Conformal invariance and microscopic sensitivity in cosmic inflation

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Inflation embedded in supergravity

As we have emphasized earlier, inflation is a coarse phenomenon in classical general relativity. In principle it should therefore not be too difficult to embed inflationary models within a unifying theory of quantum gravity such as string theory or its supergravity description at low energies. Nevertheless, inflation turns out to depend sensitively on the microscopic description of the theory. Although this is a blessing if we ever want to observationally verify our ideas about the microscopic structure of our universe, it also means that we have to be very careful in neglecting any part of the theory that we do not (yet) completely understand. By restricting ourselves to the part of the model we have control over, we could be throwing away the baby with the bath water. Although it is usual practice and often plain necessity to consider inflation in a controlled environment, one makes implicit, and possibly unrealistic, assumptions on the unknown parts of the theory in the way it (does not) contribute to the inflationary dynamics. As a result, the predictive power of the theory and its chance to be compared with observations from the early universe, are limited.

In this chapter we will see, in the context of supergravity, how hidden sectors affect the carefully controlled physics of any model for inflation. This will be a useful illustration of the sensitivity of inflation to unknown physics and of the importance to compare inflation observationally with a *complete* description of nature. The chapter is based on [151] and [152].

4.1 Introduction

The construction of realistic models of slow-roll inflation in supergravity is a long-standing puzzle. Supersymmetry can alleviate the finetuning necessary to obtain slow-roll inflation — if one assumes that the inflaton is a modulus of the supersymmetric ground state — but cannot solve it completely. This is most clearly seen in the supergravity η-problem: if the inflaton is a lifted modulus, then its mass in the inflationary background is proportional to the supersymmetry breaking scale. Therefore, the slow-roll parameter $\eta \simeq V''/V$ generically equals unity rather than a small number [153].

We will show here, however, that the η-problem is more serious than a simple hierarchy problem. In the conventional mode of study, the inflaton sector is always a subsector of the full supergravity theory presumed to describe our universe. When the inflationary subsector of the supergravity is studied *an sich*, tuning a few parameters of the Lagrangian to order 10^{-2} will generically solve the problem. We will clarify that this split of the supergravity sector into an inflationary sector and other hidden sectors implicitly makes the assumption that all the other sectors are in a "supersymmetric" ground state: i.e. if the inflaton sector —which must break supersymmetry— is decoupled, the ground state of the remaining sectors is supersymmetric. If this is not the case, the effect on the η-parameter or on the inflationary dynamics in general can be large, even if the sypersymmetry breaking scale in the hidden sector is small. Blind truncation in supergravities to the inflaton sector alone, if one does not know whether other sectors preserve supersymmetry, is therefore an inconsistent approach towards slow-roll supergravity inflation. Coupling the truncated sector back in completely spoils the naïve solution found. This result, together with recent qualitatively similar findings for sequestered supergravities (where only the potential has a two-sector structure) [154], provides strong evidence that to find true slow-roll inflation in supergravity one needs to know the global ground state of the system. The one obvious class of models where sector-mixing is not yet considered is the newly discovered manifest embedding of single field inflationary models in supergravity [155, 156]. If these models are also sensitive to hidden sectors, it would arguably certify the necessity of a global analysis for cosmological solutions in supergravity and string theory.

We will obtain our results on two-sector supergravities by an explicit calculation. The gravitational coupling between the hidden and the inflaton sectors is universal, which can be described by a simple F-term scalar supergravity theory. As in most discussions on inflationary supergravity theories, we will ignore D-terms as one expects its vacuum expectation value to be zero throughout the early universe [30]. Including D-terms (which themselves always need to be accompanied by F-terms [88]) only complicates the F-term analysis, which is where the η-problem resides. Furthermore, although true inflationary dynamics ought to be described in a fully kinetic description [52], we can already make our point by simply considering the mass eigenmodes of the system. In a strict slow-roll and slow-turn approximation the mass eigenmodes of the system determine the dynamics of the full system.

Specifically we shall show the following for two-sector supergravities where the sectors are distinguished by independent R-symmetry invariant Kähler functions:

- Given a naïve supergravity solution to the η-problem, this solution is only consistent if the other sector is in its supersymmetric ground state.

- If it is not in its ground state, then the scalar fields of that sector cannot be static but *must* evolve cosmologically as well.

- In order for the naïve solution to still control the cosmological evolution these fields must move very slowly. This translates in the requirement that the contribution to the first slow-roll parameter of the hidden sector must be much smaller than the contribution from the naïve inflaton sector, $\epsilon_{\text{hidden}} \ll \epsilon_{\text{naïve}}$.

- There are two ways to ensure that ϵ_{hidden} is small: Either the supersymmetry breaking scale in the hidden sector is very small or a particular linear combination of first and second derivatives of the generalized Kähler function is small.

  - In the latter case, one finds that the second slow-roll parameter $\eta_{\text{naïve}}$ receives a very large correction $\eta_{\text{true}} - \eta_{\text{naïve}} \gg \eta_{\text{naïve}}$, unless the supersymmetry breaking scale in the hidden sector is small. This returns us to the first case.
  - In the first case, one finds that the hidden sector always contains a light mode, because in a supersymmetry breaking (almost) stabilized supergravity sector there is always a mode that scales with the scale of supersymmetry breaking. This light mode will overrule the naïve single field inflationary dynamics.

Thus for *any* nonzero supersymmetry breaking scale in the hidden sector — even when this scale is very small — the true mass eigenmodes of the system are linear combinations of the hidden sector fields and the inflaton sector fields. We compute these eigenmodes. By assumption, the true value of the slow-roll parameter η is the smallest of these eigenmodes. Depending on the values of the supersymmetry breaking scale and the naïve lowest mass eigenstate in the hidden sector, we find that

1. The new set of mass eigenmodes can have closely spaced eigenvalues, and thus the initial assumption of single field inflation is incorrect. Then a full multi-field re-analysis is required.

2. The relative change of the value of η from the naïve to the true solution can be quantified and shows that for a supersymmetry breaking hidden sector, the naïve model is only reliable if the naïve lowest mass eigenstate in the hidden sector is much larger than the square of the scale of hidden sector supersymmetry breaking divided by the inflaton mass. This effectively excludes all models where the hidden sector has (nearly) massless modes.

3. The smallest eigenmode can be dominantly determined by the hidden sector, and thus the initial assumption that the cosmological dynamics is constrained to the inflaton sector is incorrect. Again a full multi-field re-analysis is required.

One concludes that in general one needs to know/assume the ground states and the lowest mass eigenstates of *all* the hidden sectors to reliably find a slow-roll inflationary supergravity.

The structure of this chapter is the following. Section 4.2 explains how sectors are coupled in supergravity. To make contact with global supersymmetry models, we consider the no-gravity limit of a multi-sector supergravity model. As we will see, decoupling in this limit turns out to be more delicate than just taking the simple $M_{pl} \to \infty$ limit. We begin the discussion on the effects of having multiple sectors in section 4.3 with the result that in a stabilized supergravity sector there always is a mode that scales with the scale of supersymmetry breaking. In section 4.4 the η-problem in a single sector theory is discussed and we consider the effect of a hidden sector qualitatively and quantitatively. The quantitative result is analyzed in section 4.5 both in terms of effective parameters and direct supergravity parameters. As a notable example of our result, we show that if the hidden sector is the standard model, where its supersymmetry breaking is not caused by the inflaton sector but otherwise, spoils the naïve slow-roll solution in the putative inflaton sector. The chapter is supplemented with two appendices in which some of the longer formulae are given.

4.2 Canonical coupling in supergravity

We shall start by arguing how two sectors are gravitationally coupled in supergravity. We will seek for a minimal (universal) coupling between sectors. It has an interesting interpretation in terms of the superpotentials, which multiply rather than add as in globally supersymmetric minimally coupled systems. As a result, the zero-gravity limit from multi-sector supergravities to decoupled multi-sector global supersymmetry theories is more subtle than the usual $M_{pl} \to \infty$ limit. To be able to embed the supersymmetry objects into a multi-sector supergravity theory, we will consider a possible decoupling limit with non-canonical scaling of the superpotential couplings. This limit will later be used to apply our general results to a standard model-like globally supersymmetric hidden sector in section 4.5.3

4.2.1 Maximal decoupling in supergravity

Multiple sectors are a common feature in supergravity cosmology and phenomenology. These sectors are necessary to either incorporate inflation or supersymmetry breaking or are a consequence of string model-building. In particular to study inflation, it is desirable to separate the dynamics of all fields that do not contribute to the exponential expansion of the universe from the inflaton fields that do. Since gravity is the weakest possible interaction, the inflationary sector is assumed to only couple gravitationally to an unknown hidden sector that may also break supersymmetry by itself. Whereas it is natural for a rigid supersymmetric theory to be separated into several sectors, the restrictive structure of supergravity forces the different sectors to couple not only non-locally through graviton exchange but also directly. For this reason embedding supersymmetric theories as sectors into a supergravity can be notoriously difficult, see e.g. [88, 89, 157–162].

Though multiple sector supergravities are a long studied subject, the context of cosmology has seriously sharpened the question. In supergravity models of inflation, it is commonly noted that one seeks a consistent truncation of the scalar sector. This is necessary but not sufficient. Even with a consistent truncation one may have dominating instabilities towards the naïvely non-dynamical sectors, that can move them away from their supersymmetric critical points. One needs either a symmetry constraint or an energy barrier to constrain the dynamics to the putative inflaton sector.

During inflation, supersymmetry is broken and although it is frugal to consider scenarios where the inflaton sector is also responsible for phenomenological supersymmetry breaking (see e.g. [163–165]), this need not be so. For instance, in a generic gauge-mediation scenario, the mechanism responsible for supersymmetry breaking need not involve the fields that drive inflation. This example immediately shows that the generic cosmological set-up must be able to account for a sector that breaks supersymmetry *independently* of the inflationary dynamics.

This consideration is our starting point. We consider a multiple-sector supergravity that decouples in the strictest sense in the limit $\mathrm{M_{pl}} \to \infty$. In this limit the action must then be the sum of two independent functions

$$S[\phi,\overline{\phi},q,\overline{q}] = S[\phi,\overline{\phi}] + S[q,\overline{q}], \tag{4.1}$$

such that the path integral factorizes.[1] ϕ and q denote the fields in the two sectors respectively. In the following, we will take the indices $\{i, \bar{\jmath}\}$ to run over the ϕ-fields, while $\{a, \bar{b}\}$ denote the fields in the q-sector. Later, we will take the ϕ-fields to drive inflation, while the q-fields reside in another sector which is naïvely assumed not to take part in the inflationary dynamics and is hence called the hidden sector.

For a globally supersymmetric field theory with a standard kinetic term, a decoupled action can be achieved by demanding that the independent Kähler and superpotentials sum as well,

$$K_{\text{susy}}(\phi, \bar{\phi}, q, \bar{q}) = K^{(1)}(\phi, \bar{\phi}) + K^{(2)}(q, \bar{q}), \quad W_{\text{susy}}(\phi, q) = W^{(1)}(\phi) + W^{(2)}(q). \quad (4.2)$$

By contrast, in supergravity complete decoupling in the sense of (4.1) appears to be impossible, even in principle. Even with block diagonal kinetic terms from a sum of Kähler potentials, the more complicated form of the supergravity potential (3.28) implies that there are many *direct* couplings between the two sectors. It raises the immediate question: if the low-energy $\mathrm{M_{pl}} \to \infty$ globally supersymmetric model must consist of decoupled sectors, what is the relation between $K_{\text{sugra}}, W_{\text{sugra}}$ and $K_{\text{susy}}, W_{\text{susy}}$, or vice versa given a globally supersymmetric model described by $K_{\text{susy}}, W_{\text{susy}}$, what is the best choice for $K_{\text{sugra}}, W_{\text{sugra}}$ such that the original theory can be recovered in the limit $\mathrm{M_{pl}} \to \infty$?

