Conformal invariance and microscopic sensitivity in cosmic inflation

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Motivation

1.1 On being a string cosmologist

Our universe had a beginning —or at least, so it seems. With the technological advances of previous decades and the incredible achievements in the theoretical understanding of nature's workings, we are tantalizingly close to answering existential questions that are as old as mankind itself. *"Where do we come from?"*, *"What are we made of?"* These questions, that have always been of a philosophical nature, are now almost in reach of observational verification. Science is painting us a picture of our universe, which we can understand from as early as 10^{-10} seconds after the supposed beginning, the *big bang*. Since its beginning the universe has been expanding, which led to a cooling down of the initial hot, dense state. This allowed atoms to combine to molecules and stars to be born, finally leading to the universe we observe today. Our current knowledge of the cosmic evolution of our universe is an astonishing feat, but there is always the quest to learn more. Driven by curiosity, we are trying to push our knowledge to within the first 10^{-10} seconds. This tour de force requires a perfect orchestrated collaboration between observational achievements and theoretical advances.

To understand observations, we need a theoretical foundation. For observations from the early universe, we need to acquire knowledge about nature's working from the smallest to the largest energy scales or equivalently on all distance scales. In the early beginning of the universe, the typical energy of particles was beyond imagination, as the whole universe was tightly packed into a hot, energetic plasma. Such energies probe microscopic particle interactions that happen at the tiniest of scales. Without an accurate understanding of the processes happening at the highest energies, it is impossible to correctly interpret any observation from the primordial epoch and to truly understand how our universe came to be.

Vice versa, to verify theoretical predictions, we need an observational foundation. This is a rather obvious statement. The backbone of physics is its solid foundation in the form of verifiable and falsifiable claims, following from a consistent, coherent and ideally complete theoretical framework. However, given the progression of our theoretical understanding into ever more energetic realms of physics, achieving the required regime observationally is easier said than done. For example, the physics of particle interactions is uniformly described in one all-coordinating theory, called the *standard model*. It correctly describes processes of typical energies up to about 10^3 GeV and is currently under scrutiny precisely around this threshold energy at CERN's state-of-the-art Large Hadron Collider. However, the standard model misses one vital piece in its description of nature, as it does not contain the gravitational theory of general relativity. A unified theory of both gravity and particle physics would be an enormous achievement of our fundamental understanding of nature. However, since it is not expected that a unified theory of both gravity and particle physics interactions will appear below 10^{19} GeV —the scale where quantum effects are comparable to classical gravity—, testing such a theory will not happen in the foreseeable future with current terrestrial technology. Our only hope is to observationally explore the realm where such high energies occur naturally: astrophysics and cosmology. Without a way of probing processes at such high energies, we risk to bring any theoretical advance in this direction to a stop.

The lack of observations has not stopped us from formulating a candidate theory of quantum gravity, *string theory*. In this theory, fundamental interactions happen between strings rather than point particles. It is possibly a unifying theory for general relativity and the standard model, although its very nature, i.e. characteristic energy scales of 10^{19} GeV, currently prevents the theory from making testable predictions. Nevertheless a consistent combined theory of quantum mechanics and gravity must exist and at the moment string theory is the only real candidate.

This is where cosmology may help, especially since we now have good reasons to believe that the earliest evolution of the universe is described by a period of accelerated expansion, called *inflation*. Inflation is capable of solving many of the issues of a standard expanding universe. However, inflation, which is a generic phase of accelerated expansion, turns out to be surprisingly sensitive to its microscopic description, which is largely unknown. With a fundamental theory of everything without observational means to test it and with an observationally satisfying mechanism in need of a microscopic description, *string inflation* seems to provide a unique opportunity to address two problems at once. A string theory description of inflation would provide a means to probe microscopic physics by cosmological observations, while simultaneously providing a means to microscopically gain insight in the structure of the dawn of time.

1.2 This thesis

In this thesis we present multiple, complementary approaches to the understanding of string inflation. As explained, the sensitivity of inflation to the microscopic details of the theory from which it originates, provides a great opportunity to probe this microscopics observationally. However, it also means that describing a consistent microscopic theory in which inflation can be embedded, is very hard. In this thesis we will study to what extent this sensitivity might hinder theoretical model building and what are possible approaches to circumvent these issues.

A fundamental ingredient in the approaches we present is *conformal invariance*, or (local) scale invariance. A theory that is scale invariant describes a system which looks the same on all length scales. This might be a surprising starting point to describe our universe, which certainly does not seem to be scale invariant, but in the mathematical description of inflation, conformal invariance plays an exceptionally important role. In this thesis we consider two (different) ways in which conformal invariance might prove crucial in our understanding of the primordial inflationary epoch.

A review of relevant aspects of conformal field theory is presented in chapter 3, explaining its relevance to string theory. There, we also present the three different guises in which string theory will be employed throughout the different chapters. Its *worldsheet* description; the low energy effective *supergravity* description derived from it; as well as the *holographic* conformal field theory description of an accelerating universe that arises from string theoretical considerations, are all addressed. These form the background material to chapters 5, 4 and 6 respectively, where we will explore the consequences for string inflation within each of the different approaches.

In chapter 4 we consider supergravity descriptions of inflation. Supergravity appears as the low energy limit of (super)string theory. Since string theory itself is not a fully understood theory and it is generally very hard to use it for explicit calculations, supergravity has become an interesting framework to reconcile inflation within stringlike physics [1, 2]. Due to the sensitive nature of inflation, it has proven to be very difficult to find models that are stable under quantum corrections. We study the sensitivity of inflation to possible additional physics such as the standard model, in a general supergravity set-up.

In chapter 5 we consider a new approach to string inflation, based on the (defining) worldsheet description of string theory. The worldsheet theory is described by a conformal field theory. In general, both the inflationary dynamics of a four-dimensional spacetime as well as other internal/matter sectors are encoded in the total worldsheet conformal field theory. We probe the interaction between the two sectors by enforcing a nearly marginal perturbation of the internal sector theory. Assuming worldsheet conformal invariance to be maintained for the total theory throughout the inflationary evolution, the deformation of the internal sector induces a response in the form of a deformation of the inflationary sector, which can be understood as a time evolution in the four-dimensional spacetime. Insisting on a slow-roll inflationary phase, this imposes constraints on the allowed deformations of the internal sector. In this way, observations from the inflationary era might be used to constrain the underlying string theory.

The final approach to obtain a fundamental understanding of the inflationary epoch is presented in chapter 6, in which we study the two- and three-point correlation functions of density perturbations generated during inflation. Since inflation is described by a quasi-de Sitter evolution, these correlation functions are expected to be constrained by the (broken) conformal isometry group of de Sitter spacetime at asymptotically late times. The approach is truly orthogonal to all other approaches, as it uses the symmetries of de Sitter space in a way that, tantalizingly, borrows heavily from a revolutionary insight from string theory: holographic duality. The origin of this holographic duality is rooted in string theory and it is one of the theory's most surprising predictions, maximizing the power that underlies the string theory surface. A true understanding of the duality and the microscopic origin behind it would provide an incredible improvement in the understanding of the origin of our universe. The investigations in chapter 6 of the symmetry constraints on the two- and three-point functions provide a first step in the investigation to what extent this duality might apply to the inflationary era.

Given the emphasis earlier in this motivation on the necessity of an amalgamation of observations and theory, this work admittedly contains only theoretical studies of a possible stringy origin of inflation. However, in all cases there is a clear observational component behind the initial motivation of the study, which permeates each approach. Therefore, to put our results into the required cosmological context, we present an introduction to inflationary cosmology in chapter 2. With this background, we hope to have made clear how this work can provide new and interesting insights in the development of a string theoretical description that underlies the inflationary, primordial stages of our universe.

2

Cosmic evolution of our universe

The understanding of the evolution of our universe is one of the unabashed successes of modern physics. To a large extent, the evolution of our universe is understood extremely well, from a few moments after its birth to very close to the present day. In this chapter we will review standard big bang cosmology, both theoretically as well as observationally. We will explain how inflation can solve some of the problems associated to the big bang paradigm and, since it is the measurements of the infant stage of the universe that provide the observational backbone for string cosmology, in what way measurements of the infant universe can be related to microscopic models of inflation.

2.1 A short history of big bang cosmology

2.1.1 Theoretical development of a dynamic universe

Modern cosmology is less than 100 years old. Before Einstein had developed his theory of general relativity [3, 4], Newtonian mechanics did not invite to study the universe as a whole. Surely, mankind probably always had an interest for the stars and galaxies that appear on the night sky, but in Newtonian theory this merely results in the study of these objects *within* a fixed arena, not of the black sky itself. The universe itself only features as the stage in which extraterrestrial physical phenomena occur. With the advent of general relativity this all changes, as spacetime itself inevitably becomes dynamic.

To describe the universe as a whole, we rely on the *cosmological principle*, the assumption that no place in the universe is special and that it is the same from any vantage point. This is an extrapolated version of the Copernican principle, that our planet nor our solar system nor our galaxy is the center of the universe. Rather, the laws of physics are the same throughout the universe and no observer can distinguish a preferred location. Consequently, the universe should be homogeneous and isotropic on large scales, a fundamental assumption which enabled Friedmann-Lemaître-Robertson-Walker [5–8] to propose a model for cosmic evolution within general relativity. Homogeneity and isotropy of the cosmological principle translate into the mathematical statement that the metric of spacetime is maximally symmetric in its spatial part,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left(\frac{\mathrm{d}r^2}{1 - kr^2} + r^2 \mathrm{d}\Omega^2 \right),$$

where the scale factor $a(t)$ describes the overall (spatial) scale of the universe and k corresponds to positively curved, negatively curved or flat spatial slices for $k = 1, -1, 0$ respectively.[1] The matter content must be taken homogeneous and isotropic too, specified by a perfect fluid energy-momentum tensor that only depends on the energy density ρ and pressure p of the fluid,

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}.$$

With these expressions for the metric and energy-momentum tensor, Einstein's equations, including the cosmological constant as a matter contribution, reduce to the Friedmann equations

$$H^2 = \frac{\kappa^2}{3}\rho - \frac{k}{a^2}, \tag{2.1a}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}(\rho + 3p), \tag{2.1b}$$

where the Hubble parameter, $H = \frac{\dot{a}}{a}$, determines the rate of expansion. The reduced Planck mass $\mathrm{M}_{\mathrm{pl}}^{-2} = \kappa^2 = 8\pi G_N$ acts as the gravitational coupling constant; we will often use natural units, where $\kappa^2 = 1$.

One can combine these equations into the continuity equation

$$\dot{\rho} = -3H(\rho + p),$$

[1] The universe is spatially flat to a very high precision. For this reason, we will be mainly concerned with the metric in case of a flat spatial slicing.

which also follows from energy conservation, $\nabla_\mu T^\mu{}_\nu = 0$. If one specifies the equation of state between energy density and pressure by writing $p = w\rho$, the continuity equation can be integrated to express the evolution of the energy density as a function of the evolution of the scale factor a,

$$\rho \propto a^{-3(1+w)}.$$

In general the evolution will be a mixture of different non-interacting fluids, which are traditionally distinguished as being (pressureless) non-relativistic matter ($w = 0$), relativistic matter and radiation ($w = \frac{1}{3}$) or a contribution from the cosmological constant ($w = -1$). To denote the matter content of the universe, one often uses the dimensionless quantity $\Omega(t) = \frac{1}{3H^2}\rho$. In terms of Ω, the first Friedmann equation, (2.1a), can be written as $\Omega - 1 = k/(aH)^2$.

2.1.2 Observational confirmation and new challenges

Equation (2.1a) clearly implies that a static universe, $\dot{a} = 0$, only occurs for very specific values of the energy density and spatial curvature. Hence, the cosmological principle in combination with general relativity, seems to tell us that we live in a dynamic universe. Historically, at first a non-static universe was merely a theoretically predicted possibility within general relativity, based on the assumption that the universe is homogeneous and isotropic. By now both the expansion of the universe as well as its homogeneity and isotropy are well established by observations.

Already at the end of the 1920s, very soon after the theory of general relativity and the proposed FLRW solution, first evidence of an expanding universe was obtained by Hubble [9]. He famously discovered that the spectrum of stars is redshifted proportionally to their distance to us. Subsequent experiments have refined his findings to a rate of expansion given by $H_0 = 70.2 \pm 1.4$ $(km/s)/Mpc$ for the present era [10]. A second confirmation of the FLRW model was developed during the following years, when physicists realized that an expanding universe must have had a very hot and very dense early beginning, emerging from an initial singularity called the *big bang*. Such a beginning implies that the universe was so hot that nuclei could not have existed and must have formed as the universe cooled. This epoch is called big bang nucleosynthesis, which ended when the universe cooled down further. The estimated relative production of light elements from protons and neutrons during the epoch of big bang nucleosynthesis accounts for the observed abundances to a very great precision [11] and is therefore also a clear confirmation of the expanding universe model.

The most important confirmation, particularly useful for present-day observations, is the existence of the *cosmic microwave background* (CMB) radiation. In the epoch after big bang nucleosynthesis, electrons were still energetic enough to escape from the pull of ionized nuclei. Only when the universe expanded further and consequently cooled down further, did neutral hydrogen become stable. At this moment of recombination, photons no longer encountered (charged) free electrons and nuclei from which they would scatter. Ever since, they have therefore been traveling (mostly) freely through a neutral universe. As the universe kept expanding, these photons cooled down, redshifting towards microwave radiation, at which frequencies we observe them now. The first observation of the microwave photons was in 1965, when Penzias and Wilson observed an excess microwave background noise in their radio antenna [12], which was quickly realized [13] to be the predicted cosmic microwave background radiation [14, 15].

With following improved observations, the CMB is now the most-precisely measured black body spectrum in nature [16], having a temperature of 2.73 K isotropically across the sky, implying recombination happened approximately 380 000 years after the big bang. The fact that for each local patch across the sky, the variation in the temperature of the CMB is only a remarkable 1 in 10^5 means we have now observationally justified the earlier made assumption of the homogeneity and isotropy of the universe. Each photon on the *surface of last scattering* has the same temperature to an astonishing precision, confirming that recombination and the subsequent expansion happened homogeneously throughout the universe.

Combining different cosmological observations, such as CMB observations [10], the formation of large-scale structures [17, 18], the recessions of type Ia supernovae [19, 20], observed mass distributions through gravitational lensing [21] and the study of peculiar motion of galaxies and clusters [22, 23], we now have an increasingly precise understanding of the content and dynamics of our universe and its evolution after the first fraction of a second. Observationally a huge improvement has been obtained in the last 10–20 years, mainly because of improved measurements of the CMB, which made it possible to estimate the parameters of the FLRW model with ever greater accuracy and firmly established cosmology as a “precision science”. We now know we live in a spatially flat universe $\Omega_0 = 1.002 \pm 0.011$, which expands at an accelerated rate [10, 19, 20, 24]. However, this success-story has brought with it a number of new puzzles directly emergent from the data.

One is the discovery of the current accelerated expansion of our universe. It earned its discoverers [19, 20] the 2011 Nobel Prize, which is a recognition of the enormous advances that observational cosmology has seen in the last two decades. However, theoretically the reason for this accelerated expansion is far from clear. At the moment the expansion is accounted for by a dominant energy contribution coming from *dark energy*, such as the cosmological constant Λ or some other energy component which has an equation of state $w < -\frac{1}{3}$. Except for its name, dark energy remains a largely unknown form of "stuff". We do not know its origin nor its precise characteristics. From the observations it is clear that the universe is currently dominated by dark energy. It constitutes 74% of today's total energy budget. The remaining 26% of the energy decomposition consists of matter, $w = 0$, although only about 4% (of the total budget) is ordinary visible matter. This means another 22% of the total energy budget is yet unaccounted for. All observations [10, 17, 18, 21–23] indicate the presence of *some* sort of (invisible) matter, dubbed (cold) *dark matter*, a second puzzle.

The ΛCDM-model derives its name from the dominant contributions in our universe, the cosmological constant Λ and cold dark matter. Although we have good indications that these contributions are really there, for the moment their precise nature eludes understanding. For this reason a large branch of present day cosmology focusses on the nature and characteristics of the dominant contributions to the energy decomposition of our universe. However, in this thesis we will focus on yet another mystery of the current cosmological model. This mystery does not focus on the content of our present day universe, but rather on how it has all come to be.

2.2 Cosmic inflation

2.2.1 Initial conditions

Our universe seems to be very special in the way it is very sensitive to its precise initial conditions. In principle the need for such precise initial conditions is not a problem, since cosmology is not claiming to provide a full explanation for the cosmic evolution *including* its starting point. We only need to be able to evolve the universe from a set of given initial conditions to the present day. However, the level of precision for the initial conditions is so high, that one starts wondering why we happen to live in *this* universe. If the initial conditions were only slightly different, standard big bang evolution would lead to a significantly different universe. For this reason it is unsatisfactory to simply take the required initial conditions as a given, without the slightest wondering why it had to be these initial conditions. The strong dependence on the actual initial conditions weakens any claim done by cosmologists, as one can seemingly evolve to *any* universe by simply starting from marginally different initial conditions. For most cosmologists such a sensitive and unstable situation begs for an explanation. Such an explanation exists, it is provided by the inflationary paradigm [25–27].[2]

The problem with the initial conditions consists of two separate problems, called the *horizon problem* and the *flatness problem*. In short, the horizon problem is the observation that the CMB is far more homogeneous than one should naïvely expect. In general, any inhomogeneity will grow bigger and bigger, through gravitational interaction. Indeed, the amount of inhomogeneities today is larger than that in the CMB, but similarly we also expect that the amount of inhomogeneities was even smaller at any time before the CMB. Since the CMB has inhomogeneities of the order of 10^{-5}, one wonders how smooth exactly the initial conditions must have been to provide the smoothness of the CMB. Most importantly, in the standard big bang cosmology, the CMB is homogeneous even across regions which could have never been in causal contact at the time of last scattering. Decoupling occurred 380 000 years after the big bang and so the present-day full-sky observation of the CMB consists of multiple patches, each only 380 000 light years across, in which photons were in causal contact.

Let us explain how this compares with the current causally connected patch. Mathematically we define the *particle horizon* as the (comoving) size of a causally connected region. From (2.1b), it is given by

$$\frac{1}{aH} \propto a^{\frac{1}{2}(1+3w)}. \tag{2.2}$$

The expression takes into account that the universe expands while the light is propagating through space. If the universe would expand too quickly, such that the photons can not “keep up”, the particle horizon decreases. However, for an evolution dominated by ordinary matter, $w \geq 0$, the particle horizon grows with time. This means that, for example those CMB photons that enter the particle horizon now, were not causally connected *before*. Specifically, they were not causally connected at the time the CMB was produced. Yet, the CMB spectrum is consistent over all length scales with a uniform black body spectrum having a homogeneous temperature to 1 part in 10^5. How can the CMB be so homogeneous even across all causally disconnected regions?

The other problem with initial conditions, the flatness problem, is the observation that the universe is incredibly close to being spatially flat, $\Omega_0 = 1.002 \pm 0.011$ [10, 24]. From $\Omega - 1 = k/(aH)^2$ and (2.2) it follows that for ordinary matter, $w \geq 0$, any deviation away from flatness, $\Omega = 1$, can only be growing. In fact, by taking a derivative of the first Friedmann equation and using (2.1b), we can derive a differential equation

[2] Several excellent books and lecture notes provide a detailed introduction to inflation, cf. [28–31].

for Ω,

$$a\frac{d\Omega}{da} = (1 + 3w)\Omega(\Omega - 1),$$

which tells us that the critical value $\Omega = 1$ is an unstable fixed point for $w > -\frac{1}{3}$. This means that in order to find $\Omega_0 = 1.002$ today, the initial conditions must have been exponentially closer to $\Omega = 1$. How can the universe be so spatially flat if all matter is desperately trying to push it away from flatness?

The horizon problem and the flatness problem have a common origin; in both cases the inconsistency arises because of the growing nature of the particle horizon $(aH)^{-1}$ for ordinary matter or equivalently,

$$\frac{d}{dt}(aH)^{-1} = -\frac{\ddot{a}}{(aH)^2} = \frac{\rho}{6aH^2}(1 + 3w) > 0,$$

because ordinary cosmology is dominated by matter, $w = 0$, or radiation, $w = \frac{1}{3}$. As the intermediate result shows, this is equivalent with an expanding but decelerating universe. Therefore an obvious solution would be to look for a period of *accelerated* expansion, $\ddot{a} > 0$, dominated by some form of matter with $w < -\frac{1}{3}$. Although the current vacuum energy dominated era, with $w = -1$, meets the requirements and seems to make the problem less urgent, the universe has only recently entered the vacuum energy dominated epoch. Radiation and matter dominated for most of its history. To solve the problems with initial conditions, we should consider an accelerating phase *before* the current big bang paradigm, which should at least last for about 60 e-folds to solve the flatness and horizon problems [32, 33]. This phase is called *cosmic inflation*. It is specified by the need to explain the initial conditions, but only in a very coarse manner. Any epoch which is dominated by some matter-component having $w < -\frac{1}{3}$ will be capable of solving the big bang problems. However, the requirement $w < -\frac{1}{3}$ is difficult to meet with ordinary matter and radiation, because it requires a negative pressure. This makes the search for the true microscopic nature of the inflationary epoch a worthwhile and interesting endeavor.

2.2.2 Accelerated expansion

No ordinary matter has negative pressure, but it was the insight of [25] that "order parameter" physics can easily account for this, by considering a (single) scalar field (the order parameter) coupled to gravity,

$$S = \frac{1}{2}\int d^4x\,\sqrt{g}\,\left[R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V(\phi)\right]. \tag{2.3}$$

The scalar field is a matter source to the gravitational field, appearing on the right hand side of Einstein's equation, with

$$\rho = \frac{1}{2}\dot{\phi}^2 - V, \qquad p = \frac{1}{2}\dot{\phi}^2 + V,$$

under the assumption that $\phi(t, \boldsymbol{x}) = \phi(t)$ is spatially homogeneous. In a regime in which the potential dominates over the kinetic energy of the field, the equation of state can indeed be negative. The limiting case $w = -1$ is reached by assuming a stationary scalar field. In that situation, the field equations for the field ϕ and for the scale factor a of a (flat) FLRW ansatz for the metric,

$$0 = \ddot{\phi} + 3H\dot{\phi} + V'(\phi), \tag{2.4a}$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{2.4b}$$

tell us that $V(\phi)$ should be constant and equal to $3H^2$ and the scale factor is exponentially growing, $a(t) = e^{Ht}$. The resulting accelerated expansion is that of a de Sitter universe,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + e^{2Ht}\mathrm{d}\boldsymbol{x}^2,$$

corresponding to a maximally symmetric universe with positive cosmological constant $\Lambda > 0$, just as in the current epoch.

An inflationary epoch being driven by a constant (positive) potential $V = \Lambda > 0$ is too simplistic, in that there is no dynamical way for inflation to end. This can be resolved by allowing the field $\phi(t)$ to be dynamical [25–27]. To still maintain a handle on the equations of motion, it is useful to consider a dynamical situation which is still very close to de Sitter, i.e. to study a nearly constant Hubble parameter or equivalently a slowly varying field $\phi(t)$ that is potential energy dominated, $\dot{\phi}^2 \ll V$. In order to quantify the slowness of the variation, we define the slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2} = 2\left(\frac{H'}{H}\right)^2 = \frac{1}{2}\left(\frac{\dot{\phi}}{H}\right)^2, \qquad \eta = 2\frac{H''}{H} = -\frac{\ddot{\phi}}{H\dot{\phi}}, \tag{2.5}$$

where $'$ indicates a derivative of $H(\phi)$ with respect to the field ϕ. The equalities in these expressions are a consequence of (2.4), which imply that $2H' = -\dot{\phi}$. Via $\ddot{a} = aH^2(1 - \epsilon)$, it is clear that the universe undergoes accelerated expansion if and only if ϵ is smaller than unity,

$$\ddot{a} > 0 \Leftrightarrow \epsilon < 1. \tag{2.6}$$

$\epsilon = \eta = 0$ corresponds to the pure de Sitter phase. When both slow-roll parameters are taken to be small but non-vanishing, the field equations (2.4) resemble a quasi-de Sitter phase,

$$H^2(t) \approx \frac{1}{3}V(\phi) \approx \text{constant}, \qquad a \approx e^{H(t)t}, \qquad \dot{\phi} \approx \frac{-V'}{3H(t)} \approx 0.$$

The approximation $\epsilon, |\eta| \ll 1$ is called the *slow-roll approximation*. This name arises in the context of a single scalar field driving the acceleration. The definition of ϵ and η makes clear, however, that the existence of acceleration or quasi-de Sitter evolution is not tied to the existence of a scalar field. Nevertheless, almost all models use a scalar field description and often use a different set of *potential slow-roll parameters* ϵ_V, η_V. These are related to the previous ones *in the slow-roll approximation* by

$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \approx \epsilon, \qquad \eta_V = \frac{V''}{V^2} \approx \epsilon + \eta. \tag{2.7}$$

The potential slow-roll parameters express inflation as a slowly rolling field on a flat potential and have as an advantage over ϵ and η that they provide a direct connection between the potential and the dynamics of the system. However, this connection only holds when the slow-roll approximation is assumed, whereas the field equations can be expressed in terms of ϵ and η exactly. The latter set of parameters is therefore better suited to set up a consistent approximation scheme [34] and are, in this respect, preferred over ϵ_V and η_V. In the slow-roll approximation, $\epsilon, |\eta| \ll 1$, the use of the potential slow-roll parameters may be more convenient. Since there are clear indications that the slow-roll approximation is indeed satisfied during inflation, both sets of slow-roll parameters can be used almost interchangeably.

In summary, inflation is a coarse phenomenon that happens if and only if $\epsilon < 1$. *Realistic* inflation has as additional requirement that $\epsilon, |\eta| \ll 1$ or equivalently $\epsilon_V, |\eta_V| \ll 1$ [10, 35] and describes a quasi-de Sitter evolution.

2.3 Seeds of structure

2.3.1 Primordial perturbations

In the description of inflation above, we have assumed the inflaton field $\phi(t, \boldsymbol{x})$ to be spatially homogeneous $\phi(t, \boldsymbol{x}) = \phi(t)$. This assumption is justified by the observed homogeneity in the universe, but we know it cannot be the end of the story, since we have also observed small anisotropies in the CMB [35, 36]. The inflationary paradigm provides a satisfying explanation for the origin of these anisotropies [37–43]. Like we expect around any classical field, the inflaton is subject to small quantum fluctuations

$$\phi(t, \boldsymbol{x}) = \overline{\phi}(t) + \delta\phi(t, \boldsymbol{x}),$$

which in this case parameterize small deviations from spatial homogeneity in the inflaton field. Qualitatively the consequences are easily understood. Through Einstein's equations, small variations in the inflaton field ϕ generate perturbations in the geometry of spacetime, which lead to gravitional wells and voids in which slight relative overdensities and underdensities of the matter distribution start to form. As a result, CMB photons experience slightly different redshifts and this we observe in our measurements of the CMB.

Quantitatively we also understand the transition from one type of perturbation to the other, providing a powerful bridge between observation and theory. Several excellent books and review papers have been written about this rich topic [28, 29, 44–46]. Here, we only present the very basics in order to provide a flavor of why the theoretical calculations in this thesis are relevant for observations. From observations, we have direct access to the relative temperature anisotropies $\delta T(\hat{\boldsymbol{n}})$ in each direction $\hat{\boldsymbol{n}}$ in the sky. Traditionally the information is encoded in terms of multipole moments a_{lm}, that result from expanding $\delta T(\hat{\boldsymbol{n}})$ on the orthonormal set of spherical harmonic functions $Y_{lm}(\hat{\boldsymbol{n}})$,

$$\delta T(\hat{\boldsymbol{n}}) = \sum_{lm} a_{lm} Y_{lm}(\hat{\boldsymbol{n}}).$$

From these coefficients we can then build an angular n-point function

$$\langle a_{l_1 m_1} \dots a_{l_n m_n} \rangle.$$

In principle, the average is an ensemble average over multiple universes, but since we have only access to one universe, the statistical uncertainty is instead controlled by a (weighted) angular average over the m_j-modes [28, 45]. The multipole modes a_{lm} of the temperature (differences) $\delta T(\hat{\boldsymbol{n}})$ in the CMB are sourced by the primordial scalar curvature perturbations ζ. They are related via a *transfer function* $\Delta_l(k)$,

$$a_{lm} = 4\pi(-i)^l \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \Delta_l(k) \zeta_{\boldsymbol{k}} Y_{lm}(\hat{\boldsymbol{k}}). \tag{2.8}$$

The transfer function is the solution to a set of coupled differential equations, resulting from Einstein's equations and Boltzmann's equations for the interactions among different types of fluids [28]. It can be computed numerically [47], once the background cosmology and the initial spectrum for ζ are specified. Conversely, by scanning over

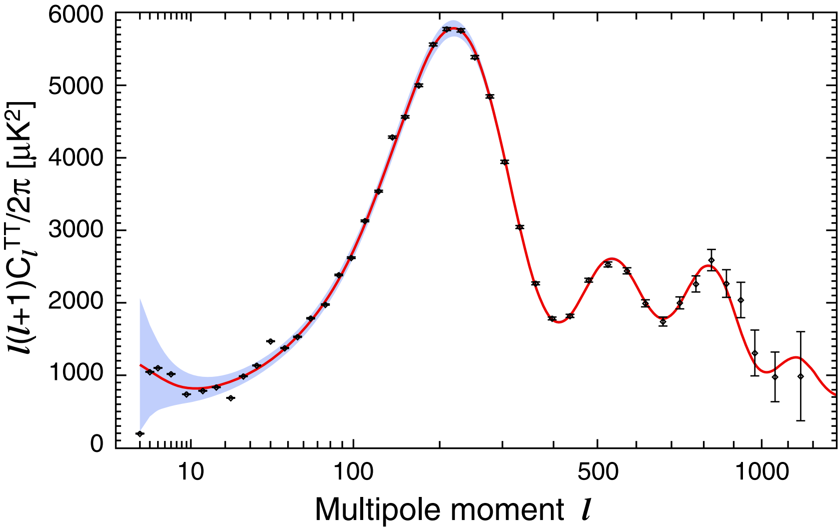


Figure 2.1: *The power spectrum of the temperature anisotropies, expressed in terms of the multipole coefficients* $C_l = \frac{1}{2l+1}\sum_m \langle a^*_{lm} a_{lm}\rangle$*. The curve represents a* Λ*CDM best fit to the 7-year WMAP data with a nearly scale invariant power spectrum* $P_\zeta \sim k^{n_s-1}$ *with* $n_s \approx 0.96$ *[35].*

many results, we can fit the parameters of the background cosmology as well as the primordial spectrum of perturbations to the CMB data.

The upshot of the preceding paragraph is clear: the n-point function of primordial perturbations $\langle \zeta_{\boldsymbol{k}_1} \dots \zeta_{\boldsymbol{k}_n}\rangle$ is directly related to the correlations of observationally accessible temperature fluctuations, $\langle \delta T(\hat{\boldsymbol{n}}_1)\dots\delta T(\hat{\boldsymbol{n}}_n)\rangle$. Via (2.8), the temperature two-point function, given in terms of the multipole coefficients $C_l = \frac{1}{2l+1}\sum_m \langle a^*_{lm} a_{lm}\rangle$, is related to the primordial two-point function

$$C_l = 4\pi \int \frac{\mathrm{d}k}{k} P_\zeta(k)\Delta_l(k)^2. \tag{2.9}$$

Here, the relation is given in terms of the power spectrum $P_\zeta(k)$ of the primordial curvature perturbations. For any quantum operator $\hat{f}$, the power spectrum $P_f(k)$ is defined via

$$\langle \hat{f}_{\boldsymbol{k}} \hat{f}_{\boldsymbol{k}'}\rangle = \delta(\boldsymbol{k}+\boldsymbol{k}')\frac{16\pi^5}{k^3}P_f(k).$$

In recent years, the two-point function of temperature anisotropies C_l has been observed to very high precision by the WMAP collaboration [35], cf. figure 2.1. In order to produce the temperature spectrum as shown in figure 2.1, the primordial power spectrum $P_\zeta(k)$ should be almost constant over all scales. On phenomenological grounds, such a scale invariant primordial power spectrum was already proposed by Harrison and Zel'dovich [48, 49]. As we will calculate shortly, one of the great successes of inflation, in addition to solving the flatness and horizon problem, is that it provides a very natural explanation for such a nearly scale invariant spectrum. The precision with which theory matches observation in the CMB temperature two-point function and the way inflation provides us with an explanation for its peaks and valleys [50] and for the underlying scale invariance, lends incredible credence to the existence of a primordial inflationary epoch. With new investigations that focus on subleading effects in the power spectrum, such as small oscillations on top of the near scale invariance [51, 52], more and more details about the inflationary epoch will hopefully soon be revealed.

Another way to probe deeper into the nature of inflation is by studying higher order n-point functions. Similar to the two-point function, the three-point function of temperature anisotropies can be expressed in terms of the three-point functions of the primordial curvature perturbations [44], called the primordial *bispectrum*,

$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = (2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B_\zeta(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3).$$

The bispectrum is the leading contribution to *non-Gaussian* effects in the spectrum of the CMB, which would be a pure Gaussian distribution if only the two-point functions were non-vanishing [53]. From momentum conservation, its dependence on the three momenta defines a triangular shape. Because different inflationary models predict a peak for different triangular shapes, the shape of non-Gaussianities is an interesting tool to distinguish between models [45, 46, 54].

The simplest inflationary scenario, single field slow-roll inflation with canonical kinetic energy in a Bunch-Davies vacuum, predicts only non-Gaussianities that are too small to be observable [55–57], cf. (2.21). Therefore, current literature is focussed on any possible observation of non-Gaussianities, as it may indicate a variety of violations of the assumptions, favoring e.g. multi-field inflation [58, 59], non-canonical kinetic terms [60–62], non-standard initial states [62–66] or a different scenario for inflation altogether [67, 68]. Developments into this direction are very exciting, especially with the preliminary indication that such non-Gaussianities may be present in the upcoming release of data by the Planck mission [10, 69], but are beyond the scope of this work. We will limit ourselves to the three-point function of primordial curvature perturbations in single field slow-roll inflation with canonical kinetic terms. Even though these non-Gaussianities are beyond the observable level in any near future experiment, from the structure of the correlation functions of even this simplest inflationary model, we can learn a lot about the nature of the early universe.

2.3.2 The power spectrum

Let us now explain the origin of the primordial spectrum of density perturbations from inflation. As in other places in physics, we can calculate the small fluctuations to the inflationary evolution using perturbation theory,

$$\phi(t, \boldsymbol{x}) = \overline{\phi}(t) + \delta\phi(t, \boldsymbol{x}), \qquad g_{\mu\nu}(t, \boldsymbol{x}) = \overline{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t, \boldsymbol{x}).$$

In cosmological perturbation theory, such a split is not well-defined, since a coordinate transformation may redefine the background fields $\overline{\phi}$ and $\overline{g}_{\mu\nu}$. Therefore one has to be careful to consider only true perturbations and to distinguish these from induced perturbations caused by coordinate redefinitions. Before, in the FLRW ansatz with a homogeneous field $\phi(t)$, this dependence on the coordinate choice did not pose a problem, since we had a clear preferred choice in which the metric looks homogeneous and isotropic. Once perturbations are allowed, such a preferred choice no longer exists, leaving only gauge-invariant statements meaningful. From the scalar perturbations $\delta\phi$ and the curvature perturbation Ψ, defined by $R^{(3)} = \frac{4}{a^2}\nabla^2\Psi$ with $R^{(3)}$ the curvature of the spatial slices, we can construct the gauge-invariant object

$$\zeta = \Psi + \frac{H}{\dot{\overline{\phi}}}\delta\phi. \tag{2.10}$$

Not surprisingly, two popular gauge choices exist in which to calculate the scalar curvature perturbations produced during inflation: the spatially flat gauge $\Psi = 0$ and the comoving gauge $\delta\phi = 0$ [28, 37–43].

In the spatially flat gauge, $\Psi = 0$, one can first simply consider the perturbations of a scalar field in a (flat) de Sitter background. The gauge invariant perturbations ζ are directly obtained from the fluctuations in the field, via $\zeta = (H/\dot{\overline{\phi}})\delta\phi$. At the end of the calculation, the generalization to quasi-de Sitter backgrounds is straightforward. To compute the power spectrum of $\delta\phi$, the two field expectation value, we need to solve its equations of motion, quantize the system and compute the expectation value. Let us choose a massless field for simplicity. With some rewriting, $v_{\boldsymbol{k}} = a\delta\phi_{\boldsymbol{k}}$, of the Fourier modes of the fluctuations $\delta\phi(t, \boldsymbol{x}) = (2\pi)^{-3/2} \int \mathrm{d}^3\boldsymbol{k}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\delta\phi_{\boldsymbol{k}}(t)$, the scalar field equation for the fluctuations,

$$\delta\ddot{\phi} - \frac{1}{a^2}\nabla^2\delta\phi + 3H\delta\dot{\phi} = 0,$$

reduces to

$$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right) v_k = 0 \tag{2.11}$$

in Fourier-space, where a prime $'$ denotes differentiation with respect to conformal time $\tau = -1/(aH)$ and where $k = |\boldsymbol{k}|$. A solution to this equation is

$$v_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right). \tag{2.12}$$

On subhorizon scales, $k \gg aH$ or equivalently $k|\tau| \gg 1$, the modes oscillate, while on superhorizon scales, $k \ll aH$ or $k|\tau| \ll 1$, the fluctuations $\delta\phi_k = v_k/a$ are frozen out at a constant value $|\delta\phi_k| = H/\sqrt{2k^3}$. The conservation on superhorizon scales is very convenient. It enables one to calculate the fluctuations at horizon exit, knowing that they will not change until the modes re-enter the horizon. After horizon re-entry, the transfer function $\Delta_l(k)$ relates the primordial fluctuations with the temperature anisotropies.

The classical dynamics can be quantized by promoting the solution (2.12) to a quantum operator

$$\hat{v}_{\boldsymbol{k}} = v_k(\tau)\hat{a}_{\boldsymbol{k}} + v^*_{-k}(\tau)\hat{a}^\dagger_{-\boldsymbol{k}}, \tag{2.13}$$

where $\hat{a}_{\boldsymbol{k}}$ and $\hat{a}^\dagger_{-\boldsymbol{k}}$ are the usual creation and annihilation operators of the set of harmonic oscillators described by (2.11), with commutation relation $[\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}] = (2\pi)^3\delta(\boldsymbol{k} - \boldsymbol{k}')$. The commutation relation imposes a normalization of the modes v_k. Together with the choice for a Bunch-Davies vacuum, $\hat{a}_{\boldsymbol{k}}|0\rangle$, —defined by the requirement that it is equal to the Minkowski vacuum in the far past [70]— this imposes sufficient boundary conditions to uniquely determine (2.12) as the solution of the second order differential equation (2.11). Using (2.12) we can compute the power spectrum of the $\delta\phi_k$ perturbations in the superhorizon limit, $P_{\delta\phi}(k) = \left(\frac{H}{2\pi}\right)^2$, which is equal to the value at horizon crossing, $k \approx aH$. As a result, we can easily generalize the de Sitter calculation to the slow-roll situation in which the Hubble parameter varies slightly or when the field is massive. In that case, different modes exit the horizon at slightly different times $k = a(t)H(t)$. Using the relation between $\delta\phi$ and ζ, the power spectrum of the gauge invariant curvature perturbations generated during slow-roll inflation is, in units $\mathrm{M_{pl}} = 1$,

$$P_\zeta(k) = \frac{1}{8\pi^2}\frac{H^2}{\epsilon}\bigg|_{k=aH}, \tag{2.14}$$

which has to be evaluated at horizon crossing. In the slow-roll regime, this is completely controlled by the effective value of H at horizon crossing. The power spectrum is scale invariant in the de Sitter limit. Departure from scale invariance is defined in terms of the spectral index $n_s - 1 = \mathrm{d}\log P/\mathrm{d}\log k$. Using the relations $\mathrm{d}\log H/\mathrm{d}t = -H\epsilon$ and $\mathrm{d}\log\epsilon/\mathrm{d}t = 2H(\epsilon - \eta)$ and the relation between $k = a(t)H(t)$ and t at horizon exit, $\mathrm{d}\log k/\mathrm{d}t = H - H\epsilon$, the spectral index n_s is given by

$$n_s - 1 = 2\eta - 4\epsilon \tag{2.15}$$

to first order in slow-roll.

The same result can be obtained by a calculation in the comoving gauge, $\delta\phi = 0$, as is explained in [57]. It is convenient to work in the ADM formalism, in which the metric is parameterized via a lapse function N and shift vector N^i [71],

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t).$$

The slow-roll action (2.3) is then given by

$$S = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{h}\,[NR^{(3)} - 2NV + N^{-1}(E_{ij}E^{ij} - E^2 + (\dot\phi - N^i\partial_i\phi)^2) - Nh^{ij}\partial_i\phi\partial_j\phi], \tag{2.16}$$

where $E_{ij} = \frac{1}{2}(\dot h_{ij} - \nabla_i N_j - \nabla_j N_i)$ and $E = E^i_{\ i}$. Spatial indices can be raised and lowered by h_{ij} and ∇_i is the covariant derivative of this spatial metric. In the comoving gauge, the scalar perturbations to the metric are given by writing

$$h_{ij} = a^2 e^{2\zeta}\delta_{ij} \approx a^2(1 + 2\zeta)\delta_{ij} \tag{2.17}$$

to first order in ζ. The field fluctuations $\delta\phi$ are zero, which means that all spatial derivatives on $\phi(t, \boldsymbol{x})$ vanish. The power of the ADM formalism is that the equations of motion for the Lagrange multipliers N and N^i are simply constraint equations, the hamiltonian and momentum constraints. Solving these constraints perturbatively in terms of ζ,

$$N = 1 + \frac{\dot\zeta}{H} + \ldots, \qquad N_i = \partial_i\left(-\frac{\zeta}{H} + \epsilon\frac{a^2}{H}\partial^{-2}\dot\zeta\right) + \ldots, \tag{2.18}$$

and substituting the result back into the action, then gives the action solely in terms of ζ. In order to find the quadratic action for ζ, it is sufficient to solve N and N^i only to first order in ζ, as the quadratic piece of N and N^i multiplies the zeroth order constraint equation which vanishes for a background solution satisfying the equations of motion [57]. Performing this procedure up to quadratic order gives

$$S^{(2)} = \int \mathrm{d}t\mathrm{d}^3\boldsymbol{x}\, a^3\epsilon\left[\dot\zeta^2 - a^{-2}(\partial_i\zeta)^2\right] = \frac{1}{2}\int \mathrm{d}\tau\mathrm{d}^3\boldsymbol{x}\left[w'^2 + \frac{z''}{z}w^2 - (\partial_i w)^2\right],$$

where $w = z\zeta$ and $z = a\sqrt{2\epsilon}$, which has as equation of motion in Fourier-space,

$$w_k'' + \left(k^2 - \frac{z''}{z}\right) w_k = 0.$$

To lowest order in slow-roll $\frac{z''}{z} \approx \frac{a''}{a} \approx \frac{2}{\tau^2}$ and we find exactly the same differential equation as (2.11). Hence, from (2.12) we read off that the power spectrum of w_k in the superhorizon limit is $P_w(k) = a^2H^2/4\pi^2$ and again we find $P_\zeta(k) = \frac{1}{z^2}P_w(k)\big|_{k=aH} = \frac{1}{8\pi^2}\frac{H^2}{\epsilon}\big|_{k=aH}$. Although the calculation is technically more involved in the comoving gauge, the advantage is that one directly uses the variable of interest ζ. It is this gauge invariant object that is conserved on superhorizon scales [40, 57].

2.3.3 Non-Gaussianities

The procedure to find the bispectrum B_ζ of primordial curvature perturbations in slow-roll inflation was laid down in [57]. In the comoving gauge, it is a direct generalization of the calculation of the two-point function, expanding (2.16) up to third order in ζ. Again it suffices to solve the hamiltonian and momentum constraints up to first order in ζ, cf. (2.18). The third order terms again multiply the constraint equations at zeroth order, while the second order terms multiply the constraint equations to first order, which vanish by the first order solution (2.18) [57, 62]. Substituting (2.18) into (2.16) and keeping cubic contributions, gives

$$S^{(3)} = \int \mathrm{d}t\mathrm{d}^3\boldsymbol{x} \left(a^3\epsilon^2\left[\dot{\zeta}^2\zeta + a^{-2}(\partial_i\zeta)^2\zeta - 2\dot{\zeta}\partial_i\zeta\partial_i\partial^{-2}\dot{\zeta}\right] + f(\zeta)\left.\frac{\delta L}{\delta\zeta}\right|_{(1)} + \dots\right). \tag{2.19}$$

The ellipsis contain terms that are of higher order in the slow-roll approximation. They are omitted to keep the calculation simple, with the justification that only —if any— the leading order contributions are likely to be observable [61]. The term prior to the ellipsis is proportional to the first order equations of motion and can therefore be removed by field redefinitions.

