-Perot is limited to only $F = 5400$ for a wavelength of $\lambda = 1064$ nm [15] and even less for visible wavelengths. This is a real drawback in many applications.

Stable resonators with *spherical* mirrors (first proposed in 1956) are much less affected by these limitations and can achieve a much higher finesse, up to $F = 1 \times 10^6$ [16]. For completeness we note that unstable resonators, which also comprise spherical mirrors, are lossy by their geometry and can never achieve a high finesse. The spherical shape of the mirrors (in a stable resonator) compensates for diffraction [17] and the resonator is less sensitive to spatially extended imperfections as the modes on the mirrors are more compact. For resonators with state-of-the-art spherical mirrors, the finesse is eventually limited (if not by transmission of the mirrors) by the power loss per round-trip due to the area-integrated roughness-induced scatter. This so-called total integrated scatter (TIS) of the resonator scales inversely with m^2 [18], so that the finesse scales as m^2. This is obviously a significantly more relaxed requirement than that for a planar cavity, where the finesse scales linearly with m. Another advantage that favors spherical resonators over planar ones is that spherical mirrors can be manufactured more precisely than planar ones.

Just as their planar counterpart, resonators comprising spherical mirrors can be operated in both the angular and the spectral domain. Again, fringes appear for illumination with a wide beam, addressing many transverse modes in the cavity. Spherical aberration of the mirrors makes a description of the fringes more complicated than for a planar resonator [19] and reduces the finesse for the higher-order fringes [20]. The "quadratic" influence of imperfections is also observed for a resonator with spherical mirrors operated in the spectral domain.

The initial goal of this Thesis was to demonstrate chaos in an open two-mirror resonator. Two requirements have to be fulfilled to obtain chaos within the context of geometrical (*i.e.* ray) optics. Firstly, exponential sensitivity of the evolution of the intra-cavity ray to the initial conditions is required, and, secondly, the ray has to remain confined inside the resonator for a sufficient time to produce mixing. We have designed a bifocal mirror that, in combination with a conventional concave mirror, forms a resonator with an unstable inner and a stable outer part ("inner" and "outer" refer here to the transverse coordinate). The unstable part provides for the exponential sensitivity, whereas the stable part provides for the mixing. We note that although the resonator comprises an unstable part, the resonator is stable in an overall sense. In order to achieve chaos in this overall stable cavity, we need, as mentioned above, a long residence time of the light in the cavity. This implies that the finesse must be as large as possible and thus requires a solid understanding of the imperfections of a Fabry-Perot. In fact, this has become the main theme of this thesis.

Another motivation for a thorough understanding of imperfections is that the unstable part of the bifocal resonator acts as a sort of "macro"-imperfection and produces, in combination with the stable part, a challenging and complex physical system. For full appreciation and understanding of this system, it is necessary to be able to distinguish phenomena unique for this configuration from effects also present in conventional resonators, comprising two

standard spherical mirrors. We decided to investigate first the effect of imperfect mirrors, *i.e.*, roughness-induced scattering and aberrations, on the performance of a conventional stable resonator. By aberrations, we mean the deviation of the actual wavefront from the spherical reference wavefront; these deviations may be caused by a combination of the spherical shape of the mirror and the nonparaxial transverse excursion of the ray through the resonator.

A central and important theme in our analysis is the concept of frequency-degeneracy (first introduced by Herriot [21] in 1964), where the ray and wave description of light in a resonator are intimately linked. In the ray picture, frequency-degeneracy means that a ray retraces itself after an integer number of N round-trips through the cavity. In the wave picture, frequency-degeneracy imposes that resonances in the spectrum overlap in N clumps of modes within a free spectral range.

The contents of this Thesis is organized as follows:
In Chapter 2, we characterize the roughness-induced scattering of a single mirror by means of its angular distribution (BRDF) and total scattered power (TIS). We also describe the effect of scattering on the performance of a conventional resonator, comprising two mirrors. We demonstrate and discuss how the losses affect the cavity finesse, measured in both time and spectral domain, as well as the average power throughput.

In Chapter 3, scattering is shown to produce mode coupling close to frequency-degenerate points. This effect has drastic consequences which are analyzed in the spatial, spectral, and time domain. A numerical simulation helps us to quantify the number of coupled modes. The effect of mode beating on cavity ring-down is pointed out as well.

In Chapter 4, a scanning cavity is injected on-axis with a compact ("pencil") beam. Although we inject locally, fringes appear over the concave mirror aperture, at least close to frequency-degenerate points. We claim that these fringe are caused by light scattered out of the on-axis beam into resonant orbits. In our resonator, spectral and spatial properties are intimately linked and cannot be separated. We demonstrate how an analysis of the observed fringe pattern yields a method to accurately determine aberrations.

In Chapter 5, we measure the deviations from paraxiality in a folded 3-mirror resonator, a result from earlier attempts to show chaos in an open resonator. We quantify this by accurately measuring the Gouy phase of subsequent higher-order modes around frequency-degeneracy. The experimental results are supported by a ray-tracing simulation.

In Chapter 6, a connection is established between a wave and ray description of aberrations, used in Chapter 4 and 5. The connection is based on Fermat's principle in a frequency-degenerate resonator. We derive and compare the cavity length reductions needed to maintain frequency-degeneracy for higher-order modes or, equivalently, larger transverse displacements.

In Chapter 7, we report on mirrors that are not fabricated by traditional grinding and polishing, but by diamond-machining. The diamond chisel makes circular grooves on the substrate, and causes a different type of scatter than the more random defects introduced in traditionally produced mirrors. We investigate the influence of this production method on the multiple interferences in the resonator and show that a reasonable finesse can still be obtained.

In Chapter 8, the eigenmodes of a resonator with one diamond-machined bifocal mirror are discussed. The central convex part of the bifocal mirror breaks the full quadratic profile of the mirror and imposes Laguerre-Gaussian eigenmodes on the resonator. The observed mode

profiles are compared with analytically calculated eigenmodes and a numerical simulation is performed to model the bifocal mirror.

In Chapter 9, we investigate the behavior of a bifocal cavity, consisting out of a stable outer and an unstable inner resonator, which is expected to show the onset of chaos. We demonstrate the coupling of two resonators based on transmission spectra and patterns, and report on the ability of the configuration to fulfill the basic requirements to obtain chaos.