The conclusion of this section is that the scaling implied by the explicit factors of $\mathrm{M_{pl}}$ in the supergravity potential (3.28) is an incomplete answer to this question. The direct communication between the sectors, controlled by $\mathrm{M_{pl}}$, has serious consequences for both the ground state structure (solutions to the equation of motion, i.e. the cosmological dynamics) and the interactions between the two sectors. To be explicit, the first guess at how the rigid supersymmetry and supergravity Kähler potentials and superpotentials are related

$$K_{\text{sugra}}(\phi, \bar{\phi}, q, \bar{q}) = K^{(1)}_{\text{susy}}(\phi, \bar{\phi}) + K^{(2)}_{\text{susy}}(q, \bar{q}) + \ldots, \quad (4.3a)$$

$$W_{\text{sugra}}(\phi, q) = W^{(1)}_{\text{susy}}(\phi) + W^{(2)}_{\text{susy}}(q) + \ldots, \quad (4.3b)$$

with ... indicating Planck-suppressed terms and possibly a constant term, does not define a sensible way of splitting up the action in multiple sectors. This definition is not invariant under Kähler transformations in each sector separately and is valid only in a specific Kähler frame or, say, gauge dependent [166]. Another way to understand

[1] As example we consider the simplest case, a model with uncharged scalar supermultiplets $\xi^I = (\phi^i, q^a)$ that are singlets under all symmetries. Gauge interactions and global symmetries will not change this general argument provided the two sectors are not mixed by symmetries or coupled by gauge fields. Therefore, we will also ignore D-terms in the supergravity potential below.

the problem is to realize that the definition (4.3) does not lead to a Kähler metric and mass matrix that can be made block diagonal in the same basis [167], and thus there is no sense of “independent” sectors. Moreover, (4.3) suffers from the drawback that the ground states of the full theory are no longer the product of the ground states of the individual sectors, except when both (rather than only one) ground states are supersymmetric [168, 169] (see also [166, 167, 170]). This directly follows from considering the extrema of the supergravity potential[2]

$$\nabla_a V = \frac{\mathrm{D}_a W}{W} V + e^{K/\mathrm{M}_{\mathrm{pl}}^2}|W|^2 \left(\nabla_a \left(\frac{\mathrm{D}_b W}{W} \right) \frac{\mathrm{D}^b \overline{W}}{\overline{W}} + \frac{1}{\mathrm{M}_{\mathrm{pl}}^2} \frac{\mathrm{D}_a W}{W} + \nabla_a \left(\frac{\mathrm{D}_j W}{W} \right) \frac{\mathrm{D}^j \overline{W}}{\overline{W}} \right), \tag{4.4a}$$

$$\begin{aligned} \nabla_a \nabla_i V = {} & \frac{\mathrm{D}_i W}{W} \nabla_a V + \frac{\mathrm{D}_a W}{W} \nabla_i V - \frac{\mathrm{D}_a W}{W} \frac{\mathrm{D}_i W}{W} V + \mathrm{D}_a \left(\frac{\mathrm{D}_i W}{W} \right) \left(V + \frac{2}{\mathrm{M}_{\mathrm{pl}}^2} e^{K/\mathrm{M}_{\mathrm{pl}}^2} |W|^2 \right) \\ & + e^{K/\mathrm{M}_{\mathrm{pl}}^2}|W|^2 \left(\nabla_a \nabla_i \left(\frac{\mathrm{D}_j W}{W} \right) \frac{\mathrm{D}^j \overline{W}}{\overline{W}} + \nabla_i \nabla_a \left(\frac{\mathrm{D}_b W}{W} \right) \frac{\mathrm{D}^b \overline{W}}{\overline{W}} \right). \end{aligned} \tag{4.4b}$$

Supersymmetric ground states, for which the covariant derivatives of W vanish on the solution, $\mathrm{D}_i W = 0$ and $\mathrm{D}_a W = 0$, are still product solutions. But for Kähler and superpotentials that sum (4.3), even if only one sector is in a non-supersymmetric ground state, by which we mean $\mathrm{D}_a W = 0$, $\mathrm{D}_i W \neq 0$, we can neither conclude that sector 2 is in a minimum, for which $\nabla_a V$ would vanish, nor that the condition for sector 1 to be in a local ground state is independent of the sector 2 fields q^a, which would mean that $\nabla_a \nabla_i V = 0$. The former is only true when

$$\nabla_a \left(\frac{\mathrm{D}_j W}{W} \right) \frac{\mathrm{D}^j \overline{W}}{\overline{W}} = 0. \tag{4.5}$$

The second requires, in addition,

$$\nabla_a \nabla_i \left(\frac{\mathrm{D}_j W}{W} \right) \frac{\mathrm{D}^j \overline{W}}{\overline{W}} + \nabla_i \nabla_a \left(\frac{\mathrm{D}_b W}{W} \right) \frac{\mathrm{D}^b \overline{W}}{\overline{W}} = 0, \tag{4.6}$$

[2] To derive (4.4b) note that, since $\mathrm{D}W/W$ is Kähler invariant and since the Levi-Civita connection ∇ of the field space manifold does not get cross-contributions in a product manifold,

$$\nabla_a \frac{\mathrm{D}_i W}{W} = \partial_a \frac{\mathrm{D}_i W}{W} = \mathrm{D}_a \frac{\mathrm{D}_i W}{W}.$$

and also sharpens the first condition (4.5) to[3]

$$\mathrm{D}_a \frac{\mathrm{D}_i W}{W} = 0. \tag{4.7}$$

Equations (4.5–4.7) are conditions for decoupling which apply not only to the ground state of the full system but also to other critical points of the potential, for instance along an inflationary valley. Generically these conditions are not met on the solution (the second derivative need not vanish at an extremum; recall that $\mathrm{D}_a W$ does not vanish identically but only on the solution). Hence, generically the ground states of hidden sectors mix and this spoils many cosmological supergravity scenarios that truncate the action to one or the other sector (see e.g. [171] and references therein). It is this issue that is particularly relevant for inflationary model building, where a very weak coupling between the inflaton sector and all other sectors has to persist over an entire *trajectory* in field space where the expectation values of the fields are changing with time (see e.g. [52, 172–174]). At the same time, one is interested in the generic situation in which *both* sectors may contribute to supersymmetry breaking.[4]

[3]These conditions are merely sufficient not necessary. However, it is clear that the restrictive nature of supergravity enforces conditions on the unknown sectors for the system to be separate.

[4]This situation has to be contrasted to phenomenological models appropriate for studying gravity mediated supersymmetry breaking, such as an ansatz [175]

$$\begin{aligned} K(\phi,\overline{\phi},q,\overline{q}) &= K^{(0)}(\phi,\overline{\phi}) + q^a\overline{q}^{\overline{b}}K^{(1,1)}_{a\overline{b}}(\phi,\overline{\phi}) + q^a q^b K^{(2,0)}_{ab}(\phi,\overline{\phi}) + \overline{q}^{\overline{a}}\overline{q}^{\overline{b}}K^{(0,2)}_{\overline{a}\overline{b}}(\phi,\overline{\phi}) + \ldots, \\ W(\phi,q) &= W^{(0)}(\phi) + q^a q^b W^{(1)}_{ab}(\phi) + \ldots, \end{aligned}$$

or equivalently, if $W \neq 0$,

$$G(\phi,\overline{\phi},q,\overline{q}) = G^{(0)}(\phi,\overline{\phi}) + q^a\overline{q}^{\overline{b}}G^{(1,1)}_{a\overline{b}}(\phi,\overline{\phi}) + q^a q^b G^{(2,0)}_{ab}(\phi,\overline{\phi}) + \overline{q}^{\overline{a}}\overline{q}^{\overline{b}}G^{(0,2)}_{\overline{a}\overline{b}}(\phi,\overline{\phi}) + \ldots.$$

In models like these, it is understood that $\dot{q} = 0$ and the q-sector can remain in its supersymmetric critical point throughout the evolution of the supersymmetry breaking fields. For inflation, such an expectation is unrealistic, as the supersymmetry preserving sector can become unstable during the inflationary dynamics, see e.g. a recent discussion of the case in which the inflaton field ϕ is solely responsible for supersymmetry breaking during inflation ([165] and references therein). In this relatively simple case, and except for very fine-tuned situations, the generic scenario appears to be that one or more of the q-fields are destabilized somewhere along the inflationary trajectory and they trigger an exit from inflation (in other words, they become "waterfall" fields, and inflation is of the hybrid kind [176]). This implies that the pattern of supersymmetry breaking today is not related to the one during inflation, and also, since the waterfall fields are forced away from their supersymmetric critical points, that supersymmetry is broken by both sectors as the universe evolves towards the current vacuum.

4.2.2 Natural multi-sector supergravities

There is a natural way to construct supergravity potentials for which the ground states (and critical points) do separate better. This obvious combination of superpotentials automatically satisfies (4.5–4.7) and hence does ensure that if one of the ground states is supersymmetric, the ground state of the other sector is a decoupled field theory ground state whether it breaks supersymmetry or not. This is if we choose a product of superpotentials, keeping the sum of Kähler potentials as before,

$$K_{\text{sugra}}(\phi,\overline{\phi},q,\overline{q}) = K^{(1)}_{\text{sugra}}(\phi,\overline{\phi}) + K^{(2)}_{\text{sugra}}(q,\overline{q}), \quad W_{\text{sugra}}(\phi,q) = \frac{1}{\mathrm{M}_{\text{pl}}^3} W^{(1)}_{\text{sugra}}(\phi) W^{(2)}_{\text{sugra}}(q). \tag{4.8}$$

This is well-known [177–179] and has recently been emphasized in the context of cosmology [166, 167, 170, 171, 173, 174, 180, 181]. This ansatz conforms to the more natural description of supergravities in terms of the Kähler invariant function (3.29) that can be defined if W is non-zero in the region of interest.[5] In turn, the Kähler function underlies a better description of multiple sectors in supergravity, where G is a simple sum of independent functions

$$G(\phi,\overline{\phi},q,\overline{q}) = G^{(1)}(\phi,\overline{\phi}) + G^{(2)}(q,\overline{q}). \tag{4.9}$$

It is invariant under Kähler transformations in each sector separately [166–169, 182] and thus defines a sensible way of splitting up the action in multiple sectors. As a result, this split guarantees that a BPS solution in one particular sector is a BPS solution of the full theory. It is the simplest ansatz that still allows some degree of calculational control when both sectors break supersymmetry —as well as optimizing decoupling along the inflationary trajectory. One of the simplest models of hybrid inflation in supergravity, F-term inflation [183, 184], is in this class.

The sum of Kähler functions (4.9) implies the conventional separation of the Kähler potentials, but it constitutes a class of minimally coupled scenarios due to the multiplicative nature of the superpotentials put forward above. Let us illustrate the importance of this multiplicative superpotential in the situation in which the hidden sector resides in a supersymmetric vacuum, i.e. $\partial_a V(q_0) = 0$ and $\partial_a G^{(2)}(q_0) = 0$. We write the superpotential of the hidden sector as $W^{(2)}(q) = W^{(2)}_0 + W^{(2)}_{\text{dyn}}(q - q_0)$. The second term in this expression is what determines the potential for fluctuations around the minimum of the hidden sector, while the first constant term is just an overall contribution and hence not interesting for the internal hidden sector dynamics at

[5] We expect this condition to hold around a supersymmetry breaking vacuum with almost vanishing cosmological constant. It also holds in many models of supergravity inflation, although a notable exception is [155, 156].

energies much less than the Planck scale. However, for the gravitational dynamics and the remaining ϕ-sector this "vacuum energy contribution" $W_0^{(2)} = \langle W^{(2)} \rangle$ is of crucial importance as it sets the scale of the potential

$$V = e^{K^{(2)}/\mathrm{M}_{\mathrm{pl}}^2} |W_0^{(2)}|^2 e^{G^{(1)}} \left(G_i^{(1)} G^{(1)i} - 3\mathrm{M}_{\mathrm{pl}}^2 \right) \mathrm{M}_{\mathrm{pl}}^{-4}, \tag{4.10}$$

which is evaluated at $q = q_0$ such that all terms depending on $W_{\mathrm{dyn}}^{(2)}$ vanish. The normal practice of setting $W_0^{(2)}$ to zero as an overall contribution to the hidden sector is neglecting the fact that gravity also feels the constant part of the potential energy, as opposed to field theory. The inflationary sector feels the presence of the hidden sector through this coupling and as such it may be more intuitive to regard $W_0^{(2)}$ to contain information about the inflationary sector rather than the hidden sector. Making a similar split in $W^{(1)}$, the constant part $W_0^{(1)}$ is the overall contribution to the hidden sector due to the inflaton sector.