Once the third order action is known, the three-point function can be calculated. As was emphasized in [57, 61], the three-point function is an expectation value, defined with respect to the vacuum $|\mathrm{in}\rangle$ of the interacting theory at a given time,

$$\langle \mathrm{in}|\zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}|\mathrm{in}\rangle.$$

As in ordinary quantum field theory [72], the vacuum of the interacting theory can be obtained from an evolution of the free vacuum $|0\rangle$ using the interaction hamiltonian

$S^{(3)} = -\int d\tau H_{\rm int}(\zeta^{(2)})$. The interaction hamiltonian depends on the quantum operator $\zeta^{(2)}$ corresponding to the solution of the free theory (2.12). In a cosmological context, this procedure is summarized in the "in-in"-formalism [57, 61, 73–75], which results into

$$\langle \zeta_{\boldsymbol{k}_1}(\tau)\zeta_{\boldsymbol{k}_2}(\tau)\zeta_{\boldsymbol{k}_3}(\tau)\rangle = -i\int d\tau' \langle 0| \left[\zeta^{(2)}_{\boldsymbol{k}_1}(\tau)\zeta^{(2)}_{\boldsymbol{k}_2}(\tau)\zeta^{(2)}_{\boldsymbol{k}_3}(\tau), H_{\rm int}(\tau')\right] |0\rangle. \qquad (2.20)$$

With the interaction hamiltonian defined by $S^{(3)}$ and the free field solution $\zeta^{(2)}$ given by (2.12), the above prescription yields the bispectrum of primordial curvature perturbations produced during slow-roll inflation. To leading order in the slow-roll expansion, it is given by [57]

$$\begin{aligned} B_\zeta(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= (2\pi)^4 \left(P_\zeta\right)^2 \frac{1}{k_1^3 k_2^3 k_3^3}\left(A_\epsilon + A_\eta\right) + \dots, \qquad (2.21)\\ A_\epsilon &= \epsilon\left(\frac{1}{8}\sum_{j=1}^{3} k_j^3 + \frac{1}{8}\sum_{j\neq l} k_j k_l^2 + \frac{1}{k_t}\sum_{j<l} k_j^2 k_l^2\right),\\ A_\eta &= \eta\left(-\frac{1}{4}\sum_{j=1}^{3} k_j^3\right), \end{aligned}$$

where $k_t = k_1 + k_2 + k_3$ and where P_ζ is the power spectrum evaluated when the modes cross the horizon, under the assumption that this happens almost simultaneously for all modes.

Equation (2.21) depends only on the two leading order slow-roll parameters ϵ, η. It can be generalized to other inflationary scenarios with more parameters. For example, multi-field slow-roll inflation has a set of multi-dimensional slow-roll parameters [59] and the bispectrum of the most general single field scenario depends on a set of five parameters [62]. Equation (2.21) can also be calculated directly from the field equations, as was done in [76]. In that case, one directly solves the second order equation for $\delta\phi$, rather than using the "in-in"-formalism to calculate the three-point function in the interacting theory from the solutions of the free theory. The result can be written as

$$\begin{aligned} B_\zeta(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= (2\pi)^4 \left(P_\zeta\right)^2 \frac{1}{k_1^3 k_2^3 k_3^3}\left(A_\epsilon + A_\eta + A_\xi\right) + \dots, \qquad (2.22)\\ A_\xi &= \xi_V^2 \frac{1}{4}\left((-1 + \gamma + \log[-k_t\tau_*])\sum_{j=1}^{3} k_j^3 + k_1 k_2 k_3 - \sum_{j\neq l} k_j k_l^2\right), \end{aligned}$$

where γ is Euler's constant and τ_* is the (conformal) time of horizon crossing. Compared to (2.21), this result includes a contribution proportional to the higher order (potential) slow-roll parameter

$$\xi_V^2 = \frac{V'V'''}{V^2}. \tag{2.23}$$

It is the contribution coming from an interaction term $V'''\delta\phi^3$ in the action, as calculated by [77, 78].

The calculation of [77, 78] is actually a much simpler calculation, because the field under consideration acts as a (massless) spectator field in an expanding de Sitter background, i.e. the field is not responsible for driving the accelerated expansion. The specific form A_ξ of (2.22) corresponds to this bispectrum of a massless scalar spectator field, as the gauge invariant density perturbation can in many aspects be thought of as a massless scalar field (i.e. its solution to the equation of motion, cf. (2.20) and (2.11)). However, the perturbations of the inflaton field are also coupled to gravity and the gauge invariant curvature perturbations obtain contributions both from the fluctuations of the inflaton field as well as from metric perturbations. As argued in [57] the $V'''\delta\phi^3$-contribution to the bispectrum resides within the ... of (2.19), indicating higher order slow-roll contributions, and is neglected in that calculation. The result (2.22) confirms this expectation and explicitly shows that the contribution from a direct interaction between the scalar fields is second order in slow-roll.

Strictly speaking, by including the ξ_V^2-contribution, one should also include the other contributions that are second order in slow-roll, i.e. those proportional to ϵ^2, η^2 and $\epsilon\eta$. Since these effects will be beyond the observable threshold, the effort of correctly combining all higher order slow-roll contributions is not a relevant exercise at this time, although first results into this direction are known [79]. For our purposes, the appearance of the ξ_V^2-proportional term and in particular of the momentum structure given by $\log[-k_t\tau_*]$ in (2.22) is interesting from a more fundamental point of view, as will be further discussed in chapter 6.

3

Describing nature at its tiniest

Phenomenologically, inflation is a very successful theory that is compatible with all observations. However, its microscopic origin is not very clear and ideally one would like to have an embedding of inflation into a more fundamental framework. Maintaining the slow-roll condition, especially over 60 e-folds, turns out to be an extremely delicate exercise that is easily disturbed by the very quantum effects that provide the origin of density fluctuations. To understand inflation and the origin of our cosmos, we need to have an accurate description of the workings of nature at the smallest scales.

The quest to find out nature's workings at an ever more precise level is the tale of the history of physics, a progression that has happened in steps. Nature has been so kind to us that in order to understand a certain macroscopic phenomenon, we do not need to know the (full) details of the microscopic details within. An effective description of the phenomenon, in which the microscopic degrees of freedom decouple, is often sufficient to completely understand the relevant behavior. At a certain stage however, the details do become important and one should refine the fundamental theory. By successively focussing on the details of a given theory, we have come to understand more and more about nature. The current fundamental theory, which is believed to unify all known particles and interactions, is *string theory*.

In this chapter we will discuss the effective field theory description alluded to above, in particular the role played by *conformally invariant* theories. Furthermore we will discuss the relation between string theory and conformal invariance and we consider two additional aspects of string theory: supergravity and holography.[1]

[1] The material presented in this chapter can also be found in many terrific books and reviews. The information on renormalization and conformal field theory can be found in [72, 80–84]. String theory books and lecture notes include [85–87]. Supersymmetry and supergravity is discussed by [88, 89] and an

3.1 Conformal field theory

3.1.1 Nature at different energy scales

Renormalization and effective field theory

In technical terms, our stepwise progression into the details of nature's workings is understood through renormalization of the quantum field theory that is used to describe the world around us. Historically, renormalization was invented as a procedure of mathematical tricks, in order to extract finite answers from the divergent expressions [92], but Wilson's interpretation of renormalization in terms of coarse graining has played a key role in the conceptual understanding of renormalization and effective field theory [93–96]. The remarkable conclusion of renormalization is that the renormalized, *physical* coupling constants, i.e. the strengths of the interactions between particles, depend on the energy scale t at which a given process happens. In the Wilsonian context, this dependence is understood as the only remaining effect of the unknown underlying microphysics. The power of the renormalization group, however, is that the way the couplings run does not depend on the microscopic physics.

The scale dependence of the couplings has immediate consequences for the observability of different interactions. In a d-dimensional theory, a coupling constant u multiplying an operator of mass dimension Δ, has itself mass dimension $d - \Delta$, where the mass typically is of order of the cut-off scale Λ, used to regularize the theory. Hence, using the momentum-scale t of a given process to make a dimensionless quantity, the coupling scales as $u \sim \left(\frac{t}{\Lambda}\right)^{\Delta-d}$. This simple argument based on dimensional analysis shows that operators can be split into 3 categories: *relevant* operators with $\Delta < d$, whose coupling constants become increasingly important at low energy scales $t \ll \Lambda$, *marginal* operators with $\Delta = d$, whose coupling constants are scale independent, and *irrelevant* operators with $\Delta > d$, whose coupling constants are irrelevant at low energies, but all the more important at high energies. Therefore at low energies, an *effective field theory* in terms of only $\Delta \leq d$ operators is a sufficient description of nature, as long as one probes the theory at energies below the fundamental cut-off scale Λ [93–96]. This argument explains why nature is insensitive to microscopic details when it is only observed at a macroscopic level. All the details of the microscopic theory can be captured in terms of just a finite number of (relevant and marginal) coupling constants, which survive the small t-limit. It also implies that we are hard pressed to deduce anything about the tiniest scales in nature with our everyday, low energy experiments [97]. It is for this reason that we need to probe

introduction to holography is given in [90, 91].

high energy scales $t > \Lambda$, such as in the opportunity given by (indirect) observations from the inflationary epoch. By probing beyond the regime of validity of the effective field theory, we hope to find out what kind of irrelevant operators reside within the underlying, more fundamental theory.

Callan-Symanzik equation

The arguments using dimensional analysis give a first, qualitative indication of why a physical process depends on the energy scale t at which it occurs. The quantitative formalism to understand the precise energy dependence of a quantum field theory has been worked out by Callan and Symanzik [98–101], which results into the *Callan-Symanzik equation*, a differential equation that governs the energy dependence of n-point correlation functions. To derive it, we consider an n-point correlation function $G_0^{(n)}(p_j; u_0)$ of an operator O, given in terms of its bare coupling u_0 and depending on the momenta p_j of the operators. After regularization and renormalization, imposing renormalization group conditions at a certain scale μ, the correlation functions can also be expressed in terms of the renormalized coupling $u(\mu)$,

$$G^{(n)}(p_j; u(\mu), \mu) = Z^{-n/2} G_0^{(n)}(p_j; u_0),$$

where Z is the field rescaling factor $O \to Z^{-1/2} O$. The Callan-Symanzik equation results from the observation that the original, bare correlation function $G_0^{(n)}$ cannot depend on the choice of renormalization scale μ. This imposes a consistency condition on the renormalized n-point function $G^{(n)}$, which determines uniquely its dependence on the energy scale μ,

$$0 = Z^{-n/2} \mu \frac{\mathrm{d}}{\mathrm{d}\mu} G_0^{(n)}(p_j; u_0) = \left(\mu \frac{\partial}{\partial \mu} + \beta(u) \frac{\partial}{\partial u} + n\gamma(u) \right) G^{(n)}(p_j; u(\mu), \mu). \tag{3.1}$$

The *β function*, $\beta(u)$, and *anomalous dimension* $\gamma(u)$ of the operator O are defined through the use of the chain-rule,

$$\beta(u) = \mu \frac{\partial u}{\partial \mu}, \qquad \gamma(u) = \frac{1}{2} \frac{\mu}{Z} \frac{\partial Z}{\partial \mu}. \tag{3.2}$$

β function and anomalous dimension

The dependence of the renormalized n-point function $G^{(n)}$ on the renormalization scale μ, specified by β and γ through the Callan-Symanzik equation (3.1), automatically dictates the dependence of the theory on the physical scale t [102]. The mass dimension of a correlation function $G^{(n)}(x_j; u, \mu)$ of n operators $O(x_j)$ with scaling dimension Δ_0 is $n\Delta_0$ [72]. Extracting a momentum-conserving $\delta^{(d)}(p_1 + \ldots + p_n)$ function, the corresponding Fourier transformed correlation function has mass dimension $\Delta_n^{(p)} = n(\Delta_0 - d) + d$, which can be written in terms of the renormalization scale μ and some function of dimensionless ratios p_j/μ,

$$G^{(n)}(p_j; u, \mu) = \mu^{\Delta_n^{(p)}} f\left(\frac{p_j}{\mu}\right).$$

We can rescale all momenta with a common factor of t. It sets the energy at which the physical process is probed. Using the relation between t and μ,

$$t\frac{\partial}{\partial t} G^{(n)}(tp_j; u, \mu) = \left(-\mu\frac{\partial}{\partial\mu} + \Delta_n^{(p)}\right) G^{(n)}(tp_j; u, \mu),$$

the Callan-Symanzik equation is written completely in terms of the overall momentum dependence t,

$$\left(t\frac{\partial}{\partial t} - \beta(u)\frac{\partial}{\partial u} - (\Delta_n^{(p)} + n\gamma(u))\right) G^{(n)}(tp_j; u, \mu) = 0. \tag{3.3}$$

In this form, the Callan-Symanzik equation fixes the dependence of the correlation function with physical rescalings. A general solution to this equation is [72],

$$G^{(n)}(tp_j; u, \mu) = G^{(n)}(p_j; \tilde{u}(t; u), \mu) \exp\left(\int_{t'=1}^{t'=t} \mathrm{d}\log t' \left[\Delta_n^{(p)} + n\gamma(\tilde{u}(t'; u))\right]\right), \tag{3.4}$$

in terms of the function $\tilde{u}(t; u)$ defined through the differential equation (3.2),

$$t\frac{\partial}{\partial t}\tilde{u}(t; u) = \beta(\tilde{u}(t; u)), \qquad \tilde{u}(1; u) = u. \tag{3.5}$$

Usually, once a solution for $\tilde{u}(t; u)$ is found, it is denoted with $u(t)$ and simply referred to as the *running coupling* of the theory.

For a free field theory, with $\beta = \gamma = 0$, the solution (3.4) indeed reproduces the correct scaling $G^{(n)}(tp_j; u, \mu) \sim t^{\Delta_n^{(p)}}$. When β and γ are nonzero, the momentum dependence changes. In a given theory, the β function can be calculated by computing the counterterms in a renormalization procedure. The differential equation (3.5) then determines the running of the coupling constant $u = u(t)$ as a function of the energy scale t at which the specific process is considered. As such, the renormalization group equations can be seen as a *flow* on the space of coupling constants of the theory: depending on the sign of $\beta(u)$, coupling constants are increasingly dominant or less and less important along the renormalization group flow. As we will see shortly, close to a free field theory, $u \approx 0$, the β functions determine a flow that indeed approximates the anticipated $u \sim t^{\Delta-d}$ behavior. However, once $u \neq 0$, the β function might change and as a result the precise scaling behavior of $u(t)$ will also change. Whether operators are *truly* relevant, marginal or irrelevant therefore depends on whether $\beta < 0, \beta = 0$ or $\beta > 0$.

A nonzero anomalous dimension γ changes the scaling behavior of the correlation function and induces a change in the scaling dimension of the operators $\mathcal{O}$ inside the correlation function. This is best seen around a non-trivial fixed point of the theory, where $\beta(u_*) = 0$ and $u(t)$ is constant $u(t) = u_* \neq 0$. Then the solution (3.4) yields

$$G^{(n)}(tp_j; u_*, \mu) = t^{\Delta_n^{(p)} + n\gamma(u_*)} G^{(n)}(p_j; u_*, \mu).$$

Again, the n-point function scales with a power law of t, but now the exponent is different than the usual $\Delta_n^{(p)}$. Tracing back to the scaling dimension of the operator $\mathcal{O}(x_j)$, it appears as if its scaling dimension Δ_0 is changed to

$$\Delta = \Delta_0 + \gamma, \tag{3.6}$$

which explains why γ is called the anomalous dimension.

3.1.2 Field theory without a scale

Conformal transformations

It is clear that theories with vanishing β functions play a special role in the study of renormalization group flow on the space of theories. Such a scale invariant theory acts as a fixed point for the renormalization group flow: the coupling constants are scale invariant and remain scale invariant. As such, they form an ideal starting point to study the renormalization group flow perturbatively. Before explaining the perturbative approach, let us consider the *conformal field theories* themselves [83, 84].

Conformal field theories are invariant under conformal transformations, i.e. transformations $x \mapsto x'(x)$ such that the metric $h_{\alpha\beta}$ changes with an overall spacetime dependent factor,

$$h'_{\alpha\beta}(x') = \Lambda(x) h_{\alpha\beta}(x).$$

Together with the standard Lorentz transformations, they form a group, the conformal group, whose transformations in dimensions $d > 2$ are translations, dilations, rotations[2] and special conformal transformations,

$$x'^{\alpha} = x^{\alpha} + a^{\alpha}, \qquad x'^{\alpha} = \lambda x^{\alpha}, \tag{3.7a}$$

$$x'^{\alpha} = L^{\alpha}{}_{\beta} x^{\beta}, \qquad x'^{\alpha} = \frac{x^{\alpha} - b^{\alpha} x^2}{1 - 2b^{\alpha} x_{\alpha} + b^2 x^2}, \tag{3.7b}$$

respectively. The first three transformations together form the Poincaré group extended with dilations, i.e. the symmetry group of scale invariant theories. Hence, conformal invariance, or *local* scale invariance, implies scale invariance, which is why they form a good starting point to study renormalization group fixed points.

Rotations and special conformal transformations shall not play a large role in this thesis and we shall focus on translations and dilations. On a field O, these two transformations act as $O'(x) = (1 - iG_a \omega_a(x))O(x)$, where ω_a is the infinitesimal parameter of the transformation and the generators G_a are given by

$$G_{T,\alpha} = -i\partial_{\alpha}, \qquad G_D = -i\left(x^{\alpha}\partial_{\alpha} + \Delta\right),$$

respectively. Δ is the scaling dimension of the field O, $O'(\lambda x) = \lambda^{-\Delta} O(x)$. The conserved current associated with translational symmetry is the stress-energy tensor $j_{T,\alpha\beta} = T_{\alpha\beta}$. Canonically it can be expressed through a standard Noether procedure, i.e. under translations $x'^{\alpha} = x^{\alpha} + a^{\alpha}$ the action changes infinitesimally

$$\delta S = \int \mathrm{d}^d x \, \sqrt{h} \, T^c_{\alpha\beta} \nabla^{\alpha} a^{\beta}.$$

The disadvantage of this definition is that $T^c_{\alpha\beta}$ will not necessarily be symmetric. Therefore, a new, improved stress-energy tensor can be defined [83, 98] which plays the same role as $T^c_{\alpha\beta}$ and which is symmetric. Another way to define the stress-energy tensor is by considering a dynamical metric $h_{\alpha\beta}$ for the theory. Under the diffeomorphism $x'^{\alpha} = x^{\alpha} + a^{\alpha}(x)$ the metric changes as a tensor, $\delta h^{\alpha\beta} = \nabla^{\alpha} a^{\beta} + \nabla^{\beta} a^{\alpha}$. Hence in an invariant theory, the metric itself must transform opposite to this,

$$\delta S = -\frac{1}{2} \int \mathrm{d}^d x \, \sqrt{h} \, T_{\alpha\beta} \delta h^{\alpha\beta}. \tag{3.8}$$

A manifestly symmetric stress-energy tensor can therefore also be obtained via

$$T_{\alpha\beta} = -\frac{2}{\sqrt{h}} \frac{\delta S}{\delta h^{\alpha\beta}}, \tag{3.9}$$

[2] Although we will partly be interested in conformal field theories that are of Lorentzian signature, we can always Wick-rotate to a Euclidian signature. For this reason, the inner products in this section are always taken to be Euclidean. Moreover, when studying conformal field theory, the metric is fixed, which for many purposes may assumed to be flat.

where the normalization depends on convention. In particular the stress-energy tensor in string theory often contains factors of π in its definition [85, 86].

Under an infinitesimal conformal transformation $x'^{\alpha} = x^{\alpha} + \epsilon^{\alpha}(x)$, which can be shown to satisfy the conformal Killing equation,

$$\nabla_{\alpha}\epsilon_{\beta} + \nabla_{\beta}\epsilon_{\alpha} = \frac{2}{d}\nabla_{\gamma}\epsilon^{\gamma}h_{\alpha\beta}, \tag{3.10}$$

the action is invariant,

$$0 = \delta S = -\frac{1}{d}\int \mathrm{d}^d x\, \sqrt{h}\, T^{\alpha}{}_{\alpha}\nabla_{\beta}\epsilon^{\beta}, \tag{3.11}$$

if the stress-energy tensor is traceless,

$$\Theta = T^{\alpha}{}_{\alpha} = 0.$$

Hence, a theory with a traceless stress-energy tensor is conformally invariant. One might be tempted to think that the reverse is also true, but since $\epsilon(x)$ has to satisfy (3.10), it is not an arbitrary function. Nevertheless, for most conformal field theories, the stress-energy tensor can indeed be made traceless by a procedure similar to the one used to make it symmetric [83]. In those cases, the stress-energy tensor is related to the dilational current,

$$j^{\alpha}_{D} = T^{\alpha}{}_{\beta}x^{\beta},$$

and tracelessness follows from (translational and) scale invariance.

Correlation functions in a conformal field theory

The symmetries in a conformal field theory impose powerful constraints on the functional dependence of n-point correlation functions, particularly the two- and three-point functions. For example, translational and rotational invariance of the theory tell us that the dependence on the arguments x_a can only appear via $|x_a - x_b|$. Including dilational invariance and special conformal transformations, the two- and three-point functions of operators O_a with scaling dimension Δ_a are fixed to have the form [83]

$$\langle O_1(x_1)O_2(x_2)\rangle = \frac{N\delta_{\Delta_1\Delta_2}}{x_{12}^{2\Delta_1}}, \tag{3.12a}$$

$$\langle O_1(x_1)O_2(x_2)O_3(x_3)\rangle = \frac{C^1_{23}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3}\, x_{23}^{-\Delta_1+\Delta_2+\Delta_3}\, x_{13}^{\Delta_1-\Delta_2+\Delta_3}}, \tag{3.12b}$$

where $x_{ab} = |x_a - x_b|$. The overall coefficients N and C^1_{23} are not fixed by any symmetry constraints. N determines the overall normalization of the field O_a. C^1_{23} is called the *OPE coefficient* because it is the coefficient in the operator product expansion, an expansion similar to a Taylor expansion that relates the product of two operators $O_a(x)$ and $O_b(y)$ to the other operators in the theory in the limit $x \to y$,

$$O_b(x)O_c(y) = \sum_a C^a_{bc}|x-y|^{\Delta_a-\Delta_b-\Delta_c} O_a\left(\frac{x+y}{2}\right).$$

Higher order n-point functions are less constrained than the two- and three-point correlation functions. For example, the four-point function can have an arbitrary functional dependence on *cross ratios*,

$$\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle = F\left[\frac{x_{12}x_{34}}{x_{13}x_{24}}, \frac{x_{12}x_{34}}{x_{23}x_{14}}\right]\prod_{a<b}^{4} x_{ab}^{\sum_c \Delta_c/3-\Delta_a-\Delta_b}.$$

Conformal invariance in two dimensions

Conformal symmetry is particularly powerful in two dimensions. This is clear from the condition (3.10) on the infinitesimal parameter $\epsilon^\alpha(x)$, which on a flat metric, $h_{\alpha\beta} = \delta_{\alpha\beta}$, reduces to the Cauchy-Riemann equations

$$\partial_1\epsilon_1 = \partial_2\epsilon_2, \qquad \partial_1\epsilon_2 = -\partial_2\epsilon_1.$$

This suggests we should actually express the transformation parameter $\epsilon(z) = \epsilon^1 + i\epsilon^2$ and $\bar{\epsilon}(\bar{z}) = \epsilon^1 - i\epsilon^2$ in terms of the complex coordinates $z = x^1 + ix^2$, $\bar{z} = x^1 - ix^2$. The functions $\epsilon(z)$ and $\bar{\epsilon}(\bar{z})$ are otherwise unconstrained, giving an infinite set of symmetry generators $z' = z+\epsilon(z)$, $\bar{z}' = \bar{z}+\bar{\epsilon}(\bar{z})$, rather than the finite set given by (the infinitesimal version of) (3.7).

By momentarily promoting x^1 and x^2 to elements in $\mathbb{C}$, the transformation between x^α and $z, \bar{z}$ is a coordinate transformation of independent coordinates. The symmetry algebras for $\epsilon(z)$ and $\bar{\epsilon}(\bar{z})$ are then independent copies of the same algebra. Only at the end of a calculation is the reality condition $\bar{z} = z^*$ imposed.

Mathematically the restrictive power of two-dimensional conformal symmetry is equivalent to the conditions imposed on (anti)-holomorphic functions in complex analysis. Since complex analysis is such a rich and well-developed branch of mathematics, many of the techniques can be applied successfully to two-dimensional conformal field theory [83, 84]. Although part of this thesis deals with conformal invariance in two dimensions, its remarkable structure is not heavily or actively built upon. For this reason, we do not elaborate much further on the special two-dimensional case.

3.1.3 Conformal perturbation theory

Weyl anomaly coefficients

In the previous section we have mostly been interested in the classical behavior of conformal field theories. When considering quantum field theories with conformal invariance, the situation becomes more involved. To make sense out of a quantum field theory, its expressions need to be regularized and renormalized, thereby automatically introducing a scale into the classically scale invariant theory [83]. For this reason, it might be that a classically conformally invariant theory loses its conformality as a quantum field theory. The departure from conformal invariance, can be expressed in terms of a violation of the hallmark of a conformal field theory, i.e. the trace of the stress-energy tensor is no longer vanishing,

$$\Theta = -\beta(u)\frac{\delta\mathcal{L}}{\delta u}. \tag{3.13}$$

The notation of the coefficients, $\beta(u)$, is no accident, as they are closely related to the renormalization group β functions (3.2) [103–106]. Conceptually this is easy to understand. Only a quantum field theory with a vanishing β function will remain a fixed point for the renormalization group flow, without the introduction of any new scale into the theory. All relevant and irrelevant operators induce a renormalization of their couplings and therefore a scale dependence.

Technically the relation may be seen by the effect of an explicit scale transformation $x'^{\alpha} = e^{\omega}x^{\alpha}$ on the coupling $\delta u = \omega\beta(u)$, where $\beta(u)$ now really is the renormalization group β function [72]. As a result the action changes as

$$\delta S = \int \mathrm{d}^d x\, \delta\mathcal{L} = \int \mathrm{d}^d x\, \omega\beta(u)\frac{\delta\mathcal{L}}{\delta u}.$$

Compared to the definition (3.8) of the stress-energy tensor in terms of a scale transformation of the metric $h'_{\alpha\beta}(x') = e^{-2\omega}h_{\alpha\beta}(x)$,

$$\delta S = -\frac{1}{2}\int \mathrm{d}^d x\, T_{\alpha\beta}\delta h^{\alpha\beta} = -\int \mathrm{d}^d x\, \Theta\omega, \tag{3.14}$$

the renormalization group β functions appear as coefficients in

$$\int \mathrm{d}^d x\, \Theta = -\int \mathrm{d}^d x \beta(u)\frac{\delta\mathcal{L}}{\delta u}.$$

Hence, the renormalization group β functions and the *Weyl anomaly coefficients* β appearing in (3.13) are related in the same way as global and local scale invariance are related. The requirements $\int \Theta = 0$ and $\Theta = 0$ for global and local scale invariant theories respectively are directly transferred to the β functions. Although this relation can and is used [107–109] to simplify the computation of Weyl anomaly coefficients, we will not emphasize the distinction.

In this section we have specifically restricted ourselves to flat metrics only. As we will see when we turn our attention to string theory, conformal symmetry on a curved space introduces another source for Weyl anomaly, due to the curvature scale that is introduced.

Computation of the β functions

A conformal field theory, with vanishing β functions and traceless stress-energy tensor, is a fixed point for the renormalization group flow. To study the flow perturbatively, we consider a perturbation of the conformal field theory S_0 by operators O_a with coupling u^a and dimension $\Delta_{0,a}$,

$$S_u = S_0 + \int \mathrm{d}^d x\, u^a O_a(x). \tag{3.15}$$

The trace of the stress-energy tensor is

$$\Theta = -\beta^a(u) O_a. \tag{3.16}$$

The coefficients $\beta^a(u)$ can be calculated perturbatively by considering the Callan-Symanzik equation (3.1). The anomalous dimension γ appearing in the Callan-Symanzik equation actually becomes a matrix of anomalous dimensions γ^a_b for the multi-operator case under consideration. It can be related to the β functions [110] via[3]

$$\gamma^a_b(u) = \frac{\partial \beta^a}{\partial u^b} - (\Delta_{0,b} - d)\delta^a_b. \tag{3.17}$$

Writing $\beta^a(u) = A^a + B^a_b u^b + \ldots$ and remembering that u^a has mass dimension $\mu^{d-\Delta_{0,a}}$, applying the Callan-Symanzik equation to the partition function of the perturbed theory,

$$Z = \langle e^{-\int \mathrm{d}^d x\, u^a O_a} \rangle_0 = \langle 1 - \int \mathrm{d}^d x\, u^a O_a + \ldots \rangle_0,$$

enables us to compute $\beta(u)$ recursively as a perturbation series in u [86, 110–112]. To compute the higher order coefficients of $\beta(u)$, an operator product expansion is necessary, whose singularities have to be regularized. The regularization scheme

[3] We note that different conventions compared to [110, 111] are used.

dependence is carefully explained in [111]. In the limit where the operators are nearly marginal, $|\Delta_{0,a} - d| \ll 1$, the result in the Zamolodchikov scheme [110, 111] is

$$\beta^a(u) = (\Delta_{0,a} - d)u^a + 2\pi C^a_{bc} u^b u^c + \dots \tag{3.18}$$

to second order in u. In the first term, there is no summation over the a-index.

As we see, conformal perturbation theory enables us to study the renormalization group flow around a conformal fixed point. The flow is determined by the β functions, which can be expressed in terms of the scaling dimension $\Delta_{0,a}$ of the operator $\mathcal{O}_a$ in the unperturbed conformal field theory, as long as the deviation from the fixed point is small, $u^a \ll 1$ and the operators under consideration are nearly marginal $|\Delta_a - d| \ll 1$.

3.2 String theory

3.2.1 Worldsheet physics

Strings in a flat background

The previous section mostly dealt with conformal field theories with a fixed flat metric. In essence, string theory is the study of two-dimensional conformal field theory with a dynamical, and hence curved, metric. The motivation to study such a theory follows from a direct generalization of the first quantization description of a point particle. Similar to a point particle, the classical trajectory of a string is determined by minimizing its worldvolume, which is called a worldsheet for a one-dimensional extended object. The string is described by embedding the worldsheet, with coordinates $\sigma^\alpha = (\sigma^0, \sigma^1)$ into the d-dimensional target spacetime $\sigma \mapsto x^\mu(\sigma)$. In a flat target spacetime, the worldsheet area is minimized by the minimization of the Polyakov action

$$S[x, h] = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{h}\, h^{\alpha\beta} \eta_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu. \tag{3.19}$$

α' is a coupling constant for the two-dimensional field theory, which determines the string's tension. Both the fields x^μ and the worldsheet metric $h_{\alpha\beta}$ are dynamical objects. As a two-dimensional field theory, the Polyakov action describes d scalar fields x^μ coupled to two-dimensional gravity. Varying the action with respect to the metric $h_{\alpha\beta}$ tells us that the two-dimensional stress-energy tensor $T_{\alpha\beta}$ should vanish. The equations of motion from a variation with respect to the fields x^μ yield a free wave equation, determining the string's propagation in target spacetime. The latter variation also specifies boundary conditions, allowing both open and closed string solutions [85, 86].

The Polyakov action is invariant under several symmetries. It is invariant under d-dimensional Poincaré transformations,

$$x'^{\mu}(\sigma) = \Lambda^{\mu}{}_{\nu} x^{\nu}(\sigma) + a^{\mu}, \qquad h'_{\alpha\beta}(\sigma) = h_{\alpha\beta}(\sigma),$$

and under two-dimensional reparameterizations $\sigma \mapsto \sigma'(\sigma)$,

$$x'^{\mu}(\sigma') = x^{\mu}(\sigma), \qquad h'_{\alpha\beta}(\sigma') = \frac{\partial\sigma^{\gamma}}{\partial\sigma'^{\alpha}} \frac{\partial\sigma^{\delta}}{\partial\sigma'^{\beta}} h_{\gamma\delta}(\sigma).$$

Most importantly, and very specific to the two-dimensional nature of the worldsheet, it is invariant under *Weyl transformations*,

$$x'^{\mu}(\sigma) = x^{\mu}(\sigma), \qquad h'_{\alpha\beta}(\sigma) = e^{2\omega(\sigma)} h_{\alpha\beta}(\sigma),$$

which are, again, local scale transformations of the theory. From (3.14) it is clear that the stress-energy tensor is traceless if and only if a theory is invariant under Weyl transformations, explaining the terminology for the coefficients in (3.13). In two dimensions, the Weyl symmetry is special, as it ensures that all three metric modes can be gauged away. This also shows why there are no gravitational dynamics in two dimensions.

The Weyl invariance of the worldsheet action is reminiscent of conformal invariance. The relation can best be seen by noting that the symmetries of the Polyakov action are gauge symmetries. Gauge symmetries describe a redundancy in the theory, introduced for mathematical convenience but at the same time introducing more degrees of freedom than just the physical ones. In the Polyakov action the redundancy can be removed by fixing a gauge, $h_{\alpha\beta} = \eta_{\alpha\beta}$. After gauge fixing, the (Wick)-rotated action,

$$S[x] = \frac{1}{4\pi\alpha'} \int \mathrm{d}^2\sigma \, \delta^{\alpha\beta} \partial_{\alpha} x^{\mu} \partial_{\beta} x^{\nu} \eta_{\mu\nu}, \tag{3.20}$$

is conformally invariant. Conformal transformations describe a residual gauge symmetry, a particular combination of diffeomorphisms that can be undone by a Weyl transformation [87].

Weyl invariance and background dynamics

Before and after gauge fixing, string theory can equivalently be described by a Weyl invariant worldsheet action with dynamical metric or by a two-dimensional conformal field theory with a fixed metric respectively. As these are gauge symmetries, it is important that invariance is maintained both at the classical as well as at the quantum level. An anomalous gauge symmetry would introduce a dependence on the gauge choice, promoting unphysical degrees of freedom. For a conformal field theory the risk of a quantum anomaly is not improbable. We have already seen that, even if a theory is classically locally scale invariant, renormalization effects can easily introduce anomalous contributions to the stress-energy tensor at the quantum level. Therefore, imposing conformal invariance on the worldsheet even at the quantum level, introduces severe constraints on the possible worldsheet theory.

One of the quantum excitations of the solutions x^μ of the Polyakov action (3.19) is the spacetime graviton. This introduces a quantum deviation from the flat Minkowski metric $\eta_{\mu\nu}$ through which the strings propagate. Building a full coherent target spacetime metric $g_{\mu\nu}$ from such gravitons, the string's trajectory is determined by the spacetime curvature determined from the two-dimensional worldsheet action. At the same time, conformal invariance dictates what kind of conformal theory, including its quantum excitations such as the graviton, is allowed. The subtle interplay between the string's propagation in the background metric and the background dynamics built up from string excitations is a non-trivial consistency check on the two-dimensional worldsheet. Weyl (or, equivalently, conformal) invariance is at the heart of this consistency of string theory. The relation between background dynamics and Weyl invariance is one of the best understood and most studied features of string theory, going back to the advent of the theory in the early 1980s [85–87, 113–117]. We will now explain how Weyl invariance determines the dynamics of the background fields and as a result how general relativity follows from string theory.

Strings in a curved background

Strings moving in a curved background target spacetime metric can be described by the Euclidean action

$$S[x,h] = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\,\sqrt{h}\,\left[h^{\alpha\beta}g_{\mu\nu}(x)\partial_\alpha x^\mu\partial_\beta x^\nu + \alpha'\Phi(x)R^{(2)}\right], \tag{3.21}$$

where $g_{\mu\nu}(x)$ is the target spacetime metric and $\Phi(x)$ is the dilaton field. The action (3.21) is a straightforward generalization of (3.19), promoting the flat spacetime metric $\eta_{\mu\nu}$ to the (dynamical) metric $g_{\mu\nu}(x)$ of the curved background spacetime. The dilaton contribution gives rise to a weight factor in the path integral sum over all geometries. When the dilaton Φ is constant, it multiplies the topologically invariant Euler number of the worldsheet, that counts the genus of the two-dimensional Riemann surface. The vacuum expectation value of the dilaton is therefore directly related to the *string coupling constant* $g_s = e^{\Phi_0}$ which determines the likelihood of strings joining or splitting. Usually one also includes a contribution from the anti-symmetric Kalb-Ramond field $B_{\mu\nu}(x)$, which will be assumed to vanish throughout this thesis.

Equation (3.21) can be obtained from an exponentiation of the massless quantum excitations of the string. In a flat Minkowski target spacetime metric, the massless excitations of the string decompose into three irreducible representations of the Poincaré algebra: a traceless symmetric representation giving rise to the graviton, an anti-symmetric representation leading to $B_{\mu\nu}$ and a trace, i.e. singlet, representation for the dilaton Φ. Each of the excitations is described by a corresponding vertex operator, inserting the required excitation in the far past, via the state-operator correspondence. The vertex operators may be combined into the fields $g_{\mu\nu}(x)$ and $\Phi(x)$, which give rise to (3.21) after exponentiation of the vertex operators [85, 86, 118].

As expected after our plea for Weyl invariance of the worldsheet theory, the first term in the action (3.21) is (classically) Weyl invariant. The generalization from (3.19) by promoting the Minkowski metric $\eta_{\mu\nu}$ to a general metric $g_{\mu\nu}(x)$ has no effect on the Weyl invariance of the theory. As a two-dimensional theory, it is just a change in the functional of couplings in front of the kinetic terms for the scalar fields. The way in which this is done is known as a *nonlinear σ model*. The second term of (3.21) is all the more surprising, as it already breaks Weyl invariance at the classical level for a non-constant dilaton profile $\Phi(x)$. However, it is necessary to include the dilaton in order to take into account the full multiplet of massless quantum string excitations. For this reason, the term appearing in the action was introduced by [119] and was shown to behave consistently with the other massless degrees of freedom. As we will see shortly, the tree level Weyl variation of the dilaton can be combined with the one-loop Weyl anomalies arising from the other terms [120]. The additional factor of α' helps ordering the different contributions to the breaking of Weyl invariance.

To preserve Weyl invariance at the quantum level, we again impose a vanishing trace of the stress-energy tensor Θ of the two-dimensional worldsheet tensor. Expanding Θ in terms of the operators appearing in (3.21),

$$\Theta = -\frac{1}{2\alpha'}\beta^g_{\mu\nu}h^{\alpha\beta}\partial_\alpha x^\mu\partial_\beta x^\nu - \frac{1}{2}\beta^\Phi R^{(2)},$$

the stress-energy tensor is traceless if the β functions,

$$\beta^g_{\mu\nu} = \alpha' R_{\mu\nu} + 2\alpha'\nabla_\mu\nabla_\nu\Phi + O(\alpha'^2), \tag{3.22a}$$

$$\beta^\Phi = \frac{d-26}{6} - \frac{\alpha'}{2}\nabla^2\Phi + \alpha'\nabla_\mu\Phi\nabla^\mu\Phi + O(\alpha'^2), \tag{3.22b}$$

vanish. The β functions are given up to first order in α'. To compute them, one has to combine several contributions, which we consider individually.

The first term in β^Φ is a pure quantum anomaly, first calculated in the context of string theory by [116]. In general, a conformal field theory with a curved metric $h_{\alpha\beta}$ has an anomaly proportional to the two-dimensional Ricci scalar

$$\Theta = -\frac{c}{12}R^{(2)}. \tag{3.23}$$

The constant of proportionality is called the *central charge*, as it appears as a central charge in the quantum algebra of the generators of conformal transformations. This anomaly tells us that, in the quantum theory, the trace of the stress-energy tensor no longer vanishes and conformal invariance is broken, i.e. the theory has become scale dependent. Since in string theory the conformal symmetry follows from the local Weyl *gauge* symmetry, the metric is dynamical and the only way to ensure an anomaly-free quantum theory is to consider conformal field theories which have central charge $c = 0$. The value of $c = d - 26$ in β^Φ can be understood by the study of the path integral of the string worldsheet [116]. Due to the gauge redundancy in the path integral measure, one has to be careful to not overcount the number of (inequivalent) physical configurations. This can be done by the use of a Faddeev-Popov determinant, which can be written in terms of a ghost action. The ghost action for Weyl transformations is a conformal field theory with central charge $c = -26$. This is why any worldsheet action, with the ghost action left implicit, has to have central charge $c = 26$. The curved Polyakov action (3.21) achieves this geometrically by considering d scalar fields, which explains the much emphasized critical dimension for string theory. However, the curved Polyakov action only serves as a motivational starting point for string theory. In principle, any conformal field theory with central charge $c = 26$ would describe some solution to string theory, emphasizing it is not the dimension but the central charge that is critical.

The other terms in (3.22) are conceptually more straightforward to understand, but technically still quite involved to compute [85, 86, 115, 117]. The metric profile $g_{\mu\nu}(x)$ and dilaton profile $\Phi(x)$ act as coupling functionals to the operators in (3.21). Renormalizing these coupling constants will lead to β functions much in the same way as we explained previously. The Ricci tensor $R_{\mu\nu}$ of the target spacetime and the second term of β^Φ arise due to these renormalization effects. The remaining terms are due to classical breaking of Weyl invariance by the dilaton term, which appear at the same order as the renormalization effects from the other terms as predicted.

It can be shown explicitly that the β functions (3.22) are proportional to the equations of motion for the background fields, $g_{\mu\nu}(x)$ and $\Phi(x)$, that one would compute in string perturbation theory [105, 121]. The stringy degrees of freedom, i.e. excitations with a mass proportional to α', do not play a role, as the worldsheet perturbation is derived in the limit of small α', or in other words for strings with a very large tension. In this limit, stringy excitations cost a lot of energy, which justifies the name "low energy equations of motion" for (3.22). In fact, (3.22) can be integrated to a *low energy effective action*,

$$S[g,\Phi] = \frac{1}{2\kappa_0^2}\int \mathrm{d}^d x\,\sqrt{g}\,e^{-2\Phi}\left[-\frac{2(d-26)}{3\alpha'} + R + 4\,(\nabla\Phi)^2 + O(\alpha')\right]. \tag{3.24}$$

By a field redefinition $\Phi^{\text{new}} = \Phi^{\text{old}} - \Phi_0$ and $g_{\mu\nu}^{\text{new}} = e^{-4\Phi^{\text{new}}/(d-2)} g_{\mu\nu}^{\text{old}}$, the action can be written in a more familiar form,

$$S[g,\Phi] = \frac{1}{2\kappa^2}\int \mathrm{d}^d x\,\sqrt{g}\left[-\frac{2(d-26)}{3\alpha'}e^{4\Phi/(d-2)} + R - \frac{4}{d-2}\,(\nabla\Phi)^2 + O(\alpha')\right], \tag{3.25}$$

where $\kappa = \kappa_0 e^{\Phi_0} = \sqrt{8\pi G_N}$ is the gravitational coupling constant. Equation (3.25) describes a scalar field Φ coupled to Einstein gravity in d dimensions, showing explicitly that string theory is a theory of spacetime quantum gravity.

The relation between the two-dimensional worldsheet action and how it describes general relativity in the d-dimensional target spacetime is an intricate result. The fact that we can express the equations of motion given by (3.22) in terms of a target spacetime action guarantees that the equations are mutually consistent [120]. Crucial for the inner consistency is the interdependence among the β functions. The dilaton β function β^{Φ} acts as the central charge of the full nonlinear σ model. Although it looks like an x-dependent quantity, it is really a c-number due to the vanishing of $\beta^g_{\mu\nu}$ (and $\beta^B_{\mu\nu}$ if we would not have set $B_{\mu\nu}$ to zero to begin with) [117, 120]. It is this central charge, or rather the combination $\beta^{\Phi} - g^{\mu\nu}\beta^g_{\mu\nu}$, that effectively acts as the integrand for the low energy effective action (3.24). For the first order equations (3.22) we can verify these statements explicitly, but it can be proven to hold on general grounds for all order α'-corrections [107, 108, 122]. The possibility to interpret the conditions set by worldsheet Weyl invariance as a spacetime low energy effective action is one of the most remarkable results from string theory.

3.2.2 Supergravity

A super symmetry in our universe

In the previous section we considered the bosonic string, i.e. a string whose worldsheet theory is defined in terms of bosonic scalar fields x^{μ} only. It is a very interesting theory to study the relation between the worldsheet and spacetime theories, but it is unsure to what extent this version of string theory can describe our universe. Apart from the massless quantum excitations we have just considered, the bosonic string also contains a tachyonic mode, indicating that the theory suffers from an instability [85]. To remove the tachyon from the spectrum, the worldsheet is extended to a *superstring* theory, in which worldsheet bosons and fermions are related by a symmetry called *supersymmetry*. Similar to the previous discussion, the superstring worldsheet theory defines a low energy effective field theory for the background fields on the target spacetime. Because the value of the central charge c of a worldsheet theory with superconformal symmetry is $c = \frac{3d}{2} - 15$, the spacetime theory is a ten-dimensional theory.