Using the minimal coupling scenario (4.9), the two-sector action (3.27) reads

$$S = \mathrm{M}_{\mathrm{pl}}^2 \int \mathrm{d}^4 x \sqrt{g} \left[\frac{1}{2} R - g^{\mu\nu} (G_{i\bar{\jmath}}^{(1)} \partial_\mu \phi^i \partial_\nu \bar{\phi}^{\bar{\jmath}} + G_{a\bar{b}}^{(2)} \partial_\mu q^a \partial_\nu \bar{q}^{\bar{b}}) - V \mathrm{M}_{\mathrm{pl}}^2 \right], \tag{4.11}$$

with

$$V(\phi, \bar{\phi}, q, \bar{q}) = e^{G^{(1)} + G^{(2)}} \left(G_i^{(1)} G^{(1)i} + G_a^{(2)} G^{(2)a} - 3 \right). \tag{4.12}$$

We will often allow ourselves to drop the sector label from G in the remainder, since $G_\phi^{(1)} = G_\phi$ and similarly for q. For a short overview of relevant conventions and identities in supergravity, we refer the reader to appendix 4.A. For later calculational convenience, we have given (4.11) and (4.12) in terms of the dimensionless scalar fields $\xi^I = (\phi^i, q^a)$ and functions V, G, K and W. However, before we start the exploration of the inflationary consequences of a coupling such as (4.11), we will momentarily keep the M_{pl}-dependence explicit (and quantities dimensionful) and study the no-gravity limit $\mathrm{M}_{\mathrm{pl}} \to \infty$ to see how the supergravity sectors decouple.

4.2.3 Zero-gravity decoupling limit

Given that we have just argued that a product of superpotentials is a more natural framework to discuss hidden sector supergravities, the obvious question arises how to recover a decoupled *sum* of potentials for a globally supersymmetric theory in the limit where gravity decouples, i.e. in which

$$V_{\mathrm{sugra}} = e^{K/\mathrm{M}_{\mathrm{pl}}^2} \left(|\mathrm{D}W|^2 - \frac{3|W|^2}{\mathrm{M}_{\mathrm{pl}}^2} \right) \;\to\; V_{\mathrm{susy}} = \sum_n |\partial_n W^{(n)}|^2.$$

For a two-sector supergravity defined by equations (4.8) one would not find this answer, if one takes the standard decoupling limit $\mathrm{M}_{\mathrm{pl}} \to \infty$ with both $K = K^{(1)} + K^{(2)}$ and $W = \mathrm{M}_{\mathrm{pl}}^{-3} W^{(1)} W^{(2)}$ fixed[6]. Instead, the product structure of the superpotential introduces a cross-coupling between sectors,

$$V_{\mathrm{eff}} = \frac{1}{\mathrm{M}_{\mathrm{pl}}^3}\left(|W^{(2)}|^2|\partial_\alpha W^{(1)}|^2 + |W^{(1)}|^2|\partial_i W^{(2)}|^2\right) \neq V_{\mathrm{susy}},$$

whose behavior under the limit $\mathrm{M}_{\mathrm{pl}} \to \infty$ is best examined at the level of the superpotential.

Supergravity is sensitive to the expectation value $W_0 = \langle W \rangle$ of W, which relates the scale of supersymmetry breaking to the expectation value of the potential, i.e. the cosmological constant

$$\Lambda^2 \mathrm{M}_{\mathrm{pl}}^2 = \langle V \rangle \sim \langle \mathrm{D}W^2 \rangle - \frac{3}{\mathrm{M}_{\mathrm{pl}}^2}\langle W^2 \rangle = m_{\mathrm{susy}}^4 - 3\frac{W_0^2}{\mathrm{M}_{\mathrm{pl}}^2}.$$

The vacuum expectation value cannot vanish in a supersymmetry breaking vacuum with (nearly) zero cosmological constant, such as our universe. Therefore, in the following we assume $\langle W \rangle \neq 0$ in the region of interest. Instead of the usual way to incorporate it, $W_{\mathrm{sugra}} = W_0 + W_{\mathrm{dyn}}$ with $W_{\mathrm{dyn}} = W_{\mathrm{susy}} + \ldots$, we include the vacuum expectation value for a two-sector product superpotential by writing

$$\begin{aligned} W(\phi, q) &= \frac{1}{\mathrm{M}_{\mathrm{pl}}^3} W^{(1)} W^{(2)} = \frac{1}{\mathrm{M}_{\mathrm{pl}}^3}\left(W_0^{(1)} + W_{\mathrm{dyn}}^{(1)}(\phi)\right)\left(W_0^{(2)} + W_{\mathrm{dyn}}^{(2)}(q)\right) \\ &= \frac{1}{\mathrm{M}_{\mathrm{pl}}^3}\left(W_0^{(1)} W_0^{(2)} + W_0^{(2)} W_{\mathrm{dyn}}^{(1)}(\phi) + W_0^{(1)} W_{\mathrm{dyn}}^{(2)}(q) + W_{\mathrm{dyn}}^{(1)}(\phi) W_{\mathrm{dyn}}^{(2)}(q)\right). \end{aligned} \tag{4.13}$$

This is physically equivalent to a sum of superpotentials except for the last term. Note again that if one uses the standard scaling, $\frac{\phi}{\mathrm{M}_{\mathrm{pl}}} \to 0$, $\frac{q}{\mathrm{M}_{\mathrm{pl}}} \to 0$ with all couplings in

[6]Strictly speaking the decoupling limit sends $\mathrm{M}_{\mathrm{pl}} \to \infty$ while keeping the fields ϕ, q fixed with $W^{(n)}/\mathrm{M}_{\mathrm{pl}}^3$ a holomorphic function of $\phi/\mathrm{M}_{\mathrm{pl}}$ or $q/\mathrm{M}_{\mathrm{pl}}$ and $K^{(n)}/\mathrm{M}_{\mathrm{pl}}^2$ a real function of $\phi/\mathrm{M}_{\mathrm{pl}}, \overline{\phi}/\mathrm{M}_{\mathrm{pl}}$ or $q/\mathrm{M}_{\mathrm{pl}}, \overline{q}/\mathrm{M}_{\mathrm{pl}}$. The limit zooms in to the origin so K must be assumed to be non-singular there. Formally the decoupling limit does not exist otherwise. Physically it means that one is taking the decoupling limit with respect to an a priori determined ground state, around which K and W are expanded. If K is non-singular at the origin, the overall factor $e^{K/\mathrm{M}_{\mathrm{pl}}^2}$ yields an overall constant as $\mathrm{M}_{\mathrm{pl}} \to \infty$, which may be set to unity, i.e. the constant part of K vanishes. In the decoupling limit, both K and W may then be written as polynomials. Letting the coefficients in W and K scale as their canonical scaling dimension such that W has mass dimension three and K has mass dimension two, then gives the rule of thumb that both K and W are held fixed as $\mathrm{M}_{\mathrm{pl}} \to \infty$.

$W^{(\mathrm{total})}$ having the canonical scaling dimensions, this last term contains renormalizable couplings involving the scalar partner of the goldstino, and these are not Planck-suppressed: if supersymmetry is broken by the ϕ sector, terms of the form ϕq^2 are renormalizable and would survive the $\mathrm{M}_{\mathrm{pl}} \to \infty$ limit, leading to a direct coupling between the two sectors.[7] If both sectors break supersymmetry then mass-mixing terms ϕq also survive. All such (relevant) terms are of course absent if none of the two sectors break supersymmetry, but this is not the case we are interested in. One would have expected that these cross-couplings naturally vanish in the decoupling limit.

The point of this section is simply to remark that the realization that each of the superpotentials $W^{(n)} = W_0^{(n)} + W_{\mathrm{dyn}}^{(n)}$ contains a constant term can resolve this conundrum by assuming a non-standard scaling for the constituent parts $W_0^{(n)}$, $W_{\mathrm{dyn}}^{(n)}$. To achieve a decoupling we need that the cross term $W_{\mathrm{dyn}}^{(1)} W_{\mathrm{dyn}}^{(2)}$, which contains the coupling between the two sectors, scales away in the limit $\mathrm{M}_{\mathrm{pl}} \to \infty$. As a result the first term in (4.13) has to diverge, because its product with the cross term should remain finite. In particular we can choose an overall scaling

$$W = \frac{1}{\mathrm{M}_{\mathrm{pl}}^3}(\underbrace{W_0^{(1)} W_0^{(2)}}_{\sim \mathrm{M}_{\mathrm{pl}}^{3+r}} + \underbrace{W_0^{(1)} W_{\mathrm{dyn}}^{(2)}}_{\sim \mathrm{M}_{\mathrm{pl}}^{3}} + \underbrace{W_0^{(2)} W_{\mathrm{dyn}}^{(1)}}_{\sim \mathrm{M}_{\mathrm{pl}}^{3}} + \underbrace{W_{\mathrm{dyn}}^{(1)} W_{\mathrm{dyn}}^{(2)}}_{\sim \mathrm{M}_{\mathrm{pl}}^{3-r}}), \tag{4.14}$$

with $r > 0$. Let us account for dimensions by introducing an extra scale m_Λ such that

$$\begin{aligned} W_0^{(1)} &= m_\Lambda^{\frac{3-r}{2}-A} \mathrm{M}_{\mathrm{pl}}^{\frac{3+r}{2}+A}, & W_{\mathrm{dyn}}^{(1)} &= \mathrm{M}_{\mathrm{pl}}^3 \frac{W_{\mathrm{susy}}^{(1)}}{W_0^{(2)}}, \\ W_0^{(2)} &= m_\Lambda^{\frac{3-r}{2}+A} \mathrm{M}_{\mathrm{pl}}^{\frac{3+r}{2}-A}, & W_{\mathrm{dyn}}^{(2)} &= \mathrm{M}_{\mathrm{pl}}^3 \frac{W_{\mathrm{susy}}^{(2)}}{W_0^{(1)}}, \end{aligned} \tag{4.15}$$

with $W_{\mathrm{susy}}^{(n)}$ fixed as $\mathrm{M}_{\mathrm{pl}} \to \infty$. Formally one can choose an inhomogeneous scaling with $A \neq 0$, but as we shall see it has no real consequences. For any A it is easily seen

[7] For a product of superpotentials we can always choose a Kähler gauge *at every point* with $\langle K \rangle = \langle \partial_\phi K \rangle = \langle \partial_q K \rangle = 0$ without mixing the superpotentials. In that case F-term supersymmetry breaking is given by the linear terms in the expansion of $W^{(1)}$ and $W^{(2)}$: $\langle \mathrm{D}_\phi W \rangle \sim \langle \partial_\phi W^{(1)} \rangle$, $\langle \mathrm{D}_q W \rangle \sim \langle \partial_q W^{(2)} \rangle$.

that with this scaling,

$$\begin{aligned}
\mathrm{D}_i W &= \partial_i W^{(1)}_{\mathrm{susy}} + \frac{m_\Lambda^{r-3}}{\mathrm{M}_{\mathrm{pl}}^r} W^{(2)}_{\mathrm{susy}} \partial_i W^{(1)}_{\mathrm{susy}} \\
&\quad + \frac{\partial_i K^{(1)}}{\mathrm{M}_{\mathrm{pl}}^2}\left(m_\Lambda^{3-r}\mathrm{M}_{\mathrm{pl}}^r + W^{(1)}_{\mathrm{susy}} + W^{(2)}_{\mathrm{susy}} + \frac{m_\Lambda^{r-3}}{\mathrm{M}_{\mathrm{pl}}^r} W^{(1)}_{\mathrm{susy}} W^{(2)}_{\mathrm{susy}} \right) \to \partial_i W^{(1)}_{\mathrm{susy}},
\end{aligned}$$

in the limit $\mathrm{M}_{\mathrm{pl}} \to \infty$ if and only if $0 < r < 2$ and thus

$$V_{\mathrm{sugra}} = e^{K/\mathrm{M}_{\mathrm{pl}}^2}\left(|\mathrm{D}W|^2 - \frac{3|W|^2}{\mathrm{M}_{\mathrm{pl}}^2} \right) \to \sum_n |\partial_n W^{(n)}_{\mathrm{susy}}|^2 - 3m_\Lambda^{2(3-r)}\mathrm{M}_{\mathrm{pl}}^{2(r-1)} + O\left(\frac{1}{\mathrm{M}_{\mathrm{pl}}}\right).$$

For $r < 1$ the manifestly constant term in the potential vanishes as well and we recover the strict decoupled field theory result, with the gravitino mass going to zero as $m_{3/2} = \langle W\rangle \mathrm{M}_{\mathrm{pl}}^{-2} = m_\Lambda^{3-r}\mathrm{M}_{\mathrm{pl}}^{r-2} = \frac{m^2_{\mathrm{susy}}}{\sqrt{3}\mathrm{M}_{\mathrm{pl}}}$. We see that the gravitino mass is independent of r in physical scales.

The parameter r should not be larger than unity for the new decoupling limit to be well defined. For the special case $r = 1$ [177], the potential has an additional overall "cosmological" constant. For a generic non-gravitational field theory in which $\mathrm{M}_{\mathrm{pl}} \to \infty$ this is just an overall shift of the potential, which we can arbitrarily remove since it does not change the physics. Nevertheless from a formal point of view, we know that absolute ground state energy of a globally supersymmetric theory equals zero, as a result of the supersymmetry algebra $\{Q, Q\} = H$. For this reason it is more natural to restrict the value of r to the range $0 < r < 1$.

It may appear that we have changed the canonical renormalization group scaling of the theory. This is not quite true. For the interacting terms in the potential, it is the coefficients in the product $W_0^{(2)} W^{(1)}_{\mathrm{dyn}} = W^{(1)}_{\mathrm{susy}}$ that ought to obey canonical renormalization group scaling. This precisely corresponds to holding $W^{(n)}_{\mathrm{susy}}$ fixed as $\mathrm{M}_{\mathrm{pl}} \to \infty$ (see footnote 6). On the other hand, the scaling of the constant term in the potential has changed from its canonical value. However, this is very natural in a supersymmetric theory. The constant term, $\prod_n W_0^{(n)}$, equals the ground state energy. Precisely supersymmetric theories can "naturally" explain non-canonical scaling of the cosmological constant (at the loop level; the scaling of the bare ground state energy can be different in every model). A non-integer power is strange but $r = 1$ is certainly a viable option in a supersymmetry-breaking ground state: it is the natural scaling in theories with higher supersymmetry [185] when combined with a subleading $\log(\mathrm{M}_{\mathrm{pl}}/m_{\mathrm{susy}})$ breaking. Our engineering analysis only focuses on power-law scaling and these can always have subleading logarithms. ($r = 2$ would correspond to the cosmological constant for a spontaneously broken $\mathcal{N} = 1$ theory due to mass splitting).

The novel scaling in (4.15) can be readily generalized to an arbitrary number of sectors. For s sectors, writing $W^{(n)} = W_0^{(n)} + W_{\text{dyn}}^{(n)}$ for each sector, the superpotential $W = \frac{1}{\mathrm{M}_{\text{pl}}^{3(s-1)}} \prod_{n=1}^{s} W^{(n)}$ becomes

$$W = \frac{1}{\mathrm{M}_{\text{pl}}^{3(s-1)}} \left[\prod_{n=1}^{s} W_0^{(n)} + \sum_{m=1}^{s} \left(W_{\text{dyn}}^{(m)} \prod_{n \neq m}^{s} W_0^{(n)} \right) + \sum_{l>m}^{s} \left(W_{\text{dyn}}^{(m)} W_{\text{dyn}}^{(l)} \prod_{n \neq l,m}^{s} W_0^{(n)} \right) + \ldots \right].$$

In this expression, we want the last term before the ... and all terms on the ... to scale away as $\mathrm{M}_{\text{pl}}^{-r}$ or stronger under $\mathrm{M}_{\text{pl}} \to \infty$, where $r > 0$. The second term(s) should be constant. As a consequence the first term will scale as $\mathrm{M}_{\text{pl}}^{r}$. Assuming a scaling that is homogeneous across sectors, this implies

$$W_0^{(n)} \sim \mathrm{M}_{\text{pl}}^{\frac{3(s-1)+r}{s}}, \qquad W_{\text{dyn}}^{(n)} \sim \mathrm{M}_{\text{pl}}^{\frac{(3-r)(s-1)}{s}},$$

for each of the $n \in \{1, \ldots, s\}$. With this scaling, a general term consisting of t dynamical superpotentials and $s - t$ constant parts, scales as

$$\frac{W_{\text{dyn}}^{t} W_0^{s-t}}{\mathrm{M}_{\text{pl}}^{3(s-1)}} \sim \mathrm{M}_{\text{pl}}^{r(1-t)},$$

and as constructed any term containing dynamical interactions between sectors, $t \geq 2$, is Planck-suppressed. To ensure a vanishing constant term as in equation (4.2.3), r is again limited to the range $0 < r < 1$.

4.3 Zero mass mode for a stabilized sector

Anticipating the situation for an inflationary scenario we now analyze the mass spectrum of a stabilized q-sector in a de Sitter background. For Minkowski spaces it is known that the lightest mass in a stabilized sector scales with the supersymmetry breaking vacuum expectation value G_a [186]. Here we extend the analysis to de Sitter vacua as the zeroth order approximation of slow-roll inflation. Already in this zeroth order approach we will show that a similar light mode develops in the stabilized sector. Throughout this discussion we assume that the potential V is kept positive by the presence of the "inflationary" sector. In the next section we show that this result can be translated directly into an inflationary setting, where this light mode will affect the slow-roll dynamics.

Given that we insist the q-sector to be stabilized, we have $\partial_a V = 0$. In terms of the Kähler function $G(\phi, \overline{\phi}, q, \overline{q})$ this means

$$(\nabla_a G_b)G^b = -G_a(1 + e^{-G}V).$$

If the q-ground state breaks supersymmetry, i.e. $G_a \neq 0$, we may rewrite it in terms of the supersymmetry breaking direction $f_a = G_a/\sqrt{G^b G_b}$,

$$(\nabla_a G_b)f^b = -f_a(1 + e^{-G}V).$$

For simplicity we will assume that the q-sector consists of only a single complex scalar field q, in which case we may write this equation as

$$\nabla_q G_q = -G_{q\overline{q}}(1 + e^{-G}V)\widehat{G}_q^2. \tag{4.16}$$

A hat $\widehat{z}$ on a complex number denotes the "phase"-part of the number, $z = |z|\widehat{z} = |z|e^{i\arg(z)}$. As such $\widehat{G}_q = \sqrt{G^{q\overline{q}}}f_q$. Note that in an arbitrary supersymmetric configuration $G_a = 0$ there are no restrictions on $\nabla_a G_b$, but on a supersymmetry broken configuration this is no longer true. Were one to turn on supersymmetry breaking, one would first have to reach a surface in parameter space where this restriction can be imposed at the onset of supersymmetry breaking.

We will now compute the mass spectrum for the two modes of the complex scalar field q, at the hypersurface defined by (4.16). The mass modes are given by the eigenvalues of the matrix

$$M^2 = \begin{pmatrix} V^q{}_q & V^q{}_{\overline{q}} \\ V^{\overline{q}}{}_q & V^{\overline{q}}{}_{\overline{q}} \end{pmatrix},$$

which in our case means

$$m_q^{\pm} = \left(V^q{}_q \pm |V^q{}_{\overline{q}}|\right) = G^{q\overline{q}}\left(V_{q\overline{q}} \pm |V_{qq}|\right). \tag{4.17}$$

Expanding the second derivatives of the potential (cf. appendix 4.B) to first order in $|G_q|$, these eigenvalues are

$$m_q^- = e^G G^{q\overline{q}}\mathrm{Re}\{(\nabla_q\nabla_q G_q)\widehat{G^q}^3\}|G^q| + O(|G_q|^2), \tag{4.18a}$$

$$m_q^+ = e^G\left[2(2 + e^{-G}V)(1 + e^{-G}V) - G^{q\overline{q}}\mathrm{Re}\{(\nabla_q\nabla_q G_q)\widehat{G^q}^3\}|G^q|\right] + O(|G_q|^2). \tag{4.18b}$$

We see from (4.18a) that in the limit of vanishing supersymmetry breaking the lightest mass mode becomes massless, just as in the case of Minkowski space [186].[8] It is

[8] The result can also be extended to hold for anti-de Sitter vacua. However, for $-2 < e^{-G}V < -1$, also a tachyonic mode develops.

important to note that this result depends crucially on taking the limit G_q to zero in the supersymmetry breaking direction. When supersymmetry is restored and both $G_q = 0$ and $G_{\overline{q}} = 0$, the phases of these vectors have no meaning. In fact, we see that then a new degree of freedom arises: $\nabla_q G_q$ becomes unrestricted which allows one to choose the masses freely.

The geometrical picture is that there is a whole plane of supersymmetric solutions where arbitrary masses are allowed. However, when supersymmetry is broken, the supersymmetry breaking direction has to align with its complex conjugate fixing one point on this plane where supersymmetry can be broken. In this point, the lightest mode becomes massless.

4.4 Two-sector inflation in supergravity

Generally, when inflation is described in supergravity, realistic matter resides in a hidden sector.[9] Supergravities descending from string theory often have additional hidden sectors as well. These sectors are always gravitationally coupled. In the previous section we have seen that for de Sitter vacua the hidden sector develops a light direction. In this section we will consider how this light mode of the hidden sector can affect the naïve dynamics of the inflationary sector. We will show that despite the weakness of gravity, these effects can be large. Realistic slow-roll inflation is characterized by small numbers, the slow-roll parameters ϵ and η, and even small absolute changes to these numbers can be of the order of 100% in relative terms.

We will first briefly review the η-problem in the context of single field inflation in supergravity. Then we will explain what effects are to be expected when including an additional (hidden) sector. The section ends with calculating the relevant objects to determine the true dynamics of the full system.

4.4.1 Inflation and the η-problem in supergravity

In single scalar field models of inflation the spectrum of density perturbations is characterized by the two slow-roll parameters ϵ and η. To ensure that this spectrum matches the observed near scale invariance, both $\epsilon \ll 1$ and $\eta \ll 1$. Inflationary supergravity in its simplest form consists of a single complex scalar field, the inflaton, whose potential is generated by F-terms (3.28). The definition of η may be phrased

[9] The supersymmetric partners of the standard model are not good inflaton candidates, as these partners are charged under the standard model gauge group and gauge fields taking part in inflation would lead to topological defects [eg. 187, 188]. The exception could be a gravitationally non-minimally coupled Higgs field [eg. 189, 190].

as the lightest direction of the mass matrix in units of the Hubble rate $3H^2 = V$, i.e. η is the smallest eigenvalue of the matrix, cf. equation (2.7), [191]

$$\widetilde{N} = \frac{1}{V}\begin{pmatrix} \nabla^i\nabla_j V & \nabla^i\nabla_{\bar{j}} V \\ \nabla^{\bar{i}}\nabla_j V & \nabla^{\bar{i}}\nabla_{\bar{j}} V \end{pmatrix},$$

where the tilde on $\widetilde{N}$ indicates that this value of η is defined with respect to the inflaton sector only.[10] From the second ϕ-derivative of V,

$$V_{i\bar{j}} = G_{i\bar{j}}V + G_i V_{\bar{j}} + G_{\bar{j}}V_i - G_i G_{\bar{j}}V + e^G\left[R_{\bar{i}\bar{j}k\bar{l}}G^k G^{\bar{l}} + G^{k\bar{l}}\nabla_i G_k \nabla_{\bar{j}} G_{\bar{l}} + G_{i\bar{j}}\right],$$

we see that a natural value for η is $V^i{}_j/V \sim \nabla^i G_j \sim 1$ is unity. Therefore, we must tune G_i, $\nabla_i G_j$ and $R_{\bar{i}\bar{j}k\bar{l}}$ so that $V^i{}_j = O(10^{-3})V$. The necessity of this tuning is known as the η-problem.

As shown in [194], successful inflation is achievable if one tunes the Kähler function G such that

$$R_{\bar{i}\bar{j}k\bar{l}} f^i f^{\bar{j}} f^k f^{\bar{l}} \lesssim \frac{2}{3}\frac{1}{1+\gamma},$$

where $\gamma = e^{-G}V/3$ is inversely proportional to an overall mass scale $m_{3/2} = e^{G/2}$, which is related to the gravitino mass and $R_{\bar{i}\bar{j}k\bar{l}}$ is the Riemann tensor of the inflaton sector. As $f^i f_i = 1$, the above equation defines the normalized sectional curvature along the direction of supersymmetry breaking. The constraint becomes stronger as $\gamma \gg 1$, thus as $H \gg m_{3/2}$. When the bound is met, one can always tune η to be small by tuning G_i, $\nabla_i G_j$ and $R_{\bar{i}\bar{j}k\bar{l}}$.