In order to relate to our four-dimensional spacetime, the spacetime has to be compactified on an internal six-dimensional manifold [123]. The internal manifold is a compact manifold, which is too small for us to detect at low energies, giving rise to an effective four-dimensional action for the spacetime theory after compactification. In this thesis we will study superstring theory only through its four-dimensional low energy effective action, except for a short excursion in chapter 5 where we discuss how to possibly generalize the result of that chapter to open strings, the D-branes that they end on and the background RR fields sourced by the D-branes. At the level of the low energy effective action, supersymmetry remains a fundamental aspect for the theory, since the spacetime bosons and fermions are also invariant under the supersymmetry transformations [86].

The low energy effective action of superstring theory is an example of a *supergravity* theory, but the framework of supergravity is more general than just the supergravity theories arising from superstring theory. In the 1970s supersymmetry was discovered as a way to regulate UV-divergences in phenomenological particle physics models [124–127]. As gauge symmetries were particularly popular at the time for the successful way in which they describe particle physics, it was only a natural step to consider a theory which is invariant under local supersymmetry [128]. The surprising result is that such a theory necessarily incorporates gravity [88], hence the name "supergravity". Initial hope that supergravity theory might be a "theory of everything", unifying particle physics theories with general relativity, soon proved incorrect, because supergravity is not renormalizable. Therefore, in the beginning of the 1980s superstring theory started to replace supergravity as the new candidate theory for quantum gravity [85]. Nevertheless, through the relation between worldsheet superstring theory and its supergravity low energy effective theory, supergravity models have never really left the stage, providing an interesting playground at the effective field theory level for the study of quantum gravity.

Super dynamics

As with any symmetry, supersymmetrically invariant theories are constrained. The bosons and fermions of the theory have to reside in supermultiplets, which are irreducible representations of the supersymmetry algebra. To ensure that the algebra is closed off-shell as well, each supermultiplet also contains an auxiliary field. Conventionally for the chiral supermultiplets, i.e. the simplest four-dimensional supermultiplet that contains a scalar, the auxiliary field is denoted by F^I, where I is an index running over the number of chiral supermultiplets. Similarly, gauge vector supermultiplets have an auxiliary field denoted by D^A, where A runs over the number of vector supermultiplets. These auxiliary fields do not have a kinetic term in the action and therefore contain no propagating (physical) degrees of freedom. It turns out that potentials in supersymmetric field theories are precisely generated by integrating out these non-dynamical fields [88, 89]. Supersymmetry and supergravity potentials therefore naturally fall into two categories. The scalar potentials built from F are called F-terms, those built from the D-fields are called D-terms. In this thesis we will be concerned with the scalars ξ^I of the chiral multiplets. We will assume they are neutral under the gauge group, allowing us to concentrate on the F-terms.

In global supersymmetry the action for the complex scalars ξ^I in the chiral supermultiplets can be written as

$$S = -\int \mathrm{d}^4x\,\sqrt{g}\left[g^{\mu\nu}K_{I\bar{J}}(\xi,\bar{\xi})\nabla_\mu\xi^I\nabla_\nu\bar{\xi}^{\bar{J}} + V(\xi,\bar{\xi})\right]. \tag{3.26}$$

Supersymmetry has restricted the kinetic term to be a nonlinear σ model describing a Kähler manifold. The *Kähler potential* $K(\xi,\bar{\xi})$ is a real function which completely specifies the metric $G_{I\bar{J}}(\xi,\bar{\xi})$ of the target manifold,

$$G_{I\bar{J}} = \partial_I\partial_{\bar{J}}K \equiv K_{I\bar{J}}, \qquad G_{IJ} = G_{\bar{I}\bar{J}} = 0.$$

The F-term potential V is determined by the holomorphic *superpotential* $W(\xi)$ [127] via

$$V = K^{I\bar{J}}W_I\overline{W}_{\bar{J}},$$

where we denote derivatives with respect to the fields ξ^I and $\bar{\xi}^{\bar{J}}$ with a subscript, e.g. $W_I = \frac{\partial}{\partial\xi^I}W$. In supersymmetric theories, supersymmetry is broken precisely if the vacuum expectation value for F^I is non-vanishing, which via the equations of motion for F in the original action

$$F^I = K^{I\bar{J}}\overline{W}_{\bar{J}},$$

implies that supersymmetry is broken if and only if $W_I = \partial_I W = 0$ [88, 127].

In supergravity, an important change happens to the potential. The action for the complex scalars ξ^I in the chiral supermultiplets can now be written as

$$S = \int \mathrm{d}^4 x \sqrt{g} \left[\frac{\mathrm{M}_{\mathrm{pl}}^2}{2} R - g^{\mu\nu} K_{I\bar{J}}(\xi, \bar{\xi}) \nabla_\mu \xi^I \nabla_\nu \bar{\xi}^{\bar{J}} - V(\xi, \bar{\xi}) \right]. \tag{3.27}$$

Again the nonlinear σ model target manifold is restricted to be a Kähler manifold with Kähler potential $K(\xi, \bar{\xi})$, but the F-term potential is now given by

$$V = e^{K/\mathrm{M}_{\mathrm{pl}}^2} \left(K^{I\bar{J}} \mathrm{D}_I W \overline{\mathrm{D}_J W} - \frac{3}{\mathrm{M}_{\mathrm{pl}}^2} W \overline{W} \right), \tag{3.28}$$

where $\mathrm{D}_I W$ denotes the Kähler covariant derivative

$$\mathrm{D}_I W = \partial_I W - \frac{\partial_I K}{\mathrm{M}_{\mathrm{pl}}^2} W.$$

The supergravity action is invariant under Kähler transformations

$$K(\xi, \bar{\xi}) \to K(\xi, \bar{\xi}) + f(\xi) + \bar{f}(\bar{\xi}), \qquad W(\xi) \to e^{-f(\xi)/\mathrm{M}_{\mathrm{pl}}^2} W(\xi),$$

by an arbitrary holomorphic function $f(\xi)$, which suggests to rewrite the theory in terms of one real, Kähler invariant function $G(\xi, \bar{\xi})$ that is related to the Kähler potential and superpotential via

$$G(\xi, \bar{\xi}) = K(\xi, \bar{\xi}) + \mathrm{M}_{\mathrm{pl}}^2 \log\left(\frac{W(\xi)}{\mathrm{M}_{\mathrm{pl}}^3} \right) + \mathrm{M}_{\mathrm{pl}}^2 \log\left(\frac{\overline{W}(\bar{\xi})}{\mathrm{M}_{\mathrm{pl}}^3} \right). \tag{3.29}$$

This definition is only valid for $W \neq 0$. A vanishing superpotential is a fixed point under Kähler transformations and deserves special treatment. Throughout this thesis we will therefore assume that $W \neq 0$. In terms of the *Kähler function* $G(\xi, \bar{\xi})$, the F-term potential reads

$$V = e^{G/\mathrm{M}_{\mathrm{pl}}^2} \left(G^{I\bar{J}} G_I G_{\bar{J}} - 3\mathrm{M}_{\mathrm{pl}}^2 \right) \mathrm{M}_{\mathrm{pl}}^2. \tag{3.30}$$

Since

$$F^I = e^{G/2\mathrm{M}_{\mathrm{pl}}^2} G^{I\bar{J}} G_{\bar{J}}$$

in supergravity theories, supersymmetry is broken if and only if $G_I = \frac{\mathrm{D}_I W}{W} = 0$ [88].

The action (3.27) provides an interesting starting point for the purpose of inflationary model building. Ideally, one would like to study inflation directly from the superstring theory point of view, but since superstring theory is not yet fully known, our investigations are restricted to its low energy effective limit. The supergravity theories that appear as the low energy effective action of superstring theory are a subset of all supergravity theories. However, in the literature this distinction is not always made, because we first need to focus on the characteristic effects and possible issues in supergravity in general. For example, from (3.30) we can already infer one of these generalities, that a quasi-de Sitter phase with $V > 0$ requires supersymmetry to be broken during inflation. Although string inspired work can be found throughout the supergravity literature [1, 2], the construction of a model completely rooted in a consistent superstring theory set-up is still to be found. Until such a model exist, the rich but yet restricted character of supergravity make it an interesting framework for the study of inflation in quantum gravity.

3.2.3 Holography

Gauge/gravity duality

A final ingredient we shall need for the studies following, is holography and the AdS/CFT-correspondence. The discovery, about fifteen years ago, that string theory realizes the holographic principle, is a major development in theoretical physics. The holographic principle is a (crazy) hypothesis that the physics of a d-dimensional gauge theory can also be described by a $d+1$-dimensional theory with gravity and vice versa [129, 130]. The motivation for such a hypothesis derives from black hole physics, in which all the information of the black hole can be encoded by way of its event horizon.

Inspired by the work of others in this direction [131–135], a conjectured realization of two dual theories was constructed by [136]. In this realization we consider a system of D-branes[4] in a flat background geometry. This configuration has two distinct limits, each with its own description. One description considers the supergravity approximation around the branes, which is that of an anti-de Sitter AdS_5-geometry. The other description decouples the interacting brane-bulk system, leaving only the

[4] The known examples of the holographic duality are all advanced constructs in superstring theory. As a result, they contain elements not explained in this text elsewhere. The particular system in [136] is a set of N parallel $D3$-branes in a ten-dimensional flat background, which one can view as the supergravity limit of a type IIB superstring in an $AdS_5 \times S^5$-background on the one hand or as a decoupled brane-bulk system on the other hand, with the gauge theory on the brane being a four-dimensional $\mathcal{N} = 4$ superconformal $SU(N)$ Yang-Mills theory.

gauge theory on the brane, a specific four-dimensional conformal field theory. Since both descriptions originate from the same system, they present a dual description of the same physics [90, 91, 136]. We say that the theory in the bulk is dual to the conformal field theory living on its boundary, because the brane resides at the boundary of the anti-de Sitter space. The holographic duality in this construction is called the *AdS/CFT-correspondence* as it is a duality between anti-de Sitter geometry and conformal field theory.

Since a d-dimensional field theory has one dimension less than a d+1-dimensional gravity theory, the natural question arises how holography manages to encode the additional dimension of the bulk theory into the boundary theory. The example of [136] provides a clear indication of how this happens. The (Euclidean) AdS_{d+1}-metric is given by

$$\mathrm{d}s^2 = \mathrm{d}y^2 + e^{-2y/R}\mathrm{d}\boldsymbol{x}^2,$$

or

$$\mathrm{d}s^2 = \frac{R^2}{z^2}\left(\mathrm{d}\boldsymbol{x}^2 + \mathrm{d}z^2\right),$$

in Poincaré coordinates, where R is the anti-de Sitter-radius and where the boundary is located at $z = 0$. It is invariant under a scale transformation $\boldsymbol{x} \to \lambda\boldsymbol{x}$, $z \to \lambda z$. The d coordinates $\boldsymbol{x}$ are naturally identified with the coordinates of the conformal field theory, setting $z = 0$. The interpretation of the additional coordinate z becomes clear when we consider a scale transformation $\boldsymbol{x} \to \lambda\boldsymbol{x}$ in the field theory as well. The theory is scale invariant when such a scale transformation is accompanied by a rescaling of the energy scale $\mu \to \lambda^{-1}\mu$ [137]. Hence, the additional coordinate of the gravity theory corresponds to the energy scale in the gauge theory,

$$z \sim \frac{1}{\mu},$$

and the direction towards the interior of the bulk corresponds to a renormalization group flow from high energies to low energies. This immediately suggests that the AdS/CFT correspondence could be generalized to a bulk theory that is asymptotically anti-de Sitter with a dual gauge theory that approaches a conformal fixed point in the ultraviolet [137, 138]. Renormalization of the ultraviolet divergences of the gauge theory is completely understood in terms of regularizing and renormalizing the large distance, i.e. near-boundary, behavior of the bulk theory [138–142].

An important aspect of AdS/CFT is that the two limits where either the field theory or the gravitational description arises, correspond to opposite limits of the intrinsic CFT coupling constant [136, 143]. This means that the strongly coupled physics of one theory is equivalently described by the weakly coupled dual theory. On the one hand, the strong/weak-aspect of the duality makes it very difficult to verify a conjectured holographic correspondence, since a perturbative approach can only work for one of the two dual theories at a time. Making use of protecting symmetries of the theory, it is possible to match some of the properties of each of the two systems, indicating that the conjecture might hold. On the other hand, once a correspondence between theories has been (reasonably) established, the strong/weak duality provides a truly powerful approach to understand strongly coupled physics, by considering the weakly coupled dual theory.

The holographic correspondence is conjectured to hold for more general gauge and gravity theories than the AdS/CFT-correspondence of [136]. Finding other examples is difficult, but possible [90, 137]. As said, the hallmark strong/weak-duality of dual theories gives ample motivation to search for holographic examples, for the unique orthogonal approach the duality provides to the study of strongly coupled systems. Particularly relevant for cosmology would be if a correspondence between de Sitter space and some gauge theory is found. In principle dS/CFT should be closely related to AdS/CFT, as both gravity theories have a great resemblance [144–146]. This is immediate at the level of their symmetries, which in both cases is $\mathrm{O}(1,d)$ for a d-dimensional spacetime. In practice it proves difficult to actually find an explicit realization of the dS/CFT-correspondence. Nevertheless, the possibility of having a holographic description of (quasi)-de Sitter geometry provides the motivation behind chapter 6 of this thesis. In particular, in that chapter we will see to what extent conformal invariance dictates the correlation functions of the gravity theory.

Correlation functions

The real power of the AdS/CFT-correspondence is the precise quantitative dictionary, described in [147, 148], between the two perspectives. In these descriptions, a Euclideanized version of the gravity theory is considered. A field $\phi(z,\boldsymbol{x})$ in the bulk of the $d+1$-dimensional gravity theory has an asymptotic value $\phi_0(\boldsymbol{x})$ on the d-dimensional boundary, which acts as a coupling constant for an operator $O(\boldsymbol{x})$ of the boundary field theory. The duality is then summarized by the statement that the partition functions Z_{CFT} and Z_{AdS} are equal,

$$Z_{AdS}[\phi(\phi_0)] = Z_{CFT}[\phi_0] = \left\langle e^{-\int \mathrm{d}^d\boldsymbol{x}\,\phi_0 O}\right\rangle_{CFT}. \tag{3.31}$$

The partition function $Z_{AdS}[\phi(\phi_0)] = \int_{\phi_0}\mathcal{D}\phi e^{-S_{AdS}(\phi)}$ is evaluated in the semiclassical limit, i.e. a classical solution for the field $\phi(z,\boldsymbol{x})$ is found subject to the boundary condition $\phi_0(\boldsymbol{x})$ around which the action is perturbed. n-point correlation functions of the operators in the conformal field theory can then be calculated in the usual way through functional differentiation with respect to the boundary conditions,

$$\langle O(\boldsymbol{x}_1)\ldots O(\boldsymbol{x}_n)\rangle = \frac{\delta}{\delta\phi_0(\boldsymbol{x}_1)}\cdots\frac{\delta}{\delta\phi_0(\boldsymbol{x}_n)} Z_{AdS}\left[\phi(\phi_0)\right]\Big|_{\phi_0=0}, \tag{3.32}$$

which act as sources to the operators.

To get finite answers in the matching of the asymptotic values for the bulk fields ϕ with the boundary couplings ϕ_0, the boundary fields are renormalized, which leads to a relation between the scaling dimension of the operator O to which ϕ_0 couples and the mass m of the bulk field [137, 147, 148],

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2R^2}, \tag{3.33}$$

where R is again the anti-de Sitter curvature radius. A massless field m corresponds to a marginal operator $\Delta = d$. With the identification given above, the (physical degrees of freedom of the) metric field $g_{\mu\nu}(z, \boldsymbol{x})$ corresponds to the stress-energy tensor operator $T_{\alpha\beta}(\boldsymbol{x})$ of the conformal field theory. The stress-energy tensor is a marginal operator that is always part of the conformal field theory, which is why the bulk theory always has to include gravity [137].

As an illustrative example of how the correspondence works, we consider an interacting massive scalar field ϕ in $d+1$-dimensional AdS, with action

$$S_{AdS} = \frac{1}{2}\int \mathrm{d}^d\boldsymbol{x}\mathrm{d}z\,\sqrt{g}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2 + \frac{\lambda}{3}\phi^3\right]. \tag{3.34}$$

We have to solve the classical equation of motion subject to the boundary condition $\phi_0(\boldsymbol{x})$. This can be achieved conveniently by first finding the Green's function for the equation of motion of the quadratic part of the action [148, 149],

$$K_\Delta(z, \boldsymbol{x}, \boldsymbol{x}') = \frac{\Gamma(\Delta)}{\pi^{\frac{d}{2}}\Gamma(\Delta - \frac{d}{2})}\left(\frac{z}{z^2 + (\boldsymbol{x} - \boldsymbol{x}')^2}\right)^{\Delta},$$

where Δ is related to the mass m via (3.33). The function $K_\Delta(z, \boldsymbol{x}, \boldsymbol{x}')$ is called the bulk-to-boundary propagator, which has to be normalized such that it is regular in the interior and provides the required singular behavior for $z \to 0$. The classical (homogeneous) solution is then automatically determined by the boundary value $\phi_0(\boldsymbol{x})$ via

$$\phi(z, \boldsymbol{x}) = \int \mathrm{d}^d\boldsymbol{x}'\, K_\Delta(z, \boldsymbol{x}, \boldsymbol{x}')\phi_0(\boldsymbol{x}').$$

To find the three-point function of the dual operator O (3.32), we can substitute this expression into (3.34) and find

$$\begin{aligned}\langle O(\boldsymbol{x}_1)O(\boldsymbol{x}_2)O(\boldsymbol{x}_3)\rangle &= \frac{\delta}{\delta\phi_0(\boldsymbol{x}_1)}\frac{\delta}{\delta\phi_0(\boldsymbol{x}_2)}\frac{\delta}{\delta\phi_0(\boldsymbol{x}_3)}Z_{AdS}\left[\phi(\phi_0)\right]\Big|_{\phi_0=0} \\ &= -\lambda\int\frac{\mathrm{d}^d\boldsymbol{x}\mathrm{d}z}{z^{d+1}}K_\Delta(z,\boldsymbol{x},\boldsymbol{x}_1)K_\Delta(z,\boldsymbol{x},\boldsymbol{x}_2)K_\Delta(z,\boldsymbol{x},\boldsymbol{x}_3).\end{aligned} \tag{3.35}$$

Since this is a three-point correlation function in a conformal field theory, it should be of the form (3.12b). One can explicitly verify that this is so and determine the coefficient from the explicit form of the bulk-to-boundary propagator [149, 150],

$$\langle O(\boldsymbol{x}_1)O(\boldsymbol{x}_2)O(\boldsymbol{x}_3)\rangle = \frac{\lambda a(\Delta)}{(x_{12}x_{23}x_{13})^\Delta}, \tag{3.36a}$$

$$a(\Delta) = -\frac{\Gamma\left(\frac{1}{2}(3\Delta-d)\right)\Gamma\left(\frac{\Delta}{2}\right)^3}{2\pi^d\Gamma\left(\Delta-\frac{d}{2}\right)^3}. \tag{3.36b}$$

The matching of (3.35) with (3.12b) is a necessary requirement for the correspondence to hold. It is an explicit check that the anti-de Sitter space is constrained by the same symmetries as the conformal field theory. In general, for a duality to hold, the theories need to be invariant under the same symmetries. This is of course not a sufficient condition. Nevertheless, it is interesting to see what one can already derive based solely upon symmetry arguments. We will take the latter approach in our study of a hypothesized dS/CFT-correspondence in chapter 6.

4

Inflation embedded in supergravity

As we have emphasized earlier, inflation is a coarse phenomenon in classical general relativity. In principle it should therefore not be too difficult to embed inflationary models within a unifying theory of quantum gravity such as string theory or its supergravity description at low energies. Nevertheless, inflation turns out to depend sensitively on the microscopic description of the theory. Although this is a blessing if we ever want to observationally verify our ideas about the microscopic structure of our universe, it also means that we have to be very careful in neglecting any part of the theory that we do not (yet) completely understand. By restricting ourselves to the part of the model we have control over, we could be throwing away the baby with the bath water. Although it is usual practice and often plain necessity to consider inflation in a controlled environment, one makes implicit, and possibly unrealistic, assumptions on the unknown parts of the theory in the way it (does not) contribute to the inflationary dynamics. As a result, the predictive power of the theory and its chance to be compared with observations from the early universe, are limited.

In this chapter we will see, in the context of supergravity, how hidden sectors affect the carefully controlled physics of any model for inflation. This will be a useful illustration of the sensitivity of inflation to unknown physics and of the importance to compare inflation observationally with a *complete* description of nature. The chapter is based on [151] and [152].

4.1 Introduction

The construction of realistic models of slow-roll inflation in supergravity is a long-standing puzzle. Supersymmetry can alleviate the finetuning necessary to obtain slow-roll inflation — if one assumes that the inflaton is a modulus of the supersymmetric ground state — but cannot solve it completely. This is most clearly seen in the supergravity η-problem: if the inflaton is a lifted modulus, then its mass in the inflationary background is proportional to the supersymmetry breaking scale. Therefore, the slow-roll parameter $\eta \simeq V''/V$ generically equals unity rather than a small number [153].

We will show here, however, that the η-problem is more serious than a simple hierarchy problem. In the conventional mode of study, the inflaton sector is always a subsector of the full supergravity theory presumed to describe our universe. When the inflationary subsector of the supergravity is studied *an sich*, tuning a few parameters of the Lagrangian to order 10^{-2} will generically solve the problem. We will clarify that this split of the supergravity sector into an inflationary sector and other hidden sectors implicitly makes the assumption that all the other sectors are in a "supersymmetric" ground state: i.e. if the inflaton sector —which must break supersymmetry— is decoupled, the ground state of the remaining sectors is supersymmetric. If this is not the case, the effect on the η-parameter or on the inflationary dynamics in general can be large, even if the sypersymmetry breaking scale in the hidden sector is small. Blind truncation in supergravities to the inflaton sector alone, if one does not know whether other sectors preserve supersymmetry, is therefore an inconsistent approach towards slow-roll supergravity inflation. Coupling the truncated sector back in completely spoils the naïve solution found. This result, together with recent qualitatively similar findings for sequestered supergravities (where only the potential has a two-sector structure) [154], provides strong evidence that to find true slow-roll inflation in supergravity one needs to know the global ground state of the system. The one obvious class of models where sector-mixing is not yet considered is the newly discovered manifest embedding of single field inflationary models in supergravity [155, 156]. If these models are also sensitive to hidden sectors, it would arguably certify the necessity of a global analysis for cosmological solutions in supergravity and string theory.

We will obtain our results on two-sector supergravities by an explicit calculation. The gravitational coupling between the hidden and the inflaton sectors is universal, which can be described by a simple F-term scalar supergravity theory. As in most discussions on inflationary supergravity theories, we will ignore D-terms as one expects its vacuum expectation value to be zero throughout the early universe [30]. Including D-terms (which themselves always need to be accompanied by F-terms [88]) only complicates the F-term analysis, which is where the η-problem resides. Furthermore, although true inflationary dynamics ought to be described in a fully kinetic description [52], we can already make our point by simply considering the mass eigenmodes of the system. In a strict slow-roll and slow-turn approximation the mass eigenmodes of the system determine the dynamics of the full system.

Specifically we shall show the following for two-sector supergravities where the sectors are distinguished by independent R-symmetry invariant Kähler functions:

- Given a naïve supergravity solution to the η-problem, this solution is only consistent if the other sector is in its supersymmetric ground state.

- If it is not in its ground state, then the scalar fields of that sector cannot be static but *must* evolve cosmologically as well.

- In order for the naïve solution to still control the cosmological evolution these fields must move very slowly. This translates in the requirement that the contribution to the first slow-roll parameter of the hidden sector must be much smaller than the contribution from the naïve inflaton sector, $\epsilon_{\text{hidden}} \ll \epsilon_{\text{naïve}}$.

- There are two ways to ensure that ϵ_{hidden} is small: Either the supersymmetry breaking scale in the hidden sector is very small or a particular linear combination of first and second derivatives of the generalized Kähler function is small.

  - In the latter case, one finds that the second slow-roll parameter $\eta_{\text{naïve}}$ receives a very large correction $\eta_{\text{true}} - \eta_{\text{naïve}} \gg \eta_{\text{naïve}}$, unless the supersymmetry breaking scale in the hidden sector is small. This returns us to the first case.
  - In the first case, one finds that the hidden sector always contains a light mode, because in a supersymmetry breaking (almost) stabilized supergravity sector there is always a mode that scales with the scale of supersymmetry breaking. This light mode will overrule the naïve single field inflationary dynamics.

Thus for *any* nonzero supersymmetry breaking scale in the hidden sector — even when this scale is very small — the true mass eigenmodes of the system are linear combinations of the hidden sector fields and the inflaton sector fields. We compute these eigenmodes. By assumption, the true value of the slow-roll parameter η is the smallest of these eigenmodes. Depending on the values of the supersymmetry breaking scale and the naïve lowest mass eigenstate in the hidden sector, we find that

1. The new set of mass eigenmodes can have closely spaced eigenvalues, and thus the initial assumption of single field inflation is incorrect. Then a full multi-field re-analysis is required.

2. The relative change of the value of η from the naïve to the true solution can be quantified and shows that for a supersymmetry breaking hidden sector, the naïve model is only reliable if the naïve lowest mass eigenstate in the hidden sector is much larger than the square of the scale of hidden sector supersymmetry breaking divided by the inflaton mass. This effectively excludes all models where the hidden sector has (nearly) massless modes.

3. The smallest eigenmode can be dominantly determined by the hidden sector, and thus the initial assumption that the cosmological dynamics is constrained to the inflaton sector is incorrect. Again a full multi-field re-analysis is required.

One concludes that in general one needs to know/assume the ground states and the lowest mass eigenstates of *all* the hidden sectors to reliably find a slow-roll inflationary supergravity.

The structure of this chapter is the following. Section 4.2 explains how sectors are coupled in supergravity. To make contact with global supersymmetry models, we consider the no-gravity limit of a multi-sector supergravity model. As we will see, decoupling in this limit turns out to be more delicate than just taking the simple $M_{pl} \to \infty$ limit. We begin the discussion on the effects of having multiple sectors in section 4.3 with the result that in a stabilized supergravity sector there always is a mode that scales with the scale of supersymmetry breaking. In section 4.4 the η-problem in a single sector theory is discussed and we consider the effect of a hidden sector qualitatively and quantitatively. The quantitative result is analyzed in section 4.5 both in terms of effective parameters and direct supergravity parameters. As a notable example of our result, we show that if the hidden sector is the standard model, where its supersymmetry breaking is not caused by the inflaton sector but otherwise, spoils the naïve slow-roll solution in the putative inflaton sector. The chapter is supplemented with two appendices in which some of the longer formulae are given.

4.2 Canonical coupling in supergravity

We shall start by arguing how two sectors are gravitationally coupled in supergravity. We will seek for a minimal (universal) coupling between sectors. It has an interesting interpretation in terms of the superpotentials, which multiply rather than add as in globally supersymmetric minimally coupled systems. As a result, the zero-gravity limit from multi-sector supergravities to decoupled multi-sector global supersymmetry theories is more subtle than the usual $M_{pl} \to \infty$ limit. To be able to embed the supersymmetry objects into a multi-sector supergravity theory, we will consider a possible decoupling limit with non-canonical scaling of the superpotential couplings. This limit will later be used to apply our general results to a standard model-like globally supersymmetric hidden sector in section 4.5.3

4.2.1 Maximal decoupling in supergravity

Multiple sectors are a common feature in supergravity cosmology and phenomenology. These sectors are necessary to either incorporate inflation or supersymmetry breaking or are a consequence of string model-building. In particular to study inflation, it is desirable to separate the dynamics of all fields that do not contribute to the exponential expansion of the universe from the inflaton fields that do. Since gravity is the weakest possible interaction, the inflationary sector is assumed to only couple gravitationally to an unknown hidden sector that may also break supersymmetry by itself. Whereas it is natural for a rigid supersymmetric theory to be separated into several sectors, the restrictive structure of supergravity forces the different sectors to couple not only non-locally through graviton exchange but also directly. For this reason embedding supersymmetric theories as sectors into a supergravity can be notoriously difficult, see e.g. [88, 89, 157–162].

Though multiple sector supergravities are a long studied subject, the context of cosmology has seriously sharpened the question. In supergravity models of inflation, it is commonly noted that one seeks a consistent truncation of the scalar sector. This is necessary but not sufficient. Even with a consistent truncation one may have dominating instabilities towards the naïvely non-dynamical sectors, that can move them away from their supersymmetric critical points. One needs either a symmetry constraint or an energy barrier to constrain the dynamics to the putative inflaton sector.

During inflation, supersymmetry is broken and although it is frugal to consider scenarios where the inflaton sector is also responsible for phenomenological supersymmetry breaking (see e.g. [163–165]), this need not be so. For instance, in a generic gauge-mediation scenario, the mechanism responsible for supersymmetry breaking need not involve the fields that drive inflation. This example immediately shows that the generic cosmological set-up must be able to account for a sector that breaks supersymmetry *independently* of the inflationary dynamics.

This consideration is our starting point. We consider a multiple-sector supergravity that decouples in the strictest sense in the limit $\mathrm{M_{pl}} \to \infty$. In this limit the action must then be the sum of two independent functions

$$S[\phi, \overline{\phi}, q, \overline{q}] = S[\phi, \overline{\phi}] + S[q, \overline{q}], \tag{4.1}$$

such that the path integral factorizes.[1] ϕ and q denote the fields in the two sectors respectively. In the following, we will take the indices $\{i, \bar{\jmath}\}$ to run over the ϕ-fields, while $\{a, \bar{b}\}$ denote the fields in the q-sector. Later, we will take the ϕ-fields to drive inflation, while the q-fields reside in another sector which is naïvely assumed not to take part in the inflationary dynamics and is hence called the hidden sector.

For a globally supersymmetric field theory with a standard kinetic term, a decoupled action can be achieved by demanding that the independent Kähler and superpotentials sum as well,

$$K_{\text{susy}}(\phi, \bar{\phi}, q, \bar{q}) = K^{(1)}(\phi, \bar{\phi}) + K^{(2)}(q, \bar{q}), \quad W_{\text{susy}}(\phi, q) = W^{(1)}(\phi) + W^{(2)}(q). \tag{4.2}$$

By contrast, in supergravity complete decoupling in the sense of (4.1) appears to be impossible, even in principle. Even with block diagonal kinetic terms from a sum of Kähler potentials, the more complicated form of the supergravity potential (3.28) implies that there are many *direct* couplings between the two sectors. It raises the immediate question: if the low-energy $\mathrm{M_{pl}} \to \infty$ globally supersymmetric model must consist of decoupled sectors, what is the relation between $K_{\text{sugra}}, W_{\text{sugra}}$ and $K_{\text{susy}}, W_{\text{susy}}$, or vice versa given a globally supersymmetric model described by $K_{\text{susy}}, W_{\text{susy}}$, what is the best choice for $K_{\text{sugra}}, W_{\text{sugra}}$ such that the original theory can be recovered in the limit $\mathrm{M_{pl}} \to \infty$?

The conclusion of this section is that the scaling implied by the explicit factors of $\mathrm{M_{pl}}$ in the supergravity potential (3.28) is an incomplete answer to this question. The direct communication between the sectors, controlled by $\mathrm{M_{pl}}$, has serious consequences for both the ground state structure (solutions to the equation of motion, i.e. the cosmological dynamics) and the interactions between the two sectors. To be explicit, the first guess at how the rigid supersymmetry and supergravity Kähler potentials and superpotentials are related

$$K_{\text{sugra}}(\phi, \bar{\phi}, q, \bar{q}) = K^{(1)}_{\text{susy}}(\phi, \bar{\phi}) + K^{(2)}_{\text{susy}}(q, \bar{q}) + \dots, \tag{4.3a}$$

$$W_{\text{sugra}}(\phi, q) = W^{(1)}_{\text{susy}}(\phi) + W^{(2)}_{\text{susy}}(q) + \dots, \tag{4.3b}$$

with ... indicating Planck-suppressed terms and possibly a constant term, does not define a sensible way of splitting up the action in multiple sectors. This definition is not invariant under Kähler transformations in each sector separately and is valid only in a specific Kähler frame or, say, gauge dependent [166]. Another way to understand

[1] As example we consider the simplest case, a model with uncharged scalar supermultiplets $\xi^I = (\phi^i, q^a)$ that are singlets under all symmetries. Gauge interactions and global symmetries will not change this general argument provided the two sectors are not mixed by symmetries or coupled by gauge fields. Therefore, we will also ignore D-terms in the supergravity potential below.

the problem is to realize that the definition (4.3) does not lead to a Kähler metric and mass matrix that can be made block diagonal in the same basis [167], and thus there is no sense of "independent" sectors. Moreover, (4.3) suffers from the drawback that the ground states of the full theory are no longer the product of the ground states of the individual sectors, except when both (rather than only one) ground states are supersymmetric [168, 169] (see also [166, 167, 170]). This directly follows from considering the extrema of the supergravity potential[2]

$$\nabla_a V = \frac{\mathrm{D}_a W}{W} V + e^{K/\mathrm{M}_{\mathrm{pl}}^2}|W|^2 \left(\nabla_a \left(\frac{\mathrm{D}_b W}{W} \right) \frac{\mathrm{D}^b \overline{W}}{\overline{W}} + \frac{1}{\mathrm{M}_{\mathrm{pl}}^2} \frac{\mathrm{D}_a W}{W} + \nabla_a \left(\frac{\mathrm{D}_j W}{W} \right) \frac{\mathrm{D}^j \overline{W}}{\overline{W}} \right), \tag{4.4a}$$

$$\begin{aligned}\nabla_a \nabla_i V = {} & \frac{\mathrm{D}_i W}{W} \nabla_a V + \frac{\mathrm{D}_a W}{W} \nabla_i V - \frac{\mathrm{D}_a W}{W} \frac{\mathrm{D}_i W}{W} V + \mathrm{D}_a \left(\frac{\mathrm{D}_i W}{W} \right) \left(V + \frac{2}{\mathrm{M}_{\mathrm{pl}}^2} e^{K/\mathrm{M}_{\mathrm{pl}}^2} |W|^2 \right) \\ & + e^{K/\mathrm{M}_{\mathrm{pl}}^2} |W|^2 \left(\nabla_a \nabla_i \left(\frac{\mathrm{D}_j W}{W} \right) \frac{\mathrm{D}^j \overline{W}}{\overline{W}} + \nabla_i \nabla_a \left(\frac{\mathrm{D}_b W}{W} \right) \frac{\mathrm{D}^b \overline{W}}{\overline{W}} \right).\end{aligned} \tag{4.4b}$$

Supersymmetric ground states, for which the covariant derivatives of W vanish on the solution, $\mathrm{D}_i W = 0$ and $\mathrm{D}_a W = 0$, are still product solutions. But for Kähler and superpotentials that sum (4.3), even if only one sector is in a non-supersymmetric ground state, by which we mean $\mathrm{D}_a W = 0$, $\mathrm{D}_i W \neq 0$, we can neither conclude that sector 2 is in a minimum, for which $\nabla_a V$ would vanish, nor that the condition for sector 1 to be in a local ground state is independent of the sector 2 fields q^a, which would mean that $\nabla_a \nabla_i V = 0$. The former is only true when

$$\nabla_a \left(\frac{\mathrm{D}_j W}{W} \right) \frac{\mathrm{D}^j \overline{W}}{\overline{W}} = 0. \tag{4.5}$$

The second requires, in addition,

$$\nabla_a \nabla_i \left(\frac{\mathrm{D}_j W}{W} \right) \frac{\mathrm{D}^j \overline{W}}{\overline{W}} + \nabla_i \nabla_a \left(\frac{\mathrm{D}_b W}{W} \right) \frac{\mathrm{D}^b \overline{W}}{\overline{W}} = 0, \tag{4.6}$$

[2] To derive (4.4b) note that, since $\mathrm{D}W/W$ is Kähler invariant and since the Levi-Civita connection ∇ of the field space manifold does not get cross-contributions in a product manifold,

$$\nabla_a \frac{\mathrm{D}_i W}{W} = \partial_a \frac{\mathrm{D}_i W}{W} = \mathrm{D}_a \frac{\mathrm{D}_i W}{W}.$$

and also sharpens the first condition (4.5) to[3]

$$\mathrm{D}_a \frac{\mathrm{D}_i W}{W} = 0. \tag{4.7}$$

Equations (4.5–4.7) are conditions for decoupling which apply not only to the ground state of the full system but also to other critical points of the potential, for instance along an inflationary valley. Generically these conditions are not met on the solution (the second derivative need not vanish at an extremum; recall that $\mathrm{D}_a W$ does not vanish identically but only on the solution). Hence, generically the ground states of hidden sectors mix and this spoils many cosmological supergravity scenarios that truncate the action to one or the other sector (see e.g. [171] and references therein). It is this issue that is particularly relevant for inflationary model building, where a very weak coupling between the inflaton sector and all other sectors has to persist over an entire *trajectory* in field space where the expectation values of the fields are changing with time (see e.g. [52, 172–174]). At the same time, one is interested in the generic situation in which *both* sectors may contribute to supersymmetry breaking.[4]

[3] These conditions are merely sufficient not necessary. However, it is clear that the restrictive nature of supergravity enforces conditions on the unknown sectors for the system to be separate.

[4] This situation has to be contrasted to phenomenological models appropriate for studying gravity mediated supersymmetry breaking, such as an ansatz [175]

$$\begin{aligned} K(\phi,\overline{\phi},q,\overline{q}) &= K^{(0)}(\phi,\overline{\phi}) + q^a\overline{q}^{\overline{b}}K^{(1,1)}_{a\overline{b}}(\phi,\overline{\phi}) + q^a q^b K^{(2,0)}_{ab}(\phi,\overline{\phi}) + \overline{q}^{\overline{a}}\overline{q}^{\overline{b}}K^{(0,2)}_{\overline{a}\overline{b}}(\phi,\overline{\phi}) + \dots, \\ W(\phi,q) &= W^{(0)}(\phi) + q^a q^b W^{(1)}_{ab}(\phi) + \dots, \end{aligned}$$

or equivalently, if $W \neq 0$,

$$G(\phi,\overline{\phi},q,\overline{q}) = G^{(0)}(\phi,\overline{\phi}) + q^a\overline{q}^{\overline{b}}G^{(1,1)}_{a\overline{b}}(\phi,\overline{\phi}) + q^a q^b G^{(2,0)}_{ab}(\phi,\overline{\phi}) + \overline{q}^{\overline{a}}\overline{q}^{\overline{b}}G^{(0,2)}_{\overline{a}\overline{b}}(\phi,\overline{\phi}) + \dots.$$

In models like these, it is understood that $\dot{q} = 0$ and the q-sector can remain in its supersymmetric critical point throughout the evolution of the supersymmetry breaking fields. For inflation, such an expectation is unrealistic, as the supersymmetry preserving sector can become unstable during the inflationary dynamics, see e.g. a recent discussion of the case in which the inflaton field ϕ is solely responsible for supersymmetry breaking during inflation ([165] and references therein). In this relatively simple case, and except for very fine-tuned situations, the generic scenario appears to be that one or more of the q-fields are destabilized somewhere along the inflationary trajectory and they trigger an exit from inflation (in other words, they become "waterfall" fields, and inflation is of the hybrid kind [176]). This implies that the pattern of supersymmetry breaking today is not related to the one during inflation, and also, since the waterfall fields are forced away from their supersymmetric critical points, that supersymmetry is broken by both sectors as the universe evolves towards the current vacuum.

4.2.2 Natural multi-sector supergravities

There is a natural way to construct supergravity potentials for which the ground states (and critical points) do separate better. This obvious combination of superpotentials automatically satisfies (4.5–4.7) and hence does ensure that if one of the ground states is supersymmetric, the ground state of the other sector is a decoupled field theory ground state whether it breaks supersymmetry or not. This is if we choose a product of superpotentials, keeping the sum of Kähler potentials as before,

$$K_{\rm sugra}(\phi,\overline{\phi},q,\overline{q}) = K^{(1)}_{\rm sugra}(\phi,\overline{\phi}) + K^{(2)}_{\rm sugra}(q,\overline{q}), \qquad W_{\rm sugra}(\phi,q) = \frac{1}{\mathrm{M}_{\rm pl}^3} W^{(1)}_{\rm sugra}(\phi) W^{(2)}_{\rm sugra}(q). \tag{4.8}$$

This is well-known [177–179] and has recently been emphasized in the context of cosmology [166, 167, 170, 171, 173, 174, 180, 181]. This ansatz conforms to the more natural description of supergravities in terms of the Kähler invariant function (3.29) that can be defined if W is non-zero in the region of interest.[5] In turn, the Kähler function underlies a better description of multiple sectors in supergravity, where G is a simple sum of independent functions

$$G(\phi,\overline{\phi},q,\overline{q}) = G^{(1)}(\phi,\overline{\phi}) + G^{(2)}(q,\overline{q}). \tag{4.9}$$

It is invariant under Kähler transformations in each sector separately [166–169, 182] and thus defines a sensible way of splitting up the action in multiple sectors. As a result, this split guarantees that a BPS solution in one particular sector is a BPS solution of the full theory. It is the simplest ansatz that still allows some degree of calculational control when both sectors break supersymmetry —as well as optimizing decoupling along the inflationary trajectory. One of the simplest models of hybrid inflation in supergravity, F-term inflation [183, 184], is in this class.

The sum of Kähler functions (4.9) implies the conventional separation of the Kähler potentials, but it constitutes a class of minimally coupled scenarios due to the multiplicative nature of the superpotentials put forward above. Let us illustrate the importance of this multiplicative superpotential in the situation in which the hidden sector resides in a supersymmetric vacuum, i.e. $\partial_a V(q_0) = 0$ and $\partial_a G^{(2)}(q_0) = 0$. We write the superpotential of the hidden sector as $W^{(2)}(q) = W^{(2)}_0 + W^{(2)}_{\rm dyn}(q - q_0)$. The second term in this expression is what determines the potential for fluctuations around the minimum of the hidden sector, while the first constant term is just an overall contribution and hence not interesting for the internal hidden sector dynamics at

[5]We expect this condition to hold around a supersymmetry breaking vacuum with almost vanishing cosmological constant. It also holds in many models of supergravity inflation, although a notable exception is [155, 156].

energies much less than the Planck scale. However, for the gravitational dynamics and the remaining ϕ-sector this "vacuum energy contribution" $W_0^{(2)} = \langle W^{(2)} \rangle$ is of crucial importance as it sets the scale of the potential

$$V = e^{K^{(2)}/\mathrm{M}_{\mathrm{pl}}^2} |W_0^{(2)}|^2 e^{G^{(1)}} \left(G_i^{(1)} G^{(1)i} - 3\mathrm{M}_{\mathrm{pl}}^2 \right) \mathrm{M}_{\mathrm{pl}}^{-4}, \tag{4.10}$$

which is evaluated at $q = q_0$ such that all terms depending on $W_{\mathrm{dyn}}^{(2)}$ vanish. The normal practice of setting $W_0^{(2)}$ to zero as an overall contribution to the hidden sector is neglecting the fact that gravity also feels the constant part of the potential energy, as opposed to field theory. The inflationary sector feels the presence of the hidden sector through this coupling and as such it may be more intuitive to regard $W_0^{(2)}$ to contain information about the inflationary sector rather than the hidden sector. Making a similar split in $W^{(1)}$, the constant part $W_0^{(1)}$ is the overall contribution to the hidden sector due to the inflaton sector.

Using the minimal coupling scenario (4.9), the two-sector action (3.27) reads

$$S = \mathrm{M}_{\mathrm{pl}}^2 \int \mathrm{d}^4 x \sqrt{g} \left[\frac{1}{2} R - g^{\mu\nu} (G_{i\bar{\jmath}}^{(1)} \partial_\mu \phi^i \partial_\nu \bar{\phi}^{\bar{\jmath}} + G_{a\bar{b}}^{(2)} \partial_\mu q^a \partial_\nu \bar{q}^{\bar{b}}) - V \mathrm{M}_{\mathrm{pl}}^2 \right], \tag{4.11}$$

with

$$V(\phi, \bar{\phi}, q, \bar{q}) = e^{G^{(1)} + G^{(2)}} \left(G_i^{(1)} G^{(1)i} + G_a^{(2)} G^{(2)a} - 3 \right). \tag{4.12}$$

We will often allow ourselves to drop the sector label from G in the remainder, since $G_\phi^{(1)} = G_\phi$ and similarly for q. For a short overview of relevant conventions and identities in supergravity, we refer the reader to appendix 4.A. For later calculational convenience, we have given (4.11) and (4.12) in terms of the dimensionless scalar fields $\xi^I = (\phi^i, q^a)$ and functions V, G, K and W. However, before we start the exploration of the inflationary consequences of a coupling such as (4.11), we will momentarily keep the M_{pl}-dependence explicit (and quantities dimensionful) and study the no-gravity limit $\mathrm{M}_{\mathrm{pl}} \to \infty$ to see how the supergravity sectors decouple.