Finding a suitably tuned supergravity potential from a (UV-complete) string theoretical set-up has proven to be incredibly difficult [195, 196], but possible [197–199]. Currently, in models with correctly tuned slow-roll parameters it is typically assumed that the "hidden sector" does not affect the finetuning of parameters. The subject of this chapter is to examine whether such an assumption is justified and hence how relevant tuned models are that only consider the inflationary sector.

4.4.2 Stability of the hidden sector during inflation

Having reviewed the η-problem in single sector supergravity theories, we will now consider if and how the fields in the hidden sector can affect the inflationary evolution. From the diagonalization of the kinetic terms in (4.11) the distinction between ϕ-fields

[10] A careful definition based on the kinetic behavior of the inflaton field is done in [192, 193]. In the slow-roll, slow-turn limit, it reduces to the definition of η given here.

and q-fields is explicit, leading naturally to an inflationary and a hidden sector. We will again assume these sectors to both consist of only one complex scalar field, ϕ and q respectively. The argument we shall present can already be made in a two-field system. It carries through to multi-field models because the field ϕ is viewed as the inflaton in an effective single field inflationary model, while the field q can be seen as the lightest mode in the hidden sector. Following the usual practice [1, 2, and references therein], we assume that inflation is solved by tuning the inflationary sector only, including obtaining satisfactory values for the slow-roll parameters from a phenomenological viewpoint. As a result all data in the inflationary sector are fixed and known. Contrarily, the hidden sector is left unspecified and the restrictions we find on it are a function of model specific parameters of the inflaton sector only.

To ensure that the hidden sector does not take part in the inflationary dynamics, one generally assumes that the fields in the hidden sector are stabilized in a ground state at a constant field value $q = q_0$ throughout inflation

$$\partial_q V\Big|_{q_0} = 0 \tag{4.19}$$

and, hence, are not dynamical. Clearly an extremum for the hidden sector is obtained if $G_q = 0$, i.e. when the ground state of the hidden sector preserves supersymmetry. As was shown in detail in [155, 156, 166–170, 174], when $G_q = 0$ the ground state of the hidden sector decouples gravitationally from the inflationary sector and the inflationary sector truly determines the inflationary evolution without any contributions from the hidden sector. The stability of the extremum of the hidden sector, however, depends on the inflationary trajectory and a stable extremum might develop into an instability, leading to a waterfall for the hidden sector fields and, as a result, to the end of inflation, as discussed in [166, 170].

The case we examine here is when supersymmetry is broken in the hidden sector, $G_q \neq 0$. The first thing to note is that the stability assumption (4.19) cannot be met anymore. In supergravity the position $q = q_0$ of the minimum of the potential is given by

$$V_q = G_q V(\phi, \overline{\phi}, q, \overline{q}) + e^{G(\phi,\overline{\phi},q,\overline{q})} \left((\nabla_q G_q) G^q + G_q \right) = 0,$$

which shows that for $G_q \neq 0$ the ground state q_0 depends on the inflaton field ϕ, through $V(\phi, \overline{\phi}, q, \overline{q})$ and $G(\phi, \overline{\phi}, q, \overline{q})$. In the situation of unbroken supersymmetry, $G_q = 0$, all ϕ-dependence drops out, but for $G_q \neq 0$ we see that it is impossible to keep the position of the minimum constant during inflation. As the inflaton ϕ rolls down the inflaton direction, the "stabilized" hidden scalar q will change its value. It is clear that the assumption of a vanishing $V_q = 0$ for all q is incompatible with $G_q \neq 0$ and we should therefore abandon it. This in turn means that the hidden sector field

q must be dynamical, through its equation of motion. Since we still want to identify the field ϕ as the inflaton in the sense that it drives the cosmological dynamics, we have to assume that q moves very little. We must therefore also assume a slow-roll, slow-turn approximation to the solution of the q equation of motion

$$\dot{q} = \frac{G^{q\bar{q}} V_{\bar{q}}}{3H}.$$

The statement that the cosmological dynamics is driven by the ϕ-sector means that $||\dot{q}|| \ll ||\dot{\phi}||$, where $||\dot{q}|| \equiv \sqrt{G_{q\bar{q}}\dot{q}\dot{\bar{q}}}$, etc. Through both slow-roll equations of motion this equates to $||V_q|| \ll ||V_\phi||$ or $\epsilon_q \ll \epsilon_\phi$,

As the hidden sector has now become dynamical, we have to treat the system as a multi-field inflationary model. Since it is impossible to diagonalize the Kähler transformations and mass matrix simultaneously, the fields will mix in the case of a hidden sector with broken supersymmetry [166]. In the next section we will study the consequences of this mixing by explicitly diagonalizing the mass matrix of the full two-field system. From the result we shall find three possible effects on the inflationary dynamics.

First, the lightest masses of fields from the different sectors can be too close together. It is obvious that one cannot consider an effective single field model if this is the case, since for the dynamics to be independent of initial conditions, the lightest field needs to be much lighter than the other fields. When the masses of the two fields are similar, both of them contribute to the dynamics, resulting into a multi-field rather than a single field inflationary scenario. As is known from the literature, a multi-field inflationary model will produce effects such as isocurvature modes [eg. 67, 200–213], features in the power spectrum [eg. 52, 214–216] and non-Gaussianities [isocurvature models and eg. 58, 59, 217–224], pointing to a qualitatively different model.

Second, a change of the true value of η can occur. We have assumed the inflaton sector to be tuned in such a way that it agrees with observed values for the slow-roll parameters. If the effects of the hidden sector on the total dynamics are such that η will change significantly, the initial naïve tuning would be of no meaning and one would have to start the tuning process all over again after the hidden sector has been added. Again we note that there is no contribution in the case of unbroken supersymmetry in the hidden sector, since we shall show that the contribution to η from the hidden sector is mostly determined by the cross terms in the mass matrix,

$$V_{\phi q} = G_\phi V_q + G_q V_\phi - G_\phi G_q V,$$

which vanish when $G_q = 0$.

Third, a complete change of the sector that determines η is possible. It is possible that the eventual η-parameter is still within the limits of its naïve tuned value, satisfying the second bound, but instead it is determined by the hidden sector rather than the inflationary sector. Any initial control obtained by tuning the inflationary sector is superseded by the sheer coincidental configuration of the hidden sector.

4.4.3 The mass matrix of a two-sector system

To investigate when effects from the hidden sector are to be expected, we need to calculate the eigenvalues of the mass matrix of the full two-field system. Since we assume the inflationary evolution to be in the slow-roll, slow-turn regime, the dynamics is completely potential energy dominated. The mass matrix of the full two-field system determines which directions are stable or steep, as characterized by the eigenvalues of this matrix. Normalizing by $1/V$ to obtain the value of η directly, the matrix we want to diagonalize is the 4×4-matrix

$$N = \frac{1}{V}\begin{pmatrix} \nabla^I \nabla_J V & \nabla^I \nabla_{\bar{J}} V \\ \nabla^{\bar{I}} \nabla_J V & \nabla^{\bar{I}} \nabla_{\bar{J}} V \end{pmatrix}. \tag{4.20}$$

Equation (4.20) is to be evaluated at a point near $q_0 = q_0(\phi_0)$, where q_0 is such that $\partial_q V(q_0) = 0$, with ϕ_0 indicating the beginning of inflation. As is clear from the discussion of section 4.4.2 we cannot truly expect the hidden sector to be stabilized throughout the inflationary evolution. Nevertheless we may consider $\partial_q V(q_0) = 0$ at a certain point $q_0 = q_0(\phi_0)$, with $\|\partial_q V\| \ll \|\partial_\phi V\|$ around q_0 in accordance with the restriction $\epsilon_q \ll \epsilon_\phi$.

The mass matrix is Hermitian and, considering again a two-field system, can be put in the form

$$N = \frac{1}{V}\begin{pmatrix} \nabla^\phi V_\phi & \nabla^\phi V_{\bar{\phi}} & \nabla^\phi V_q & \nabla^\phi V_{\bar{q}} \\ \nabla^{\bar{\phi}} V_\phi & \nabla^{\bar{\phi}} V_{\bar{\phi}} & \nabla^{\bar{\phi}} V_q & \nabla^{\bar{\phi}} V_{\bar{q}} \\ \nabla^q V_\phi & \nabla^q V_{\bar{\phi}} & \nabla^q V_q & \nabla^q V_{\bar{q}} \\ \nabla^{\bar{q}} V_\phi & \nabla^{\bar{q}} V_{\bar{\phi}} & \nabla^{\bar{q}} V_q & \nabla^{\bar{q}} V_{\bar{q}} \end{pmatrix},$$

by a coordinate transformation. Diagonalizing the full matrix in general is involved. Therefore, we adopt the strategy to diagonalize the two sectors separately and then pick the lightest modes only. The first step yields

$$N = \begin{pmatrix} \frac{1}{V}(V^\phi{}_\phi - |V^\phi{}_{\bar{\phi}}|) & 0 & A_{11} & A_{12} \\ 0 & \frac{1}{V}(V^\phi{}_\phi + |V^\phi{}_{\bar{\phi}}|) & A_{21} & A_{22} \\ \bar{A}_{11} & \bar{A}_{21} & \frac{1}{V}(V^q{}_q - |V^q{}_{\bar{q}}|) & 0 \\ \bar{A}_{12} & \bar{A}_{22} & 0 & \frac{1}{V}(V^q{}_q + |V^q{}_{\bar{q}}|) \end{pmatrix},$$

with

$$A = \frac{1}{2V}\begin{pmatrix} -\widehat{V_{\overline{\phi\phi}}} & \widehat{V_{\overline{\phi\phi}}} \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} V^{\phi}{}_{q} & V^{\phi}{}_{\bar{q}} \\ V^{\bar{\phi}}{}_{q} & V^{\bar{\phi}}{}_{\bar{q}} \end{pmatrix} \begin{pmatrix} -\widehat{V_{\overline{qq}}} & \widehat{V_{\overline{qq}}} \\ 1 & 1 \end{pmatrix}.$$

Here, the first matrix is the inverse of the similarity transformation of the ϕ-sector and the last matrix diagonalizes the q-sector.

In general the eigenmodes in the individual sectors will be different, one always being smaller than the other. Dynamically the most relevant direction is the lightest mode of each sector, but by restricting to these light directions, one assumes a hierarchy already within the sectors. For the inflationary sector this is phenomenologically justified if we assume that inflation is described by a single field, where we know that $V^{\phi}{}_{\phi}$ and $V^{\phi}{}_{\bar{\phi}}$ combine such that a light mode appears with mass ηV, much lighter than the other mass modes. For the hidden sector we will simply assume that a large enough hierarchy between mass modes exists. This will simplify matters without weakening our result. By including only the lightest mode of the hidden sector, we can already show that the true dynamics is in many cases not correctly described by the naïve inflaton sector. Our case would only be more strongly supported if we would include the heavy mode of the hidden sector, but this is technically more involved. Projecting on the light directions, we get a submatrix of light mass modes

$$N_{\text{light}} = \begin{pmatrix} \lambda_{\phi} & A_{11} \\ \overline{A}_{11} & \lambda_{q} \end{pmatrix},$$

with

$$\lambda_{\phi} = \frac{1}{V}\left(V^{\phi}{}_{\phi} - |V^{\phi}{}_{\bar{\phi}}|\right) = \frac{G^{\phi\bar{\phi}}}{V}(V_{\phi\bar{\phi}} - |V_{\phi\phi}|), \tag{4.21a}$$

$$\lambda_{q} = \frac{1}{V}\left(V^{q}{}_{q} - |V^{q}{}_{\bar{q}}|\right) = \frac{G^{q\bar{q}}}{V}(V_{q\bar{q}} - |V_{qq}|), \tag{4.21b}$$

$$A_{11} = \frac{G^{\phi\bar{\phi}}}{2V}\left(\widehat{V_{\overline{qq}}}\,\widehat{V_{\phi\phi}}\,V_{\bar{\phi}q} - \widehat{V_{\overline{qq}}}\,V_{\phi q} + V_{\phi\bar{q}} - \widehat{V_{\phi\phi}}\,V_{\overline{\phi q}}\right). \tag{4.21c}$$

The eigenvalues of this two-field system are given by

$$\mu_{\pm} = \frac{1}{2}\left(\lambda_{\phi} + \lambda_{q}\right) \pm \frac{1}{2}\sqrt{\left(\lambda_{q} - \lambda_{\phi}\right)^{2} + 4|A_{11}|^{2}}. \tag{4.22}$$

Since $\mu_{-} < \mu_{+}$ the second slow-roll parameter for the full system is given by $\eta = \mu_{-}$.