4.2.3 Zero-gravity decoupling limit

Given that we have just argued that a product of superpotentials is a more natural framework to discuss hidden sector supergravities, the obvious question arises how to recover a decoupled *sum* of potentials for a globally supersymmetric theory in the limit where gravity decouples, i.e. in which

$$V_{\mathrm{sugra}} = e^{K/\mathrm{M}_{\mathrm{pl}}^2} \left(|\mathrm{D}W|^2 - \frac{3|W|^2}{\mathrm{M}_{\mathrm{pl}}^2} \right) \quad \to \quad V_{\mathrm{susy}} = \sum_n |\partial_n W^{(n)}|^2.$$

For a two-sector supergravity defined by equations (4.8) one would not find this answer, if one takes the standard decoupling limit $\mathrm{M_{pl}} \to \infty$ with both $K = K^{(1)} + K^{(2)}$ and $W = \mathrm{M_{pl}^{-3}} W^{(1)} W^{(2)}$ fixed[6]. Instead, the product structure of the superpotential introduces a cross-coupling between sectors,

$$V_{\text{eff}} = \frac{1}{\mathrm{M_{pl}^3}} \left(|W^{(2)}|^2 |\partial_\alpha W^{(1)}|^2 + |W^{(1)}|^2 |\partial_i W^{(2)}|^2 \right) \neq V_{\text{susy}},$$

whose behavior under the limit $\mathrm{M_{pl}} \to \infty$ is best examined at the level of the superpotential.

Supergravity is sensitive to the expectation value $W_0 = \langle W \rangle$ of W, which relates the scale of supersymmetry breaking to the expectation value of the potential, i.e. the cosmological constant

$$\Lambda^2 \mathrm{M_{pl}^2} = \langle V \rangle \sim \langle \mathrm{D}W^2 \rangle - \frac{3}{\mathrm{M_{pl}^2}} \langle W^2 \rangle = m_{\text{susy}}^4 - 3 \frac{W_0^2}{\mathrm{M_{pl}^2}}.$$

The vacuum expectation value cannot vanish in a supersymmetry breaking vacuum with (nearly) zero cosmological constant, such as our universe. Therefore, in the following we assume $\langle W \rangle \neq 0$ in the region of interest. Instead of the usual way to incorporate it, $W_{\text{sugra}} = W_0 + W_{\text{dyn}}$ with $W_{\text{dyn}} = W_{\text{susy}} + \ldots$, we include the vacuum expectation value for a two-sector product superpotential by writing

$$\begin{aligned} W(\phi, q) &= \frac{1}{\mathrm{M_{pl}^3}} W^{(1)} W^{(2)} = \frac{1}{\mathrm{M_{pl}^3}} \left(W_0^{(1)} + W_{\text{dyn}}^{(1)}(\phi) \right) \left(W_0^{(2)} + W_{\text{dyn}}^{(2)}(q) \right) \\ &= \frac{1}{\mathrm{M_{pl}^3}} \left(W_0^{(1)} W_0^{(2)} + W_0^{(2)} W_{\text{dyn}}^{(1)}(\phi) + W_0^{(1)} W_{\text{dyn}}^{(2)}(q) + W_{\text{dyn}}^{(1)}(\phi) W_{\text{dyn}}^{(2)}(q) \right). \end{aligned} \tag{4.13}$$

This is physically equivalent to a sum of superpotentials except for the last term. Note again that if one uses the standard scaling, $\frac{\phi}{\mathrm{M_{pl}}} \to 0$, $\frac{q}{\mathrm{M_{pl}}} \to 0$ with all couplings in

[6] Strictly speaking the decoupling limit sends $\mathrm{M_{pl}} \to \infty$ while keeping the fields ϕ, q fixed with $W^{(n)}/\mathrm{M_{pl}^3}$ a holomorphic function of $\phi/\mathrm{M_{pl}}$ or $q/\mathrm{M_{pl}}$ and $K^{(n)}/\mathrm{M_{pl}^2}$ a real function of $\phi/\mathrm{M_{pl}}, \bar{\phi}/\mathrm{M_{pl}}$ or $q/\mathrm{M_{pl}}, \bar{q}/\mathrm{M_{pl}}$. The limit zooms in to the origin so K must be assumed to be non-singular there. Formally the decoupling limit does not exist otherwise. Physically it means that one is taking the decoupling limit with respect to an a priori determined ground state, around which K and W are expanded. If K is non-singular at the origin, the overall factor $e^{K/\mathrm{M_{pl}^2}}$ yields an overall constant as $\mathrm{M_{pl}} \to \infty$, which may be set to unity, i.e. the constant part of K vanishes. In the decoupling limit, both K and W may then be written as polynomials. Letting the coefficients in W and K scale as their canonical scaling dimension such that W has mass dimension three and K has mass dimension two, then gives the rule of thumb that both K and W are held fixed as $\mathrm{M_{pl}} \to \infty$.

$W^{(\text{total})}$ having the canonical scaling dimensions, this last term contains renormalizable couplings involving the scalar partner of the goldstino, and these are not Planck-suppressed: if supersymmetry is broken by the ϕ sector, terms of the form ϕq^2 are renormalizable and would survive the $\mathrm{M_{pl}} \to \infty$ limit, leading to a direct coupling between the two sectors.[7] If both sectors break supersymmetry then mass-mixing terms ϕq also survive. All such (relevant) terms are of course absent if none of the two sectors break supersymmetry, but this is not the case we are interested in. One would have expected that these cross-couplings naturally vanish in the decoupling limit.

The point of this section is simply to remark that the realization that each of the superpotentials $W^{(n)} = W_0^{(n)} + W_{\text{dyn}}^{(n)}$ contains a constant term can resolve this conundrum by assuming a non-standard scaling for the constituent parts $W_0^{(n)}$, $W_{\text{dyn}}^{(n)}$. To achieve a decoupling we need that the cross term $W_{\text{dyn}}^{(1)} W_{\text{dyn}}^{(2)}$, which contains the coupling between the two sectors, scales away in the limit $\mathrm{M_{pl}} \to \infty$. As a result the first term in (4.13) has to diverge, because its product with the cross term should remain finite. In particular we can choose an overall scaling

$$W = \frac{1}{\mathrm{M_{pl}^3}} (\underbrace{W_0^{(1)} W_0^{(2)}}_{\sim \mathrm{M_{pl}^{3+r}}} + \underbrace{W_0^{(1)} W_{\text{dyn}}^{(2)}}_{\sim \mathrm{M_{pl}^{3}}} + \underbrace{W_0^{(2)} W_{\text{dyn}}^{(1)}}_{\sim \mathrm{M_{pl}^{3}}} + \underbrace{W_{\text{dyn}}^{(1)} W_{\text{dyn}}^{(2)}}_{\sim \mathrm{M_{pl}^{3-r}}}), \tag{4.14}$$

with $r > 0$. Let us account for dimensions by introducing an extra scale m_Λ such that

$$\begin{aligned} W_0^{(1)} &= m_\Lambda^{\frac{3-r}{2} - A} \mathrm{M_{pl}^{\frac{3+r}{2} + A}}, & W_{\text{dyn}}^{(1)} &= \mathrm{M_{pl}^3} \frac{W_{\text{susy}}^{(1)}}{W_0^{(2)}}, \\ W_0^{(2)} &= m_\Lambda^{\frac{3-r}{2} + A} \mathrm{M_{pl}^{\frac{3+r}{2} - A}}, & W_{\text{dyn}}^{(2)} &= \mathrm{M_{pl}^3} \frac{W_{\text{susy}}^{(2)}}{W_0^{(1)}}, \end{aligned} \tag{4.15}$$

with $W_{\text{susy}}^{(n)}$ fixed as $\mathrm{M_{pl}} \to \infty$. Formally one can choose an inhomogeneous scaling with $A \neq 0$, but as we shall see it has no real consequences. For any A it is easily seen

[7] For a product of superpotentials we can always choose a Kähler gauge *at every point* with $\langle K \rangle = \langle \partial_\phi K \rangle = \langle \partial_q K \rangle = 0$ without mixing the superpotentials. In that case F-term supersymmetry breaking is given by the linear terms in the expansion of $W^{(1)}$ and $W^{(2)}$: $\langle \mathrm{D}_\phi W \rangle \sim \langle \partial_\phi W^{(1)} \rangle$, $\langle \mathrm{D}_q W \rangle \sim \langle \partial_q W^{(2)} \rangle$.

that with this scaling,

$$\mathrm{D}_i W = \partial_i W^{(1)}_{\text{susy}} + \frac{m_\Lambda^{r-3}}{\mathrm{M}_{\text{pl}}^r} W^{(2)}_{\text{susy}} \partial_i W^{(1)}_{\text{susy}} \\ + \frac{\partial_i K^{(1)}}{\mathrm{M}_{\text{pl}}^2} \left(m_\Lambda^{3-r} \mathrm{M}_{\text{pl}}^r + W^{(1)}_{\text{susy}} + W^{(2)}_{\text{susy}} + \frac{m_\Lambda^{r-3}}{\mathrm{M}_{\text{pl}}^r} W^{(1)}_{\text{susy}} W^{(2)}_{\text{susy}} \right) \to \partial_i W^{(1)}_{\text{susy}},$$

in the limit $\mathrm{M}_{\text{pl}} \to \infty$ if and only if $0 < r < 2$ and thus

$$V_{\text{sugra}} = e^{K/\mathrm{M}_{\text{pl}}^2} \left(|\mathrm{D}W|^2 - \frac{3|W|^2}{\mathrm{M}_{\text{pl}}^2} \right) \to \sum_n |\partial_n W^{(n)}_{\text{susy}}|^2 - 3 m_\Lambda^{2(3-r)} \mathrm{M}_{\text{pl}}^{2(r-1)} + O\left(\frac{1}{\mathrm{M}_{\text{pl}}} \right).$$

For $r < 1$ the manifestly constant term in the potential vanishes as well and we recover the strict decoupled field theory result, with the gravitino mass going to zero as $m_{3/2} = \langle W \rangle \mathrm{M}_{\text{pl}}^{-2} = m_\Lambda^{3-r} \mathrm{M}_{\text{pl}}^{r-2} = \frac{m^2_{\text{susy}}}{\sqrt{3}\mathrm{M}_{\text{pl}}}$. We see that the gravitino mass is independent of r in physical scales.

The parameter r should not be larger than unity for the new decoupling limit to be well defined. For the special case $r = 1$ [177], the potential has an additional overall “cosmological” constant. For a generic non-gravitational field theory in which $\mathrm{M}_{\text{pl}} \to \infty$ this is just an overall shift of the potential, which we can arbitrarily remove since it does not change the physics. Nevertheless from a formal point of view, we know that absolute ground state energy of a globally supersymmetric theory equals zero, as a result of the supersymmetry algebra $\{Q, Q\} = H$. For this reason it is more natural to restrict the value of r to the range $0 < r < 1$.

It may appear that we have changed the canonical renormalization group scaling of the theory. This is not quite true. For the interacting terms in the potential, it is the coefficients in the product $W_0^{(2)} W^{(1)}_{\text{dyn}} = W^{(1)}_{\text{susy}}$ that ought to obey canonical renormalization group scaling. This precisely corresponds to holding $W^{(n)}_{\text{susy}}$ fixed as $\mathrm{M}_{\text{pl}} \to \infty$ (see footnote 6). On the other hand, the scaling of the constant term in the potential has changed from its canonical value. However, this is very natural in a supersymmetric theory. The constant term, $\prod_n W_0^{(n)}$, equals the ground state energy. Precisely supersymmetric theories can “naturally” explain non-canonical scaling of the cosmological constant (at the loop level; the scaling of the bare ground state energy can be different in every model). A non-integer power is strange but $r = 1$ is certainly a viable option in a supersymmetry-breaking ground state: it is the natural scaling in theories with higher supersymmetry [185] when combined with a subleading $\log(\mathrm{M}_{\text{pl}}/m_{\text{susy}})$ breaking. Our engineering analysis only focuses on power-law scaling and these can always have subleading logarithms. ($r = 2$ would correspond to the cosmological constant for a spontaneously broken $\mathcal{N} = 1$ theory due to mass splitting).

The novel scaling in (4.15) can be readily generalized to an arbitrary number of sectors. For s sectors, writing $W^{(n)} = W_0^{(n)} + W_{\text{dyn}}^{(n)}$ for each sector, the superpotential $W = \frac{1}{\mathrm{M}_{\text{pl}}^{3(s-1)}} \prod_{n=1}^{s} W^{(n)}$ becomes

$$W = \frac{1}{\mathrm{M}_{\text{pl}}^{3(s-1)}} \left[\prod_{n=1}^{s} W_0^{(n)} + \sum_{m=1}^{s} \left(W_{\text{dyn}}^{(m)} \prod_{n \neq m}^{s} W_0^{(n)} \right) + \sum_{l>m}^{s} \left(W_{\text{dyn}}^{(m)} W_{\text{dyn}}^{(l)} \prod_{n \neq l,m}^{s} W_0^{(n)} \right) + \ldots \right].$$

In this expression, we want the last term before the ... and all terms on the ... to scale away as $\mathrm{M}_{\text{pl}}^{-r}$ or stronger under $\mathrm{M}_{\text{pl}} \to \infty$, where $r > 0$. The second term(s) should be constant. As a consequence the first term will scale as $\mathrm{M}_{\text{pl}}^{r}$. Assuming a scaling that is homogeneous across sectors, this implies

$$W_0^{(n)} \sim \mathrm{M}_{\text{pl}}^{\frac{3(s-1)+r}{s}}, \qquad W_{\text{dyn}}^{(n)} \sim \mathrm{M}_{\text{pl}}^{\frac{(3-r)(s-1)}{s}},$$

for each of the $n \in \{1, \ldots, s\}$. With this scaling, a general term consisting of t dynamical superpotentials and $s - t$ constant parts, scales as

$$\frac{W_{\text{dyn}}^{t} W_0^{s-t}}{\mathrm{M}_{\text{pl}}^{3(s-1)}} \sim \mathrm{M}_{\text{pl}}^{r(1-t)},$$

and as constructed any term containing dynamical interactions between sectors, $t \geq 2$, is Planck-suppressed. To ensure a vanishing constant term as in equation (4.2.3), r is again limited to the range $0 < r < 1$.

4.3 Zero mass mode for a stabilized sector

Anticipating the situation for an inflationary scenario we now analyze the mass spectrum of a stabilized q-sector in a de Sitter background. For Minkowski spaces it is known that the lightest mass in a stabilized sector scales with the supersymmetry breaking vacuum expectation value G_a [186]. Here we extend the analysis to de Sitter vacua as the zeroth order approximation of slow-roll inflation. Already in this zeroth order approach we will show that a similar light mode develops in the stabilized sector. Throughout this discussion we assume that the potential V is kept positive by the presence of the "inflationary" sector. In the next section we show that this result can be translated directly into an inflationary setting, where this light mode will affect the slow-roll dynamics.

Given that we insist the q-sector to be stabilized, we have $\partial_a V = 0$. In terms of the Kähler function $G(\phi, \overline{\phi}, q, \overline{q})$ this means

$$(\nabla_a G_b) G^b = -G_a(1 + e^{-G} V).$$

If the q-ground state breaks supersymmetry, i.e. $G_a \neq 0$, we may rewrite it in terms of the supersymmetry breaking direction $f_a = G_a / \sqrt{G^b G_b}$,

$$(\nabla_a G_b) f^b = -f_a(1 + e^{-G} V).$$

For simplicity we will assume that the q-sector consists of only a single complex scalar field q, in which case we may write this equation as

$$\nabla_q G_q = -G_{q\overline{q}}(1 + e^{-G} V)\widehat{G}_q^2. \tag{4.16}$$

A hat $\widehat{z}$ on a complex number denotes the "phase"-part of the number, $z = |z|\widehat{z} = |z| e^{i \arg(z)}$. As such $\widehat{G}_q = \sqrt{G^{q\overline{q}}} f_q$. Note that in an arbitrary supersymmetric configuration $G_a = 0$ there are no restrictions on $\nabla_a G_b$, but on a supersymmetry broken configuration this is no longer true. Were one to turn on supersymmetry breaking, one would first have to reach a surface in parameter space where this restriction can be imposed at the onset of supersymmetry breaking.

We will now compute the mass spectrum for the two modes of the complex scalar field q, at the hypersurface defined by (4.16). The mass modes are given by the eigenvalues of the matrix

$$M^2 = \begin{pmatrix} V^q{}_q & V^q{}_{\overline{q}} \\ V^{\overline{q}}{}_q & V^{\overline{q}}{}_{\overline{q}} \end{pmatrix},$$

which in our case means

$$m_q^{\pm} = \left(V^q{}_q \pm |V^q{}_{\overline{q}}| \right) = G^{q\overline{q}} \left(V_{q\overline{q}} \pm |V_{qq}| \right). \tag{4.17}$$

Expanding the second derivatives of the potential (cf. appendix 4.B) to first order in $|G_q|$, these eigenvalues are

$$m_q^- = e^G G^{q\overline{q}} \mathrm{Re}\{(\nabla_q \nabla_q G_q)\widehat{G^q}^3\}|G^q| + O(|G_q|^2), \tag{4.18a}$$

$$m_q^+ = e^G \left[2(2 + e^{-G} V)(1 + e^{-G} V) - G^{q\overline{q}} \mathrm{Re}\{(\nabla_q \nabla_q G_q)\widehat{G^q}^3\}|G^q| \right] + O(|G_q|^2). \tag{4.18b}$$

We see from (4.18a) that in the limit of vanishing supersymmetry breaking the lightest mass mode becomes massless, just as in the case of Minkowski space [186].[8] It is

[8] The result can also be extended to hold for anti-de Sitter vacua. However, for $-2 < e^{-G} V < -1$, also a tachyonic mode develops.

important to note that this result depends crucially on taking the limit G_q to zero in the supersymmetry breaking direction. When supersymmetry is restored and both $G_q = 0$ and $G_{\overline{q}} = 0$, the phases of these vectors have no meaning. In fact, we see that then a new degree of freedom arises: $\nabla_q G_q$ becomes unrestricted which allows one to choose the masses freely.

The geometrical picture is that there is a whole plane of supersymmetric solutions where arbitrary masses are allowed. However, when supersymmetry is broken, the supersymmetry breaking direction has to align with its complex conjugate fixing one point on this plane where supersymmetry can be broken. In this point, the lightest mode becomes massless.

4.4 Two-sector inflation in supergravity

Generally, when inflation is described in supergravity, realistic matter resides in a hidden sector.[9] Supergravities descending from string theory often have additional hidden sectors as well. These sectors are always gravitationally coupled. In the previous section we have seen that for de Sitter vacua the hidden sector develops a light direction. In this section we will consider how this light mode of the hidden sector can affect the naïve dynamics of the inflationary sector. We will show that despite the weakness of gravity, these effects can be large. Realistic slow-roll inflation is characterized by small numbers, the slow-roll parameters ϵ and η, and even small absolute changes to these numbers can be of the order of 100% in relative terms.

We will first briefly review the η-problem in the context of single field inflation in supergravity. Then we will explain what effects are to be expected when including an additional (hidden) sector. The section ends with calculating the relevant objects to determine the true dynamics of the full system.

4.4.1 Inflation and the η-problem in supergravity

In single scalar field models of inflation the spectrum of density perturbations is characterized by the two slow-roll parameters ϵ and η. To ensure that this spectrum matches the observed near scale invariance, both $\epsilon \ll 1$ and $\eta \ll 1$. Inflationary supergravity in its simplest form consists of a single complex scalar field, the inflaton, whose potential is generated by F-terms (3.28). The definition of η may be phrased

[9] The supersymmetric partners of the standard model are not good inflaton candidates, as these partners are charged under the standard model gauge group and gauge fields taking part in inflation would lead to topological defects [eg. 187, 188]. The exception could be a gravitationally non-minimally coupled Higgs field [eg. 189, 190].

as the lightest direction of the mass matrix in units of the Hubble rate $3H^2 = V$, i.e. η is the smallest eigenvalue of the matrix, cf. equation (2.7), [191]

$$\widetilde{N} = \frac{1}{V}\begin{pmatrix} \nabla^i\nabla_j V & \nabla^i\nabla_{\bar{j}} V \\ \nabla^{\bar{\imath}}\nabla_j V & \nabla^{\bar{\imath}}\nabla_{\bar{j}} V \end{pmatrix},$$

where the tilde on $\widetilde{N}$ indicates that this value of η is defined with respect to the inflaton sector only.[10] From the second ϕ-derivative of V,

$$V_{i\bar{j}} = G_{i\bar{j}}V + G_i V_{\bar{j}} + G_{\bar{j}}V_i - G_i G_{\bar{j}} V + e^G \left[R_{i\bar{j}k\bar{l}} G^k G^{\bar{l}} + G^{k\bar{l}} \nabla_i G_k \nabla_{\bar{j}} G_{\bar{l}} + G_{i\bar{j}} \right],$$

we see that a natural value for η is $V^i{}_j/V \sim \nabla^i G_j \sim 1$ is unity. Therefore, we must tune G_i, $\nabla_i G_j$ and $R_{i\bar{j}k\bar{l}}$ so that $V^i{}_j = O(10^{-3})V$. The necessity of this tuning is known as the η-problem.

As shown in [194], successful inflation is achievable if one tunes the Kähler function G such that

$$R_{i\bar{j}k\bar{l}} f^i f^{\bar{j}} f^k f^{\bar{l}} \lesssim \frac{2}{3}\frac{1}{1+\gamma},$$

where $\gamma = e^{-G}V/3$ is inversely proportional to an overall mass scale $m_{3/2} = e^{G/2}$, which is related to the gravitino mass and $R_{i\bar{j}k\bar{l}}$ is the Riemann tensor of the inflaton sector. As $f^i f_i = 1$, the above equation defines the normalized sectional curvature along the direction of supersymmetry breaking. The constraint becomes stronger as $\gamma \gg 1$, thus as $H \gg m_{3/2}$. When the bound is met, one can always tune η to be small by tuning G_i, $\nabla_i G_j$ and $R_{i\bar{j}k\bar{l}}$.

Finding a suitably tuned supergravity potential from a (UV-complete) string theoretical set-up has proven to be incredibly difficult [195, 196], but possible [197–199]. Currently, in models with correctly tuned slow-roll parameters it is typically assumed that the "hidden sector" does not affect the finetuning of parameters. The subject of this chapter is to examine whether such an assumption is justified and hence how relevant tuned models are that only consider the inflationary sector.

4.4.2 Stability of the hidden sector during inflation

Having reviewed the η-problem in single sector supergravity theories, we will now consider if and how the fields in the hidden sector can affect the inflationary evolution. From the diagonalization of the kinetic terms in (4.11) the distinction between ϕ-fields

[10] A careful definition based on the kinetic behavior of the inflaton field is done in [192, 193]. In the slow-roll, slow-turn limit, it reduces to the definition of η given here.

and q-fields is explicit, leading naturally to an inflationary and a hidden sector. We will again assume these sectors to both consist of only one complex scalar field, ϕ and q respectively. The argument we shall present can already be made in a two-field system. It carries through to multi-field models because the field ϕ is viewed as the inflaton in an effective single field inflationary model, while the field q can be seen as the lightest mode in the hidden sector. Following the usual practice [1, 2, and references therein], we assume that inflation is solved by tuning the inflationary sector only, including obtaining satisfactory values for the slow-roll parameters from a phenomenological viewpoint. As a result all data in the inflationary sector are fixed and known. Contrarily, the hidden sector is left unspecified and the restrictions we find on it are a function of model specific parameters of the inflaton sector only.

To ensure that the hidden sector does not take part in the inflationary dynamics, one generally assumes that the fields in the hidden sector are stabilized in a ground state at a constant field value $q = q_0$ throughout inflation

$$\partial_q V\Big|_{q_0} = 0 \tag{4.19}$$

and, hence, are not dynamical. Clearly an extremum for the hidden sector is obtained if $G_q = 0$, i.e. when the ground state of the hidden sector preserves supersymmetry. As was shown in detail in [155, 156, 166–170, 174], when $G_q = 0$ the ground state of the hidden sector decouples gravitationally from the inflationary sector and the inflationary sector truly determines the inflationary evolution without any contributions from the hidden sector. The stability of the extremum of the hidden sector, however, depends on the inflationary trajectory and a stable extremum might develop into an instability, leading to a waterfall for the hidden sector fields and, as a result, to the end of inflation, as discussed in [166, 170].

The case we examine here is when supersymmetry is broken in the hidden sector, $G_q \neq 0$. The first thing to note is that the stability assumption (4.19) cannot be met anymore. In supergravity the position $q = q_0$ of the minimum of the potential is given by

$$V_q = G_q V(\phi, \overline{\phi}, q, \overline{q}) + e^{G(\phi,\overline{\phi},q,\overline{q})} \left((\nabla_q G_q) G^q + G_q \right) = 0,$$

which shows that for $G_q \neq 0$ the ground state q_0 depends on the inflaton field ϕ, through $V(\phi, \overline{\phi}, q, \overline{q})$ and $G(\phi, \overline{\phi}, q, \overline{q})$. In the situation of unbroken supersymmetry, $G_q = 0$, all ϕ-dependence drops out, but for $G_q \neq 0$ we see that it is impossible to keep the position of the minimum constant during inflation. As the inflaton ϕ rolls down the inflaton direction, the "stabilized" hidden scalar q will change its value. It is clear that the assumption of a vanishing $V_q = 0$ for all q is incompatible with $G_q \neq 0$ and we should therefore abandon it. This in turn means that the hidden sector field

q must be dynamical, through its equation of motion. Since we still want to identify the field ϕ as the inflaton in the sense that it drives the cosmological dynamics, we have to assume that q moves very little. We must therefore also assume a slow-roll, slow-turn approximation to the solution of the q equation of motion

$$\dot{q} = \frac{G^{q\bar{q}} V_{\bar{q}}}{3H}.$$

The statement that the cosmological dynamics is driven by the ϕ-sector means that $||\dot{q}|| \ll ||\dot{\phi}||$, where $||\dot{q}|| \equiv \sqrt{G_{q\bar{q}}\dot{q}\dot{\bar{q}}}$, etc. Through both slow-roll equations of motion this equates to $||V_q|| \ll ||V_\phi||$ or $\epsilon_q \ll \epsilon_\phi$,

As the hidden sector has now become dynamical, we have to treat the system as a multi-field inflationary model. Since it is impossible to diagonalize the Kähler transformations and mass matrix simultaneously, the fields will mix in the case of a hidden sector with broken supersymmetry [166]. In the next section we will study the consequences of this mixing by explicitly diagonalizing the mass matrix of the full two-field system. From the result we shall find three possible effects on the inflationary dynamics.

First, the lightest masses of fields from the different sectors can be too close together. It is obvious that one cannot consider an effective single field model if this is the case, since for the dynamics to be independent of initial conditions, the lightest field needs to be much lighter than the other fields. When the masses of the two fields are similar, both of them contribute to the dynamics, resulting into a multi-field rather than a single field inflationary scenario. As is known from the literature, a multi-field inflationary model will produce effects such as isocurvature modes [eg. 67, 200–213], features in the power spectrum [eg. 52, 214–216] and non-Gaussianities [isocurvature models and eg. 58, 59, 217–224], pointing to a qualitatively different model.

Second, a change of the true value of η can occur. We have assumed the inflaton sector to be tuned in such a way that it agrees with observed values for the slow-roll parameters. If the effects of the hidden sector on the total dynamics are such that η will change significantly, the initial naïve tuning would be of no meaning and one would have to start the tuning process all over again after the hidden sector has been added. Again we note that there is no contribution in the case of unbroken supersymmetry in the hidden sector, since we shall show that the contribution to η from the hidden sector is mostly determined by the cross terms in the mass matrix,

$$V_{\phi q} = G_\phi V_q + G_q V_\phi - G_\phi G_q V,$$

which vanish when $G_q = 0$.

Third, a complete change of the sector that determines η is possible. It is possible that the eventual η-parameter is still within the limits of its naïve tuned value, satisfying the second bound, but instead it is determined by the hidden sector rather than the inflationary sector. Any initial control obtained by tuning the inflationary sector is superseded by the sheer coincidental configuration of the hidden sector.

4.4.3 The mass matrix of a two-sector system

To investigate when effects from the hidden sector are to be expected, we need to calculate the eigenvalues of the mass matrix of the full two-field system. Since we assume the inflationary evolution to be in the slow-roll, slow-turn regime, the dynamics is completely potential energy dominated. The mass matrix of the full two-field system determines which directions are stable or steep, as characterized by the eigenvalues of this matrix. Normalizing by $1/V$ to obtain the value of η directly, the matrix we want to diagonalize is the 4×4-matrix

$$N = \frac{1}{V}\begin{pmatrix} \nabla^I \nabla_J V & \nabla^I \nabla_{\bar{J}} V \\ \nabla^{\bar{I}} \nabla_J V & \nabla^{\bar{I}} \nabla_{\bar{J}} V \end{pmatrix}. \tag{4.20}$$

Equation (4.20) is to be evaluated at a point near $q_0 = q_0(\phi_0)$, where q_0 is such that $\partial_q V(q_0) = 0$, with ϕ_0 indicating the beginning of inflation. As is clear from the discussion of section 4.4.2 we cannot truly expect the hidden sector to be stabilized throughout the inflationary evolution. Nevertheless we may consider $\partial_q V(q_0) = 0$ at a certain point $q_0 = q_0(\phi_0)$, with $||\partial_q V|| \ll ||\partial_\phi V||$ around q_0 in accordance with the restriction $\epsilon_q \ll \epsilon_\phi$.

The mass matrix is Hermitian and, considering again a two-field system, can be put in the form

$$N = \frac{1}{V}\begin{pmatrix} \nabla^\phi V_\phi & \nabla^\phi V_{\bar{\phi}} & \nabla^\phi V_q & \nabla^\phi V_{\bar{q}} \\ \nabla^{\bar{\phi}} V_\phi & \nabla^{\bar{\phi}} V_{\bar{\phi}} & \nabla^{\bar{\phi}} V_q & \nabla^{\bar{\phi}} V_{\bar{q}} \\ \nabla^q V_\phi & \nabla^q V_{\bar{\phi}} & \nabla^q V_q & \nabla^q V_{\bar{q}} \\ \nabla^{\bar{q}} V_\phi & \nabla^{\bar{q}} V_{\bar{\phi}} & \nabla^{\bar{q}} V_q & \nabla^{\bar{q}} V_{\bar{q}} \end{pmatrix},$$

by a coordinate transformation. Diagonalizing the full matrix in general is involved. Therefore, we adopt the strategy to diagonalize the two sectors separately and then pick the lightest modes only. The first step yields

$$N = \begin{pmatrix} \frac{1}{V}(V^\phi{}_\phi - |V^\phi{}_{\bar{\phi}}|) & 0 & A_{11} & A_{12} \\ 0 & \frac{1}{V}(V^\phi{}_\phi + |V^\phi{}_{\bar{\phi}}|) & A_{21} & A_{22} \\ \bar{A}_{11} & \bar{A}_{21} & \frac{1}{V}(V^q{}_q - |V^q{}_{\bar{q}}|) & 0 \\ \bar{A}_{12} & \bar{A}_{22} & 0 & \frac{1}{V}(V^q{}_q + |V^q{}_{\bar{q}}|) \end{pmatrix},$$

with

$$A = \frac{1}{2V}\begin{pmatrix} -\widehat{V_{\overline{\phi\phi}}} & \widehat{V_{\overline{\phi\phi}}} \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} V^{\phi}{}_{q} & V^{\phi}{}_{\bar{q}} \\ V^{\bar{\phi}}{}_{q} & V^{\bar{\phi}}{}_{\bar{q}} \end{pmatrix} \begin{pmatrix} -\widehat{V_{\overline{qq}}} & \widehat{V_{\overline{qq}}} \\ 1 & 1 \end{pmatrix}.$$

Here, the first matrix is the inverse of the similarity transformation of the ϕ-sector and the last matrix diagonalizes the q-sector.

In general the eigenmodes in the individual sectors will be different, one always being smaller than the other. Dynamically the most relevant direction is the lightest mode of each sector, but by restricting to these light directions, one assumes a hierarchy already within the sectors. For the inflationary sector this is phenomenologically justified if we assume that inflation is described by a single field, where we know that $V^{\phi}{}_{\phi}$ and $V^{\phi}{}_{\bar{\phi}}$ combine such that a light mode appears with mass ηV, much lighter than the other mass modes. For the hidden sector we will simply assume that a large enough hierarchy between mass modes exists. This will simplify matters without weakening our result. By including only the lightest mode of the hidden sector, we can already show that the true dynamics is in many cases not correctly described by the naïve inflaton sector. Our case would only be more strongly supported if we would include the heavy mode of the hidden sector, but this is technically more involved. Projecting on the light directions, we get a submatrix of light mass modes

$$N_{\text{light}} = \begin{pmatrix} \lambda_{\phi} & A_{11} \\ \overline{A}_{11} & \lambda_{q} \end{pmatrix},$$

with

$$\lambda_{\phi} = \frac{1}{V}\left(V^{\phi}{}_{\phi} - |V^{\phi}{}_{\bar{\phi}}|\right) = \frac{G^{\phi\bar{\phi}}}{V}(V_{\phi\bar{\phi}} - |V_{\phi\phi}|), \tag{4.21a}$$

$$\lambda_{q} = \frac{1}{V}\left(V^{q}{}_{q} - |V^{q}{}_{\bar{q}}|\right) = \frac{G^{q\bar{q}}}{V}(V_{q\bar{q}} - |V_{qq}|), \tag{4.21b}$$

$$A_{11} = \frac{G^{\phi\bar{\phi}}}{2V}\left(\widehat{V_{\overline{qq}}}\widehat{V_{\phi\phi}}V_{\bar{\phi}q} - \widehat{V_{\overline{qq}}}V_{\phi q} + V_{\phi\bar{q}} - \widehat{V_{\phi\phi}}V_{\overline{\phi q}}\right). \tag{4.21c}$$

The eigenvalues of this two-field system are given by

$$\mu_{\pm} = \frac{1}{2}\left(\lambda_{\phi} + \lambda_{q}\right) \pm \frac{1}{2}\sqrt{\left(\lambda_{q} - \lambda_{\phi}\right)^{2} + 4|A_{11}|^{2}}. \tag{4.22}$$

Since $\mu_{-} < \mu_{+}$ the second slow-roll parameter for the full system is given by $\eta = \mu_{-}$.

4.5 Dynamics due to the hidden sector

In the slow-roll and slow-turn approximation, the mass modes $\mu_\pm$ from (4.22) determine the dynamics of the full system. In general the true dynamics will deviate from the naïve single sector evolution. As explained in section 4.4.2 it is necessary to put constraints on the full system for the true dynamics to still (largely) agree with the initial naïve dynamics. We will quantify these constraints in terms of the hidden sector light mode λ_q and the dynamical cross coupling $|A_{11}|$ between sectors. The results are graphically summarized in figures 4.1 and 4.2. In section 4.5.2 and figure 4.3 we will discuss the result again but then interpreted from the viewpoint of supergravity. Finally we will explain that a simple application of these bounds implies that the standard model cannot be ignored during cosmological inflation, if standard model supersymmetry breaking is independent of the inflaton sector.

4.5.1 Conditions on the hidden sector data

From (4.22) we see that the light modes λ_ϕ, λ_q from the two separate sectors mix through a cross coupling $|A_{11}|$ and combine to the true eigenvalues $\mu_\pm$ of the full two-sector system. As explained in 4.4.2, for the inflaton sector to still describe the cosmological evolution and the η-parameter reliably, the three constraints it must obey are (1) the bound arising from demanding a hierarchy between $\mu_\pm$ to prevent multi-field effects, (2) the bound arising from demanding the second slow-roll parameter $\mu_- = \eta$ to not change its value too much and (3) the bound from demanding that η is mostly determined by the ϕ-sector rather than the q-sector.

To prevent multi-field effects from setting in, we take as a minimum hierarchy that μ_+ is at least five times as heavy as μ_- in units of the scale of the problem $|\mu_-|$,

$$\frac{\mu_+ - \mu_-}{|\mu_-|} > 5. \tag{4.23}$$

This bound is rather arbitrary, but clearly a hierarchy between μ_+ and μ_- must exist. Calculations in [215] show that for a mass hierarchy $\lesssim 5$ multi-field effects are typically important.

The second bound is given by the A_{11}-dependence of μ_-. The value of the second slow-roll parameter from the single field inflationary model only is $\eta_{\text{naïve}} = \lambda_\phi$. In the full two-sector system, μ_- takes over the role as the true second slow-roll parameter $\eta_{\text{true}} = \mu_-$. The contribution to the actual η-parameter from the presence of the hidden sector is therefore

$$\Delta\eta = \mu_- - \lambda_\phi = \frac{1}{2}\left[(\lambda_q - \lambda_\phi) - \sqrt{(\lambda_q - \lambda_\phi)^2 + 4|A_{11}|^2}\right], \tag{4.24}$$

which is always negative. We argue that this difference should stay within $|\Delta\eta/\lambda_\phi| < 0.1$, i.e. η should not change by more than 10%. This choice for the range of η is given by current experimental accuracy. Current experiments can only determine $n_s = 1 - 6\epsilon + 2\eta$. WMAP has a 1σ error of 6.53% [10], Planck will have an error of 0.70% [69]. For $n_s - 1$, assuming 0.96, this gives a 17.5% error on the combination of $-6\epsilon + 2\eta$, which means an uncertainty of about 10% on the value of η.

We will examine λ_q, A_{11} in units of $|\lambda_\phi|$ and exclude regions in which the hidden sector affects the tuned inflationary sector too much. The analysis is best done separately for the cases $\lambda_\phi = \eta_{\text{naïve}} > 0$ and $\lambda_\phi = \eta_{\text{naïve}} < 0$ because of the qualitative differences between these cases.

The case $\eta_{\text{naïve}} > 0$

We first examine the hierarchy bound as explained above and focus first on the situation where $\mu_- > 0$. In this case (4.23) means that we demand

$$\frac{\mu_+ - 6\mu_-}{\lambda_\phi} = \frac{1}{2}\left[-5\left(\frac{\lambda_q}{\lambda_\phi} + 1\right) + 7\sqrt{\left(\frac{\lambda_q}{\lambda_\phi} - 1\right)^2 + 4\left(\frac{|A_{11}|}{\lambda_\phi}\right)^2}\right] > 0,$$

which allows us to solve λ_q/λ_ϕ as a function of $|A_{11}|/\lambda_\phi$,

$$\left(\frac{12}{35}\right)^2\left(\frac{\lambda_q}{\lambda_\phi} - \frac{37}{12}\right)^2 + \left(\frac{2\sqrt{6}}{5}\right)^2\left(\frac{|A_{11}|}{\lambda_\phi}\right)^2 = 1.$$

This excludes everything inside the ellipse demarcating the green region in figure 4.1. The case $\mu_- < 0$ is not relevant as it is already excluded by the second bound.

For this second bound, to be somewhat more general than the observationally inspired constraint $\Delta\eta/\lambda_\phi > -0.1$, we give the bound $\Delta\eta/\lambda_\phi > -f$. Solving for λ_q this gives

$$\frac{\lambda_q}{\lambda_\phi} > 1 - f + \frac{1}{f}\left(\frac{|A_{11}|}{\lambda_\phi}\right)^2,$$

as is indicated in blue in figure 4.1. Note that since the true value of η is always lower than $\eta_{\text{naïve}}$ (see [225] for some specific examples), a change in η of 100% means that η changes sign from its naïve value. This shows that we were justified to only consider positive μ_- in the hierarchy bound earlier.

The third bound is given by a λ_q-dominance in μ_-. Since λ_ϕ and λ_q are treated on equal footing in μ_-, the true η is dominantly determined by the smallest eigenvalue,

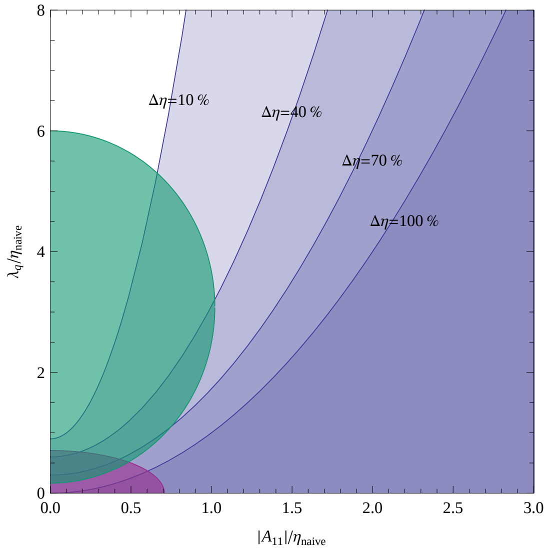


Figure 4.1: *Bounds from a dynamical hidden sector for $\eta_{\text{naïve}} > 0$. The multi-field constraint excludes an ellipse near the λ_q-axis (shaded in green). The bound from having too much effect on η excludes large $|A_{11}|$ (shaded with increasing intensities of blue for larger deviations). Around $\lambda_q = A_{11} = 0$ the hidden sector mode λ_q rather than λ_ϕ determines η, excluding that region as well (shaded in purple).*

which is not necessarily λ_ϕ. When $\lambda_\phi \gg \lambda_q$ and $\lambda_\phi \gg |A_{11}|$ we see immediately that the true $\eta = \mu_-$ is determined by λ_q and is *independent* of λ_ϕ,

$$\mu_- = \frac{1}{2}\left[(\lambda_q + \lambda_\phi) - \lambda_\phi\left(1 - \frac{\lambda_q}{\lambda_\phi} + O\left(\frac{\lambda_q^2}{\lambda_\phi^2}, \frac{|A_{11}|^2}{\lambda_\phi^2}\right)\right)\right].$$

It is clear that this arguments excludes the lower left corner of parameter space. We will take the bound to be $1/\sqrt{2}$ such that $\left(\lambda_q/\lambda_\phi\right)^2, \left(|A_{11}|/\lambda_\phi\right)^2 < 1/2 \ll 1$, the radius of convergence of this Taylor expansion. Contrarily to the somewhat debatable bounds imposed by $\Delta\eta/\lambda_\phi$, the points within this circle are truly excluded because they violate one of the core assumptions in the approach, viz. that the ϕ-sector is responsible for all cosmological dynamics including determining the value of η. The circle

$$\left(\frac{\lambda_q}{\lambda_\phi}\right)^2 + \left(\frac{|A_{11}|}{\lambda_\phi}\right)^2 = \frac{1}{2},$$

is indicated as the purple region in the figure.

In figure 4.1 we have indicated in which regions of λ_q/λ_ϕ- and $|A_{11}|/\lambda_\phi$-parameter space the effects of a hidden sector can be rightfully ignored. We have shown that all negative values of λ_q are excluded and only in the region with large λ_q/λ_ϕ and small $|A_{11}|/\lambda_\phi$ there are no large effects from the hidden sector. This result is qualitatively easily understood, as the hidden sector with broken supersymmetry will still decouple if the masses in the hidden sector are truly large. We argue that this possibility is too easily assumed to be the case in the literature without considering the actual hidden constraints it imposes on the hidden sector. These hidden assumptions should be mentioned explicitly and one should show that they can be obtained.

The case $\eta_{\text{naïve}} < 0$

In the case that $\lambda_\phi = \eta_{\text{naïve}}$ is negative, the last bound of section 4.5.1 does not impose any condition on $\lambda_q/|\lambda_\phi|, |A_{11}|/|\lambda_\phi|$-parameter space. When $\lambda_\phi < 0$, i.e. when $\lambda_\phi = -|\lambda_\phi|$, the eigenvalues can be written as

$$\mu_\pm = \frac{|\lambda_\phi|}{2}\left[\left(\frac{\lambda_q}{|\lambda_\phi|} - 1\right) \pm \sqrt{\left(\frac{\lambda_q}{|\lambda_\phi|} + 1\right)^2 + 4\left|\frac{A_{11}}{\lambda_\phi}\right|^2}\right],$$

which means that μ_- is not determined by λ_q to first order in $\lambda_q/|\lambda_\phi|$ but by λ_ϕ as should be,

$$\mu_- = \frac{|\lambda_\phi|}{2}\left[\left(\frac{\lambda_q}{|\lambda_\phi|} - 1\right) - \left(1 + \frac{\lambda_q}{|\lambda_\phi|} + \dots\right)\right].$$

However, by the hierarchy bound the small $\lambda_q/|\lambda_\phi|$-regime does get excluded. Since μ_- is always negative in this case,

$$\mu_- \leq \frac{|\lambda_\phi|}{2}\left[\left(\frac{\lambda_q}{|\lambda_\phi|} - 1\right) - \left|\frac{\lambda_q}{|\lambda_\phi|} + 1\right|\right] = -|\lambda_\phi|,$$

equation (4.23) translates into

$$\frac{\mu_+ + 4\mu_-}{|\lambda_\phi|} = \frac{1}{2}\left[5\left(\frac{\lambda_q}{|\lambda_\phi|} - 1\right) - 3\sqrt{\left(\frac{\lambda_q}{|\lambda_\phi|} + 1\right)^2 + 4\left|\frac{A_{11}}{\lambda_\phi}\right|^2}\right] > 0.$$

This excludes everything beneath the upper branch of the hyperbola given by the line

$$\frac{\lambda_q}{|\lambda_\phi|} > \frac{17}{8} + \frac{1}{8}\sqrt{15^2 + 28\left|\frac{A_{11}}{\lambda_\phi}\right|^2},$$

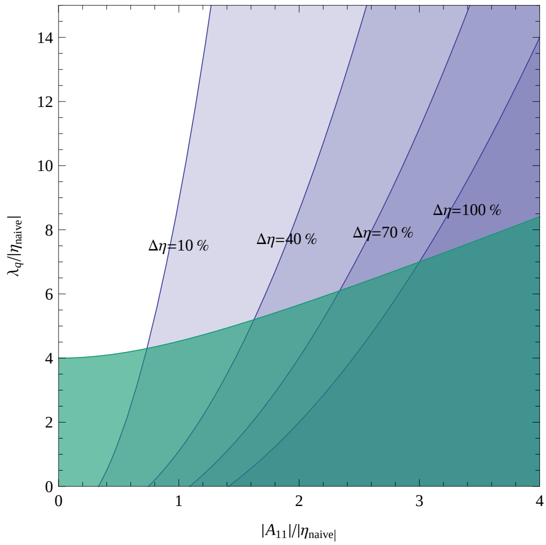


Figure 4.2: *Bounds from a dynamical hidden sector for $\eta_{\text{naïve}} < 0$. The multi-field bound excludes a hyperbola starting at $\lambda_q = 4|\lambda_\phi|$ and, in particular, small λ_q (shaded in green). The bound from having too much effect on η excludes the large $|A_{11}|$-region (shaded with increasing intensities of blue for larger deviations), but leaves open in particular the full range of λ_q.*

which is shaded green region in figure 4.2.