4.5 Dynamics due to the hidden sector

In the slow-roll and slow-turn approximation, the mass modes $\mu_\pm$ from (4.22) determine the dynamics of the full system. In general the true dynamics will deviate from the naïve single sector evolution. As explained in section 4.4.2 it is necessary to put constraints on the full system for the true dynamics to still (largely) agree with the initial naïve dynamics. We will quantify these constraints in terms of the hidden sector light mode λ_q and the dynamical cross coupling $|A_{11}|$ between sectors. The results are graphically summarized in figures 4.1 and 4.2. In section 4.5.2 and figure 4.3 we will discuss the result again but then interpreted from the viewpoint of supergravity. Finally we will explain that a simple application of these bounds implies that the standard model cannot be ignored during cosmological inflation, if standard model supersymmetry breaking is independent of the inflaton sector.

4.5.1 Conditions on the hidden sector data

From (4.22) we see that the light modes λ_ϕ, λ_q from the two separate sectors mix through a cross coupling $|A_{11}|$ and combine to the true eigenvalues $\mu_\pm$ of the full two-sector system. As explained in 4.4.2, for the inflaton sector to still describe the cosmological evolution and the η-parameter reliably, the three constraints it must obey are (1) the bound arising from demanding a hierarchy between $\mu_\pm$ to prevent multi-field effects, (2) the bound arising from demanding the second slow-roll parameter $\mu_- = \eta$ to not change its value too much and (3) the bound from demanding that η is mostly determined by the ϕ-sector rather than the q-sector.

To prevent multi-field effects from setting in, we take as a minimum hierarchy that μ_+ is at least five times as heavy as μ_- in units of the scale of the problem $|\mu_-|$,

$$\frac{\mu_+ - \mu_-}{|\mu_-|} > 5. \tag{4.23}$$

This bound is rather arbitrary, but clearly a hierarchy between μ_+ and μ_- must exist. Calculations in [215] show that for a mass hierarchy $\lesssim 5$ multi-field effects are typically important.

The second bound is given by the A_{11}-dependence of μ_-. The value of the second slow-roll parameter from the single field inflationary model only is $\eta_{\text{naïve}} = \lambda_\phi$. In the full two-sector system, μ_- takes over the role as the true second slow-roll parameter $\eta_{\text{true}} = \mu_-$. The contribution to the actual η-parameter from the presence of the hidden sector is therefore

$$\Delta\eta = \mu_- - \lambda_\phi = \frac{1}{2}\left[(\lambda_q - \lambda_\phi) - \sqrt{(\lambda_q - \lambda_\phi)^2 + 4|A_{11}|^2}\right], \tag{4.24}$$

which is always negative. We argue that this difference should stay within $|\Delta\eta/\lambda_\phi| < 0.1$, i.e. η should not change by more than 10%. This choice for the range of η is given by current experimental accuracy. Current experiments can only determine $n_s = 1 - 6\epsilon + 2\eta$. WMAP has a 1σ error of 6.53% [10], Planck will have an error of 0.70% [69]. For $n_s - 1$, assuming 0.96, this gives a 17.5% error on the combination of $-6\epsilon + 2\eta$, which means an uncertainty of about 10% on the value of η.

We will examine λ_q, A_{11} in units of $|\lambda_\phi|$ and exclude regions in which the hidden sector affects the tuned inflationary sector too much. The analysis is best done separately for the cases $\lambda_\phi = \eta_{\text{naïve}} > 0$ and $\lambda_\phi = \eta_{\text{naïve}} < 0$ because of the qualitative differences between these cases.

The case $\eta_{\text{naïve}} > 0$

We first examine the hierarchy bound as explained above and focus first on the situation where $\mu_- > 0$. In this case (4.23) means that we demand

$$\frac{\mu_+ - 6\mu_-}{\lambda_\phi} = \frac{1}{2}\left[-5\left(\frac{\lambda_q}{\lambda_\phi} + 1\right) + 7\sqrt{\left(\frac{\lambda_q}{\lambda_\phi} - 1\right)^2 + 4\left(\frac{|A_{11}|}{\lambda_\phi}\right)^2}\right] > 0,$$

which allows us to solve λ_q/λ_ϕ as a function of $|A_{11}|/\lambda_\phi$,

$$\left(\frac{12}{35}\right)^2\left(\frac{\lambda_q}{\lambda_\phi} - \frac{37}{12}\right)^2 + \left(\frac{2\sqrt{6}}{5}\right)^2\left(\frac{|A_{11}|}{\lambda_\phi}\right)^2 = 1.$$

This excludes everything inside the ellipse demarcating the green region in figure 4.1. The case $\mu_- < 0$ is not relevant as it is already excluded by the second bound.

For this second bound, to be somewhat more general than the observationally inspired constraint $\Delta\eta/\lambda_\phi > -0.1$, we give the bound $\Delta\eta/\lambda_\phi > -f$. Solving for λ_q this gives

$$\frac{\lambda_q}{\lambda_\phi} > 1 - f + \frac{1}{f}\left(\frac{|A_{11}|}{\lambda_\phi}\right)^2,$$

as is indicated in blue in figure 4.1. Note that since the true value of η is always lower than $\eta_{\text{naïve}}$ (see [225] for some specific examples), a change in η of 100% means that η changes sign from its naïve value. This shows that we were justified to only consider positive μ_- in the hierarchy bound earlier.

The third bound is given by a λ_q-dominance in μ_-. Since λ_ϕ and λ_q are treated on equal footing in μ_-, the true η is dominantly determined by the smallest eigenvalue,

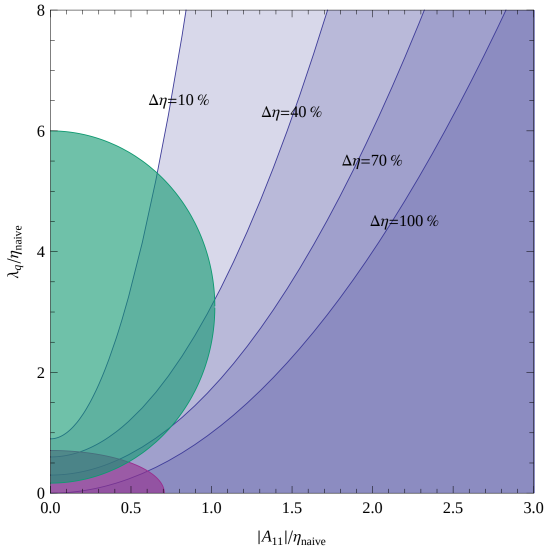

Figure 4.1: *Bounds from a dynamical hidden sector for $\eta_{\text{naïve}} > 0$. The multi-field constraint excludes an ellipse near the λ_q-axis (shaded in green). The bound from having too much effect on η excludes large $|A_{11}|$ (shaded with increasing intensities of blue for larger deviations). Around $\lambda_q = A_{11} = 0$ the hidden sector mode λ_q rather than λ_ϕ determines η, excluding that region as well (shaded in purple).*

which is not necessarily λ_ϕ. When $\lambda_\phi \gg \lambda_q$ and $\lambda_\phi \gg |A_{11}|$ we see immediately that the true $\eta = \mu_-$ is determined by λ_q and is *independent* of λ_ϕ,

$$\mu_- = \frac{1}{2}\left[(\lambda_q + \lambda_\phi) - \lambda_\phi\left(1 - \frac{\lambda_q}{\lambda_\phi} + O\left(\frac{\lambda_q^2}{\lambda_\phi^2}, \frac{|A_{11}|^2}{\lambda_\phi^2}\right)\right)\right].$$

It is clear that this arguments excludes the lower left corner of parameter space. We will take the bound to be $1/\sqrt{2}$ such that $\left(\lambda_q/\lambda_\phi\right)^2, \left(|A_{11}|/\lambda_\phi\right)^2 < 1/2 \ll 1$, the radius of convergence of this Taylor expansion. Contrarily to the somewhat debatable bounds imposed by $\Delta\eta/\lambda_\phi$, the points within this circle are truly excluded because they violate one of the core assumptions in the approach, viz. that the ϕ-sector is responsible for all cosmological dynamics including determining the value of η. The circle

$$\left(\frac{\lambda_q}{\lambda_\phi}\right)^2 + \left(\frac{|A_{11}|}{\lambda_\phi}\right)^2 = \frac{1}{2},$$

is indicated as the purple region in the figure.

In figure 4.1 we have indicated in which regions of λ_q/λ_ϕ- and $|A_{11}|/\lambda_\phi$-parameter space the effects of a hidden sector can be rightfully ignored. We have shown that all negative values of λ_q are excluded and only in the region with large λ_q/λ_ϕ and small $|A_{11}|/\lambda_\phi$ there are no large effects from the hidden sector. This result is qualitatively easily understood, as the hidden sector with broken supersymmetry will still decouple if the masses in the hidden sector are truly large. We argue that this possibility is too easily assumed to be the case in the literature without considering the actual hidden constraints it imposes on the hidden sector. These hidden assumptions should be mentioned explicitly and one should show that they can be obtained.

The case $\eta_{\text{naïve}} < 0$

In the case that $\lambda_\phi = \eta_{\text{naïve}}$ is negative, the last bound of section 4.5.1 does not impose any condition on $\lambda_q/|\lambda_\phi|, |A_{11}|/|\lambda_\phi|$-parameter space. When $\lambda_\phi < 0$, i.e. when $\lambda_\phi = -|\lambda_\phi|$, the eigenvalues can be written as

$$\mu_\pm = \frac{|\lambda_\phi|}{2}\left[\left(\frac{\lambda_q}{|\lambda_\phi|} - 1\right) \pm \sqrt{\left(\frac{\lambda_q}{|\lambda_\phi|} + 1\right)^2 + 4\left|\frac{A_{11}}{\lambda_\phi}\right|^2}\right],$$

which means that μ_- is not determined by λ_q to first order in $\lambda_q/|\lambda_\phi|$ but by λ_ϕ as should be,

$$\mu_- = \frac{|\lambda_\phi|}{2}\left[\left(\frac{\lambda_q}{|\lambda_\phi|} - 1\right) - \left(1 + \frac{\lambda_q}{|\lambda_\phi|} + \ldots\right)\right].$$

However, by the hierarchy bound the small $\lambda_q/|\lambda_\phi|$-regime does get excluded. Since μ_- is always negative in this case,

$$\mu_- \leq \frac{|\lambda_\phi|}{2}\left[\left(\frac{\lambda_q}{|\lambda_\phi|} - 1\right) - \left|\frac{\lambda_q}{|\lambda_\phi|} + 1\right|\right] = -|\lambda_\phi|,$$

equation (4.23) translates into

$$\frac{\mu_+ + 4\mu_-}{|\lambda_\phi|} = \frac{1}{2}\left[5\left(\frac{\lambda_q}{|\lambda_\phi|} - 1\right) - 3\sqrt{\left(\frac{\lambda_q}{|\lambda_\phi|} + 1\right)^2 + 4\left|\frac{A_{11}}{\lambda_\phi}\right|^2}\right] > 0.$$

This excludes everything beneath the upper branch of the hyperbola given by the line

$$\frac{\lambda_q}{|\lambda_\phi|} > \frac{17}{8} + \frac{1}{8}\sqrt{15^2 + 28\left|\frac{A_{11}}{\lambda_\phi}\right|^2},$$

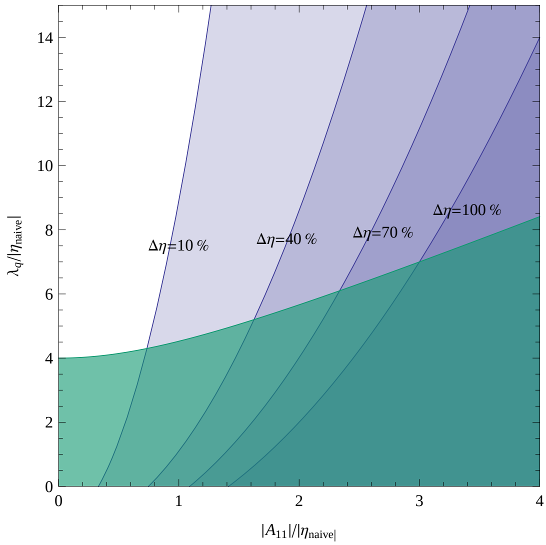

Figure 4.2: *Bounds from a dynamical hidden sector for $\eta_{\text{naïve}} < 0$. The multi-field bound excludes a hyperbola starting at $\lambda_q = 4|\lambda_\phi|$ and, in particular, small λ_q (shaded in green). The bound from having too much effect on η excludes the large $|A_{11}|$-region (shaded with increasing intensities of blue for larger deviations), but leaves open in particular the full range of λ_q.*

which is shaded green region in figure 4.2.