The final constraint on the parameter space comes from the bound on the change in η, see the previous paragraph on the $\eta_{\text{naïve}} > 0$-case for a discussion. In the blue region in figure 4.2 we have indicated the bound $|\Delta\eta/\lambda_\phi| < f$, which means

$$\frac{\lambda_q}{|\lambda_\phi|} > -1 - f + \frac{1}{f}\left|\frac{A_{11}}{\lambda_\phi}\right|^2 ,$$

for different fractions of f.

In figure 4.2 we have indicated in which regions of $\lambda_q/|\lambda_\phi|$- and $|A_{11}|/|\lambda_\phi|$-parameter space the effects of a hidden sector can be rightfully ignored after imposing both constraints. As in the case for $\eta_{\text{naïve}} > 0$, the only allowed region is for large $\lambda_q/|\lambda_\phi|$ and small $|A_{11}|/|\lambda_\phi|$. Note that all values of $\lambda_q < 4|\lambda_\phi|$ are explicitly excluded by the imposed bounds.

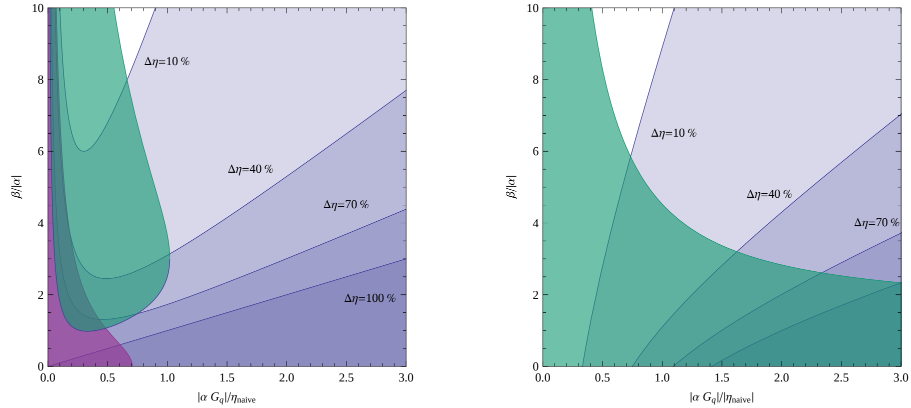


Figure 4.3: *Excluded regions for the supergravity parameter range for $|G_q|$ and β, which contains in particular $\nabla_q \nabla_q G_q$, in units of $|\eta_{\text{naïve}}|$ and $|\alpha|$, which contains ϵ_ϕ and G_ϕ. The indicated regions come from the multi-field bound (shaded in green), the correct identification of sectors (shaded in purple) and allowing only for small deviations of η (shaded in higher intensities of blue for larger deviations). The left (right) picture describes the case $\eta_{\text{naïve}} > 0$ ($\eta_{\text{naïve}} < 0$).*

4.5.2 Conditions on supergravity models

In principle, figures 4.1 and 4.2 provide all the information needed to verify whether the hidden sector of a given model may be neglected while studying the inflationary dynamics. Through equations (4.21) and the expressions for V_{IJ} as summarized in appendix 4.A, one can explicitly calculate the corresponding λ_q and A_{11} for a given model and compare them with the figures. However, we would like to have some direct intuition about the dependence of the excluded regions on the supergravity data. In this section we will investigate how much we can say about this in general without having to specify a model. The main question to answer is whether the fact that λ_q and A_{11} are determined by a supergravity theory, provides any additional constraint on which regions are obtainable to begin with. The answer to this question turns out to be that a priori supergravity is not restrictive enough to exclude any of the regions in λ_q, A_{11}-parameter space.

The easiest way to translate figures 4.1 and 4.2 in terms of supergravity data would be to simply map the regions into supergravity parameter space. Unfortunately the expressions in (4.21) are highly nonlinear and depend on too many supergravity variables to conveniently represent figures 4.1 and 4.2 in terms of supergravity data. However, for small $|G_q|$ this does turn out to be possible.

Applying the explicit expressions for V_{IJ} as found in appendix 4.A to (4.21c), yields

$$A_{11} = \alpha(\phi,\overline{\phi},q,\overline{q})|G_q|, \qquad \text{with} \tag{4.25}$$
$$\alpha(\phi,\overline{\phi},q,\overline{q}) = \frac{G^{\phi\overline{\phi}}}{2}\left(\widehat{G_{\overline{q}}} - \widehat{V_{\overline{q}\overline{q}}}\widehat{G_q}\right)\left(\left(\frac{V_\phi}{V} - G_\phi\right) - \widehat{V_{\phi\phi}}\left(\frac{V_{\overline{\phi}}}{V} - G_{\overline{\phi}}\right)\right).$$

From this equation we learn that A_{11} vanishes in the limit $G_q \to 0$, which makes sense as we know that the two sectors should decouple in the limit of restored supersymmetry. It is difficult to retrieve more information from this explicit expression of A_{11} in terms of supergravity data. In principle $A_{11}(|G_q|,\ldots)$ may be inverted to give some function $|G_q|(A_{11},\ldots)$, but this is trickier than (4.25) suggests. Although we have managed to extract one factor of G_q, the function $\alpha(\phi,\overline{\phi},q,\overline{q})$ still depends on G_q through the phases of $\widehat{V_{\overline{q}\overline{q}}}$ and $\widehat{V_{\phi\phi}}$, making it hard to perform the inversion explicitly.

The expression for λ_q looks even worse,

$$\lambda_q = \frac{G^{q\overline{q}}}{V}\left(V_{q\overline{q}} - \sqrt{V_{qq}V_{\overline{q}\overline{q}}}\right). \tag{4.26}$$

At this stage we have refrained from substituting in the expressions for $V_{q\overline{q}}$, V_{qq} and its complex conjugate. The square root clearly shows that the dependence of λ_q on $|G_q|$ and the other supergravity data is involved and difficult to invert. To get a useful expression we revert to the result of section 4.3 and consider λ_q in the small $|G_q|$-regime by performing a Taylor expansion. Copying from (4.18a), we find

$$\lambda_q = \beta(\phi,\overline{\phi},q,\overline{q})|G^q| + O(|G_q|^2), \qquad \text{with} \tag{4.27}$$
$$\beta(\phi,\overline{\phi},q,\overline{q}) = \frac{G^{q\overline{q}}}{e^{-G}V}\mathrm{Re}\{(\nabla_q\nabla_q G_q)\widehat{G^q}^3\}.$$

Having obtained the relations (4.25) and (4.27) we can now accommodate the reader with a graph of the allowed and excluded regions directly in terms of the supergravity data. For small $G_q \ll 1$ both λ_q and $|A_{11}|$ scale linearly with G_q, making it relatively easy to rewrite the bounds we found $\lambda_q/|\lambda_\phi| = \lambda_q/|\lambda_\phi|\left(|A_{11}|/|\lambda_\phi|\right)$ in terms of G_q, α and β as $\beta/|\alpha| = \beta/|\alpha|\left(|\alpha G_q|/|\lambda_\phi|\right)$. The resulting figure is depicted in 4.3. Note that α and β are still underdetermined — depending on $R_{q\overline{q}q\overline{q}}$ and $\nabla_q\nabla_q G_q$ at higher orders in $|G_q|$ — and are naturally of order 1. It is these numbers that determine where in figure 4.3 the model under investigation lies.

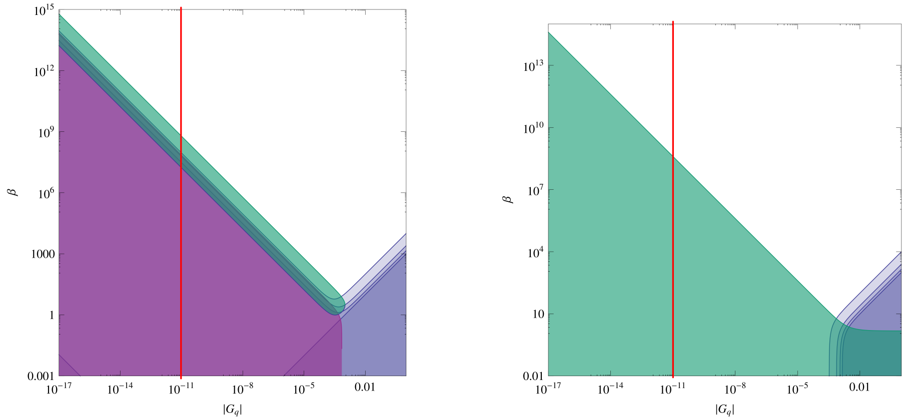


Figure 4.4: *The effects of the multi-field bound (shaded in green), the identification of the correct inflaton sector (shaded in purple) and the small deviations of η (shaded in blue) on a doubly logarithmic scale for $\eta_{\text{naïve}} > 0$ (left) and $\eta_{\text{naïve}} < 0$ (right). The approximate location of the standard model supergravity data is indicated with a red bar, showing that a large range of parameters is excluded. In this plot $\alpha = 1$ and $\lambda_\phi = \eta_{\text{naïve}} = 10^{-3}$.*

4.5.3 Inflation and the standard model of particle physics

As a simple application of the previous section, we can consider to what extent the standard model ought to be included in any reliable supergravity model for cosmological inflation. Our current understanding of nature includes a present-day supersymmetrically broken standard model after an inflationary evolution right after the big bang. As such the combined model is exactly that of a two-sector supergravity theory with an inflationary and a hidden sector whose ground state breaks supersymmetry in which it resides throughout the inflationary era.

Supersymmetry in the standard model sector can either have been broken by gravity mediation of the inflaton sector or by a mechanism in the standard model sector itself. The first situation would be a consistent approach as far as our analysis goes: as $G_q = 0$ the sector decouples from the inflationary dynamics, might be stabilized and the slow-roll parameters are reliably determined from the inflaton sector alone. Nevertheless, from the point of view of our understanding of the standard model it would be unsatisfactory to not know the precise mechanism behind its supersymmetry breaking and (complete) models describing such mechanisms would still have to be analyzed to shed light on the situation.

In the second situation, $G_q \neq 0$, we should apply the results of the previous sections. The field q may be seen as some light scalar degree of freedom in the (supersymmetrically broken) standard model. We assume the standard lore, that supersymmetry is broken in the standard model at a scale of about 1 TeV. In the F-term scalar potential, this scale enters via G_q. To determine the correct numerical value, we relate our dimensionless definition of the Kähler function to the standard dimensionful definition. Dimensionful quantities are denoted with a tilde in the following.[11] We recall from section 4.2.3 that in order to have a non-vanishing vacuum energy, the superpotential in both sectors must have a non-zero constant term $W_0^{(1)} = m_\Lambda^{(1)}/\mathrm{M_{pl}}$, $W_0^{(2)} = m_\Lambda^{(2)}/\mathrm{M_{pl}}$, which accounts for the always present gravitational coupling between the sectors. Hence, the dimensionful constant term in the total superpotential (4.13) has value $\widetilde{W}_0^{\mathrm{tot}} = W_0^{(1)} W_0^{(2)} \mathrm{M_{pl}^3} = m_\Lambda^{(1)} m_\Lambda^{(2)} \mathrm{M_{pl}}$. In contrast, the supergravity quantities $\widetilde{K}^{(2)}$ and $\widetilde{W}_{\mathrm{susy}}^{(2)} = \widetilde{W}_0^{(1)} \widetilde{W}_{\mathrm{dyn}}^{(2)}/\mathrm{M_{pl}^3}$ describing the standard model are naturally of the order of the TeV-scale, $[\widetilde{W}_{\mathrm{susy}}^{(2)}] = \mathrm{TeV}^3$, $[\partial_{\tilde{q}} \widetilde{K}^{(2)}] = \mathrm{TeV}$. We relate the scale of supersymmetry breaking $\widetilde{G_{\tilde{q}}}$ to the superpotential via

$$\widetilde{G}_{\tilde{q}} = \frac{\mathrm{M_{pl}^2}}{\widetilde{W}} \left(\partial_{\tilde{q}} \widetilde{W} + \frac{\partial_{\tilde{q}} \widetilde{K}^{(2)}}{\mathrm{M_{pl}^2}} \widetilde{W} \right),$$

which is naturally of order

$$\left[\widetilde{G}_{\tilde{q}}\right] = \frac{\mathrm{M_{pl}^2}}{m_\Lambda^{(1)} m_\Lambda^{(2)} \mathrm{M_{pl}} + \ldots} \left(\mathrm{TeV}^2 + \frac{\mathrm{TeV}}{\mathrm{M_{pl}^2}} (m_\Lambda^{(1)} m_\Lambda^{(2)} \mathrm{M_{pl}} + \ldots) \right) = \frac{\mathrm{M_{pl}} \mathrm{TeV}^2}{m_\Lambda^{(1)} m_\Lambda^{(2)}} + \mathrm{TeV} + \ldots,$$

where the ... are of subleading order. We expect that $m_\Lambda^{(1)}$, the constant term of the inflaton sector, is of order $[H] = 10^{-5}\mathrm{M_{pl}}$, while $[m_\Lambda^{(2)}] = \mathrm{TeV}$. Hence, translating back to dimensionless units, we find $G_q \sim 10^{-11}$.

Taking the kinetic gauge, i.e. a canonical Kähler metric $G_{\phi\bar{\phi}} = 1$, we can easily find the natural value of α. From (4.25) we see that α depends on ϵ_ϕ and G_ϕ via

$$\alpha \propto \sqrt{\epsilon_\phi} - G_\phi,$$

modulo some unknown but negligible phase factors. G_ϕ is of order $\sqrt{3}$ in order to have a potential $V > 0$. Since ϵ_ϕ is of order $O(10^{-3})$, the value of $|\alpha|$ is of order unity. For a rough estimate of $\eta_{\text{naïve}} \sim 10^{-3}$, we can therefore pinpoint the standard model as indicated in figure 4.4. In both cases, $\eta_{\text{naïve}} > 0$ as well as $\eta_{\text{naïve}} < 0$, the lightest

[11] E.g. in dimensionful units $[\widetilde{G}] = \mathrm{mass}^2$ and $[\widetilde{q}] = \mathrm{mass}$, while our conventions are $[G] = [q] = 0$. To relate G_q to $\widetilde{G_q}$ we can use the expression $\left[G_q\right] = \frac{[\widetilde{G_q}]}{\mathrm{M_{pl}}}$.

supersymmetric particle is too light for the single sector inflationary dynamics to truly describe the full system. Any tuned and working inflationary supergravity model in which the standard model is assumed to not take part in the cosmic evolution, requires implicit assumptions on the standard model that either the inflaton sector is responsible for standard model supersymmetry breaking through gravity mediation or the masses of the standard model scalar multiplets are unnaturally large in terms of the now independent standard model supersymmetry breaking scale.

4.6 Conclusions

We have studied the effect of hidden sectors on the finetuning of F-term inflation in supergravity, identifying a number of issues in the current methodology. Finetuning inflationary models is only valid when the neglected physics does not affect this finetuning, in which case the inflationary physics can be studied independently. As shown in figures 4.1 and 4.2 this assumption holds only under very special circumstances. The reason is that the everpresent gravitational couplings will always lead to a mixing of the hidden sectors with the inflationary sector, even in the case of the most minimally coupled action (4.11). For a hidden sector vacuum that preserves supersymmetry, the sectors decouple consistently [166–169, 182]. However, for a supersymmetry breaking vacuum the inflationary dynamics is generically altered, where the nature and the size of the change depends on the scale of supersymmetry breaking.

For a hidden sector with a low scale of supersymmetry breaking, like the standard model, the cross coupling scales with the scale of supersymmetry breaking, and is therefore typically small. Yet, as shown in section 4.3, the lightest mass of the hidden sector depends as well on the scale of supersymmetry breaking within that sector. This light mode is strongly affected by the inflationary physics and thus evolves during inflation. Therefore, any single field analysis is completely spoiled as discussed in section 4.5.3.

For massive hidden sectors, the problem is more traditional. For a small hidden sector supersymmetry breaking scale, one has a conventional decoupling as long as the lightest mass of the hidden sector is much larger than the inflaton mass. However, for large hidden sector supersymmetry breaking, this intuition fails. Then, the off-diagonal terms in the mass matrix (4.20) will lead to a large correction of the η-parameter.

To conclude, any theory that is working by only tuning the inflaton sector has made severe hidden assumptions about the hidden sector, which typically will not be easily met. Methodologically the only sensible approach is to search for inflation in a full theory, including knowledge of all hidden sectors.

4.A Some supergravity relations

For easy reference to the reader, we use this appendix to state the relevant derivatives of the supergravity potential of a two-sector system coupled via

$$G(\phi^i, \overline{\phi}^{\bar{\imath}}, q^a, \overline{q}^{\overline{a}}) = G^{(1)}(\phi^i, \overline{\phi}^{\bar{\imath}}) + G^{(2)}(q^a, \overline{q}^{\overline{a}}). \tag{4.28}$$

We use middle-alphabet indices $\{i, \bar{\imath}\}$ to denote the fields in the inflationary sector, beginning-alphabet indices $\{a, \overline{a}\}$ to denote the fields in the hidden sector and capital middle-alphabet indices $\{I, \overline{I}\}$ to denote the full system. Derivatives with respect to these fields are denoted by subscripts, e.g. $\partial_i G = G_i$ and $\partial_i \partial_j G = G_{ij}$. The Hessian $G_{I\overline{J}}$ describes the metric of the (product-) manifold parameterized by the fields. This is a Kähler manifold and hence $\nabla_I G_{\overline{J}} = G_{I\overline{J}}$.

The supergravity potential is

$$V = e^G (G_I G^I - 3) = e^G (G_{\overline{I}} G^{\overline{I}} - 3) = e^G (G_a G^a + G_i G^i - 3).$$

Its covariant derivatives are denoted with subscripts (note that this is a different convention than the one used for the Kähler function G), e.g. $\nabla_i V = \partial_i V = V_i$ and $\nabla_i \nabla_j V = V_{ij}$. In terms of derivatives of G, the first derivatives of V are given by

$$V_i = G_i V + e^G \left((\nabla_i G_j) G^j + G_i \right), \tag{4.29a}$$

$$V_{\bar{\imath}} = G_{\bar{\imath}} V + e^G \left((\nabla_{\bar{\imath}} G_{\bar{\jmath}}) G^{\bar{\jmath}} + G_{\bar{\imath}} \right), \tag{4.29b}$$

and similar expressions for V_a and $V_{\overline{a}}$. The Hessian of covariant derivatives is

$$V_{ij} = \nabla_i G_j V + G_i V_j + G_j V_i - G_i G_j V + e^G \left[(\nabla_i \nabla_j G_k) G^k + 2 \nabla_i G_j \right], \tag{4.30a}$$

$$V_{i\bar{\jmath}} = G_{i\bar{\jmath}} V + G_i V_{\bar{\jmath}} + G_{\bar{\jmath}} V_i - G_i G_{\bar{\jmath}} V + e^G \left[R_{i\bar{\jmath}k\bar{l}} G^k G^{\bar{l}} + G^{k\bar{l}} \nabla_i G_k \nabla_{\bar{\jmath}} G_{\bar{l}} + G_{i\bar{\jmath}} \right], \tag{4.30b}$$

$$\begin{aligned} V_{ia} &= \nabla_a G_i V + G_i V_a + G_a V_i - G_i G_a V + e^G \left[(\nabla_a \nabla_i G_I) G^I + \nabla_i G_a + \nabla_a G_i \right] \\ &= G_i V_a + G_a V_i - G_i G_a V, \end{aligned} \tag{4.30c}$$

$$\begin{aligned} V_{i\overline{a}} &= G_{i\overline{a}} V + G_i V_{\overline{a}} + G_{\overline{a}} V_i - G_i G_{\overline{a}} V + e^G \left[R_{I\overline{J}i\overline{a}} G^I G^{\overline{J}} + G^{I\overline{J}} \nabla_i G_I \nabla_{\overline{a}} G_{\overline{J}} + G_{i\overline{a}} \right] \\ &= G_i V_{\overline{a}} + G_{\overline{a}} V_i - G_i G_{\overline{a}} V, \end{aligned} \tag{4.30d}$$

and similar expressions for the other V_{IJ}. The equalities in (4.30c) and (4.30d) result from the specific form of the Kähler function (4.28).

4.B Mass eigenmodes in a stabilized sector

In this appendix we provide some intermediate results in the calculation of (4.18). Using the expressions as stated in appendix 4.A, to first order in $|G_q|$, the second derivatives of the potential are given by

$$V_{qq} = e^G\left[(2+e^{-G}V)\nabla_q G_q + (\nabla_q\nabla_q G_q)G^q\right] + O(|G_q|^2), \tag{4.31a}$$

$$V_{q\bar{q}} = e^G\left[G_{q\bar{q}}(1+e^{-G}V) + G^{q\bar{q}}(\nabla_q G_q)(\nabla_{\bar{q}}G_{\bar{q}})\right] + O(|G_q|^2). \tag{4.31b}$$

Using the supersymmetry breaking restriction (4.16) in (4.31), we find

$$V_{qq} = -e^G G_{q\bar{q}}\left[(2+e^{-G}V)(1+e^{-G}V)\widehat{G^q}^{-2} - G^{q\bar{q}}(\nabla_q\nabla_q G_q)G^q\right] + O(|G_q|^2), \tag{4.32a}$$

$$\begin{aligned} V_{q\bar{q}} &= e^G\left[G_{q\bar{q}}(1+e^{-G}V) + (1+e^{-G}V)^2 G^{q\bar{q}}G_{q\bar{q}}G_{q\bar{q}}\right] + O(|G_q|^2) \\ &= e^G G_{q\bar{q}}(2+e^{-G}V)(1+e^{-G}V) + O(|G_q|^2), \end{aligned} \tag{4.32b}$$

and hence

$$\begin{aligned} |V_{qq}| &= e^G G_{q\bar{q}}(2+e^{-G}V)(1+e^{-G}V)\times \\ &\quad \sqrt{1 - \frac{2G^{q\bar{q}}\mathrm{Re}\{(\nabla_q\nabla_q G_q)G^q\widehat{G^{\bar{q}}}^{-2}\}}{(2+e^{-G}V)(1+e^{-G}V)} + \frac{\left|G^{q\bar{q}}(\nabla_q\nabla_q G_q)G^q\right|^2}{(2+e^{-G}V)^2(1+e^{-G}V)^2} + O(|G_q|^2)} \\ &= e^G G_{q\bar{q}}\left[(2+e^{-G}V)(1+e^{-G}V) - G^{q\bar{q}}\mathrm{Re}\{(\nabla_q\nabla_q G_q)\widehat{G^q}^3\}|G^q|\right] + O(|G_q|^2). \end{aligned} \tag{4.33}$$

Then (4.17) is evaluated to be

$$m_q^- = e^G G^{q\bar{q}}\mathrm{Re}\{(\nabla_q\nabla_q G_q)\widehat{G^q}^3\}|G^q| + O(|G_q|^2), \tag{4.34a}$$

$$m_q^+ = e^G\left[2(2+e^{-G}V)(1+e^{-G}V) - G^{q\bar{q}}\mathrm{Re}\{(\nabla_q\nabla_q G_q)\widehat{G^q}^3\}|G^q|\right] + O(|G_q|^2). \tag{4.34b}$$

5

Worldsheet cosmology

In the previous chapter we have discussed the difficulties one faces when studying inflation in a separated but controlled environment in any supergravity theory. We have seen that there is a substantial worry that other parts of the theory will contribute to inflation in a non-negligible fashion. In this chapter we will capitalize on precisely this, employing the opportunity inflation provides to constrain unknown physics. To incorporate a complete system, we have to go back to the roots of string theory. Therefore, our approach starts from the worldsheet description of string theory, using conformal invariance to investigate the (coarse) constraints that inflation imposes on the theory. The chapter is based on [226].

5.1 Introduction

The last ten years many attempts have been made to understand inflation from a more fundamental level within string theory [1, 197, 227–230]. Cosmological observations strongly suggest an era of inflation in the early universe, and string theory, being a quantum theory of gravity with a unique UV-completion, should be able to describe this. In addition, inflation generically probes energy scales that are unobtainable in accelerator experiments, and there is a chance that string scale effects may be detectable in future cosmological observations [51, 231–236].

One of the essential characteristics of inflation is that it solves the flatness and horizon problem *within classical general relativity* [25–27]. Moreover, inflation is a very coarse phenomenon that only depends on the energy density and pressure in the universe without a need to specify any details of the matter content. In string theory the equations of motion of classical general relativity are the conditions of conformal invariance of the worldsheet string theory. As such, a string theoretic description of inflation should only depend on very generic scaling properties of the conformal field theory on the worldsheet.

Extending worldsheet descriptions of tachyon condensation scenarios [111, 237, 238], we will attempt to describe inflation with a worldsheet theory that is a combination of a spacetime and matter-part, which mix via spacetime dependent couplings $u^a(x)$ for operators O_a of an abstract internal conformal field theory. From the viewpoint of the internal conformal field theory alone such a deformation induces an internal renormalization group flow. Total conformal invariance of the combined theory can only be kept if the background fields adjust themselves in such a way that the running induced by the scaling behavior of the operators O_a of the internal conformal field theory is canceled. The renormalization group flow can therefore be seen to define the possible dependence of $u^a(x)$ on the spacetime coordinates x^μ, or in other words the β functions of the full theory determine the equations of motion for the background fields $u^a(x)$. These equations can be compared to slow-roll inflation to find conditions on the internal conformal field theory. We shall indeed find that, from the worldsheet perspective, the inflationary slow-roll parameters are completely characterized by the central charge and the scaling behavior of the couplings of the conformal field theory, in line with our expectation that inflation is a phenomenon that only depends on generic properties of the matter sector.

This is not to say that we have solved inflation in string theory. Describing strings in a time-dependent background is notoriously difficult. In a large part this is due to our lack of a background independent description of the theory. At low energies we can resort to a supergravity description, but inflation fits awkwardly in the low energy supergravity framework (η-problem, Lyth-bound, absence of de Sitter solutions [239]). As recently emphasized [240], one almost certainly needs stringy ingredients to describe accelerating backgrounds. The worldsheet approach is conceptually different from supergravity calculations, but it has its own drawbacks when trying to describe a string in a de Sitter-like background. At tree-level (in g_s), we are only able to describe small deviations from Minkowski spacetime rather than de Sitter spacetime, as is well known [241–245]. Inflationary solutions are a larger class of accelerating spacetimes than pure de Sitter, so one could optimistically hope for a better fit into string theory. Nevertheless, they are closely related to pure de Sitter and we may already anticipate problems to describe them for the same reason. Substituting the solutions to the β functions into the formal expressions, we indeed find a similar divergence due to the fact that the dilaton cannot be stabilized in tree-level string theory and with a dynamical dilaton inflation does not occur. This is of course the Fischler-Susskind phenomenon [241, 242]. This, however, is not the main point. We wish to show that, inflation being a coarse phenomenon, it only depends on coarse details of the internal conformal field theory. That we do, formally, while at the same time we recover the known Fischler-Susskind result that any tree-level string theory model is ruled out as a theory for inflation.

This chapter is structured as follows: first we describe the worldsheet set-up suitable for inflation and derive the equations of motion. We review multi-field slow-roll inflation in section 5.3, so that in section 5.4 we can state our main result. We shortly discuss the possibility to generalize the results to higher loop order. We conclude discussing the relation between our results with results known from the literature [244, 245].

5.2 Background dynamics for a generic worldsheet theory

5.2.1 Conformal perturbation of a coupled gravity and matter system

We wish to describe a realistic model of inflation in string theory, i.e. there is a $3+1$-dimensional homogeneous and isotropic cosmological spacetime which experiences accelerated expansion. Similar to phenomenological model building, we are naturally led to consider a worldsheet conformal field theory consisting of two parts: a nonlinear σ model accounting for four-dimensional gravity in combination with a matter/internal theory [111, 237]. The nonlinear sigma model is a curved bosonic string in four dimensions, $\mu, \nu \in \{0, 1, 2, 3\}$,

$$S_{\mathrm{NL}\sigma\mathrm{M}} = S_{g(x)} + S_{\Phi(x)}, \tag{5.1a}$$

$$S_{g(x)} = \frac{1}{2\pi\alpha'} \int \mathrm{d}^2 z\, g^{(S)}_{\mu\nu}(x) \partial x^\mu \bar{\partial} x^\nu, \tag{5.1b}$$

$$S_{\Phi(x)} = \frac{1}{4\pi} \int \mathrm{d}^2 z \sqrt{h}\, \Phi(x) R^{(2)}, \tag{5.1c}$$

with $g^{(S)}_{\mu\nu}$ the four-dimensional string frame metric and $h_{\alpha\beta}$ the Euclidean worldsheet metric. To keep the discussion simple we will set the Neveu-Schwarz form to zero, $B_{\mu\nu} = 0$, but we do consider the effect of the dilaton. The dilaton is a (light) scalar and is naturally a part of cosmological dynamics or any time-dependent scenario, e.g. tachyon condensation [111]. More importantly, the dilaton is closely related to the scale factor of the Einstein frame metric and as such could be driving part of the cosmological expansion.

The internal theory will be some two-dimensional conformal field theory S_0 with central charge c and (primary and descendant) operators O_a with scaling dimensions Δ_a. We purposely leave the theory unspecified. The goal of this study is to deduce what type of internal conformal field theory, i.e. which constraints on the central charge and operator dimensions and couplings, could give rise to a realistic model for inflation. Since FLRW cosmological dynamics only cares about coarse characteristics of the matter, viz. pressure and energy, we expect that only coarse information about the internal conformal field theory should be needed to deduce cosmological dynamics. Because time-dependent backgrounds must break supersymmetry, we can incorporate all the fermionic partners to x^μ and the worldsheet diff × Weyl- and supersymmetry ghosts into the internal conformal field theory.[1] The internal conformal field theory will exhibit characteristic scaling behavior under a deformation by nonzero couplings u^a to the primary operators,

$$S = S_0 + S_\Phi + S_u, \tag{5.2a}$$

$$S_\Phi = \frac{1}{4\pi}\int d^2z\,\sqrt{h}\,\Phi R^{(2)}, \tag{5.2b}$$

$$S_u = \int d^2z\, u^a O_a. \tag{5.2c}$$

This behavior is intrinsic to the internal theory and fully captured by the β functions $\overline{\beta}^a(u)$ of the couplings u^a, whose lowest order (classical) contribution is given by $(\Delta_a - 2)u^a$. We have again included the (constant part of the) dilaton Φ here as a (non x-dependent) coupling to the worldsheet curvature $R^{(2)}$ in order to easily incorporate the Weyl anomaly contribution of the internal theory. At a renormalization group fixed point of this perturbed conformal field theory, $\overline{\beta}^\Phi(u)$ will just be proportional to the central charge of the internal conformal field theory, cf. (3.23),

$$\overline{\beta}^\Phi(u) = \frac{c}{6} + O(u).$$

Due to the conformal perturbations of the internal theory, higher order effects in u will result in a "running" of $\overline{\beta}^\Phi$ [110, 246].

To obtain spacetime dynamics driven by the matter sector, we couple the internal theory plus dilaton to the Polyakov nonlinear σ model into a full worldsheet theory

[1] One could keep supersymmetry manifest in principle but it is technically far more involved: with the worldsheet supersymmetric string one needs to track the GSO projection carefully whereas the superspace Green-Schwarz string does not lend itself easily to non-supersymmetric backgrounds. Essentially all these technicalities reside in the internal sector and it is not clear what one would gain by tracking them closely.

with a cross-coupling $u^a(x)\mathcal{O}_a$ between the two sectors,[2]

$$S_{\text{tot}} = S_{g(x)} + S_0 + S_{\Phi(x)} + S_{u(x)}, \tag{5.3a}$$

$$S_{u(x)} = \int \mathrm{d}^2z\, u^a(x)\mathcal{O}_a. \tag{5.3b}$$

The couplings $u^a(x)$ to the internal conformal field theory operators $\mathcal{O}_a$ depend on the spacetime coordinates x^μ. Since a consistent string theory is described by a conformal worldsheet theory, the *full* operators $u^a(x)\mathcal{O}_a$ are assumed to be exactly marginal deformations of the theory. That is, the total theory must remain conformally invariant and the spacetime equations of motion are given by the requirement that the β functions of the full theory vanish [103, 105, 117].

The β functions of the coupling functionals $g_{\mu\nu}(x)$, $u^a(x)$ and $\Phi(x)$ are readily computed using worldsheet techniques and conformal perturbation theory [111]. We give a brief summary in appendix 5.A. Here we simply state the result,

$$0 = \frac{1}{\alpha'}\beta^g_{\mu\nu} = R_{\mu\nu} - M_{ab}(u)\nabla_\mu u^a \nabla_\nu u^b + 2\nabla_\mu\nabla_\nu\Phi, \tag{5.4a}$$

$$0 = \frac{1}{\alpha'}\beta^a = \frac{1}{\alpha'}\overline{\beta}^a(u) - \frac{1}{2}\mathrm{D}\nabla u^a + \nabla^\rho\Phi\nabla_\rho u^a, \tag{5.4b}$$

$$0 = \frac{1}{\alpha'}\beta^\Phi = U(u) - \frac{1}{2}\nabla^2\Phi + (\nabla\Phi)^2\,, \tag{5.4c}$$

where $M_{ab}(u)$ is the *positive definite* Zamolodchikov metric on the space of coupling constants [110, 111],

$$M_{ab}(u) = 4\pi^2\langle\mathcal{O}_a(\epsilon)\mathcal{O}_b(0)\rangle_u.$$

We denote its connection by K^a_{bc} and we have defined a covariant derivative [193]

[2] As in [111] we do not include cross couplings in the dilatonic sector, $\int \mathrm{d}^2z\,\Phi^a(x)\mathcal{O}_aR^{(2)}$, nor do we consider a further dependence of the spacetime metric on the internal degrees of freedom through a "warped geometry" cross coupling $\int \mathrm{d}^2z\, g^a_{\mu\nu}(x)\partial x^\mu\overline{\partial}x^\nu\mathcal{O}_a$. Conformal perturbation theory is only valid when all operators are marginal or nearly marginal, in which case the corresponding couplings describe nearly massless string excitations and the deviation away from the conformal product structure is small. When we assume the operators $\mathcal{O}_a$ to be nearly marginal, i.e. $|\Delta_a - 2| \ll 1$, the couplings Φ^a and $g^a_{\mu\nu}$ are highly irrelevant and describe very massive string excitations. As such they will fall outside the range of validity. In order to describe these more general cross couplings, a different set of operators $\mathcal{A}_\alpha$ with dimension nearly zero would have to be introduced to combine with the $\Phi^\alpha(x)R^{(2)}$- and $g^\alpha_{\mu\nu}(x)\partial x^\mu\overline{\partial}x^\nu$-operators. Considering the ubiquity of warped solutions in string inflation, it would be interesting to extend the computation below to such solutions. One should bear in mind however that almost all warped solutions other than a non-trivial dilaton involve contributions from different worldsheet topologies [102], cf. the discussion in section 5.4.2.

$D\nabla u^a$ and scalar function $U(u)$ respectively by[3]

$$D\nabla u^a = \nabla^\rho \nabla_\rho u^a + K^a_{bc} \nabla^\rho u^b \nabla_\rho u^c, \tag{5.5a}$$

$$U(u) = \frac{c_x}{6\alpha'} + \frac{1}{\alpha'}\overline{\beta}^\Phi(u). \tag{5.5b}$$

The scalar function $U(u)$ accounts for the different quantum Weyl anomalous effects. There are contributions from the central charges of the two components of the theory, $c_x = 4$ and $c \equiv 6\overline{\beta}^\Phi(0)$, and in addition there are higher order effects in u, which are collected in the non-constant parts of $\overline{\beta}^\Phi(u)$.

The actual computation of the β functions combines two methods with distinct perturbative expansions: conformal perturbation theory where u^a and $\Delta_a - 2$ are small and $\overline{\beta}^a(u) = (\Delta_a - 2)u^a + \ldots$ is known exactly, and separately the background field method where u^a can be large but $\overline{\beta}^a(u)$ and ∇u^a are required to be small. By allowing for arbitrary $\overline{\beta}^a(u)$ and $\overline{\beta}^\Phi(u)$ these methods can be combined in a mixed α'-expansion: it can be made "exact" to all orders in u^a, but only to second order in ∇u^a by capturing all u-dependence in the arbitrary unknown functions $M_{ab}(u)$, $\overline{\beta}^a(u)$ and $\overline{\beta}^\Phi(u)$. Note that $\beta^g_{\mu\nu}(u)$ only depends on ∇u^a as the two sectors of the total theory decouple when u^a is x-independent. Limiting ourselves to two derivatives is not an impediment, since inflation should be captured by a two derivative description, especially slow-roll inflation.

5.2.2 String dynamics from an action

The condition for Weyl invariance $\beta^g_{\mu\nu} = \beta^a = \beta^\Phi = 0$ determines the equations of motion for the background fields $\Phi(x)$, $g_{\mu\nu}(x)$ and $u^a(x)$. A crucial ingredient for the consistency of this interpretation is the coupling between the dilaton field $\Phi(x)$ and the other matter fields $u^a(x)$. The potential terms, $\overline{\beta}^a(u)$ and $\overline{\beta}^\Phi(u)$ in (5.4), are not independent but related via

$$M_{ab}(u)\overline{\beta}^b(u) = \partial_a \overline{\beta}^\Phi(u), \tag{5.6}$$

to all orders in u^a. This result may be derived from the fact that the conformal anomaly β^Φ is a c-number rather than an operator by the Wess-Zumino consistency

[3]For later convenience we have rescaled the metric by a factor of $4\pi^2$ compared to more conventional definitions. Furthermore, as can be read in the appendix, the M_{ab} and U used in the main text differ from the corresponding objects in conformal perturbation theory by u^a-corrections that are beyond the order of perturbation of interest to us.

condition [105, 108, 111, 117, 120] and cf. section 3.2.1. In particular β^Φ is x-independent and hence $\nabla_\mu \beta^\Phi$ vanishes. Since $\beta^g_{\mu\nu} = \beta^a = 0$, we can verify

$$0 = \nabla_\nu \beta^\Phi = \nabla_\nu \left(\beta^\Phi - \frac{1}{4} \beta^{g\mu}{}_\mu \right) = \left(\partial_a \overline{\beta}^\Phi - M_{ab} \overline{\beta}^b \right) \nabla_\nu u^a .$$

The last step follows from the explicit formulae for the β functions (5.4). Recall that the β functions are derived up to second order in ∇u^a but are *exact* in powers of zeroth derivatives of u due to the incorporation of all zeroth derivatives of u in the potential functions $\overline{\beta}^a(u)$ and $\overline{\beta}^\Phi(u)$. Whereas our result is only an effective description for the connection between the spacetime and matter sector, the matter sector itself is described exactly.

As a result of the relation (5.6) between $\overline{\beta}^\Phi(u)$ and $\overline{\beta}^a(u)$ the equations of motion can be integrated to an action

$$S_{\text{SF}} = \frac{1}{2\kappa_0^2} \int \mathrm{d}^4x \sqrt{g} e^{-2\Phi} \left[R + 4(\nabla\Phi)^2 - M_{ab} \nabla_\mu u^a \nabla^\mu u^b - 4U(u) \right]. \tag{5.7}$$

Transforming to the Einstein frame $\tilde{g}^{(E)}_{\mu\nu} = e^{\Phi_0 - \Phi} g^{(S)}_{\mu\nu} = e^{-\tilde{\Phi}} g^{(S)}_{\mu\nu}$, we obtain an action that can be directly compared to standard cosmological models,

$$S_{\text{EF}} = \frac{1}{2\kappa^2} \int \mathrm{d}^4x \sqrt{\tilde{g}} \left[\tilde{R} - 2\tilde{\nabla}_\mu \tilde{\Phi} \tilde{\nabla}^\mu \tilde{\Phi} - M_{ab} \tilde{\nabla}_\mu u^a \tilde{\nabla}^\mu u^b - 4e^{2\tilde{\Phi}} U(u) \right]. \tag{5.8}$$

Again, $\kappa = \kappa_0 e^{\Phi_0} = \sqrt{8\pi G_N}$ is the gravitational coupling. The action (5.8) is simply that of a multi-scalar field model coupled to gravity,

$$S_{\text{inflation}} = \frac{1}{\kappa^2} \int \mathrm{d}^4x \sqrt{g} \left[\frac{1}{2} R - \frac{1}{2} G_{ij} \partial^\mu \phi^i \partial_\mu \phi^j - V(\phi) \right], \tag{5.9}$$

with the potential

$$V(\phi) = 2e^{-2\Phi_0} e^{2\Phi} U(u), \tag{5.10}$$

where we have defined a multi-scalar field $\phi^i = (\Phi, u^a)^t$ and a metric on the space of fields $G_{ij} = \left(\begin{smallmatrix} 2 & 0 \\ 0 & M_{ab} \end{smallmatrix}\right)$. Since we will be working in the Einstein frame from here on, we have dropped the tilde on the spacetime metric $g_{\mu\nu}(x)$. The question we wish to investigate is whether the potential (5.10) is flat enough to provide realistic slow-roll inflation. Since $V(\phi)$ is proportional to the β function $\overline{\beta}^\Phi(u)$ of the internal sector and the central charge c_{tot} of the total theory, demanding slow-roll inflation is equivalent to a set of phenomenological constraints on the internal conformal field theory. Before we turn to this question, we quickly review slow-roll inflation in multi-field models.

5.3 Multi-field slow-roll inflation

The rapid acceleration of the universe that characterizes inflation arises when the system is potential energy dominated. Current observations favor an adiabatic slow-roll inflationary model of early universe cosmology, whose phenomenology can be described by gravity coupled to a single scalar field. The single field inflationary case was formalized in [34] and shortly explained in section 2.2. Fundamentally there is no reason to have only one scalar field. Indeed in string theory or supergravity one generically has multiple scalar fields, although its characteristic signature, isocurvature fluctuations, is at most 10% of the primordial power spectrum and is at this time not a better fit to the data [10]. The connection to the power spectrum for multi-field slow-roll inflation [26, 27, 247, 248] was formalized in [193, 212, 213]. We shall follow [193].