The final constraint on the parameter space comes from the bound on the change in η, see the previous paragraph on the $\eta_{\text{naïve}} > 0$-case for a discussion. In the blue region in figure 4.2 we have indicated the bound $|\Delta\eta/\lambda_\phi| < f$, which means

$$\frac{\lambda_q}{|\lambda_\phi|} > -1 - f + \frac{1}{f}\left|\frac{A_{11}}{\lambda_\phi}\right|^2,$$

for different fractions of f.

In figure 4.2 we have indicated in which regions of $\lambda_q/|\lambda_\phi|$- and $|A_{11}|/|\lambda_\phi|$-parameter space the effects of a hidden sector can be rightfully ignored after imposing both constraints. As in the case for $\eta_{\text{naïve}} > 0$, the only allowed region is for large $\lambda_q/|\lambda_\phi|$ and small $|A_{11}|/|\lambda_\phi|$. Note that all values of $\lambda_q < 4|\lambda_\phi|$ are explicitly excluded by the imposed bounds.

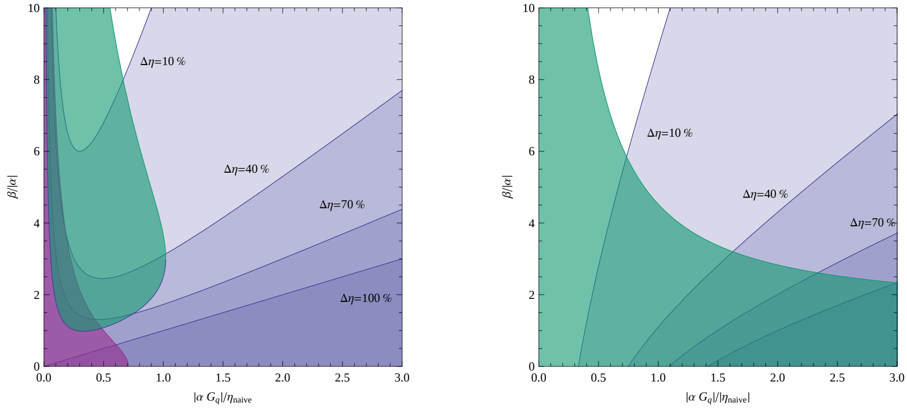

Figure 4.3: *Excluded regions for the supergravity parameter range for $|G_q|$ and β, which contains in particular $\nabla_q \nabla_q G_q$, in units of $|\eta_{\text{naïve}}|$ and $|\alpha|$, which contains ϵ_ϕ and G_ϕ. The indicated regions come from the multi-field bound (shaded in green), the correct identification of sectors (shaded in purple) and allowing only for small deviations of η (shaded in higher intensities of blue for larger deviations). The left (right) picture describes the case $\eta_{\text{naïve}} > 0$ ($\eta_{\text{naïve}} < 0$).*

4.5.2 Conditions on supergravity models

In principle, figures 4.1 and 4.2 provide all the information needed to verify whether the hidden sector of a given model may be neglected while studying the inflationary dynamics. Through equations (4.21) and the expressions for V_{IJ} as summarized in appendix 4.A, one can explicitly calculate the corresponding λ_q and A_{11} for a given model and compare them with the figures. However, we would like to have some direct intuition about the dependence of the excluded regions on the supergravity data. In this section we will investigate how much we can say about this in general without having to specify a model. The main question to answer is whether the fact that λ_q and A_{11} are determined by a supergravity theory, provides any additional constraint on which regions are obtainable to begin with. The answer to this question turns out to be that a priori supergravity is not restrictive enough to exclude any of the regions in λ_q, A_{11}-parameter space.

The easiest way to translate figures 4.1 and 4.2 in terms of supergravity data would be to simply map the regions into supergravity parameter space. Unfortunately the expressions in (4.21) are highly nonlinear and depend on too many supergravity variables to conveniently represent figures 4.1 and 4.2 in terms of supergravity data. However, for small $|G_q|$ this does turn out to be possible.

Applying the explicit expressions for V_{IJ} as found in appendix 4.A to (4.21c), yields

$$
\begin{aligned}
A_{11} &= \alpha(\phi, \overline{\phi}, q, \overline{q})|G_q|, \qquad \text{with} \qquad (4.25)\\
\alpha(\phi, \overline{\phi}, q, \overline{q}) &= \frac{G^{\phi\overline{\phi}}}{2}\left(\widehat{G_{\overline{q}}} - \widehat{V_{\overline{qq}}}\widehat{G_q}\right)\left(\left(\frac{V_\phi}{V} - G_\phi\right) - \widehat{V_{\phi\phi}}\left(\frac{V_{\overline{\phi}}}{V} - G_{\overline{\phi}}\right)\right).
\end{aligned}
$$

From this equation we learn that A_{11} vanishes in the limit $G_q \to 0$, which makes sense as we know that the two sectors should decouple in the limit of restored supersymmetry. It is difficult to retrieve more information from this explicit expression of A_{11} in terms of supergravity data. In principle $A_{11}(|G_q|, \ldots)$ may be inverted to give some function $|G_q|(A_{11}, \ldots)$, but this is trickier than (4.25) suggests. Although we have managed to extract one factor of G_q, the function $\alpha(\phi, \overline{\phi}, q, \overline{q})$ still depends on G_q through the phases of $\widehat{V_{\overline{qq}}}$ and $\widehat{V_{\phi\phi}}$, making it hard to perform the inversion explicitly.

The expression for λ_q looks even worse,

$$
\lambda_q = \frac{G^{q\overline{q}}}{V}\left(V_{q\overline{q}} - \sqrt{V_{qq}V_{\overline{qq}}}\right). \qquad (4.26)
$$

At this stage we have refrained from substituting in the expressions for $V_{q\overline{q}}$, V_{qq} and its complex conjugate. The square root clearly shows that the dependence of λ_q on $|G_q|$ and the other supergravity data is involved and difficult to invert. To get a useful expression we revert to the result of section 4.3 and consider λ_q in the small $|G_q|$-regime by performing a Taylor expansion. Copying from (4.18a), we find

$$
\begin{aligned}
\lambda_q &= \beta(\phi, \overline{\phi}, q, \overline{q})|G^q| + O(|G_q|^2), \qquad \text{with} \qquad (4.27)\\
\beta(\phi, \overline{\phi}, q, \overline{q}) &= \frac{G^{q\overline{q}}}{e^{-G}V}\mathrm{Re}\{(\nabla_q\nabla_q G_q)\widehat{G^q}^3\}.
\end{aligned}
$$

Having obtained the relations (4.25) and (4.27) we can now accommodate the reader with a graph of the allowed and excluded regions directly in terms of the supergravity data. For small $G_q \ll 1$ both λ_q and $|A_{11}|$ scale linearly with G_q, making it relatively easy to rewrite the bounds we found $\lambda_q/|\lambda_\phi| = \lambda_q/|\lambda_\phi|\left(|A_{11}|/|\lambda_\phi|\right)$ in terms of G_q, α and β as $\beta/|\alpha| = \beta/|\alpha|\left(|\alpha G_q|/|\lambda_\phi|\right)$. The resulting figure is depicted in 4.3. Note that α and β are still underdetermined — depending on $R_{q\overline{q}q\overline{q}}$ and $\nabla_q\nabla_q G_q$ at higher orders in $|G_q|$ — and are naturally of order 1. It is these numbers that determine where in figure 4.3 the model under investigation lies.

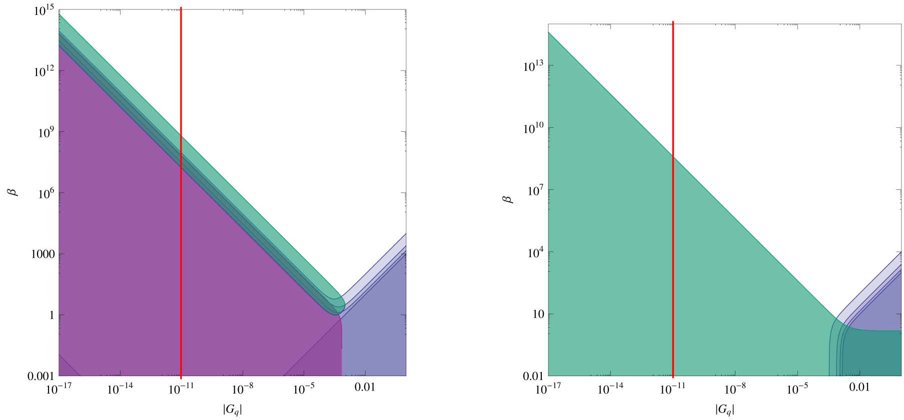

Figure 4.4: *The effects of the multi-field bound (shaded in green), the identification of the correct inflaton sector (shaded in purple) and the small deviations of η (shaded in blue) on a doubly logarithmic scale for $\eta_{\text{naïve}} > 0$ (left) and $\eta_{\text{naïve}} < 0$ (right). The approximate location of the standard model supergravity data is indicated with a red bar, showing that a large range of parameters is excluded. In this plot $\alpha = 1$ and $\lambda_\phi = \eta_{\text{naïve}} = 10^{-3}$.*

4.5.3 Inflation and the standard model of particle physics

As a simple application of the previous section, we can consider to what extent the standard model ought to be included in any reliable supergravity model for cosmological inflation. Our current understanding of nature includes a present-day supersymmetrically broken standard model after an inflationary evolution right after the big bang. As such the combined model is exactly that of a two-sector supergravity theory with an inflationary and a hidden sector whose ground state breaks supersymmetry in which it resides throughout the inflationary era.

Supersymmetry in the standard model sector can either have been broken by gravity mediation of the inflaton sector or by a mechanism in the standard model sector itself. The first situation would be a consistent approach as far as our analysis goes: as $G_q = 0$ the sector decouples from the inflationary dynamics, might be stabilized and the slow-roll parameters are reliably determined from the inflaton sector alone. Nevertheless, from the point of view of our understanding of the standard model it would be unsatisfactory to not know the precise mechanism behind its supersymmetry breaking and (complete) models describing such mechanisms would still have to be analyzed to shed light on the situation.