Minimally coupled multi-field inflation is described by the action (5.9), where $V(\phi)$ is the scalar potential and G_{ij} is the *positive definite* metric on the space of scalar fields. For a flat, homogeneous and isotropic FLRW universe, the independent equations of motion for the generic multi-field action (5.9) are[4]

$$H^2 = \frac{1}{3}\left(\frac{1}{2}G_{ij}\dot{\phi}^i\dot{\phi}^j + V\right), \tag{5.11a}$$

$$0 = \mathrm{D}\dot{\phi}^i + 3H\dot{\phi}^i + g^{ij}\partial_j V, \tag{5.11b}$$

where Γ^i_{jk} are the connection coefficients for the metric G_{ij} and where we define

$$\mathrm{D}\dot{\phi}^i = \ddot{\phi}^i + \Gamma^i_{jk}\dot{\phi}^j\dot{\phi}^k,$$

similar to (5.5a). The field equations (5.11) completely determine the dynamics of the model, but are difficult to solve exactly. Therefore, we again consider the slow-roll approximation using the slow-roll parameters [193],

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta^i = \frac{\mathrm{D}\dot{\phi}^i}{H|\dot{\phi}|}. \tag{5.12}$$

The vector $\boldsymbol{\eta}$ can be decomposed in components parallel $\eta^{\parallel}$ and perpendicular $\eta^{\perp}$ to the field velocity $\dot{\boldsymbol{\phi}}$. Define

$$\boldsymbol{e}_1^i = \frac{\dot{\phi}^i}{|\dot{\phi}|}, \qquad \boldsymbol{e}_2^i = \frac{\mathrm{D}\dot{\phi}^i - \frac{\mathrm{D}\dot{\phi}\cdot\dot{\phi}}{|\dot{\phi}|^2}\dot{\phi}^i}{\left|\mathrm{D}\dot{\phi} - \frac{\mathrm{D}\dot{\phi}\cdot\dot{\phi}}{|\dot{\phi}|^2}\dot{\phi}\right|},$$

[4]There is another equation of motion, $\dot{H} = -|\dot{\phi}|^2/2$, from the spatial part of the variation with respect to the metric, but this also follows from (5.11).

then

$$\eta^{\parallel} = \boldsymbol{e}_1 \cdot \boldsymbol{\eta} = \frac{\mathrm{D}\dot{\phi}\cdot\dot{\phi}}{H|\dot{\phi}|^2}, \qquad \eta^{\perp} = \boldsymbol{e}_2 \cdot \boldsymbol{\eta} = \frac{\left|\mathrm{D}\dot{\phi} - \frac{\mathrm{D}\dot{\phi}\cdot\dot{\phi}}{|\dot{\phi}|^2}\dot{\phi}\right|}{H|\dot{\phi}|}, \tag{5.13}$$

and

$$\eta^i = \eta^{\parallel}\boldsymbol{e}_1^i + \eta^{\perp}\boldsymbol{e}_2^i.$$

Recall that the parameter ϵ is a direct measure for inflation [34],

$$\ddot{a} > 0 \qquad \Leftrightarrow \qquad \epsilon < 1.$$

ϵ and η together quantify the relative energy contributions of kinetic and potential energy. One can reexpress (5.11) in terms of the slow-roll parameters,

$$H^2 = \frac{V}{3}(1 - \frac{1}{3}\epsilon)^{-1}, \tag{5.14a}$$

$$\dot{\phi}^i + \frac{1}{\sqrt{3V}} g^{ij}\partial_j V = -\frac{1}{3}\sqrt{\frac{2}{3}}\frac{\sqrt{\epsilon V}}{1-\frac{1}{3}\epsilon}\left(\eta^i + \frac{\epsilon\frac{\dot{\phi}^i}{|\dot{\phi}|}}{1+\sqrt{1-\frac{1}{3}\epsilon}}\right). \tag{5.14b}$$

As it is given here, equation (5.14) is exact. It shows precisely which approximation is made by assuming that "potential energy strictly dominates over kinetic energy", which is often the explanation behind slow-roll inflation. Using (5.14) one could obtain results at any order in slow-roll [34, 193]. Limiting ourselves to first order in the approximation, in which ϵ, $\sqrt{\epsilon}\eta^{\parallel}$, $\sqrt{\epsilon}\eta^{\perp} \ll 1$, equation (5.14) reduces to

$$H^2 = \frac{1}{3}V,$$
$$\dot{\phi}^i = -\frac{1}{\sqrt{3V}} g^{ij}\partial_j V.$$

The second equation tells us that slow-roll approximation implies *gradient flow*. Using these equations we see that in the slow-roll approximation

$$\dot{H} = \frac{1}{6\sqrt{\frac{V}{3}}}\partial_i V\dot{\phi}^i = -\frac{\sqrt{3}}{6\sqrt{V}}\frac{1}{\sqrt{3V}} g^{ij}\partial_i V\partial_j V = -\frac{1}{6V}|\nabla V|^2, \tag{5.15a}$$

$$\mathrm{D}\dot{\phi}^i = \partial_t\left(-\frac{1}{\sqrt{3V}} g^{ij}\partial_j V\right) + \Gamma^i_{jk}\frac{1}{3V} g^{jl} g^{km}\partial_l V\partial_m V = \frac{1}{6}\nabla^i\frac{|\nabla V|^2}{V}, \tag{5.15b}$$

and hence in the slow-roll regime,

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{2}\frac{|\nabla V|^2}{V^2}, \tag{5.16a}$$

$$\eta^i = \frac{\mathrm{D}\dot{\phi}^i}{H|\dot{\phi}|} = \frac{1}{2\,|\nabla V|}\nabla^i \frac{|\nabla V|^2}{V}, \tag{5.16b}$$

$$\eta^{\|} = \frac{\mathrm{D}\dot{\phi}\cdot\dot{\phi}}{H|\dot{\phi}|^2} = \frac{-1}{2|\nabla V|^2}\nabla V\cdot\nabla\frac{|\nabla V|^2}{V} = \epsilon - \frac{\nabla^i V\nabla^j V\nabla_i\nabla_j V}{V|\nabla V|^2}, \tag{5.16c}$$

$$\eta^{\perp} = \frac{\left|\mathrm{D}\dot{\phi} - \frac{\mathrm{D}\dot{\phi}\cdot\dot{\phi}}{|\dot{\phi}|^2}\dot{\phi}\right|}{H|\dot{\phi}|} = \frac{1}{2|\nabla V|}\sqrt{\left|\nabla\frac{|\nabla V|^2}{V}\right|^2 - \frac{\left(\nabla V\cdot\nabla\frac{|\nabla V|^2}{V}\right)^2}{|\nabla V|^2}}$$

$$= \sqrt{\frac{1}{4|\nabla V|^2}\left|\nabla\frac{|\nabla V|^2}{V}\right|^2 - (\eta^{\|})^2}. \tag{5.16d}$$

5.4 Inflation from the worldsheet

5.4.1 Slow-roll parameters for tree-level worldsheet string theory

We are now in a position to address our question: how do we describe slow-roll inflation in terms of worldsheet dynamics? That is, we need to verify that the potential $V(\Phi,u) = 2\left(\frac{\kappa_0}{\kappa}\right)^2 e^{2\Phi}U(u)$ is capable of driving a slowly rolling inflaton field. We shall assume the spacetime part of the worldsheet theory to describe an accelerating (i.e. inflationary) flat, homogeneous and isotropic FLRW universe, $g^{(E)} = \mathrm{diag}(-1, a^2(t), a^2(t), a^2(t))$, which is driven by a homogeneous dilaton $\Phi(t,\boldsymbol{x}) = \Phi(t)$ and homogeneous internal fields $u(t,\boldsymbol{x}) = u(t)$. The demand that the slow-roll parameters are small then provides restrictions on $V(\Phi,u)$ and hence, as conjectured, on the coarse characteristics of the internal conformal field theory, $c, \overline{\beta}^{\Phi}(u)$ and $\overline{\beta}^{a}(u)$. Direct calculation of (5.16) for $V(\phi) = 2\left(\frac{\kappa_0}{\kappa}\right)^2 e^{2\Phi}U(u)$ reveals

$$\epsilon = 1 + \frac{1}{2}\gamma^2, \tag{5.17a}$$

$$\eta^{\|} = -\epsilon - \frac{D}{2+\gamma^2}, \tag{5.17b}$$

$$\eta^{\perp} = \sqrt{\frac{1}{4}(2+\gamma^2)^2 + D + \frac{\frac{\gamma^b\gamma^c\nabla_a\overline{\beta}_b\nabla^a\overline{\beta}_c}{\alpha'^2U^2} - 2\gamma^2 D - 2\gamma^6 + \gamma^4}{2+\gamma^2} - (\eta^{\|})^2}, \tag{5.17c}$$

where we have defined the combinations,

$$\gamma_a(u) = \frac{M_{ab}\overline{\beta}^b}{\alpha' U} = \partial_a \log U = \partial_a \log\left[\frac{c_x}{6\alpha'} + \frac{1}{\alpha'}\overline{\beta}^\Phi\right], \tag{5.18a}$$

$$D(u) = \frac{\gamma^a\gamma^b\nabla_a\nabla_b U}{U} - \gamma^4. \tag{5.18b}$$

From (5.17a) we immediately see that $V(\Phi, u) = 2\left(\frac{\kappa_0}{\kappa}\right)^2 e^{2\Phi}U(u)$ is incapable of driving inflation: ϵ is always larger than unity. Regardless of the specific form of $\gamma_a = \partial_a \log\left[\frac{c_x}{6\alpha'} + \frac{1}{\alpha'}\overline{\beta}^\Phi\right]$, the positive definiteness of the Zamolodchikov metric M_{ab} ensures that $\gamma^2 \geq 0$.

Tracing back we see that the coefficient 1 in ϵ, characteristic of an exponential potential, is due to the dynamics of the dilaton. One could wonder whether taking Φ constant, i.e. excluding it from the cosmological dynamics, would modify the model into one which does allow for inflation. Because the field space metric G_{ij} is block diagonal, equation (5.11) implies that for a constant Φ, Φ must be stabilized at $\partial_\Phi V = 4\left(\frac{\kappa_0}{\kappa}\right)^2 e^{2\Phi}U = 0$. However, excluding $\Phi = -\infty$, the relation (5.6) precludes a constant dilaton, as U is not allowed to vanish. In our set-up, fields $u^a(x)$ that undergo a time evolution in four-dimensional spacetime are described by a renormalization group flow of the couplings, i.e. $\overline{\beta}^a \neq 0$. Equation (5.6) then implies that U cannot vanish, which forces the dilaton to be non-constant by the requirement (5.4c) of a vanishing β^Φ. Turning the argument around, suppose one magically stabilizes the dilaton at tree-level. Then $\epsilon = U^{-2}\overline{\beta}^a\overline{\beta}_a$ but $U \sim \partial_\Phi V$ which must vanish by the assumption that the dilaton is stabilized.

Within tree-level worldsheet string theory, the dilaton is therefore always part of the cosmological dynamics and its tree-level exponential potential rules out an inflationary universe.

5.4.2 Inflation from the Ramond sector, string loop corrections and inflation from open strings

Clearly to describe inflation in string theory we must have a more complicated potential for the dilaton. One guess could be to supersymmetrize the worldsheet and include RR fields, i.e. the background fields corresponding to string states with fermionic boundary conditions. Technically this is a far from trivial task, as it is not yet known how to compute β functions for RR vertex operators. However at the end of the day, even including fermionic dynamics, the resulting worldsheet theory must be of the form (5.3). On the worldsheet, the dilaton/vertex operator interactions are such that they always lead to an effective action $S = \int e^{-2\Phi}\mathcal{L}$ in the string frame [86]. Thus one always deduces equation (5.7) and the remainder of the analysis is the same.

Let us be more specific in light of the known examples of string-inspired supergravity inflation built on RR- and NS-flux compactifications [199, 249, 250]. In all global compactifications one needs O-planes to ensure tadpole cancelation. O-planes correspond to non-oriented worldsheets, which occur at higher order in the string loop expansion and are therefore not considered here. Secondly, a persistent issue in all these constructions is the stabilization of the volume modulus of the compact space. In essence this is the same absence of a potential as we exhibit for the dilaton. In current models the stabilization is thought to happen through non-perturbative D-brane effects [198, 251]. D-branes, i.e. open strings, are similarly higher order in the loop expansion.

Thus one is naturally led to consider string loop corrections or non-perturbative effects, i.e. open strings. From the worldsheet point of view these two additions roughly boil down to the same thing. Both are obtained by including more general worldsheet topologies than just the spherical worldsheet of tree-level string theory. The corrections from including closed string loops could convert ϵ into a more sensible expression. We can expect this based on the well-known dilaton tadpoles of Fischler-Susskind [241, 242]. Our results are an extension of the Fischler-Susskind result that to obtain a worldsheet description of strings in a de Sitter space, there must be a one loop (in g_s) contribution to the dilaton to have vanishing β functions, i.e. to satisfy the equations of motion. Slow-roll inflation is in essence an adiabatic continuation of de Sitter space to a slowly varying vacuum energy.

It is interesting to see what happens if we suppose that the higher loop contributions allow us to consistently stabilize the dilaton at weak coupling independent of the value of u^a. Then one finds the slow-roll parameters

$$\epsilon = \frac{\overline{\beta}^a\overline{\beta}_a}{2(\overline{\beta}^\Phi + \frac{c_x}{6})^2}, \tag{5.19a}$$

$$\eta^{\|} = \epsilon - \frac{\overline{\beta}^a\overline{\beta}^b\nabla_a\overline{\beta}_b}{(\overline{\beta}^\Phi + \frac{c_x}{6})\overline{\beta}^c\overline{\beta}_c}, \tag{5.19b}$$

$$\eta^{\perp} = \sqrt{\frac{1}{4\overline{\beta}^c\overline{\beta}_c}\left|\nabla_a\frac{\overline{\beta}^b\overline{\beta}_b}{(\overline{\beta}^\Phi + \frac{c_x}{6})}\right|^2 - (\eta^{\|})^2}. \tag{5.19c}$$

The dilaton stabilization needs to be such that $\alpha' U = \overline{\beta}^\Phi + \frac{c_x}{6}$ is no longer proportional to $\partial_\Phi V$ and hence the above expressions make sense. Of course dilaton stabilization at weak coupling has its own problems [244, 245].

The inclusion of open strings, in addition to the closed strings considered here, may yield more promising results for describing worldsheet theories on inflationary backgrounds. In the supergravity literature the usefulness/necessity of open string corrections has already been recognized [1, 197, 198, 227–230, 251].[5] Open strings have been extensively investigated from a low energy effective field theory point of view, e.g. DBI inflation, and all known viable supergravity inflationary models have an open string component.

5.5 Conclusions

Inflation does not care about anything but very coarse features of the matter sector, only its pressure and energy. This suggests that in string theory inflation is determined by coarse features of the internal conformal field theory on the worldsheet. Qualitatively this is what we find. At the same time our result shows that it is not possible to have an inflationary cosmology described by a tree-level string worldsheet. The exponential potential for the dilaton ensures that ϵ is strictly larger than unity, completely independent of the internal conformal field theory. At first sight this conclusion may be puzzling, as inflation is a classical phenomenon and one therefore may expect tree-level string theory to be sufficient for a consistent description. Nevertheless the result simply recovers that de Sitter backgrounds arise only at one-loop level in worldsheet string theory through the Fischler-Susskind mechanism [241–243]. For inflation to occur, the dilaton must be stabilized through such higher loop effects. If this stabilization happens at weak coupling, then inflation is possible with slow-roll parameters that only depend on the β functions of the internal conformal field theory.

In a way Fischler-Susskind and the result here are special cases of Dine-Seiberg runaway [244, 245]: within string theory one cannot probe a nearby vacuum from the original vacuum because in string perturbation theory, as currently understood, all higher order corrections are larger than the first order — string theory is either free or strongly coupled. Whereas the result in [244, 245] is obtained by general reasoning, Fischler-Susskind specifically attempt to describe a de Sitter cosmology from a Minkowski worldsheet, and we attempt to obtain inflation. We can be even more explicit: in our tree-level analysis the time-dependent process of inflation requires on the one hand a non-constant dilaton to satisfy the equations of motion, while on the

[5] In supergravity language open strings add D-terms in addition to F-term potentials. The closed string worldsheet only captures a dilaton type F-term inflation.

other hand only a constant dilaton makes sense observationally. In the tree-level limit we therefore have found a clear inconsistency of the approach. A strong coupling analysis is necessary to realize inflation within string theory. The reader should be aware that we have *not* ruled out a non-constant dilaton scenario at all, we simply have found out that a zeroth order weak coupling approach is insufficient to describe inflation. In the strong coupling regime the dilaton may turn out to be non-constant after all.

It is interesting to note that our result confirms a conjecture in [245], that a cosmological solution in which the world is slowly sliding to its free Minkowski vacuum cannot be studied from this final state. From the reasoning in [244, 245] this appears to be a perfectly fine solution, if unlikely. Our result confirms their expectation that such a slow-roll inflationary scenario is not possible within tree-level worldsheet string theory.

To conclude: we have provided a proof of principle that the coarse characteristics of the internal conformal field theory determine whether and how inflation occurs, by expressing the slow-roll parameters in terms of the β functions of the internal conformal field theory. As de Sitter-like solutions only arise at one-loop in a Minkowski string worldsheet, a necessary requirement for real and realistic worldsheet models of string inflation is to include higher order string loop corrections to the analysis. This remains subject to further investigation.

5.A Calculating the β functions

In this appendix we will review the calculation of the β functions (5.4) of the total theory (5.3). For more details concerning this calculation we refer to [111].

5.A.1 Conformal perturbation theory

For a general conformal field theory that is perturbed by adding operators to the action,

$$S = S_0 + \int \mathrm{d}^2 z\, u^I O_I,$$

the β functions β^I for the couplings u^I can be defined as the coefficients of the trace of the stress-energy tensor

$$\Theta = -\pi \beta^I O_I, \tag{5.20}$$

where the factor of π is convenient within a string theory context. In the Zamolodchikov renormalization group scheme these can be computed in an expansion in u^I

with $\Delta_I - 2$ small [86, 111, 252],

$$\beta^I = (\Delta_I - 2)u^I + 2\pi C^I_{JK} u^J u^K + O(u^3), \tag{5.21}$$

where C^I_{JK} are the OPE coefficients defined via

$$O_J(y)O_K(z) = \sum_I C^I_{JK} |y-z|^{\Delta_I - \Delta_J - \Delta_K} O_I\left(\frac{y+z}{2}\right).$$

In the coupled system $CFT_x \otimes CFT_O$ that is deformed by the term $S_{u(x)} = \int u^a(x) O_a$ as described in the main text, the operators in (5.20) are the three (types of) operators,

$$O_g^{\mu\nu} = \frac{1}{2\pi\alpha'}\partial x^\mu \bar{\partial} x^\nu, \qquad O_a, \qquad O_\Phi = \frac{1}{8\pi}R^{(2)},$$

which couple to the coupling functionals $g_{\mu\nu}(x)$, $u^a(x)$ and $\Phi(x)$ respectively. By a Fourier transform these coupling functionals may be seen as an infinite set of coupling constants $g_{\mu\nu}(p)$, $u^a(k)$ and $\Phi(q)$ that couple to the dressed operators $O_p^{\mu\nu} = \frac{1}{2\pi\alpha'}\partial x^\mu \bar{\partial} x^\nu e^{ip\cdot x}\mathbf{1}$, $O_{(k,a)} = O_a e^{ik\cdot x}$ and $O_q^\Phi = \frac{1}{8\pi}R^{(2)}e^{iq\cdot x}$ with dimensions

$$\Delta_p^g = 2 + \frac{\alpha'}{2}p^2, \qquad \Delta_{(k,a)} = \Delta_a + \frac{\alpha'}{2}k^2, \qquad \Delta_q^\Phi = 2 + \frac{\alpha'}{2}q^2. \tag{5.22}$$

We are not constraining the graviton momentum or dilaton momentum to be lightlike. $p^2 = 0$ and $q^2 = 0$ would be the on-shell condition for a *free* graviton and *free* dilaton, whereas we wish to consider the coupled gravity-matter system. The OPE coefficients can be readily computed to be

$$C^{(p,\mathbf{1})}_{(k_1,a)(k_2,b)} = -\frac{\alpha'}{8\pi}(k_1-k_2)_\mu(k_1-k_2)_\nu \delta(p-k_1-k_2) M_{ab}, \tag{5.23a}$$

$$C^{(k_1,a)}_{(k_2,b)(k_3,c)} = \delta(k_1-k_2-k_3) C^a_{bc}, \tag{5.23b}$$

where C^a_{bc} are the OPE coefficients of the internal conformal field theory and we have denoted the Zamolodchikov metric by $M_{ab} = 4\pi^2 C^{\mathbf{1}}_{ab}$. Applying (5.22) and (5.23) to (5.21) and Fourier-transforming back to position-space, yields

$$\frac{1}{\alpha'}\beta^g_{\mu\nu} = -\frac{1}{2}\partial^\rho\partial_\rho g_{\mu\nu} + \frac{1}{2}M_{ab}\left(u^a\partial_\mu\partial_\nu u^b - \partial_\mu u^a \partial_\nu u^b\right), \tag{5.24a}$$

$$\begin{aligned}\frac{1}{\alpha'}\beta^a &= \frac{1}{\alpha'}\left((\Delta_a - 2)u^a + 2\pi C^a_{bc}u^b u^c\right) - \frac{1}{2}\partial^\rho\partial_\rho u^a \\ &= \frac{1}{\alpha'}\bar{\beta}^a(u) - \frac{1}{2}\partial^\rho\partial_\rho u^a,\end{aligned} \tag{5.24b}$$

$$\frac{1}{\alpha'}\beta^\Phi = -\frac{1}{2}\partial^\rho\partial_\rho \Phi, \tag{5.24c}$$

where we use (5.21) in reverse to express β^a in terms of $\bar{\beta}^a$.

5.A.2 Weyl anomaly and classical dilatonic contribution

In addition to the operator effects from (5.24c), the β function for Φ receives a further contribution from the well-known Weyl anomaly, a worldsheet contribution proportional to the worldsheet curvature. Its contribution is determined by the central charge of the spacetime nonlinear σ model as well as by that from the (perturbed) internal theory as explained in the main text,

$$\Theta_{1-\text{loop}} = -\frac{c_x}{48}R^{(2)} - \frac{1}{8}\overline{\beta}^{\Phi}R^{(2)} = -\pi\left(\frac{c_x}{6} + \overline{\beta}^{\Phi}(u)\right)\frac{1}{8\pi}R^{(2)}. \tag{5.25}$$

Comparing this expression with the definition of the β functions as coefficients in the stress-energy tensor (5.20), we find a contribution $\beta^{\Phi}_{1-\text{loop}} = \frac{c_x}{6} + \overline{\beta}^{\Phi}(u) = \alpha' U(u)$ to the β function of the dilaton.

The final contribution to all of the β functions comes from the dilaton term (5.1c) in the worldsheet action, which breaks Weyl invariance already at the classical level. Due to an additional overall α'-factor compared to the other terms in the worldsheet, this classical contribution to the β functions is of the same order as loop effects from the classically Weyl invariant terms. On a curved worldsheet the easiest way to determine deviation from Weyl invariance is by calculating the trace of the stress-energy tensor via

$$\Theta = \frac{-\pi}{\sqrt{h}}\frac{\delta S}{\delta h^{\alpha\beta}}h^{\alpha\beta}.$$

This definition for Θ in terms of a variation of the worldsheet metric differs by a factor from more common definitions, which is necessary to relate the result properly with our earlier definition (5.20). One can check that this leads to the right result by looking at the metric and dilaton field, whose contributions are well-known [86, 117]. Making use of the equations of motion for x^μ,

$$\partial\bar{\partial}x^\rho = -\Gamma^\rho_{\mu\nu}\partial x^\mu\bar{\partial}x^\nu + \pi\alpha'\partial^\rho u^a O_a + \frac{\alpha'}{8}\partial^\rho\Phi R^{(2)},$$

the classical violation of Weyl invariance by the dilaton term (5.1c) is

$$\begin{aligned}\Theta_{\text{classical}} &= \left.\frac{-\pi}{\sqrt{h}}\frac{\delta S_{\Phi(x)}}{\delta h^{\alpha\beta}}h^{\alpha\beta}\right|_{h_{z\bar{z}}=1/2} = -\partial\bar{\partial}\Phi(x) = -\left(\partial_\mu\partial_\nu\Phi\partial x^\mu\bar{\partial}x^\nu + \partial_\rho\Phi\partial\bar{\partial}x^\rho\right)\\ &= -\pi\left(2\alpha'\nabla_\mu\nabla_\nu\Phi O_g^{\mu\nu} + \alpha'\nabla^\rho\Phi\nabla_\rho u^a O_a + \alpha'(\nabla\Phi)^2 O_\Phi\right).\end{aligned} \tag{5.26}$$

Again comparing with (5.26), we find contributions

$$\beta^{g}_{\text{classical}} = 2\alpha'\nabla_\mu\nabla_\nu\Phi, \tag{5.27a}$$
$$\beta^{a}_{\text{classical}} = \alpha'\partial^\rho\Phi\partial_\rho u^a, \tag{5.27b}$$
$$\beta^{\Phi}_{\text{classical}} = \alpha'\partial^\rho\Phi\partial_\rho\Phi. \tag{5.27c}$$

Therefore the full β functions read

$$\frac{1}{\alpha'}\beta^{g}_{\mu\nu} = -\frac{1}{2}\partial^\rho\partial_\rho g_{\mu\nu} + \frac{1}{2}M_{ab}\left(u^a\partial_\mu\partial_\nu u^b - \partial_\mu u^a\partial_\nu u^b\right) + 2\nabla_\mu\nabla_\nu\Phi, \tag{5.28a}$$
$$\frac{1}{\alpha'}\beta^{a} = \frac{1}{\alpha'}\overline{\beta}^{a}(u) - \frac{1}{2}\partial^\rho\partial_\rho u^a + \partial^\rho\Phi\partial_\rho u^a, \tag{5.28b}$$
$$\frac{1}{\alpha'}\beta^{\Phi} = U(u) - \frac{1}{2}\partial^\rho\partial_\rho\Phi + \partial^\rho\Phi\partial_\rho\Phi. \tag{5.28c}$$

5.A.3 Covariantization

The β functions (5.28) are (partially) non-covariant. For example, β^a is not covariant on the space of couplings $u^a(x)$. The right expression for β^a should be

$$\frac{1}{\alpha'}\beta^{a} = \frac{1}{\alpha'}\overline{\beta}^{a}(u) - \frac{1}{2}\partial^2 u^a - \frac{1}{2}K^a_{bc}(u)\partial^\rho u^b\partial_\rho u^c + \partial^\rho\Phi\partial_\rho u^a, \tag{5.29}$$

where K^a_{bc} is the connection coefficient associated to the Zamolodchikov metric M_{ab} [252]. In a general renormalization scheme it arises from contact terms in the OPE. It has not appeared explicitly in the Zamolodchikov scheme because in that scheme K^a_{bc} is already of first order in u [111], as a result of which $K^a_{bc}(u)\partial^\rho u^b(x)\partial_\rho u^c(x)$ is beyond leading order in the calculation of the β functions. In the Zamolodchikov scheme (5.29) is correct to leading order and by general covariance it holds in any renormalization scheme.

Furthermore, the terms obtained using conformal perturbation methods are not spacetime covariant at first. This is inherent to the conformal perturbation method, which uses correlation functions defined with respect to *flat* spacetime. Conformal perturbation is an expansion in $\delta g_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ which is only sensitive to the transverse traceless part of the graviton. The longitudinal and trace part of the graviton are not encoded in (nearly) marginal operators and thus fall outside conformal perturbation theory. If one corrects for this by evaluating the Weyl transformation of all terms of the coherent state of gravitons $g_{\mu\nu}(x)$, the expressions will become covariant. Covariantization is necessary because the true β functions are gravitationally only consistent when all orders and all polarizations in $\delta g_{\mu\nu}$ are taken into account.

Using background field methods one can obtain these spacetime covariant expressions [86, 111].

We propose a different method to see how the covariant expressions (5.4) may follow from the β functions derived using conformal perturbation theory methods (5.28), by relating them at the level of their action functionals.[6] We will do this only up to second order in u^a in the integrand, i.e. to first order in the equations of motion, $\beta^a = 0$, for the fields u^a. The necessity of this approximation can directly be inferred from the appearance of the non-tensorial object u^a in the integrand.

Objects from the appendix are denoted with a tilde, while quantities without a tilde refer to the fields and couplings in the main text. If we restrict ourselves to transverse traceless variations in the metric, the covariant action[7]

$$S = \int \sqrt{\tilde{g}} e^{-2\tilde{\Phi}} \left[\tilde{R} + 4(\tilde{\nabla}\Phi)^2 + \frac{1}{2}\tilde{M}_{ab}\left(u^a \tilde{\mathrm{D}}\tilde{\nabla} u^b - \tilde{\nabla} u^a \tilde{\nabla} u^b\right) - 4\tilde{U} \right], \tag{5.30}$$

generates the equations of motion given by the vanishing of (5.28), to leading order in u^a, provided

$$\frac{1}{\alpha'}\tilde{M}_{ab}\widetilde{\overline{\beta}^b} = \partial_a \tilde{U} + \frac{1}{2}\tilde{M}_{ab}u^b\tilde{U}.$$

The latter expression should be equivalent to the consistency condition (5.6), although it is probably rather involved to derive this for the non-covariant (5.28).

Being a covariantly consistent expression, we expect the action (5.30) to provide the true (spacetime and field space) covariant expressions for the β functions as we would have found by background field methods [86, 111]. The double derivative of u^a is non-standard. However, we can now consider the field redefinition

$$\tilde{\Phi} = \Phi + \frac{1}{8}\tilde{M}_{ab}u^a u^b, \qquad \tilde{g}_{\mu\nu} = e^{\frac{1}{4}\tilde{M}_{ab}u^a u^b} g_{\mu\nu}.$$

Together with the identifications

$$M_{ab} = \tilde{M}_{ab}, \qquad U = e^{\frac{1}{4}\tilde{M}_{ab}u^a u^b}\tilde{U},$$

the action (5.30) transforms to the conventional covariant action

$$S = \int \sqrt{g} e^{-2\Phi} \left[R + 4(\nabla\Phi)^2 - M_{ab}\nabla u^a \nabla u^b - 4U \right], \tag{5.31}$$

[6] In [111] this is done by way of a diffeomorphism that is not entirely clear to the authors.

[7] Note that this restriction means that the contraction of the variation of the connection in $M_{ab}u^a \tilde{\mathrm{D}}\tilde{\nabla} u^b$ does not contribute to the equations of motion. It is orthogonal to the transverse traceless fluctuations

$$\tilde{g}^{\mu\nu}\delta\tilde{\Gamma}^\rho_{\mu\nu} = -\frac{1}{2}\left(2\nabla_\mu \delta g^{\rho\mu} - g_{\mu\nu}\nabla^\rho \delta g^{\mu\nu}\right) = 0.$$

up to second order in u^a.

6

Conformal universe

Our studies into the nature of supergravity inflation and worldsheet inflation point towards an intrinsic difficulty in describing microscopic theories for inflation. In both cases, the sensitivity of inflation to the underlying details of the theory demands a level of understanding that is presently unobtainable and may remain unobtainable for the (near) future. For this reason, we are led to consider different approaches to understand the structure of inflation. Conventionally we can rely on the symmetries of the theory to better understand its behavior. For inflation, which is a quasi-de Sitter evolution, the late-time geometry has a conformal isometry group. Hence, the restrictions and structure of conformal field theory are expected to be imprinted in the late-time phenomena of the theory.

With this observation in mind, string theory provides a new approach in the manner in which these symmetry considerations can be written down. The techniques in the context of holography make full use of all of our knowledge of microscopic physics. As we have seen in section 3.2.3, in the case of the known holographic realization of anti-de Sitter spacetimes, the gravity theory can be understood from a field theory perspective, interchanging strongly coupled and weakly coupled regimes. Hopefully, knowing that anti-de Sitter and de Sitter spacetimes are closely related mathematically, a holographic study of (quasi-)de Sitter spacetimes yields similar results, although the subject still needs to be shaped and molded before its full power can be used.

In this chapter we present a small but important step towards a better understanding of a cosmological holographic duality, at the level of the constraining symmetries of inflation. We study the correlation functions of primordial curvature perturbations generated during inflation, specifically the power spectrum and the bispectrum, from a purely conformal point of view. At a technical level, many techniques that are developed in chapter 3 and put to use in chapter 5 will again prove to be useful, because again, conformal invariance and the deviation away from pure conformality are central to the analysis. This chapter can therefore be seen as yet another example displaying the breadth of the applicability of conformal invariance within physics. The chapter is based on [253].

6.1 Introduction

Phenomenologically, the inflationary paradigm provides a satisfying explanation for the initial value problems of the standard big bang model. Over the last decades, we have gathered increasing evidence for the existence of an epoch of primordial inflation, most importantly through the appearance of acoustic oscillations in the temperature anisotropies of the cosmic microwave background radiation [10]. Via a careful study of the relation between theory and observation, inflation enables us to open a new window towards the study of the structure of our universe at very high energies. As explained in chapter 2, the most accurate mapping we possess between theory and observation is that of the n-point functions of curvature perturbations. The observation of primordial gravitational waves, of features in the power spectrum of primordial density fluctuations [51, 52] or the observation of any type of non-Gaussianities [45, 46, 54] would all pave the way for a leap in our understanding of the primordial phase of the universe. It is for this reason that there is much research devoted to the structure of the two- and three-point functions of primordial density perturbations. A true understanding of the structure of the power spectrum and bispectrum may be a direct probe of new physics, once the required sensitivity is obtained observationally. To satisfy this need, different theoretical techniques have been developed in the literature for calculating the three-point function [57, 59, 62, 76, 254].

Direct calculation of these correlation functions, however, can be rather involved [56, 57, 61, 62], as the organization imposed by the slow-roll expansion does not necessarily ensure that the expressions remain tractable at intermediate steps. As such, the underlying structure behind the final result is obscured. It would certainly be welcoming to have alternative ways to derive these non-Gaussian correlation functions which emphasize strongly the symmetries of inflation. In particular, slow-roll inflation is a quasi-de Sitter expansion and as such, it is expected that the correlation functions of inflationary curvature perturbations inherit constraints from the (remnants of the) isometry group of the de Sitter phase. At late times, the isometry group of the de Sitter phase asymptotically reduces to three-dimensional Euclidian conformal symmetry, which suggests that the late-time correlation functions generated during inflation are naturally constrained by this (broken) conformal symmetry [57, 255–257]. In this chapter, we investigate the constraints of late-time de Sitter symmetry and the effect of its breaking on the bispectrum of primordial density fluctuations in single field slow-roll inflation.

Many of the techniques we use, have come of age in the context of holography and as such, our presentation and analysis have a distinct holographic flavor. Holographic duality between gravity theories and gauge theories [129, 130] is arguably the most profound and deep achievement of string theory in the last fifteen years, with the realization of the AdS/CFT-correspondence [90, 136, 147, 148]. The duality enables us to understand a physical system from a different perspective, thereby emphasizing aspects that had gone unnoticed before in the original description. For this reason, holography can provide fundamental new insights into the structure of the phenomena. When applied to critical phenomena in condensed matter systems, a holographic understanding already seems to bear fruit [258–262].

Given the close relationship to anti-de Sitter spaces, our cosmic evolution might also be described by some conformal field theory. Indeed, after the proposed AdS/CFT-correspondence, the related dS/CFT-correspondence was quickly formulated [144–146] and further investigated in the context of inflation [263–265]. However, no concrete proposal for a dS/CFT-correspondence exists and there are fundamental objections against a dS/CFT-correspondence [266–268]. Taking this into consideration, we emphasize that our viewpoint is more modest, and depends only on the *symmetries*. In our considerations, the late-time de Sitter symmetry will lead the way to a different perspective, in terms of terminology *inspired* by (A)dS/CFT [57, 263, 265]. Ultimately it is the symmetries, or the approximate lack thereof, of the late-time behavior of the observed perturbations that constrain the form of the n-point correlation functions.

In [264, 265] it was shown that the nearly scale invariant power spectrum of curvature density perturbations can be fully understood from the constrained form of two-point functions in a conformal field theory. This means that the universal behavior of the inflationary power spectrum can be explained as a critical phenomenon, suggesting that there should not be any finetuning problems. The main motivation for this chapter is to study to what extent this can be generalized to the three-point function. Since slow-roll inflation is a quasi-de Sitter evolution, the exact conformal symmetry is broken. This is understood holographically in terms of a renormalization group flow, which has been extensively studied in the context of AdS/CFT [138–142] as explained in chapter 3. The underlying symmetry imposes Ward identities on the correlation function that restrict the form of the stress-energy tensor, i.e. the holographic dual of the curvature perturbations, in terms of the correlation functions of the nearly marginal operator driving the renormalization group flow. The final result should then be obtained by finding the solution to the renormalization group equations.

The scale invariance of the power spectrum and the conformal symmetries of a de Sitter spacetime are a striking feature of the inflationary epoch. It is therefore not surprising that in recent literature active investigations are undertaken to understand the structure of the power spectrum, bispectrum and trispectrum in terms of conformal symmetries, for both scalar as well as tensor perturbations [57, 255–257, 269–278]. These studies recognize that a pure de Sitter phase is the zeroth order result in a slow-roll inflation calculation and hence, the observed correlation functions are constrained by conformal symmetry to leading order [255–257, 269]. To further understand the connection with inflation, a departure from conformal symmetry is necessary, which can be studied through consistency relations between the n-point function and the squeezed limit of $n+1$-correlation functions [57, 270–274, 279, 280] or in terms of spontaneously broken symmetries [275–278]. We provide a supplementary view by studying the departure from conformal symmetry as a renormalization group flow.

Other studies employing the strengths of holographic renormalization to the inflationary bispectrum exist [254, 281–283]. The approach undertaken in [254] provides an alternative method for calculating the three-point correlation function, which provides a valuable consistency check and a clear insight in the dS/CFT-correspondence. The techniques from AdS/CFT used by [254] are to regulate divergences in a calculation that is in essence a bulk calculation. As such it is not clear to us how the three-point function that they obtain could be found from a conformal field theory. The purpose of our study is to supplement their analysis, fully from the perspective of a boundary conformal field theory.

The study of [281–283] is a far-reaching, technically advanced understanding of a proposed dS/CFT-correspondence. The authors apply the correspondence to a free conformal field theory, thereby calculating the bispectrum of a strongly coupled gravitational theory. Our investigations are concerned with ordinary slow-roll inflation, which is already a solution in *classical* general relativity. Hence, we are forced to consider an arbitrary (strongly coupled) conformal field theory. The reason we are still capable of considering interactions between operators is that the perturbations are dictated by the renormalization group flow and conformal symmetry alone, allowing us to circumvent any expected problems regarding the strong coupling of the field theory. On the other hand, as far as our analysis goes, symmetry may not completely specify the full structure of the bispectrum, whereas [281–283] find explicit predictions.

This chapter is organized as follows. In section 6.2 we review the relation between time evolution during inflation and the energy scale of the boundary field theory, i.e. holographic renormalization in the dS/CFT-correspondence. Then, in section 6.3 we relate the power spectrum and bispectrum of primordial density fluctuations to Ward identities between the trace of the field theory stress-energy tensor and the operator dual to the inflaton field. We will use this relation in 6.4 to investigate the structure of the inflationary two- and three-point functions, focussing on the consistency condition between them and on their behavior under a renormalization of the field theory. Technical aspects concerning the Ward identities and the Fourier transform of the three-point function are summarized in appendices.

6.2 Cosmology and the dS/CFT-correspondence

6.2.1 Renormalization group flow and cosmic evolution

The discovery of an explicit realization of the holographic principle [129, 130] in anti-de Sitter geometry [136] immediately sparked the question whether other spacetime geometries could be seen to have a holographic dual as well. The holographic principle itself does not rely on the precise structure of the spacetime geometry and it would be rather unsatisfactory if no other realizations could be found. Since anti-de Sitter geometry is mathematically very similar to de Sitter space, a natural candidate for an extension of the AdS/CFT-duality is de Sitter geometry [145]. A realization of the dS/CFT-correspondence would phenomenologically be very interesting, as our own universe is observed to currently resemble de Sitter geometry [19, 20]. In theory, we could therefore enlarge our understanding of our own spacetime geometry through holographic means, by borrowing results from the mathematically related and much better understood AdS/CFT-correspondence [144, 145, 284, 285].

Not only for the present de Sitter geometry would the existence of a dS/CFT-correspondence be very interesting, but also for the primordial inflationary epoch, which follows a quasi-de Sitter evolution [146, 263]. In this chapter we will continue our study of inflation from a holographic point of view, but we only consider holography at its minimum (necessary) level, viz. that of the symmetries between the theories. We will consider what structure the asymptotic de Sitter symmetries impose on the late-time two- and three-point correlation functions. Investigations of a correspondence between other correlation functions and thereby a first indication of a more complete dS/CFT-correspondence is left for future research.

The dS/CFT-correspondence predicts a relation between cosmic evolution of the de Sitter spacetime and scale invariance in the (boundary) field theory, similar to the relation in AdS/CFT between radial coordinates in the bulk and renormalization group flow on the boundary field theory [138–142]. This relation follows from one of the isometries of the pure de Sitter geometry,

$$ds^2 = -dt^2 + a(t)^2 d\boldsymbol{x}^2,$$

in flat spatial slicing with $a(t) = e^{Ht}$. It is invariant under the combined transformation [146]

$$t \to t + \Delta t, \qquad \boldsymbol{x} \to e^{-H\Delta t}\boldsymbol{x}. \tag{6.1}$$

The parameter $\mu = Ha$ related to this time translation has dimensions of energy and is therefore taken to be the typical energy scale of the boundary theory. With this, the cosmic evolution can be seen as a reversed renormalization group flow in the field theory [146]. One identifies primordial stages of the cosmic evolution with the IR fixed point of the field theory and to study the late-time behavior of the gravity theory, one can consider the field theory around the UV fixed point.

In the context of inflation, this observation suggests a natural description for the inflationary dynamics in terms of the renormalization group flow. Inflation occurred at early times in the cosmic evolution, right after the field theory IR fixed point. It is described by the inflaton scalar field $\phi(t, \boldsymbol{x})$ coupled gravitationally to a background FLRW spacetime that is spatially flat, in accordance with the Friedmann equations. The asymptotic value $\phi_0(\boldsymbol{x})$ of the inflaton scalar field $\phi(t, \boldsymbol{x})$, for $t \to \infty$, acts in the dual conformal field theory as the coupling $u = \phi_0$ to an operator O. As a consequence of this coupling, the conformal field theory S_0, which describes the asymptotic symmetry of pure de Sitter spacetime, is perturbed

$$S_u = S_0 + \int d^3\boldsymbol{x}\, uO. \tag{6.2}$$

When the operator is non-marginal, it will induce a renormalization group flow, which in the case of the cosmic evolution is reversed and ends asymptotically in the UV fixed point of the theory.

While one can consider the asymptotic behavior of inflation from the point of view of the field theory IR fixed point [263], from the UV fixed point [265] or from the bulk gravitational IR point of view [264], it is important to realize that inflation itself actually is an epoch *along* the renormalization group flow, cf. figure 6.1. The essence of slow-roll inflation is that at every point along the inflationary flow the spacetime can be approximated by a de Sitter phase. Typically, a particular de Sitter phase is chosen as the pivot point around which the slow-roll expansion is defined [28, 30]. Similarly, at any intermediate point along the renormalization group flow,

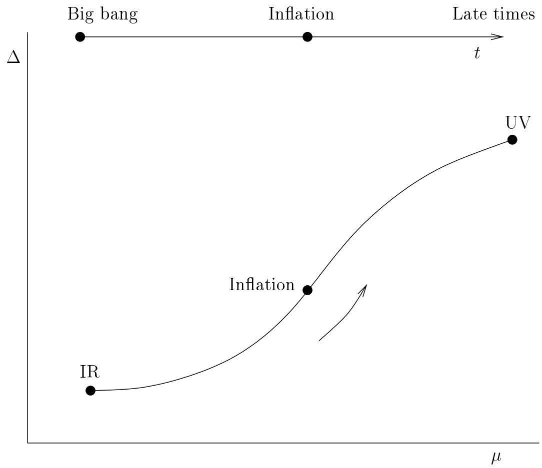


Figure 6.1: *The cosmic evolution can be seen as a reversed renormalization group flow, from the IR fixed point of the dual theory to the UV fixed point of the dual theory. Inflation occurs at a certain intermediate stage during the renormalization group flow. As is usual for inflation, a pivot point along the flow is chosen around which the slow-roll expansion can be studied. We observe the effects of inflation at late times, corresponding to the UV fixed point of the renormalization group flow.*

the dual description is approximately a conformal field theory itself, about which the effects of the flow can be expanded. When considering the correlation functions at late times, it is important to realize that the result has to be related to this intermediate renormalization group point, rather than the IR or UV fixed points.

6.2.2 Holographic slow-roll parameters

The close relation between the renormalization group flow induced by the non-marginal operator $\mathcal{O}$ and the inflationary solution for the bulk field $\phi(t, \boldsymbol{x})$ can be made technically more precise [263]. For a massive scalar field and taking $a(t) \sim e^{Ht}$ in the asymptotic limit $t \to \infty$, the equations of motion determine the asymptotic solution as $\phi = \phi_0(\boldsymbol{x})e^{\lambda_\pm Ht}$, where

$$\lambda_\pm = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - \frac{m^2}{H^2}}. \tag{6.3}$$

With the identification $\mu \propto e^{Ht}$, the invariance of the full asymptotic solution under the transformation (6.1) dictates ϕ_0 to transform as

$$\phi_0(\boldsymbol{x}) \to \phi_0(\boldsymbol{x}e^{-H\Delta t})e^{-\lambda_\pm H\Delta t} = \left(\frac{\mu}{\mu_0}\right)^{-\lambda_\pm} \phi_0\left(\frac{\mu_0}{\mu}\boldsymbol{x}\right).$$

Hence, identifying $\phi_0(\boldsymbol{x})$ and u, the operator O is seen to have scaling dimension $\Delta = 3 + \lambda_\pm$. In accordance with AdS/CFT reasoning, the λ_--solution vanishes at the boundary, i.e. at late times, and is therefore regarded as the vanishing mode. The $\lambda = \lambda_+$-solution defines a non-normalized mode, i.e. vanishing in the interior, which is sourced by the boundary field $\phi_0(\boldsymbol{x})$. Depending on the sign of λ, it describes a relevant ($\lambda < 0$) or irrelevant ($\lambda > 0$) perturbation from the field theory perspective. Via

$$\lambda = \frac{\partial \log \phi}{\partial \log a} = \frac{\partial \log u}{\partial \log \mu} = \frac{\beta}{u},$$

(6.3) is related to the β function $\beta = \frac{\partial u}{\partial \log \mu}$ of the operator O. We can define the non-marginal scaling dimension $\lambda(u)$ as

$$\lambda(u) = \frac{\partial \beta}{\partial u} + O(u), \tag{6.4}$$

in accordance with (3.17) [110]. In the limit $u \to 0$ that we consider, both definitions are equivalent and no ambiguity exists [263, 265].

In inflation, the pure de Sitter evolution is perturbed due to the varying inflaton scalar field $\phi(t, \boldsymbol{x})$. From the observation of the near scale invariant power spectrum, we know that the perturbation away from the de Sitter phase is only small, leading to a time dependent Hubble parameter $H(t)$ that is allowed to vary only slightly during the inflationary evolution. This is conveniently expressed by the requirement that the slow-roll parameters (2.5) are much smaller than unity. For $\epsilon = \eta = 0$, the evolution is that of a de Sitter spacetime.

From the field theory point of view, it will perturb away from the conformal fixed point S_0 to which the pure de Sitter phase corresponds, due to the non-marginal nature of O, i.e. $\lambda \neq 0$ to first order or, more precisely, its β function is non-vanishing, $\beta \neq 0$. It is therefore to be expected that λ and the β function of the operator O express the departure away from the pure de Sitter phase. Indeed the slow-roll parameters ϵ and η can be fully expressed in terms of the conformal field theory-data β and λ via

$$\beta = \frac{\partial u}{\partial \log \mu} = \frac{\partial \phi}{\partial \log a} = \frac{\dot{\phi}}{H} = -2\frac{H'}{H}, \tag{6.5}$$

where in the last step, the Friedmann equations are employed, which tell us that $\dot{\phi} = -2H'$. By taking multiple derivatives of this relation with respect to $\phi = u$, one readily obtains

$$\beta^2 = 2\epsilon, \qquad \epsilon = \frac{1}{2}\beta^2, \tag{6.6a}$$

$$\lambda = \epsilon - \eta, \qquad \eta = \frac{1}{2}\beta^2 - \lambda, \tag{6.6b}$$

$$\frac{\partial^2\beta}{\partial u^2} = \frac{1}{\sqrt{2\epsilon}}\left(2\xi^2 - 3\epsilon\eta + 2\epsilon^2\right), \qquad \xi^2 = \beta\left(\frac{1}{2}\frac{\partial^2\beta}{\partial u^2} + \frac{1}{8}\beta^3 - \frac{3}{4}\beta\lambda\right). \tag{6.6c}$$

We have included the second derivative $\frac{\partial^2\beta}{\partial u^2}$ and the third slow-roll parameter $\xi^2 = 2\frac{H'H'''}{H^2}$ as these will appear in later expressions.

The relation between scaling parameters of the field theory and the slow-roll parameters of the inflationary theory suggests that for the study of slow-roll inflation, for which $\epsilon, \eta, \xi \ll 1$, we can consider a nearly marginal deformation of the conformal field theory fixed point, $\beta, \lambda, \frac{\partial^2\beta}{\partial u^2} \approx 0$. With the relations (6.6), more substance has been given to the picture as presented in figure 6.1, in that the inflationary quasi-de Sitter phase can be approximated by a near conformal field theory.

Although it is tempting to also rely on techniques from conformal perturbation theory, the above relation between the β function (and its derivatives) and the slow-roll parameters does not necessarily imply the smallness of the coupling u. In fact, from the expression of $\beta(u)$ in conformal perturbation theory (3.18), we can immediately read off possible problems,

$$\beta = \lambda u + \dots . \tag{6.7}$$

If $\beta^2 = O(\epsilon)$ and $\lambda = O(\epsilon)$, it means that u itself is of order $O(\epsilon^{-1/2})$. Hence, the slow-roll expansion seems to correspond to the *large* u-regime. Drawing a parallel with expansion in dimensionful parameters, we know that we should perhaps not attach too much value to this observation, but it does emphasize a subtle mismatch between the slow-roll expansion and conformal perturbation theory. Conformal perturbation theory requires a small deviation from marginality $\lambda \ll 1$ and a small coupling $u \ll 1$. The slow-roll expansion is an expansion for small $\beta \ll 1$ and its derivatives, but has no analogue for u. For this reason, we will try to keep the use of conformal perturbation theory and the $u \to 0$-limit to a minimum, although we will not succeed in doing this everywhere. In particular, as we have seen, the $u \to 0$-limit is necessary to relate the higher order slow-roll parameters with β [263, 265].

6.3 Holographic correlation functions

6.3.1 Wavefunction expansion

The previous section suggests that the physics of the inflationary epoch in our (four-dimensional) universe resembles the (three-dimensional) physics close to a conformal fixed point. We will investigate whether this suggested resemblance can be employed at the level of the correlation functions. As was summarized in chapter 2, the form of these correlation functions is well-known from gravity calculations [28, 37–43, 57]. To study these from the holographic viewpoint, we need a dictionary between the gravity correlation functions and the correlation functions of the boundary field theory. In the spirit of the AdS/CFT-correspondence, such a relation has been provided by [57].

Quantitatively, the holographic relation between de Sitter geometry and conformal field theory is given by an identification of the partition function of the field theory with the wavefunction of the de Sitter universe with appropriate boundary conditions,

$$\Psi_{dS} = Z_{CFT}. \tag{6.8}$$

The partition function (6.8) determines the correlation functions via

$$\langle O_1 \dots O_n \rangle = \left. \frac{\delta^n \Psi_{dS}[\phi]}{\delta\phi_1 \dots \delta\phi_n} \right|_{\phi=0}.$$

Therefore the wavefunction may be trivially expanded as

$$\Psi_{dS}[\phi] = \exp\Bigg(\frac{1}{2} \int \mathrm{d}^3\boldsymbol{k}_1 \mathrm{d}^3\boldsymbol{k}_2\, \phi_{\boldsymbol{k}_1}\phi_{\boldsymbol{k}_2} \langle O_{\boldsymbol{k}_1} O_{\boldsymbol{k}_2} \rangle + \frac{1}{6} \int \mathrm{d}^3\boldsymbol{k}_1 \mathrm{d}^3\boldsymbol{k}_2 \mathrm{d}^3\boldsymbol{k}_3\, \phi_{\boldsymbol{k}_1}\phi_{\boldsymbol{k}_2}\phi_{\boldsymbol{k}_3} \langle O_{\boldsymbol{k}_1} O_{\boldsymbol{k}_2} O_{\boldsymbol{k}_3} \rangle + \dots \Bigg).$$

Using this expression, the dictionary follows immediately. The two-point function $\langle \phi_{\boldsymbol{k}_1}\phi_{\boldsymbol{k}_2} \rangle = \int \mathcal{D}\phi\, \phi_{\boldsymbol{k}_1}\phi_{\boldsymbol{k}_2} |\Psi_{dS}|^2$ can be rewritten as

$$\begin{aligned} \langle \phi_{\boldsymbol{k}_1}\phi_{\boldsymbol{k}_2} \rangle &= \int \mathcal{D}\phi\, \phi_{\boldsymbol{k}_1}\phi_{\boldsymbol{k}_2} e^{\int \mathrm{d}^3\boldsymbol{k} \mathrm{d}^3\boldsymbol{l}\, \phi_{\boldsymbol{k}}\phi_{\boldsymbol{l}} \mathrm{Re}\langle O_{\boldsymbol{k}} O_{\boldsymbol{l}} \rangle} \\ &= \frac{-1}{2\mathrm{Re}\langle O_{\boldsymbol{k}_1} O_{-\boldsymbol{k}_1} \rangle'} \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2) \int \mathcal{D}\tilde{\phi}\, \tilde{\phi}_{\boldsymbol{k}_1}\tilde{\phi}_{-\boldsymbol{k}_1} e^{-\frac{1}{2}\int \mathrm{d}^3\boldsymbol{k}\, \tilde{\phi}_{\boldsymbol{k}}\tilde{\phi}_{-\boldsymbol{k}}}, \end{aligned}$$

where we have employed the substitution of variables $\tilde{\phi}_{\boldsymbol{k}} = i\sqrt{2\mathrm{Re}\langle O_{\boldsymbol{k}} O_{-\boldsymbol{k}} \rangle'}$ and where we have assumed the path integral measure to be invariant under this substitution. A prime $'$ indicates that we consider the part of the correlation function multiplying the momentum conserving delta function. The path integral equals some number and hence the correlation functions are related via

$$\langle \phi_{\boldsymbol{k}} \phi_{-\boldsymbol{k}} \rangle' \propto \frac{-1}{\mathrm{Re}\langle O_{\boldsymbol{k}} O_{-\boldsymbol{k}} \rangle'}. \tag{6.9}$$

For the three-point function we can do a similar calculation,

$$\begin{aligned}
\langle \phi_{\boldsymbol{k}_1} \phi_{\boldsymbol{k}_2} \phi_{\boldsymbol{k}_3} \rangle &= \int \mathcal{D}\phi \, \phi_{\boldsymbol{k}_1} \phi_{\boldsymbol{k}_2} \phi_{\boldsymbol{k}_3} e^{\int \mathrm{d}^3\boldsymbol{k} \mathrm{d}^3\boldsymbol{l} \, \phi_{\boldsymbol{k}} \phi_{\boldsymbol{l}} \mathrm{Re}\langle O_{\boldsymbol{k}} O_{\boldsymbol{l}} \rangle + \frac{1}{3} \int \mathrm{d}^3\boldsymbol{k} \mathrm{d}^3\boldsymbol{l} \mathrm{d}^3\boldsymbol{m} \, \phi_{\boldsymbol{k}} \phi_{\boldsymbol{l}} \phi_{\boldsymbol{m}} \mathrm{Re}\langle O_{\boldsymbol{k}} O_{\boldsymbol{l}} O_{\boldsymbol{m}} \rangle} \\
&= \int \mathcal{D}\phi \, \phi_{\boldsymbol{k}_1} \phi_{\boldsymbol{k}_2} \phi_{\boldsymbol{k}_3} e^{\int \mathrm{d}^3\boldsymbol{k} \, \phi_{\boldsymbol{k}} \phi_{-\boldsymbol{k}} \mathrm{Re}\langle O_{\boldsymbol{k}} O_{-\boldsymbol{k}} \rangle'} \times \\
&\qquad \left(1 + \frac{1}{3} \int \mathrm{d}^3\boldsymbol{k} \mathrm{d}^3\boldsymbol{l} \mathrm{d}^3\boldsymbol{m} \, \phi_{\boldsymbol{k}} \phi_{\boldsymbol{l}} \phi_{\boldsymbol{m}} \mathrm{Re}\langle O_{\boldsymbol{k}} O_{\boldsymbol{l}} O_{\boldsymbol{m}} \rangle \right) \\
&= 2\mathrm{Re}\langle O_{\boldsymbol{k}_1} O_{\boldsymbol{k}_2} O_{\boldsymbol{k}_3} \rangle \int \mathcal{D}\phi \, \phi_{\boldsymbol{k}_1} \phi_{-\boldsymbol{k}_1} \phi_{\boldsymbol{k}_2} \phi_{-\boldsymbol{k}_2} \phi_{\boldsymbol{k}_3} \phi_{-\boldsymbol{k}_3} e^{\int \mathrm{d}^3\boldsymbol{k} \, \phi_{\boldsymbol{k}} \phi_{-\boldsymbol{k}} \mathrm{Re}\langle O_{\boldsymbol{k}} O_{-\boldsymbol{k}} \rangle'} \\
&= \frac{-\mathrm{Re}\langle O_{\boldsymbol{k}_1} O_{\boldsymbol{k}_2} O_{\boldsymbol{k}_3} \rangle}{4 \prod_{j=1}^3 \mathrm{Re}\langle O_{\boldsymbol{k}_j} O_{-\boldsymbol{k}_j} \rangle'} \int \mathcal{D}\tilde{\phi} \, \tilde{\phi}_{\boldsymbol{k}_1} \tilde{\phi}_{-\boldsymbol{k}_1} \tilde{\phi}_{\boldsymbol{k}_2} \tilde{\phi}_{-\boldsymbol{k}_2} \tilde{\phi}_{\boldsymbol{k}_3} \tilde{\phi}_{-\boldsymbol{k}_3} e^{-\frac{1}{2} \int \mathrm{d}^3\boldsymbol{k} \, \tilde{\phi}_{\boldsymbol{k}} \tilde{\phi}_{-\boldsymbol{k}}},
\end{aligned}$$

where we can approximate the exponent because $\langle OOO \rangle \ll \langle OO \rangle$. The zeroth order term in this approximation will integrate to 0 as it is an odd function. Hence

$$\langle \phi_{\boldsymbol{k}_1} \phi_{\boldsymbol{k}_2} \phi_{\boldsymbol{k}_3} \rangle \propto \frac{-\mathrm{Re}\langle O_{\boldsymbol{k}_1} O_{\boldsymbol{k}_2} O_{\boldsymbol{k}_3} \rangle}{\prod_{j=1}^3 \mathrm{Re}\langle O_{\boldsymbol{k}_j} O_{-\boldsymbol{k}_j} \rangle'}. \tag{6.10}$$

These expressions hold for any dual pair of fields and operators. In particular, the correlation functions of the curvature perturbation ζ are related to the correlation functions of the trace Θ of the stress-energy tensor via

$$\langle \zeta_{\boldsymbol{k}} \zeta_{-\boldsymbol{k}} \rangle' \propto \frac{-1}{\mathrm{Re}\langle \Theta_{\boldsymbol{k}} \Theta_{-\boldsymbol{k}} \rangle'}, \tag{6.11a}$$

$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle \propto \frac{-\mathrm{Re}\langle \Theta_{\boldsymbol{k}_1} \Theta_{\boldsymbol{k}_2} \Theta_{\boldsymbol{k}_3} \rangle}{\prod_{j=1}^3 \mathrm{Re}\langle \Theta_{\boldsymbol{k}_j} \Theta_{-\boldsymbol{k}_j} \rangle'}. \tag{6.11b}$$

6.3.2 Ward identities

The trace of the stress-energy tensor is not a standard primary operator. Therefore, to compute (6.11) we can not simply resort to the standard expressions (3.12) for correlation functions of primary operators in a conformal field theory. However, in the gravity calculation we have seen that the curvature perturbations are defined in a gauge independent way and one has the freedom to choose a gauge in which the calculation is done. The gauge invariance of the gravity theory is translated to the fact that in a scale dependent field theory one can either change the dimensionful coupling u to the operator O or one can change the metric, relating ζ to Θ accordingly [57]. This gauge relation is reflected in the field theory, since both operators are related, to leading order, via [286, 287]

$$\Theta = -\beta O, \tag{6.12}$$

where the constant of proportionality β is the Weyl anomaly coefficient. For the purpose of this thesis it is taken to be equal to the standard renormalization β function for the coupling u to the operator O, cf. section 3.1.3. Making use of this relation, the holographic two- and three-point functions can be calculated from the n-point functions of the primary operator O.

The appearance of β is no coincidence, as was already explained in chapter 3. In a quantum field theory, scale transformations are associated with the regularization and renormalization of the theory. This can be described in terms of the Callan-Symanzik renormalization group equations [99–101], where the β functions in the Callan-Symanzik equation describe the dependence of the coupling constants on the renormalization scale. Equivalently —and historically, in the derivation of the Callan-Symanzik equation— the scale dependence is described in terms of the Ward identity of scale transformations.

In gravity, gauge invariance is really important, but at the end of the day the only meaningful physical quantity is ζ. This corresponds to the trace of the stress-energy tensor Θ of the boundary field theory. In general, gauge symmetries of a theory correspond to constraints. In the case of the gravity theory, these are the hamiltonian and momentum/reparameterization constraints of the ADM formalism [256]. In a field theory, the symmetries impose constraints on the correlation function through Ward identities. It is in this way that the gauge choices are implemented in the field theory.

In our particular case, we need to find the relations between the two- and three-point function of the trace of the stress-energy tensor Θ and the operator O. As the trace of the stress-energy tensor is the Noether current of Weyl transformations, we consider the Ward identities of (multiple) trace insertions. Initially the calculation follows directly from any textbook field theory calculation, particularly [83], but once multiple trace insertions have to be taken into account, more care is required. The details of the calculation can be found in appendix 6.A. The final result is given by

$$\langle \Theta_u(\boldsymbol{x}) X \rangle_u = -u(\Delta - 3)\langle O(\boldsymbol{x}) X \rangle_u + \sum_k \delta(\boldsymbol{x} - \boldsymbol{x}_k)\Delta_k \langle X \rangle_u, \tag{6.13a}$$

$$\begin{aligned}\langle \Theta_u(\boldsymbol{x})\Theta_u(\boldsymbol{y})X\rangle_u &= u^2(\Delta-3)^2\langle O(\boldsymbol{x})O(\boldsymbol{y})X\rangle_u - u(\Delta-3)^2\delta(\boldsymbol{x}-\boldsymbol{y})\langle O(\boldsymbol{x})X\rangle_u \\ &\quad - u(\Delta-3)\sum_k \delta(\boldsymbol{x}-\boldsymbol{x}_k)\Delta_k\langle O(\boldsymbol{y})X\rangle_u - \boldsymbol{x}\leftrightarrow\boldsymbol{y} \\ &\quad + \sum_{k,l}\delta(\boldsymbol{x}-\boldsymbol{x}_k)\Delta_k\delta(\boldsymbol{y}-\boldsymbol{x}_l)\Delta_l\langle X\rangle_u, \qquad (6.13b)\end{aligned}$$

$$\begin{aligned}\langle \Theta_u(\boldsymbol{x})\Theta_u(\boldsymbol{y})\Theta_u(\boldsymbol{z})X\rangle_u &= -u^3(\Delta-3)^3\langle O(\boldsymbol{x})O(\boldsymbol{y})O(\boldsymbol{z})X\rangle_u \qquad (6.13c) \\ &\quad + u^2(\Delta-3)^3\delta(\boldsymbol{y}-\boldsymbol{z})\langle O(\boldsymbol{x})O(\boldsymbol{y})X\rangle_u + \boldsymbol{x}\leftrightarrow\boldsymbol{y} + \boldsymbol{y}\leftrightarrow\boldsymbol{z} + \ldots,\end{aligned}$$

where the correlation function $\langle\ \rangle_u$ of the trace(s) of the stress-energy tensor Θ_u is evaluated in the perturbed conformal field theory (6.2), contracted with an arbitrary product of operators $X = O(\boldsymbol{x}_1)\ldots O(\boldsymbol{x}_n)$. Δ_k is the (full) scaling dimension of the k'th operator $O(\boldsymbol{x}_k)$ inside X, which in a single field scenario are all equal. The ... contain highly local contributions that are negligible for our purposes.

The Ward identities (6.13) are valid *throughout* the renormalization group flow, i.e. for each value of u. However, to write them in a more familiar form, we rely on conformal perturbation theory. Equation (6.13a) does not yet seem to contain the familiar $\beta(u)$-dependence, but it does contain the u-dependent conformal weight Δ_u, which carries similar information [286, 287]. Along the renormalization group flow, the scaling behavior of the operator will change and also the coupling will adjust accordingly. Near a conformal fixed point, $u \to 0$, or similarly, near our quasi-conformal fixed point dual to inflation $u \to u_* = 0$, one can make the relation between Δ_u and $\beta(u)$ more precise in a conformal perturbation expansion, cf. (3.18)

$$\beta(u) = u(\Delta_0 - 3) + 2\pi C u^2 + \ldots. \qquad (6.14)$$

Hence, as an expansion in the coupling u, one recognizes the first order contribution $u(\Delta_0 - 3)$ to $\beta(u)$ in (6.13a). The higher order contribution, as obtained via conformal perturbation theory methods, is proportional to the operator product coefficient C of the operators O [110, 111]. The combination Cu results from expanding the one-point correlation function $\langle O\rangle_u$ with respect to the unperturbed theory $\langle O\rangle_0$. It is of order $O(\epsilon)$ and appears with increasing power, Cu, $(Cu)^2$, etc., for higher orders in u. Hence, although we can not be certain of the validity of conformal perturbation theory itself, the expansion of perturbed correlation functions in terms of unperturbed correlation functions is very much similar to the slow-roll expansion.

Since we are only interested in the small u-behavior around the (quasi-)conformal fixed point, the lowest order contribution to β should be sufficient for our purposes to interpret the result. We insist on writing the expression in terms of β, as it nicely emphasizes the dependence on the renormalization group flow or equivalently, the slow-roll dependence. Hence,

$$\langle \Theta_u(\boldsymbol{x}) X \rangle_u = -\beta \langle O(\boldsymbol{x}) X \rangle_0 + \sum_k \delta(\boldsymbol{x} - \boldsymbol{x}_k) \Delta_k \langle X \rangle_u \tag{6.15}$$

is the more precise version of the familiar relation (6.12). These two equations are equal up to contact terms in real space. Similarly, we may write the other Ward identities (6.13b) and (6.13c) as

$$\langle \Theta_u(\boldsymbol{x}) \Theta_u(\boldsymbol{y}) \rangle_u = \beta^2 \langle O(\boldsymbol{x}) O(\boldsymbol{y}) \rangle_0 - \beta \lambda \delta(\boldsymbol{x} - \boldsymbol{y}) \langle O(\boldsymbol{x}) \rangle_0 \tag{6.16}$$

and

$$\begin{aligned} \langle \Theta_u(\boldsymbol{x}) \Theta_u(\boldsymbol{y}) \Theta_u(\boldsymbol{z}) \rangle_u = & -\beta^3 \langle O(\boldsymbol{x}) O(\boldsymbol{y}) O(\boldsymbol{z}) \rangle_0 \\ & + \beta^2 \lambda \left[\delta(\boldsymbol{y} - \boldsymbol{z}) \langle O(\boldsymbol{x}) O(\boldsymbol{y}) \rangle_0 + \boldsymbol{x} \leftrightarrow \boldsymbol{z} + \boldsymbol{y} \leftrightarrow \boldsymbol{z} \right] + \ldots \end{aligned} \tag{6.17}$$

respectively. The ellipsis contain lower n-point functions in the conformal fixed point, which do not contribute. These relations form the starting point of the calculation of the power spectrum and bispectrum through holographic means.

6.4 Slow-roll predictions from Ward identities

Using the holographic dictionary (6.11) from [57] between the two- and three-point functions of the scalar curvature perturbations ζ and the two- and three-point functions of the stress-energy tensor Θ_u of the (near) conformal field theory, we can interpret the Ward identities (6.16–6.17) as inflationary correlation functions, with their dependence on the slow-roll parameters captured by β and λ. In this section we will investigate the prediction for the two-point and three-point correlation functions on the basis of conformal symmetry of the field theory. Special care has to be taken to correctly interpret the renormalization group flow, which takes the expressions away from their conformal fixed point and can be seen as the transcription of the slow-roll dependence. We will first consider the, known [264, 265], holographic description of the two-point function. From this we can draw important lessons for the three-point function, in particular via the consistency condition that should be satisfied in the *squeezed limit* of the three-point function. We first consider the squeezed limit of the bispectrum and then turn our attention to its full expression.

6.4.1 Two-point function

Power spectrum in the conformal fixed point

The power spectrum $\langle\zeta\zeta\rangle$ of curvature perturbations can be found from the field theory via the stress-energy tensor two-point function $\langle\Theta_u\Theta_u\rangle_u$, which in its turn is fully determined by the two-point function of the dual operator $\mathcal{O}$. As we have seen in chapter 3, in a conformal fixed point, the two-point function for an operator $\mathcal{O}$ with scaling dimension Δ is completely specified by conformal symmetry [83, 84],

$$\langle\mathcal{O}(\boldsymbol{x})\mathcal{O}(\boldsymbol{y})\rangle_0 = \frac{1}{|\boldsymbol{x}-\boldsymbol{y}|^{2\Delta}}.$$

In principle, this completely specifies the holographic power spectrum for the conformal fixed point. However, before we can compare with (2.14), we need to perform a Fourier transform. This is necessary as the constraints by conformal symmetry are naturally given in terms of the real space variables $\boldsymbol{x}_j$, whereas n-point functions in cosmology are naturally given in terms of the outgoing momenta $\boldsymbol{k}_j$. Any connection between conformal correlation functions and inflationary correlation functions is therefore necessarily obtained only after a Fourier transform. Although finding the Fourier transform for the two-point function is readily done, in general the Fourier transform leads to a technical obstruction for any quick use of the holographic correspondence [149, 150, 255]. As we will see, already for the three-point function this obstruction is difficult to overcome.

The Fourier transform of the two-point function is

$$\begin{aligned}\langle\mathcal{O}\mathcal{O}\rangle_0 \xrightarrow{\text{F.T.}} \int \mathrm{d}^3\boldsymbol{x}\mathrm{d}^3\boldsymbol{y}\,|\boldsymbol{x}-\boldsymbol{y}|^{-2\Delta}e^{i(\boldsymbol{k}\cdot\boldsymbol{x}+\boldsymbol{k}'\cdot\boldsymbol{y})} &= \int \mathrm{d}^3\boldsymbol{u}\,e^{i\boldsymbol{u}\cdot(\boldsymbol{k}+\boldsymbol{k}')}\int \mathrm{d}^3\boldsymbol{v}\,v^{-2\Delta}e^{i\boldsymbol{v}\cdot(\boldsymbol{k}_2+\boldsymbol{k}_3-\boldsymbol{k}_1)} \\ &= \delta(\boldsymbol{k}+\boldsymbol{k}')k^{2\Delta-3}\int \mathrm{d}\xi\,\xi^{-2\Delta+2}\int \mathrm{d}\theta\,\sin\theta e^{-i\xi\cos\theta} \\ &\propto \delta(\boldsymbol{k}+\boldsymbol{k}')k^{2\Delta-3}, \end{aligned} \tag{6.18}$$

up to factors of 2 and 2π. Since $\langle\mathcal{O}\rangle_0 = 0$, if we take $\beta(u)$ to be an overall constant, the stress-energy tensor correlation function in the conformal fixed point is

$$\langle\Theta_{\boldsymbol{k}}\Theta_{\boldsymbol{k}'}\rangle_0 \propto \delta(\boldsymbol{k}+\boldsymbol{k}')\beta^2k^{2\Delta_0-3}.$$

Using $\langle\zeta\zeta\rangle' \propto \frac{1}{\mathrm{Re}\langle\Theta\Theta\rangle'}$ and the expressions (6.6) for ϵ and η in the conformal fixed point,

$$\epsilon = \frac{1}{2}\beta^2 = 0, \qquad \eta = -(\Delta_0 - 3), \tag{6.19}$$

the Fourier transform of the two-point function of primary operators agrees with the standard result (2.14), where β^2 describes the singular behavior of the power spectrum as $\epsilon \to 0$ and where the spectral index is given by

$$n_s - 1 = -2(\Delta_0 - 3) = 2\eta. \tag{6.20}$$

Power spectrum in the renormalization group flow

As explained in section 6.2, away from the conformal fixed point, the perturbation by the operator O will lead to a renormalization group flow. Holographically this is understood as the deviation from the pure de Sitter phase to a (quasi-de Sitter) inflationary phase and was interpreted by [264] in the light of the known AdS/CFT holographic renormalization methods [139, 141, 142]. Conceptually, it is understood in the Wilsonian sense as a flow between theories, specified by the running of the coupling constants. Technically, the renormalization group flow is the result of the need to renormalize the operators O appearing in (6.16). At the level of the correlation functions, the differential Callan-Symanzik equation dictates the scale dependence of the correlation functions, which was introduced by the inclusion of non-marginal coupling constants. For the truly marginal stress-energy tensor Θ_u, the Callan-Symanzik equation determines its two-point function via

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial u}\right)\langle\Theta_u\Theta_u\rangle'_u = 0. \tag{6.21}$$

The μ-dependence can be traded for momentum dependence via dimensional analysis,

$$\langle\Theta_u\Theta_u\rangle'_u = k^3 F\left[\frac{k^2}{\mu^2}, u(\mu)\right],$$

telling us that

$$\left(\mu\frac{\partial}{\partial\mu} + k\frac{\partial}{\partial k}\right)\langle\Theta_u\Theta_u\rangle'_u = 3\langle\Theta_u\Theta_u\rangle'_u.$$

Following the literature [72], the Callan-Symanzik equation acting on the renormalized operators $\tilde{O} = Z(u)O$, can be solved by investigating the ansatz

$$\langle\Theta_u\Theta_u\rangle'_u = Z^2(u)\beta^2(u)k^3. \tag{6.22}$$

This seems to separate the k- and u-dependence completely, although the two are inherently related through the defining equation of the running coupling u,

$$\beta = k\frac{\partial u}{\partial k}.$$

Applying the Callan-Symanzik equation on this ansatz, the wavefunction renormalization Z is given by

$$\frac{\partial Z}{Z\partial u} + \frac{\partial \beta}{\beta \partial u} = 0,$$

which is solved by

$$Z(u) = Z_0 e^{-\int_0^u \mathrm{d}u' \, \frac{\lambda(u')}{\beta(u')}}. \tag{6.23}$$

Since $\beta = k\frac{\partial u}{\partial k}$, we can trade the integration variable for k, introducing μ as the only other scale in the problem,

$$Z(k) = Z_0 e^{\int_\mu^k \mathrm{d}\log(\frac{k'}{\mu})\,\lambda}. \tag{6.24}$$

In this form, it is clear that the wavefunction renormalization just introduces an anomalous dimension to the correlation function. For a constant $\lambda(u) = \Delta - 3$, the two-point function reads

$$\langle \Theta_u \Theta_u \rangle'_u = Z_0^2 \beta^2 k^{3+2(\Delta-3)}. \tag{6.25}$$

For $u \to 0$, this returns to the earlier found result with an exponent $2\Delta_0 - 3$. For completely arbitrary λ, the result is expressed through the integral in (6.24), which provides a possible method to go beyond the lowest order in slow-roll [265, 288].

As was mentioned in [265], and which deserves renewed emphasis, to connect the conformal correlation function with the inflationary power spectrum, one has to express the two-point function with respect to the *average Hubble flow*. The standard inflationary perturbation theory calculates correlation functions of the quantum fluctuations around the classical inflationary evolution. This evolution is driven by an almost —but not exactly— constant Hubble parameter $H(u)$. Of course, the fact that we have to consider a quasi-de Sitter phase rather than a pure de Sitter evolution is precisely expressed through the slow-roll approximation, something we have already incorporated in the holographic description by studying the renormalization group flow. Still, to correctly identify the fluctuations, we need to express the result with respect to the classical evolution. Since conformal perturbation theory only works around a fixed point of the renormalization group flow, one might wonder how we can express our results with respect to an arbitrary point on the flow, corresponding to the quasi-de Sitter phase. For the two-point function this can be remedied by isolating an explicit Hubble parameter dependence, expressed as an integrated effect of the slow-roll parameter (6.5),

$$H(u) = H_0 e^{-\frac{1}{2}\int_0^u \mathrm{d}u' \beta(u')} = H_0 e^{\frac{1}{2}\int_\mu^k \mathrm{d}\log(\frac{k'}{\mu})\beta^2}. \tag{6.26}$$

Using the expression in terms of k, we find

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle \propto \delta(\boldsymbol{k} + \boldsymbol{k}') k^{-3} \frac{H^2}{\beta^2} e^{-\int_\mu^k \mathrm{d}\log(\frac{k'}{\mu})(\beta^2 + 2\lambda)}. \tag{6.27}$$

The relation (6.6) between the slow-roll parameters and the (derivative of the) β function for constant $\beta(k)$ and $\lambda(k)$ then immediately gives

$$n_s - 1 = -\beta^2 - 2\lambda = 2\eta - 4\epsilon, \tag{6.28}$$

in agreement with (2.15).

6.4.2 Three-point function in the squeezed limit

In principle, using similar techniques, we should be able to analyze the holographic three-point function and give it a slow-roll interpretation. Before we will start this subtle endeavor, we will shape our understanding with a very useful consistency condition of the three-point function in the long wavelength limit.

The bispectrum describes the three-point correlation between three different Fourier modes of the curvature perturbation. If one of the three modes is very small, i.e. its wavelength is very long, it will leave the horizon earlier than the other two modes. This limit is called the *squeezed* limit, since the momentum conserving triangle of Fourier modes has a squeezed shape. Due to momentum conservation, the other two modes become equal in magnitude. Since one of the modes is frozen as a dynamical mode, effectively the three-point function reduces to a two-point function between the other two modes. The only effect of the long mode can be seen through the tilt $n_s - 1$ of the power spectrum, which describes the difference in horizon crossing between the modes.

This observation was first translated into a quantitative statement by [57] in the context of single field slow-roll inflation. It is known to hold for any inflationary scenario with a single clock [279], including our single field set-up. Taking the mode k_3 much smaller than the other two modes, $k_3 \ll k_1 \approx k_2$, the consistency condition in the squeezed limit is, to lowest order in k_3 [280],

$$\lim_{k_3 \to 0} \langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = -\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \langle \zeta_{\boldsymbol{k}_1} \zeta_{-\boldsymbol{k}_1} \rangle' \langle \zeta_{\boldsymbol{k}_3} \zeta_{-\boldsymbol{k}_3} \rangle' \frac{\mathrm{d}\log k_1^3 \langle \zeta_{\boldsymbol{k}_1} \zeta_{-\boldsymbol{k}_1} \rangle'}{\mathrm{d}\log k_1}. \tag{6.29}$$

Corrections to this expression of order $1/k_3$ and beyond are also investigated [270, 271, 279, 280]. Given the general applicability of the relation, such corrections provide interesting criteria to observationally test (and possibly rule out) large classes of slow-roll inflationary models once non-Gaussianities become within reach of observations. At the same time the squeezed limit provides a robust prediction theoretically, which can be used as a first check on the consistency of any particular description for (single clock) inflation.

For our holographic formula (6.11b) of the three-point function, we can explicitly verify the consistency condition in the squeezed limit. First we need to find the analogous expression of (6.17) for the Fourier transformed correlation functions. To do so, note that the Fourier transform of the second term(s) on the right hand side of (6.17) follows directly from the fact that the two-point functions in a conformal field theory can only depend on the spatial separation of the arguments,

$$\begin{aligned}\delta(\boldsymbol{x}_2 - \boldsymbol{x}_3)\langle O(\boldsymbol{x}_1)O(\boldsymbol{x}_2)\rangle_0 \xrightarrow{\text{F.T.}} &\int \mathrm{d}^3\boldsymbol{x}_1\mathrm{d}^3\boldsymbol{x}_2\mathrm{d}^3\boldsymbol{x}_3\, \delta(\boldsymbol{x}_2 - \boldsymbol{x}_3)\langle O(\boldsymbol{x}_1)O(\boldsymbol{x}_2)\rangle_0 e^{i\boldsymbol{k}_j\cdot\boldsymbol{x}_j}\\ &= 2^3\int \mathrm{d}^3\boldsymbol{u}\mathrm{d}^3\boldsymbol{v}\,\langle OO\rangle_0(v)e^{i\boldsymbol{u}\cdot(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)}e^{i\boldsymbol{v}\cdot(\boldsymbol{k}_2+\boldsymbol{k}_3-\boldsymbol{k}_1)}\\ &= 2^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\int \mathrm{d}^3\boldsymbol{v}\,\langle OO\rangle_0(v)e^{-2i\boldsymbol{v}\cdot\boldsymbol{k}_1}\\ &= \delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\langle O_{\boldsymbol{k}_1}O_{-\boldsymbol{k}_1}\rangle_0'. \end{aligned} \tag{6.30}$$

As before in performing the Fourier transform, we have not paid particular attention to the conventional factors of 2π. When we would include these, only an overall contribution to (6.17) will be obtained, when taking the squeezed limit.

Next we consider the Fourier transform $\langle O_{\boldsymbol{k}_1}O_{\boldsymbol{k}_2}O_{\boldsymbol{k}_3}\rangle_0$ of the first term of (6.17) in the limit $k_3 \to 0$,

$$\begin{aligned}\lim_{k_3\to 0}\langle O_{\boldsymbol{k}_1}O_{\boldsymbol{k}_2}O_{\boldsymbol{k}_3}\rangle_0' &= \int \mathrm{d}^3\boldsymbol{x}\langle O_{\boldsymbol{k}_1}O_{-\boldsymbol{k}_1}O(\boldsymbol{x})\rangle_0' = -\frac{\partial}{\partial u}\langle O_{\boldsymbol{k}_1}O_{-\boldsymbol{k}_1}\rangle_u' + O(u)\\ &= \frac{1}{\beta}\left(-k_1\frac{\partial}{\partial k_1} + 3 + 2\lambda\right)\langle O_{\boldsymbol{k}_1}O_{-\boldsymbol{k}_1}\rangle_u' + O(u)\\ &= \frac{-1}{\beta}\left(k_1\frac{\partial}{\partial k_1} - 3\right)\langle O_{\boldsymbol{k}_1}O_{-\boldsymbol{k}_1}\rangle_0' + \frac{2}{\beta}\lambda\langle O_{\boldsymbol{k}_1}O_{-\boldsymbol{k}_1}\rangle_0' + O(u), \end{aligned} \tag{6.31}$$

where in the second line, the Callan-Symanzik equation (3.3) is applied to the two-point function,

$$\left(k\frac{\partial}{\partial k} - \beta\frac{\partial}{\partial u} - 2\lambda - 3\right)\langle O_{\boldsymbol{k}}O_{-\boldsymbol{k}}\rangle_u' = 0.$$

Taking into account the prefactors $-\beta^3$ and β^2 in the Ward identity of three stress-energy tensors (6.17) and using $\lim_{k_3\to 0}\langle O_{\boldsymbol{k}_3} O_{-\boldsymbol{k}_3}\rangle'_0 = 0$, the Ward identity yields

$$\lim_{k_3\to 0}\langle \Theta_u(\boldsymbol{k}_1)\Theta_u(\boldsymbol{k}_2)\Theta_u(\boldsymbol{k}_3)\rangle'_u = \beta^2\left(k_1\frac{\partial}{\partial k_1} - 3\right)\langle O_{\boldsymbol{k}_1} O_{-\boldsymbol{k}_1}\rangle'_0 - 2\beta^2\lambda\langle O_{\boldsymbol{k}_1} O_{-\boldsymbol{k}_1}\rangle'_0 + \beta^2\lambda\left(2\langle O_{\boldsymbol{k}_1} O_{-\boldsymbol{k}_1}\rangle'_0 + 0\right) \tag{6.32}$$

in the squeezed limit. The local term of (6.17) exactly cancels against the $\frac{2}{\beta}\lambda$-contribution of the squeezed limit of $\langle OOO\rangle'_0$. Therefore, the squeezed limit of (6.11b) gives

$$\begin{aligned}\lim_{k_3\to 0}\langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle' &\propto \frac{\left(k_1\frac{\partial}{\partial k_1} - 3\right)\langle O_{\boldsymbol{k}_1} O_{-\boldsymbol{k}_1}\rangle'}{\beta^4\langle O_{\boldsymbol{k}_1} O_{-\boldsymbol{k}_1}\rangle'\langle O_{\boldsymbol{k}_2} O_{-\boldsymbol{k}_2}\rangle'\langle O_{\boldsymbol{k}_3} O_{-\boldsymbol{k}_3}\rangle'}\\ &= \langle \zeta_{\boldsymbol{k}_1}\zeta_{-\boldsymbol{k}_1}\rangle'^2\langle \zeta_{\boldsymbol{k}_3}\zeta_{-\boldsymbol{k}_3}\rangle'\left[-\frac{1}{\langle \zeta_{\boldsymbol{k}_1}\zeta_{-\boldsymbol{k}_1}\rangle'}\left(k_1\frac{\partial}{\partial k_1}\log\langle \zeta_{\boldsymbol{k}_1}\zeta_{-\boldsymbol{k}_1}\rangle' + 3\right)\right]\\ &= -\langle \zeta_{\boldsymbol{k}_1}\zeta_{-\boldsymbol{k}_1}\rangle'\langle \zeta_{\boldsymbol{k}_3}\zeta_{-\boldsymbol{k}_3}\rangle'\frac{\partial\log\left(k_1^3\langle \zeta_{\boldsymbol{k}_1}\zeta_{-\boldsymbol{k}_1}\rangle'\right)}{\partial\log k_1}.\end{aligned} \tag{6.33}$$

Hence, the squeezed limit consistency condition to lowest order in k_3, is immediate in the holographic description. The crucial step in the derivation is the second equality in (6.31), in which the three-point function with a zero Fourier mode is recognized as the first order contribution to the two-point function in a perturbed conformal field theory. In the squeezed limit the three-point function appears as a small perturbation of the two-point function around the conformal fixed point, leading to the tilt of the power spectrum. The other steps follow from a rewriting of this dependence, which can be seen as a slight rescaling, i.e. the infinitesimal coordinate transformation induced by the insertion of a stress-energy tensor. This interpretation is consistent with the original motivation behind the consistency condition, which observes that, once frozen, the only effect of the long wavelength mode to the bispectrum is to cause a local rescaling of the spatial distance scales, cf. (2.17) [57, 279].

A separate, independent derivation of the consistency condition (6.29) using similar ingredients has been given in [271], in which the (broken) conformal symmetry is described using a Ward identity. This Ward identity is equivalent to the Callan-Symanzik equation in our formalism, whereas our Ward identity relating Θ and O has no equivalent in the description of [271], which work directly with the gauge-invariant curvature perturbation ζ. Since Ward identities naturally relate an $n+1$-point correlation function with the variation of an n-point function, the observation in [271, 277] is that the consistency condition essentially *is* a Ward identity, applied to a particular conserved current. The current under consideration corresponds to a combination of a shift and dilational transformation, perturbing the system much in the same way as the Callan-Symanzik equation in our formalism. It would be very interesting to further investigate the connection between [271] and our work.

The consistency condition provides a powerful technique in the investigation of the structure of the correlation functions of curvature perturbations generated during inflation. Several approaches are considered in the literature to use the relation [271, 272] or possible generalizations [270] in order to restrict the n-point functions. In our approach, we use it as a consistency check and as an important guide to the full holographic bispectrum. In particular, the consistency condition explicitly shows that the local contributions in the bispectrum should combine in such a way that there is an overall contribution proportional to $n_s - 1 = 2\eta - 4\epsilon$. As we will see, in the full expression of the holographic bispectrum, the dominating slow-roll contribution is not at all obvious. With the consistency condition at our disposal, we have a strong indication where the important contributions should reside.

6.4.3 Three-point function

Bispectrum in a quasi-conformal fixed point

To understand inflationary non-Gaussianities, we now employ an analysis of the conformal three-point functions, similar to section 6.4.1. The Ward identity (6.17) consists of two contributions. The main contribution appears to come from the operator three-point function $\langle OOO\rangle_0$, but also a contact term proportional to the two-point function $\langle OO\rangle_0$ appears. Both of these contributions are again constrained by conformal symmetry, in particular [83, 84],

$$\langle O(\boldsymbol{x}_1)O(\boldsymbol{x}_2)O(\boldsymbol{x}_3)\rangle_0 = \frac{C}{(x_{12}x_{13}x_{23})^{\Delta}} + \text{contact terms}, \tag{6.34}$$

where $x_{jl} = |\boldsymbol{x}_j - \boldsymbol{x}_l|$ and C is the coefficient from the operator product expansion. The local contribution is generally not included in the literature, as it only contributes at coincident points. We have included it for completeness and wish to note that its contribution may well be relevant in the final expression.