In the second situation, $G_q \neq 0$, we should apply the results of the previous sections. The field q may be seen as some light scalar degree of freedom in the (supersymmetrically broken) standard model. We assume the standard lore, that supersymmetry is broken in the standard model at a scale of about 1 TeV. In the F-term scalar potential, this scale enters via G_q. To determine the correct numerical value, we relate our dimensionless definition of the Kähler function to the standard dimensionful definition. Dimensionful quantities are denoted with a tilde in the following.[11] We recall from section 4.2.3 that in order to have a non-vanishing vacuum energy, the superpotential in both sectors must have a non-zero constant term $W_0^{(1)} = m_\Lambda^{(1)}/\mathrm{M_{pl}}$, $W_0^{(2)} = m_\Lambda^{(2)}/\mathrm{M_{pl}}$, which accounts for the always present gravitational coupling between the sectors. Hence, the dimensionful constant term in the total superpotential (4.13) has value $\widetilde{W}_0^{\mathrm{tot}} = W_0^{(1)}W_0^{(2)}\mathrm{M_{pl}^3} = m_\Lambda^{(1)}m_\Lambda^{(2)}\mathrm{M_{pl}}$. In contrast, the supergravity quantities $\widetilde{K}^{(2)}$ and $\widetilde{W}^{(2)}_{\mathrm{susy}} = \widetilde{W}_0^{(1)}\widetilde{W}^{(2)}_{\mathrm{dyn}}/\mathrm{M_{pl}^3}$ describing the standard model are naturally of the order of the TeV-scale, $[\widetilde{W}^{(2)}_{\mathrm{susy}}] = \mathrm{TeV}^3$, $[\partial_{\tilde{q}}\widetilde{K}^{(2)}] = \mathrm{TeV}$. We relate the scale of supersymmetry breaking $\widetilde{G_{\tilde{q}}}$ to the superpotential via

$$\widetilde{G}_{\tilde{q}} = \frac{\mathrm{M_{pl}^2}}{\widetilde{W}}\left(\partial_{\tilde{q}}\widetilde{W} + \frac{\partial_{\tilde{q}}\widetilde{K}^{(2)}}{\mathrm{M_{pl}^2}}\widetilde{W}\right),$$

which is naturally of order

$$\left[\widetilde{G}_{\tilde{q}}\right] = \frac{\mathrm{M_{pl}^2}}{m_\Lambda^{(1)}m_\Lambda^{(2)}\mathrm{M_{pl}} + \ldots}\left(\mathrm{TeV}^2 + \frac{\mathrm{TeV}}{\mathrm{M_{pl}^2}}(m_\Lambda^{(1)}m_\Lambda^{(2)}\mathrm{M_{pl}} + \ldots)\right) = \frac{\mathrm{M_{pl}TeV^2}}{m_\Lambda^{(1)}m_\Lambda^{(2)}} + \mathrm{TeV} + \ldots,$$

where the ... are of subleading order. We expect that $m_\Lambda^{(1)}$, the constant term of the inflaton sector, is of order $[H] = 10^{-5}\mathrm{M_{pl}}$, while $[m_\Lambda^{(2)}] = \mathrm{TeV}$. Hence, translating back to dimensionless units, we find $G_q \sim 10^{-11}$.

Taking the kinetic gauge, i.e. a canonical Kähler metric $G_{\phi\bar{\phi}} = 1$, we can easily find the natural value of α. From (4.25) we see that α depends on ϵ_ϕ and G_ϕ via

$$\alpha \propto \sqrt{\epsilon_\phi} - G_\phi,$$

modulo some unknown but negligible phase factors. G_ϕ is of order $\sqrt{3}$ in order to have a potential $V > 0$. Since ϵ_ϕ is of order $O(10^{-3})$, the value of $|\alpha|$ is of order unity. For a rough estimate of $\eta_{\text{naïve}} \sim 10^{-3}$, we can therefore pinpoint the standard model as indicated in figure 4.4. In both cases, $\eta_{\text{naïve}} > 0$ as well as $\eta_{\text{naïve}} < 0$, the lightest

[11] E.g. in dimensionful units $[\widetilde{G}] = \mathrm{mass}^2$ and $[\widetilde{q}] = \mathrm{mass}$, while our conventions are $[G] = [q] = 0$. To relate G_q to $\widetilde{G_{\tilde{q}}}$ we can use the expression $\left[G_q\right] = \frac{[\widetilde{G_{\tilde{q}}}]}{\mathrm{M_{pl}}}$.

supersymmetric particle is too light for the single sector inflationary dynamics to truly describe the full system. Any tuned and working inflationary supergravity model in which the standard model is assumed to not take part in the cosmic evolution, requires implicit assumptions on the standard model that either the inflaton sector is responsible for standard model supersymmetry breaking through gravity mediation or the masses of the standard model scalar multiplets are unnaturally large in terms of the now independent standard model supersymmetry breaking scale.

4.6 Conclusions

We have studied the effect of hidden sectors on the finetuning of F-term inflation in supergravity, identifying a number of issues in the current methodology. Finetuning inflationary models is only valid when the neglected physics does not affect this finetuning, in which case the inflationary physics can be studied independently. As shown in figures 4.1 and 4.2 this assumption holds only under very special circumstances. The reason is that the everpresent gravitational couplings will always lead to a mixing of the hidden sectors with the inflationary sector, even in the case of the most minimally coupled action (4.11). For a hidden sector vacuum that preserves supersymmetry, the sectors decouple consistently [166–169, 182]. However, for a supersymmetry breaking vacuum the inflationary dynamics is generically altered, where the nature and the size of the change depends on the scale of supersymmetry breaking.

For a hidden sector with a low scale of supersymmetry breaking, like the standard model, the cross coupling scales with the scale of supersymmetry breaking, and is therefore typically small. Yet, as shown in section 4.3, the lightest mass of the hidden sector depends as well on the scale of supersymmetry breaking within that sector. This light mode is strongly affected by the inflationary physics and thus evolves during inflation. Therefore, any single field analysis is completely spoiled as discussed in section 4.5.3.

For massive hidden sectors, the problem is more traditional. For a small hidden sector supersymmetry breaking scale, one has a conventional decoupling as long as the lightest mass of the hidden sector is much larger than the inflaton mass. However, for large hidden sector supersymmetry breaking, this intuition fails. Then, the off-diagonal terms in the mass matrix (4.20) will lead to a large correction of the η-parameter.

To conclude, any theory that is working by only tuning the inflaton sector has made severe hidden assumptions about the hidden sector, which typically will not be easily met. Methodologically the only sensible approach is to search for inflation in a full theory, including knowledge of all hidden sectors.

4.A Some supergravity relations

For easy reference to the reader, we use this appendix to state the relevant derivatives of the supergravity potential of a two-sector system coupled via

$$G(\phi^i, \overline{\phi}^{\bar{\imath}}, q^a, \overline{q}^{\bar{a}}) = G^{(1)}(\phi^i, \overline{\phi}^{\bar{\imath}}) + G^{(2)}(q^a, \overline{q}^{\bar{a}}). \tag{4.28}$$

We use middle-alphabet indices $\{i, \bar{\imath}\}$ to denote the fields in the inflationary sector, beginning-alphabet indices $\{a, \overline{a}\}$ to denote the fields in the hidden sector and capital middle-alphabet indices $\{I, \overline{I}\}$ to denote the full system. Derivatives with respect to these fields are denoted by subscripts, e.g. $\partial_i G = G_i$ and $\partial_i \partial_j G = G_{ij}$. The Hessian $G_{I\overline{J}}$ describes the metric of the (product-) manifold parameterized by the fields. This is a Kähler manifold and hence $\nabla_I G_{\overline{J}} = G_{I\overline{J}}$.

The supergravity potential is

$$V = e^G(G_I G^I - 3) = e^G(G_{\overline{I}} G^{\overline{I}} - 3) = e^G(G_a G^a + G_i G^i - 3).$$

Its covariant derivatives are denoted with subscripts (note that this is a different convention than the one used for the Kähler function G), e.g. $\nabla_i V = \partial_i V = V_i$ and $\nabla_i \nabla_j V = V_{ij}$. In terms of derivatives of G, the first derivatives of V are given by

$$V_i = G_i V + e^G \left((\nabla_i G_j) G^j + G_i \right), \tag{4.29a}$$

$$V_{\bar{\imath}} = G_{\bar{\imath}} V + e^G \left((\nabla_{\bar{\imath}} G_{\bar{\jmath}}) G^{\bar{\jmath}} + G_{\bar{\imath}} \right), \tag{4.29b}$$

and similar expressions for V_a and $V_{\overline{a}}$. The Hessian of covariant derivatives is

$$V_{ij} = \nabla_i G_j V + G_i V_j + G_j V_i - G_i G_j V + e^G \left[(\nabla_i \nabla_j G_k) G^k + 2 \nabla_i G_j \right], \tag{4.30a}$$

$$V_{i\bar{\jmath}} = G_{i\bar{\jmath}} V + G_i V_{\bar{\jmath}} + G_{\bar{\jmath}} V_i - G_i G_{\bar{\jmath}} V + e^G \left[R_{i\bar{\jmath}k\bar{l}} G^k G^{\bar{l}} + G^{k\bar{l}} \nabla_i G_k \nabla_{\bar{\jmath}} G_{\bar{l}} + G_{i\bar{\jmath}} \right], \tag{4.30b}$$

$$\begin{aligned} V_{ia} &= \nabla_a G_i V + G_i V_a + G_a V_i - G_i G_a V + e^G \left[(\nabla_a \nabla_i G_I) G^I + \nabla_i G_a + \nabla_a G_i \right] \\ &= G_i V_a + G_a V_i - G_i G_a V, \end{aligned} \tag{4.30c}$$

$$\begin{aligned} V_{i\overline{a}} &= G_{i\overline{a}} V + G_i V_{\overline{a}} + G_{\overline{a}} V_i - G_i G_{\overline{a}} V + e^G \left[R_{I\overline{J}i\overline{a}} G^I G^{\overline{J}} + G^{I\overline{J}} \nabla_i G_I \nabla_{\overline{a}} G_{\overline{J}} + G_{i\overline{a}} \right] \\ &= G_i V_{\overline{a}} + G_{\overline{a}} V_i - G_i G_{\overline{a}} V, \end{aligned} \tag{4.30d}$$

and similar expressions for the other V_{IJ}. The equalities in (4.30c) and (4.30d) result from the specific form of the Kähler function (4.28).

4.B Mass eigenmodes in a stabilized sector

In this appendix we provide some intermediate results in the calculation of (4.18). Using the expressions as stated in appendix 4.A, to first order in $|G_q|$, the second derivatives of the potential are given by

$$V_{qq} = e^G \left[(2 + e^{-G}V)\nabla_q G_q + (\nabla_q \nabla_q G_q) G^q \right] + O(|G_q|^2), \tag{4.31a}$$

$$V_{q\bar{q}} = e^G \left[G_{q\bar{q}}(1 + e^{-G}V) + G^{q\bar{q}}(\nabla_q G_q)(\nabla_{\bar{q}} G_{\bar{q}}) \right] + O(|G_q|^2). \tag{4.31b}$$

Using the supersymmetry breaking restriction (4.16) in (4.31), we find

$$V_{qq} = -e^G G_{q\bar{q}} \left[(2 + e^{-G}V)(1 + e^{-G}V)\widehat{G^q}^{-2} - G^{q\bar{q}}(\nabla_q \nabla_q G_q) G^q \right] + O(|G_q|^2), \tag{4.32a}$$

$$\begin{aligned} V_{q\bar{q}} &= e^G \left[G_{q\bar{q}}(1 + e^{-G}V) + (1 + e^{-G}V)^2 G^{q\bar{q}} G_{q\bar{q}} G_{q\bar{q}} \right] + O(|G_q|^2) \\ &= e^G G_{q\bar{q}}(2 + e^{-G}V)(1 + e^{-G}V) + O(|G_q|^2), \end{aligned} \tag{4.32b}$$

and hence

$$\begin{aligned} |V_{qq}| &= e^G G_{q\bar{q}}(2 + e^{-G}V)(1 + e^{-G}V) \times \\ &\quad \sqrt{1 - \frac{2G^{q\bar{q}}\mathrm{Re}\{(\nabla_q \nabla_q G_q) G^q \widehat{G^{\bar{q}}}^{-2}\}}{(2 + e^{-G}V)(1 + e^{-G}V)} + \frac{\left| G^{q\bar{q}}(\nabla_q \nabla_q G_q) G^q \right|^2}{(2 + e^{-G}V)^2(1 + e^{-G}V)^2} + O(|G_q|^2)} \\ &= e^G G_{q\bar{q}} \left[(2 + e^{-G}V)(1 + e^{-G}V) - G^{q\bar{q}}\mathrm{Re}\{(\nabla_q \nabla_q G_q)\widehat{G^q}^3\}|G^q| \right] + O(|G_q|^2). \end{aligned} \tag{4.33}$$

Then (4.17) is evaluated to be

$$m_q^- = e^G G^{q\bar{q}}\mathrm{Re}\{(\nabla_q \nabla_q G_q)\widehat{G^q}^3\}|G^q| + O(|G_q|^2), \tag{4.34a}$$

$$m_q^+ = e^G \left[2(2 + e^{-G}V)(1 + e^{-G}V) - G^{q\bar{q}}\mathrm{Re}\{(\nabla_q \nabla_q G_q)\widehat{G^q}^3\}|G^q| \right] + O(|G_q|^2). \tag{4.34b}$$