Before we can compare any of the conformal structure with the inflationary bispectrum, we will need to Fourier transform these expressions. The contact terms are analyzed straightforwardly from (6.18) and (6.30),

$$\begin{aligned}\delta(\boldsymbol{x}_2 - \boldsymbol{x}_3)\langle O(\boldsymbol{x}_1)O(\boldsymbol{x}_2)\rangle_0 &\xrightarrow{\text{F.T.}} \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)\langle O_{\boldsymbol{k}_1}O_{-\boldsymbol{k}_1}\rangle_0' \\ &\propto \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)k_1^{2\Delta-3}.\end{aligned}$$

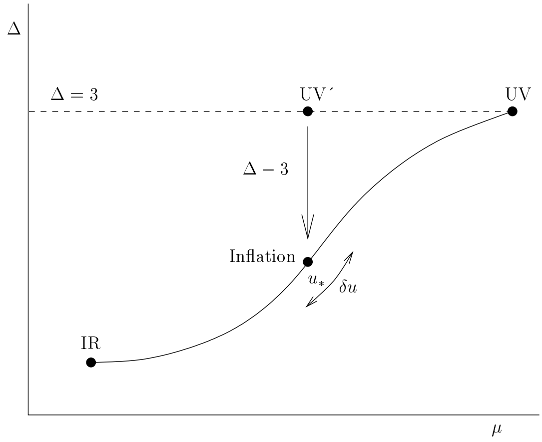


Figure 6.2: *The inflationary phase at an intermediate point in the renormalization group flow may be approximated by a conformal fixed point. The dashed line indicates a marginal renormalization group flow from one UV theory to another, for an operator with exactly marginal dimension* $\Delta = 3$*. The validity of the slow-roll approximation suggests that expressions in the quasi-fixed point can be approximated by a* $\Delta - 3$*-expansion. Around the quasi-fixed point, with coupling* u_**, the effect of inflation can be found through a further dependence on the renormalization group flow.*

For nearly marginal operators, this is a local contribution k_1^3. Adding the symmetrized terms, yields a contribution

$$Q(k_1, k_2, k_3) = k_1^3 + k_2^3 + k_3^3. \tag{6.35}$$

Fourier transforming (6.34) for arbitrary Δ is technically more involved [255] and requires some ingenuity in the analysis, cf. appendix 6.B

To understand the Fourier transform, we will make full use of the conceptual relation between slow-roll expansion and renormalization group flow, cf. section 6.2. As emphasized earlier, the dual to the inflationary phase appears as a point on (or short section of) the renormalization group flow at an intermediate stage. Because of the slow-roll expansion, at a given instance the inflationary expansion is that of a de Sitter evolution. Therefore, we can approximate the intermediate dual point on the renormalization group flow, by a nearby conformal fixed point, cf. figure 6.2. The difference between the conformal fixed point and the quasi-conformal fixed point is that the operator O does not describe a marginal renormalization group flow, i.e. $\Delta \neq 3$. Since the slow-roll expansion indicates that the operator is nearly marginal, we can approximate (6.34) as a Taylor series expansion with respect to $\lambda = \Delta - 3$,

$$\begin{aligned}\langle O(\boldsymbol{x}_1)O(\boldsymbol{x}_2)O(\boldsymbol{x}_3)\rangle_0 &\xrightarrow{\text{F.T.}} C\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)R_\Delta(k_1,k_2,k_3)\\ &= C\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\left[R_3(k_1,k_2,k_3)+\lambda R_3'(k_1,k_2,k_3)+\ldots\right].\end{aligned}$$

The evaluation of R_Δ to first order is a technical exercise, which we detail in appendix 6.B. Using the expression for three traces (6.17) and the relation (6.6) between slow-roll parameters and β and λ, the holographic prediction (6.11b) for the bispectrum in the quasi-conformal fixed point is

$$\langle\zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle' \propto \frac{1}{\epsilon^2}\frac{1}{k_1^3k_2^3k_3^3}\left[\beta C\left(R_3+(\epsilon-\eta)R_3'\right)+(\epsilon-\eta)Q\right]. \tag{6.36}$$

Next we will interpret this holographic prediction in the light of the gravitational calculation (2.22).

Local and non-local contributions to the bispectrum

The terms involving R_3 and Q have a clear interpretation. The local contribution Q seems to match precisely with the local contribution from the bispectrum (2.21),

$$\lambda Q(k_1,k_2,k_3) = (\epsilon-\eta)(k_1^3+k_2^3+k_3^3). \tag{6.37}$$

The momentum dependence as well as the parametric dependence on the slow-roll parameters agree, except for a relative factor of 2 between the ϵ- and η-terms.

The contribution $\beta C R_3$ has a clear interpretation as well. This zeroth order contribution, at $\Delta = 3$, has already been considered in [257], as a contribution from a direct three-point interaction $V'''\delta\phi^3$ [76–78, 257], cf. 6.B.3,

$$R_3(k_1,k_2,k_3) = (-1+\gamma+\log[-k_t\tau_*])\sum_{j=1}^{3}k_j^3 + k_1k_2k_3 - \sum_{j\neq l}k_jk_l^2. \tag{6.38}$$

For a massless spectator field it is the leading contribution, but as was argued in [57, 76], it appears at second order in the slow-roll expansion for the curvature perturbations. The prefactor βC can indeed be seen to be related to the third order slow-roll parameter, in agreement with (2.22). In a similar fashion to what we argued that $\lambda = \frac{\partial\beta}{\partial u}$ in the limit $u \to 0$, it is clear from the expression (6.14) of β in the conformal perturbation theory limit that the second derivative of the β function is equal to the operator product coefficient,

$$\frac{\partial^2\beta}{\partial u^2}(u) = 4\pi C + O(u).$$

Hence, from (6.6) we find

$$\beta C R_3 = \frac{1}{4\pi}\beta\frac{\partial^2\beta}{\partial u^2}R_3 = \frac{1}{2\pi}(\xi^2 - \frac{3}{2}\epsilon\eta + \epsilon^2)R_3, \tag{6.39}$$

which matches the result in (2.22).

In the holographic prediction (6.36), the final term $\lambda R_3'$ should then account for the remaining terms in (2.22). In particular, we are looking for contributions to the holographic bispectrum, which are both linear in the slow-roll parameters as well as have an interesting momentum behavior which mixes different momenta and has a $\frac{1}{k_t}$-dependence. As is clear from 6.B.4, the contribution from R_3' does contain more involved momentum dependence,

$$R_3' \subset \sum_j k_j^3(a + b\gamma)\log[-k_t\tau_*] + \frac{1}{k_t^3}\left(\sum_j k_j^6 + k_1^5k_2\log[-k_t\tau_*] + k_1^2k_3^4\mathrm{Li}_2\left[\frac{k_t}{k_t - 2k_1}\right]\right),$$

with a and b numerical constants. However, this contribution is multiplied by $\beta C\lambda$, which seems to be higher order in slow-roll, cf. [79].

From the squeezed limit we know that the latter observation is misleading. In the consistency condition we have found more terms that are linear in the slow-roll parameters than just the local term Q. Although the squeezed limit is often interpreted as a small momentum limit, $k_3 \to 0$, in principle it is a relative statement, $k_3 \ll k_1, k_2$. Hence, because of momentum conservation, the squeezed limit could also be seen as a high frequency limit $k_1 \to \infty$. In this form, the squeezed limit tells us that the high frequency dominant part of the holographic bispectrum does contain additional linear dependence on the slow-roll parameters, despite the explicit second order dependence of βC.

The question is how one could extract the "hidden" linear parametric dependence. Since the bispectrum reveals its hidden parametric dependence in the high energy regime, an obvious suggestion is to consider the counterterms of the regulated expressions. One objection might be that a counterterm is not able of producing a non-local contribution of the form $\frac{1}{k_t}$. Counterterms generically have at most polynomial dependence on the momenta, which vanish in the small frequency limit. A $\frac{1}{k_t}$-behavior looks awfully divergent in this limit. However, in the bispectrum (2.21), the numerator of the $\frac{1}{k_t}$-terms ensures that there is no divergent behavior for low frequency modes. We conclude that counterterms can produce $\frac{1}{k_t}$-terms and should therefore be studied in more detail. Although R_3' comes with explicit divergent terms, cf. (6.68), these are only homogeneous of degree 1 in the momenta and therefore do not resemble any of the terms in (2.22).

Therefore, an alternative analysis is called for. In fact, the derivation of the squeezed limit (6.31) and in particular the explicit division by $\beta \propto (\Delta - 3) = \lambda$ necessary for the cancelation against the two-point correlation function contribution of (6.17), indicates that non-analytic behavior in λ plays an essential role. This underlines the need for a Laurent series rather than a Taylor series. By analyzing the Laurent series of $\langle OOO \rangle_0$ in λ, one should be able to uncover the dominant contributions, which is the topic of future work.

The renormalization group flow

As with the power spectrum, to truly compare the inflationary bispectrum with the holographic prediction, we will need to deviate away from the quasi-de Sitter phase using the renormalization group flow. Compared to the two-point case, the sought-for change in functional dependence is different. Whereas the slow-roll result for the two-point function has slow-roll dependence in the exponent of the momentum k, the slow-roll dependence of the three-point function is usually found in the overall amplitude of the bispectrum, cf. (2.21).

To calculate $\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle$, we will have to find the renormalized expressions for $\langle \Theta_{\boldsymbol{k}_j} \Theta_{-\boldsymbol{k}_j} \rangle'_u$ and $\langle \Theta_{\boldsymbol{k}_1} \Theta_{\boldsymbol{k}_2} \Theta_{\boldsymbol{k}_3} \rangle'_u$. As we have seen in the previous section, each of the $\langle \Theta_{\boldsymbol{k}_j} \Theta_{-\boldsymbol{k}_j} \rangle'_u$ in the denominator of (6.11b), is given by

$$\langle \Theta_{\boldsymbol{k}_j} \Theta_{-\boldsymbol{k}_j} \rangle'_u = Z^2 \beta^2 k^3,$$

where $Z(u)$ is given by (6.23). The stress-energy tensor three-point function can be found by a comparable calculation. Using dimensional analysis,

$$\left(\mu \frac{\partial}{\partial \mu} + \sum_j k_j \frac{\partial}{\partial k_j} \right) \langle \Theta_u(\boldsymbol{k}_1) \Theta_u(\boldsymbol{k}_2) \Theta_u(\boldsymbol{k}_3) \rangle'_u = 3 \langle \Theta_u(\boldsymbol{k}_1) \Theta_u(\boldsymbol{k}_2) \Theta_u(\boldsymbol{k}_3) \rangle'_u,$$

we can write the Callan-Symanzik equation for the three-point function of the exactly marginal stress-energy tensor as

$$\left(k_j \frac{\partial}{\partial k_j} - 3 - \beta \frac{\partial}{\partial u} \right) \langle \Theta_u(\boldsymbol{k}_1) \Theta_u(\boldsymbol{k}_2) \Theta_u(\boldsymbol{k}_3) \rangle'_u = 0. \tag{6.40}$$

This equation determines $\tilde{Z}(u)$ in the ansatz

$$\langle \Theta_u(\boldsymbol{k}_1) \Theta_u(\boldsymbol{k}_2) \Theta_u(\boldsymbol{k}_3) \rangle'_u = -\tilde{Z}^3 \beta^3 C R(k_1, k_2, k_3) + \tilde{Z}^2 \beta^2 \lambda Q(k_1, k_2, k_3),$$

where both R and Q are homogeneous in k_1, k_2, k_3 of degree 3. This latter fact follows from the approximation we consider, in which the quasi-fixed point describing inflation is expanded with respect to a marginal conformal dimension, which ensures the degree of homogeneity of the three-point function to be 3.

As a first attempt of finding a solution for $\tilde{Z}$, we consider the case in which $\lambda(u)$ is constant. Then, $\tilde{Z}$ is again given by (6.23). Collecting results,

$$\begin{aligned}\langle\zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle' &= Z^{-6}\beta^{-6}(k_1k_2k_3)^{-3}\left(-Z^3\beta^3CR(k_1,k_2,k_3)+Z^2\beta^2\lambda Q(k_1,k_2,k_3)\right)\\ &= H^4\beta^{-4}(k_1k_2k_3)^{-3}(HZ)^{-4}\left(-Z\beta CR(k_1,k_2,k_3)+\lambda Q(k_1,k_2,k_3)\right).\end{aligned} \tag{6.41}$$

Again we have chosen to explicitly isolate the required H-dependence of the bispectrum. The overall factor $(HZ)^{-4}$ contributes additional factors of β and λ via

$$(HZ)^{-4} = e^{2\int_0^u \mathrm{d}u'\left(\beta(u')+2\frac{\lambda(u')}{\beta(u')}\right)} = 1+2\int_0^u \mathrm{d}u'\left(\beta(u')+2\frac{\lambda(u')}{\beta(u')}\right)+\dots. \tag{6.42}$$

Performing a similar expansion for $Z = e^{-\int\lambda/\beta}$, the holographic bispectrum is given by

$$\begin{aligned}B_\zeta(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) =& \frac{H^4}{\beta^4(k_1k_2k_3)^3}\left(1+2\int_0^u \mathrm{d}u'\left(\beta(u')+2\frac{\lambda(u')}{\beta(u')}\right)\right)\times\\ &\left[\lambda Q(k_1,k_2,k_3)-\beta C\left(1-\int_0^u \mathrm{d}u'\,\frac{\lambda(u')}{\beta(u')}\right)R(k_1,k_2,k_3)\right].\end{aligned} \tag{6.43}$$

This expression makes clear that a renormalization of the bispectrum will only contribute to higher order in slow-roll. Since we explicitly consider the renormalization as a variation of the slow-roll parameters, this is little surprising, but it also means that, for the time being, we do not have to consider the effect of renormalization on the bispectrum.

6.5 Conclusions

In this chapter we have considered the late-time de Sitter symmetry constraints on the two- and three-point correlation functions of curvature perturbations generated during single field slow-roll inflation. The fact that the inflationary evolution is a near-de Sitter phase is captured by considering the renormalization of the Ward identities relating the stress-energy tensor and the operators holographically dual to the inflaton field.

In the case of the power spectrum, the symmetry constraints are sufficient to retrieve the near scale invariance that is characteristic for slow-roll inflation. The fact that the power spectrum can be retrieved from the renormalization group flow, suggests a type of universality in the two-point function. If a dS/CFT-correspondence would be found, this universality contrasts with the finetuning problem for inflation, whose dynamics does seem to depend sensitively on the slow-roll parameters.

In the case of the bispectrum, our study is, as yet, inconclusive as to whether symmetries are sufficient to specify the three-point function completely. Several ingredients of the bispectrum can be seen to appear directly from our holographic study, but other ingredients, most notably the appearance of a $\frac{1}{k_t}$-term at linear order in slow-roll, remain hidden in the approach. Both ingredients do appear in one or more of the terms that contribute to the bispectrum, but the necessary combination does not appear. From the correctness of the consistency condition in the squeezed limit, we obtain tantalizing hints that counterterms and/or a Laurent expansion in λ should contain the required momentum-dependence, at the right order in slow-roll. However, a first study into the subtle regularization procedure, has not proven to be successful.

The methods we have used resemble techniques from the AdS/CFT-correspondence. If a holographic understanding of the primordial bispectrum could be found, then our analysis should be a first step into a further understanding of a possible dS/CFT-correspondence. At this stage, however, the dS/CFT-correspondence is only considered at the level of the symmetries, which is a far more general statement than the intricate details of a holographic correspondence. Moreover, one could worry in which regime we consider the field theory. Since we are set out to study a phenomenon in classical general relativity, a direct application of holography would suggest the dual theory to be strongly coupled. However, for us, this question is irrelevant, since we never consider the coupling constant of the putative dual theory and base our results solely on the restrictive power of the symmetries.

A subtle issue in our methods is the use of conformal perturbation theory. Conformal perturbation theory requires that the coupling that drives the theory away from the conformal fixed point, is small $u \ll 1$. This requirement does not seem to follow immediately from the slow-roll expansion. For this reason, it is unclear whether or when we are entitled to rely upon conformal perturbation techniques. Possibly, some of the unresolved puzzles are caused by the absence of a full understanding of the applicability of conformal perturbation theory in this context.

In conclusion, we have presented a detailed but not yet finalized understanding of how constraints from the asymptotic conformal symmetry of de Sitter space may restrict the two- and three-point functions of primordial density fluctuations generated during inflation. The three-point correlation function seems to subtly depend on regularization and renormalization, which is partly beyond the scope of this study. Clearly, it would be interesting to fully develop the necessary techniques to study the three-point function. The bulk of the ingredients essential for this analysis has been laid out in this thesis.

6.A Ward identities of multiple trace insertions

6.A.1 Ward identities for a perturbed action

The Ward identity of any symmetry generator can be derived from considering infinitesimal transformations of the correlation function,

$$\langle X \rangle_u = \frac{1}{Z} \int \mathcal{D}O \, X e^{-S_u[O]}, \tag{6.44}$$

of a product of operators $X = O(\boldsymbol{x}_1) \dots O(\boldsymbol{x}_n)$. We specifically evaluate the expectation value with respect to a *perturbed* conformal field theory,

$$S_u[O] = S_0[O] + \int \mathrm{d}^3\boldsymbol{x} \, uO(\boldsymbol{x}) + S_{\text{c.t.}}(\mu). \tag{6.45}$$

The last term is necessary to regulate any divergences, which we have introduced by turning on a scale in the form of a non-marginal operator O. The precise form of $S_{\text{c.t.}}$ is not clear at this stage, but its presence can later be used to regulate any divergences in the correlators. Under the transformation $O \to O' = O(\boldsymbol{x}) - i\omega_a G_a(\boldsymbol{x}) O(\boldsymbol{x})$, the action transforms as

$$\begin{aligned} S_u[O'] = S_u[O] &- \int \mathrm{d}^3\boldsymbol{x} \, \omega_a(\boldsymbol{x}) \partial_\mu j^\mu_a(\boldsymbol{x}) - i \int \mathrm{d}^3\boldsymbol{x} \, \omega_a(\boldsymbol{x}) G_a(\boldsymbol{x}) uO(\boldsymbol{x}) \\ &- \frac{1}{2} \int \mathrm{d}^3\boldsymbol{x} \, \omega_a(\boldsymbol{x}) G_a(\boldsymbol{x}) \omega_b(\boldsymbol{x}) G_b(\boldsymbol{x}) uO(\boldsymbol{x}) + \dots, \end{aligned}$$

to second order in ω_a, where j^μ_a is the Noether current of the transformation in the conformal field theory $S_0[O]$ at the fixed point. The last term is a contact term, which we have included because it is of second order in ω_a. It stems from the transformation $O \to O'$ in the perturbation-part of the action (6.45). In principle, the unperturbed conformal action $S_0[O]$ also obtains a contribution at second order as a result of the transformation to second order. However, this contribution is difficult to retrieve from first principles, as the Noether current of the transformation is only defined infinitesimally. It is therefore left implicit in the $\dots$, while its effect on the Ward identity will later be inferred by different means. Hence, for the moment we find

$$e^{-S_u[O']} = e^{-S_u[O]}\Big(1 - \int \mathrm{d}^3\boldsymbol{x}\, \omega_a(\boldsymbol{x})\delta L_a(\boldsymbol{x}) + \frac{1}{2}\int \mathrm{d}^3\boldsymbol{x}\mathrm{d}\boldsymbol{y}\, \omega_a(\boldsymbol{x})\omega_b(\boldsymbol{y})\delta L_a(\boldsymbol{x})\delta L_b(\boldsymbol{y})\Big)$$
$$\times\Big(1 + \frac{1}{2}\int \mathrm{d}^3\boldsymbol{x}\, \omega_a(\boldsymbol{x})G_a(\boldsymbol{x})\omega_b(\boldsymbol{x})G_b(\boldsymbol{x})uO(\boldsymbol{x})\Big),$$

up to second order, where $\delta L_a(\boldsymbol{x})$ is shorthand notation for

$$\delta L_a(\boldsymbol{x}) = -\partial_\mu j^\mu_a(\boldsymbol{x}) - iuG_a(\boldsymbol{x})O(\boldsymbol{x}).$$

Infinitesimally transforming X gives

$$X' = e^{-i\omega_a(\boldsymbol{x})G_a(\boldsymbol{x})}X = X - i\sum_k \omega_{a,k}G_{a,k}X - \frac{1}{2}\sum_{k,l}\omega_{a,k}G_{a,k}\omega_{b,l}G_{b,l}X,$$

where $\omega_{a,k} = \omega_a(\boldsymbol{x}_k)$ and $G_{a,k} = G_a(\boldsymbol{x}_k)$ acts on the k'th $O(\boldsymbol{x}_k)$ inside X.

Assuming the measure is invariant, $\mathcal{D}O' = \mathcal{D}O$, comparison of the transformed expression for $\langle X\rangle_u$ and (6.44) gives

$$0 = \int \mathrm{d}^3\boldsymbol{x}\, \omega_a(\boldsymbol{x})\langle \delta L_a(\boldsymbol{x})X\rangle_u + i\sum_k \omega_{a,k}G_{a,k}\langle X\rangle_u, \tag{6.46}$$

to first order in ω. Similarly, to second order it gives

$$0 = \int \mathrm{d}^3\boldsymbol{x}\mathrm{d}^3\boldsymbol{y}\, \omega_a(\boldsymbol{x})\omega_b(\boldsymbol{y})\langle \delta L_a(\boldsymbol{x})\delta L_b(\boldsymbol{y})X\rangle_u + 2i\sum_k \omega_{a,k}G_{a,k}\int \mathrm{d}^3\boldsymbol{x}\, \omega_b(\boldsymbol{x})\langle \delta L_b(\boldsymbol{x})X\rangle_u$$
$$- \sum_{k,l}\omega_{a,k}G_{a,k}\omega_{b,l}G_{b,l}\langle X\rangle_u + \int \mathrm{d}^3\boldsymbol{x}\, \omega_a(\boldsymbol{x})G_a(\boldsymbol{x})\omega_b(\boldsymbol{x})G_b(\boldsymbol{x})u\langle O(\boldsymbol{x})X\rangle_u$$
$$= \int \mathrm{d}^3\boldsymbol{x}\mathrm{d}^3\boldsymbol{y}\, \omega_a(\boldsymbol{x})\omega_b(\boldsymbol{y})\langle \delta L_a(\boldsymbol{x})\delta L_b(\boldsymbol{y})X\rangle_u + \sum_{k,l}\omega_{a,k}G_{a,k}\omega_{b,l}G_{b,l}\langle X\rangle_u$$
$$+ \int \mathrm{d}^3\boldsymbol{x}\, \omega_a(\boldsymbol{x})G_a(\boldsymbol{x})\omega_b(\boldsymbol{x})G_b(\boldsymbol{x})u\langle O(\boldsymbol{x})X\rangle_u. \tag{6.47}$$

6.A.2 Alternative derivation

As emphasized, these expressions have been derived with respect to the perturbed theory. Since Ward identities are usually derived with respect to an invariant theory, our approach may raise questions on its correctness. We therefore present a different calculation, which is independent from the previous one. We use the fact that

$$\langle X\rangle_u = \langle X e^{-\int \mathrm{d}^3\boldsymbol{x}\, uO(\boldsymbol{x})}\rangle_0$$

and apply it to the standard $u = 0$ Ward identities,

$$\langle \partial_\mu j_a^\mu(\boldsymbol{x}) X\rangle_0 = i \sum_k \delta(\boldsymbol{x} - \boldsymbol{x}_k) G_{a,k} \langle X\rangle_0, \tag{6.48a}$$

$$\langle \partial_\mu \partial_\nu j_a^\mu(\boldsymbol{x}) j_b^\nu(\boldsymbol{y}) X\rangle_0 = -\sum_{k,l} \delta(\boldsymbol{x} - \boldsymbol{x}_k)\delta(\boldsymbol{y} - \boldsymbol{x}_l) G_{a,k} G_{b,l} \langle X\rangle_0. \tag{6.48b}$$

Applying (6.48a) repeatedly to the Taylor expansion of the exponential yields

$$\begin{aligned}
\langle \partial_\mu j_a^\mu(\boldsymbol{x}) X\rangle_u &= \sum_n \frac{(-u)^n}{n!} \int \mathrm{d}^3\boldsymbol{x}_1 \dots \mathrm{d}^3\boldsymbol{x}_n \, \langle \partial_\mu j_a^\mu(\boldsymbol{x}) O(\boldsymbol{x}_1) \dots O(\boldsymbol{x}_n) X\rangle_0 \\
&= \sum_n \frac{(-u)^n}{n!} \int \mathrm{d}^3\boldsymbol{x}_1 \dots \mathrm{d}^3\boldsymbol{x}_n \times \\
&\qquad i \left(\sum_{k=1}^{n} + \sum_{k=X} \right) \delta(\boldsymbol{x} - \boldsymbol{x}_k) G_{a,k} \langle O(\boldsymbol{x}_1) \dots O(\boldsymbol{x}_n) X\rangle_0 \\
&= -iu G_a(\boldsymbol{x}) \langle O(\boldsymbol{x}) X\rangle_u + i \sum_k \delta(\boldsymbol{x} - \boldsymbol{x}_k) G_{a,k} \langle X\rangle_u,
\end{aligned} \tag{6.49}$$

where the summation $\sum_{k=1}^{n}$ runs over all first n operators O coming from the exponent and where $\sum_{k=X}$ runs over all remaining operators O inside X. Similarly, for the double Ward identity,

$$\begin{aligned}
\langle \partial_\mu \partial_\nu j_a^\mu(\boldsymbol{x}) j_b^\nu(\boldsymbol{y}) X\rangle_u &= \sum_n \frac{(-u)^n}{n!} \int \mathrm{d}^3\boldsymbol{x}_1 \dots \mathrm{d}^3\boldsymbol{x}_n \times \\
&\Bigg(- \left[\sum_{k\neq l}^{n} + \sum_{k=1}^{n} \sum_{l=X} + \sum_{k=X} \sum_{l=1}^{n} + \sum_{k,l=X} \right] \delta(\boldsymbol{x} - \boldsymbol{x}_k)\delta(\boldsymbol{y} - \boldsymbol{x}_l) G_{a,k} G_{b,l} \\
&\qquad - \delta(\boldsymbol{x} - \boldsymbol{y}) \sum_{k=1}^{n} \delta(\boldsymbol{x} - \boldsymbol{x}_k) G_{a,k} G_{b,k} \Bigg) \langle O(\boldsymbol{x}_1) \dots O(\boldsymbol{x}_n) X\rangle_0 \\
&= -u^2 G_a(\boldsymbol{x}) G_b(\boldsymbol{y}) \langle O(\boldsymbol{x}) O(\boldsymbol{y}) X\rangle_u + u\delta(\boldsymbol{x} - \boldsymbol{y}) G_a(\boldsymbol{x}) G_b(\boldsymbol{x}) \langle O(\boldsymbol{x}) X\rangle_u \\
&\quad + u G_a(\boldsymbol{x}) \sum_k \delta(\boldsymbol{y} - \boldsymbol{x}_k) G_{b,k} \langle O(\boldsymbol{x}) X\rangle_u + \boldsymbol{x} \leftrightarrow \boldsymbol{y} \\
&\quad - \sum_{k,l} \delta(\boldsymbol{x} - \boldsymbol{x}_k)\delta(\boldsymbol{y} - \boldsymbol{x}_l) G_{a,k} G_{b,l} \langle X\rangle_u,
\end{aligned} \tag{6.50}$$

where the summation $\sum_k$ on the right hand side of the last equation only runs over the operators inside X. We verify that

$$\langle \delta L_a(\boldsymbol{x}) X\rangle_u = -i \sum_k \delta(\boldsymbol{x}-\boldsymbol{x}_k) G_{a,k} \langle X\rangle_u, \tag{6.51a}$$

$$\begin{aligned}\langle \delta L_a(\boldsymbol{x})\delta L_b(\boldsymbol{y}) X\rangle_u &= \langle \partial_\mu \partial_\nu j_a^\mu(\boldsymbol{x}) j_b^\nu(\boldsymbol{y}) X\rangle_u - u^2 G_a(\boldsymbol{x}) G_b(\boldsymbol{y}) \langle O(\boldsymbol{x}) O(\boldsymbol{y}) X\rangle_u \\ &\quad + iu G_a(\boldsymbol{x}) \langle \partial_\nu j_b^\nu(\boldsymbol{y}) O(\boldsymbol{x}) X\rangle_u + \boldsymbol{x} \leftrightarrow \boldsymbol{y} \\ &= -\sum_{k,l} \delta(\boldsymbol{x}-\boldsymbol{x}_k)\delta(\boldsymbol{y}-\boldsymbol{x}_l) G_{a,k} G_{b,l} \langle X\rangle_u \\ &\quad - u\delta(\boldsymbol{x}-\boldsymbol{y}) G_a(\boldsymbol{x}) G_b(\boldsymbol{x}) \langle O(\boldsymbol{x}) X\rangle_u, \end{aligned} \tag{6.51b}$$

in agreement with (6.46–6.47).

6.A.3 Trace Ward identities

Returning to the integral expressions (6.46–6.47), we consider the special choice for $\omega_a = \omega(\boldsymbol{x})(1_D, -x^\nu 1_T)$ to derive the trace insertion formulae, where 1_D and 1_T means we consider the dilational and translational transformations. The combined effect of this familiar combination [98, 271, 289] yields

$$\begin{aligned}\omega_a(\boldsymbol{x}) G_a(\boldsymbol{x}) &= \omega(\boldsymbol{x}) \left(-i(x^\mu \partial_\mu + \Delta) - x^\nu(-i\partial_\nu)\right) = -i\omega(\boldsymbol{x})\Delta, \\ \omega_a(x)\delta L_a(\boldsymbol{x}) &= \omega(\boldsymbol{x})\left(-\Theta_0(\boldsymbol{x}) - u\Delta O(\boldsymbol{x})\right) = \omega(\boldsymbol{x})\left(-\Theta_u(\boldsymbol{x}) - u(\Delta-3) O(\boldsymbol{x})\right).\end{aligned}$$

In the last line we rewrite the answer in terms of the stress-energy tensor of the *perturbed* theory,

$$\Theta_u = \frac{-2}{\sqrt{h}} h^{\alpha\beta} \frac{\delta S}{\delta h^{\alpha\beta}}\bigg|_{h_{\alpha\beta}=\delta_{\alpha\beta}} = \Theta_0 + \frac{-2}{\sqrt{h}} h^{\alpha\beta}\, \frac{-1}{2}\, \sqrt{h} h_{\alpha\beta}\bigg|_{h_{\alpha\beta}=\delta_{\alpha\beta}} uO = \Theta_0 + 3uO. \tag{6.52}$$

The Ward identities can then be written as

$$\langle \Theta_u(\boldsymbol{x}) X\rangle_u = -u(\Delta-3)\langle O(\boldsymbol{x}) X\rangle_u + \sum_k \delta(\boldsymbol{x}-\boldsymbol{x}_k)\Delta_k \langle X\rangle_u \tag{6.53}$$

and

$$\begin{aligned}\langle\Theta_u(\boldsymbol{x})\Theta_u(\boldsymbol{y})X\rangle_u &= \sum_{k,l}\delta(\boldsymbol{x}-\boldsymbol{x}_k)\Delta_k\delta(\boldsymbol{y}-\boldsymbol{x}_l)\Delta_l\langle X\rangle_u + u\delta(\boldsymbol{x}-\boldsymbol{y})\Delta^2\langle O(\boldsymbol{x})X\rangle_u \\ &\quad - u(\Delta-3)\langle\Theta_u(\boldsymbol{x})O(\boldsymbol{y})X\rangle_u - \boldsymbol{x}\leftrightarrow\boldsymbol{y} - u^2(\Delta-3)^2\langle O(\boldsymbol{x})O(\boldsymbol{y})X\rangle_u \\ &= \sum_{k,l}\delta(\boldsymbol{x}-\boldsymbol{x}_k)\Delta_k\delta(\boldsymbol{y}-\boldsymbol{x}_l)\Delta_l\langle X\rangle_u \\ &\quad + u\delta(\boldsymbol{x}-\boldsymbol{y})\left(\Delta^2-2(\Delta-3)\Delta\right)\langle O(\boldsymbol{x})X\rangle_u \\ &\quad - u(\Delta-3)\sum_k\delta(\boldsymbol{x}-\boldsymbol{x}_k)\Delta_k\langle O(\boldsymbol{y})X\rangle_u - \boldsymbol{x}\leftrightarrow\boldsymbol{y} \\ &\quad + u^2(\Delta-3)^2\langle O(\boldsymbol{x})O(\boldsymbol{y})X\rangle_u, \end{aligned} \tag{6.54}$$

where Δ_k is the (full) scaling dimension of the k'th operator $O(\boldsymbol{x}_k)$ inside X.

At this stage, we have to reflect on the correctness of the expressions by performing a consistency check on the two-point function (6.54). When we perturb the conformal field theory with a purely marginal operator, $\Delta = 3$, the renormalization group flow remains in a (different) conformal field theory. The trace Θ_u of this perturbed theory is still vanishing. Hence, the two-point correlation function of the perturbed stress-energy tensor should vanish with respect to the perturbed theory. However, substituting $\Delta = 3$ into our expression,

$$\langle\Theta_u(\boldsymbol{x})\Theta_u(\boldsymbol{y})\rangle_u = u^2(\Delta-3)^2\langle O(\boldsymbol{x})O(\boldsymbol{y})\rangle_u - u\delta(\boldsymbol{x}-\boldsymbol{y})\left((\Delta-3)^2-3^2\right)\langle O(\boldsymbol{x})\rangle_u,$$

does not yield zero. Clearly in our derivation we must have missed a term of the form $-3^2u\delta(\boldsymbol{x}-\boldsymbol{y})\langle O(\boldsymbol{x})\rangle_u$. This term is a contact term and should stem from the neglected second order transformation of $S_0[O]$, which will contain a contribution from the variation of the Noether current. In [281–283] such a contribution is explicitly included for the consistency of the expressions. In our case we can infer the final result based on conceptual reasoning. We thus employ the expression

$$\begin{aligned}\langle\Theta_u(\boldsymbol{x})\Theta_u(\boldsymbol{y})X\rangle_u &= u^2(\Delta-3)^2\langle O(\boldsymbol{x})O(\boldsymbol{y})X\rangle_u - u(\Delta-3)^2\delta(\boldsymbol{x}-\boldsymbol{y})\langle O(\boldsymbol{x})X\rangle_u \\ &\quad - u(\Delta-3)\sum_k\delta(\boldsymbol{x}-\boldsymbol{x}_k)\Delta_k\langle O(\boldsymbol{y})X\rangle_u - \boldsymbol{x}\leftrightarrow\boldsymbol{y} \\ &\quad + \sum_{k,l}\delta(\boldsymbol{x}-\boldsymbol{x}_k)\Delta_k\delta(\boldsymbol{y}-\boldsymbol{x}_l)\Delta_l\langle X\rangle_u, \end{aligned} \tag{6.55}$$

for the double Ward identity.

We similarly derive the correlation function of three traces. Using either of the two methods described above to third order in ω, the Ward identity of three current insertions gives

$$0 = \int \mathrm{d}^3\boldsymbol{x}\mathrm{d}^3\boldsymbol{y}\mathrm{d}^3\boldsymbol{z}\, \omega_a(\boldsymbol{x})\omega_b(\boldsymbol{y})\omega_c(\boldsymbol{z})\langle \delta L_a(\boldsymbol{x})\delta L_b(\boldsymbol{y})\delta L_c(\boldsymbol{z}) X\rangle_u$$
$$- i \sum_{k,l,m} \omega_{a,k} G_{a,k}\omega_{b,l} G_{b,l}\omega_{c,m} G_{c,m}\langle X\rangle_u + \ldots, \qquad (6.56)$$

where the ... contain terms of order u. These terms are double contact terms and, as is clear from the main text, are not relevant for our purposes. Choosing again $\omega_a = \omega(\boldsymbol{x})(1_D, -x^\nu 1_T)$ and using the single and double trace-inserted Ward identities (6.53), (6.55), the three-point function of the stress-energy tensor reads

$$\begin{aligned}\langle \Theta_u(\boldsymbol{x})\Theta_u(\boldsymbol{y})\Theta_u(\boldsymbol{z})X\rangle_u &= -u^3(\Delta-3)^3\langle O(\boldsymbol{x})O(\boldsymbol{y})O(\boldsymbol{z})X\rangle_u \\ &\quad - u^2(\Delta-3)^2\langle \Theta_u(\boldsymbol{x})O(\boldsymbol{y})O(\boldsymbol{z})X\rangle_u - \boldsymbol{x}\leftrightarrow\boldsymbol{y} - \boldsymbol{x}\leftrightarrow\boldsymbol{z} \\ &\quad - u(\Delta-3)\langle \Theta_u(\boldsymbol{x})\Theta_u(\boldsymbol{y})O(\boldsymbol{z})X\rangle_u - \boldsymbol{z}\leftrightarrow\boldsymbol{x} - \boldsymbol{z}\leftrightarrow\boldsymbol{y} + \ldots \\ &= -u^3(\Delta-3)^3\langle O(\boldsymbol{x})O(\boldsymbol{y})O(\boldsymbol{z})X\rangle_u \\ &\quad + u^2(\Delta-3)^3\delta(\boldsymbol{y}-\boldsymbol{z})\langle O(\boldsymbol{x})O(\boldsymbol{y})X\rangle_u + \boldsymbol{x}\leftrightarrow\boldsymbol{y} + \boldsymbol{y}\leftrightarrow\boldsymbol{z} + \ldots,\end{aligned} \qquad (6.57)$$

where the ... contain highly local contributions.

6.B The Fourier transform of the three-point function

6.B.1 Feynman parameters

In a conformal field theory, the three-point correlation function of an operator O with conformal dimension Δ is determined by the symmetries to be of the form

$$\langle O(\boldsymbol{x}_1)O(\boldsymbol{x}_2)O(\boldsymbol{x}_3)\rangle_0 = \frac{C}{(x_{12}x_{13}x_{23})^\Delta},$$

in position-space. To find the momentum dependence, one has to perform a Fourier transform,

$$F_\Delta(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = C \int \mathrm{d}^3\boldsymbol{x}_1\mathrm{d}^3\boldsymbol{x}_2\mathrm{d}^3\boldsymbol{x}_3\, e^{i\sum_j \boldsymbol{k}_j\cdot\boldsymbol{x}_j}(x_{12}x_{13}x_{23})^{-\Delta}.$$

In practice, for arbitrary Δ, this quickly becomes difficult. In the appendix of [255] it is explained how the result can be written as an integral over Feynman parameters. For completeness, we shortly review this approach here.

Inserting the Fourier transform,

$$|\boldsymbol{x}|^{-\Delta} = B(\Delta)(2\pi)^{-3} \int \mathrm{d}^3\boldsymbol{p}\, |\boldsymbol{p}|^{\Delta-3} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}, \qquad \text{where } B(\Delta) = 2^{3-\Delta}\pi^{3/2}\frac{\Gamma\left(\frac{3-\Delta}{2}\right)}{\Gamma\left(\frac{\Delta}{2}\right)},$$

into F_Δ, the $\boldsymbol{x}_j$ integrals can be performed explicitly and also two of the momentum integrals can be done to give

$$F_\Delta(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = CB^3(\Delta)\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \int \mathrm{d}^3\boldsymbol{p}\, (p^2|\boldsymbol{p} - \boldsymbol{k}_1|^2|\boldsymbol{p} + \boldsymbol{k}_2|^2)^{\lambda/2}, \tag{6.58}$$

where $\lambda = \Delta - 3$. As is usual, the difficult dot-product dependence in the integrand can be rewritten with the use of Feynman parameters. Using

$$A^{-\alpha}B^{-\beta}C^{-\gamma} = \frac{\Gamma(\alpha+\beta+\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)} \int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v \times$$
$$u^{\alpha-1}(1-u)^{\beta-1}v^{\alpha+\beta-1}(1-v)^{\gamma-1}\left[uvA + (1-u)vB + (1-v)C\right]^{-\alpha-\beta-\gamma},$$

the integral in (6.58) equals

$$\frac{\Gamma(-3\lambda/2)}{\Gamma^3(-\lambda/2)} \int_0^1 \int_0^1 \mathrm{d}u\mathrm{d}v\ \left[(1-u)(1-v)\right]^{-(1+\lambda/2)} v^{-(1+\lambda)} G(u, v, \boldsymbol{k}_1, \boldsymbol{k}_2),$$

where

$$G(u, v, \boldsymbol{k}_1, \boldsymbol{k}_2) = \int \mathrm{d}^3\boldsymbol{p} \left[uvp^2 + v(1-u)|\boldsymbol{p} - \boldsymbol{k}_1|^2 + (1-v)|\boldsymbol{p} + \boldsymbol{k}_2|^2\right]^{3\lambda/2}$$
$$= \int \mathrm{d}^3\boldsymbol{p}\, (p^2 + a^2)^{3\lambda/2} = \pi^{3/2}\frac{\Gamma(-\frac{3}{2}(1+\lambda))}{\Gamma(-3\lambda/2)}(a^2)^{\frac{3}{2}(1+\lambda)}.$$

The second identity follows from a shift of the momentum $\boldsymbol{p}$. We have written a as a shorthand notation for

$$a^2 = (1-u)v(1-(1-u)v)k_1^2 + v(1-v)k_2^2 + 2v(1-u)(1-v)\boldsymbol{k}_1 \cdot \boldsymbol{k}_2,$$

which can be expressed fully in terms of the sizes of the three momenta, k_1, k_2 and k_3 due to momentum conservation. Collecting results and changing variables $u \to 1-u$, $v \to 1-v$, the Fourier transform of the three-point function is given by

$$F_\Delta(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = C\frac{2^{3-3\Delta}(2\pi)^6}{\Gamma^2(\frac{\Delta}{2})}\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)k_1^{3\Delta-6}S_\Delta(X, Y), \tag{6.59}$$

where the shape function $S_\Delta(X,Y)$ of the ratios $X = \frac{k_2^2}{k_1^2}$, $Y = \frac{k_3^2}{k_1^2}$ is given by [255]

$$\begin{aligned} S_\Delta(X,Y) &= \frac{\Gamma\left(3-\frac{3\Delta}{2}\right)}{\Gamma\left(\frac{\Delta}{2}\right)} \int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v \, \frac{[u(1-u)v]^{\frac{1}{2}-\frac{\Delta}{2}}(1-v)^{\frac{\Delta}{2}-1}}{[u(1-u)(1-v)+(1-u)vX+uvY]^{3-\frac{3\Delta}{2}}} \\ &= \frac{2}{\sqrt{\pi}}\Gamma(3-\frac{3\Delta}{2})\Gamma(\frac{3}{2}-\frac{\Delta}{2})\times \\ &\quad \int_0^1 \mathrm{d}u \, \frac{[u(1-u)]^{\frac{1}{2}-\frac{\Delta}{2}}}{[(1-u)X+uY]^{3-\frac{3\Delta}{2}}} {}_2F_1\left(3-\frac{3\Delta}{2},\frac{\Delta}{2};\frac{3}{2};Z(X,Y,u)\right). \end{aligned} \tag{6.60}$$

The hypergeometric function ${}_2F_1$ depends on u and the shape of the momentum conserving triangle via

$$Z(X,Y,u) = 1 - \frac{u(1-u)}{(1-u)X+uY}.$$

6.B.2 Bulk-boundary identity

The integral (6.60) over the hypergeometric function can not be evaluated for arbitrary values of Δ. Moreover it has divergences when Δ has integer values. Regulating the divergences is not easy, since the Feynman parameters u and v do not have a clear-cut physical meaning. This is unfortunate, since from the gravity calculation we have reasons to believe that the three-point function actually has a clean momentum dependence, which for (6.60) remains hidden in the Feynman integral. For this reason, we pursue a different approach to find the Fourier transforms. Although our technique is borrowed from AdS/CFT and is reminiscent of the actual gravity in-in calculation [57, 254], the calculation is to be understood as a pure mathematical identity, whose AdS/CFT-origin is not of particular relevance. The identity we will use is, cf. (3.36) [149, 150],

$$\begin{aligned} &\frac{a(\Delta)}{(x_{12}x_{23}x_{13})^\Delta} = \\ &\int_{z_0}^\infty \frac{\mathrm{d}z}{z^4}\mathrm{d}^3\boldsymbol{x}_1\mathrm{d}^3\boldsymbol{x}_2\mathrm{d}^3\boldsymbol{x}_3\mathrm{d}^3\boldsymbol{w}\, K_{\mathrm{bb}}(\Delta;z,|\boldsymbol{x}_1-\boldsymbol{w}|)K_{\mathrm{bb}}(\Delta;z,|\boldsymbol{x}_2-\boldsymbol{w}|)K_{\mathrm{bb}}(\Delta;z,|\boldsymbol{x}_3-\boldsymbol{w}|), \end{aligned} \tag{6.61}$$

where $a(\Delta)$ depends on Δ,

$$a(\Delta) = \frac{\Gamma\left(\frac{3}{2}(\Delta-1)\right)\Gamma\left(\frac{\Delta}{2}\right)^3}{2\pi^3\Gamma\left(\Delta-\frac{3}{2}\right)^3},$$

and where $K_{\rm bb}(\Delta; z, \boldsymbol{z}, \boldsymbol{x})$ is the bulk-boundary propagator in anti-de Sitter spacetime,

$$K_{\rm bb}(\Delta; z, \boldsymbol{z}, \boldsymbol{x}) = \frac{\Gamma(\Delta)}{\pi^{\frac{3}{2}}\Gamma(\Delta - \frac{d}{2})}\left(\frac{z}{z^2 + (\boldsymbol{z} - \boldsymbol{x})^2}\right)^{\Delta}.$$

Equation (6.61) has to be understood as a regulated expression, picking out the regular part as $z \to 0$.

We can Fourier transform the left hand side of (6.61) by Fourier transforming each of the bulk-boundary propagators [148]

$$K_{\rm bb}(\Delta; z, z_0, k) = \left(\frac{z}{z_0}\right)^{\frac{3}{2}} \frac{K_\nu(kz)}{K_\nu(kz_0)},$$

where $K_\nu(x)$ is the modified Bessel function of the second kind and $\nu = \Delta - \frac{3}{2}$. The special point z_0 is used to normalize the bulk-boundary propagator. In Fourier-space the three-point function $\langle O_\Delta(k_1)O_\Delta(k_2)O_\Delta(k_3)\rangle_0 = C\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)R_\Delta(k_1, k_2, k_3)$ is then equal to

$$R_\Delta(k_1, k_2, k_3) = \frac{1}{a(\Delta)}\int_{z_0}^{\infty} \frac{\mathrm{d}z}{z^4} K_{\rm bb}(\Delta; z, z_0, k_1)K_{\rm bb}(\Delta; z, z_0, k_2)K_{\rm bb}(\Delta; z, z_0, k_3). \quad (6.62)$$

6.B.3 The marginal case

The expression (6.62) is particularly simple for marginal operators O with $\Delta = 3$ or equivalently $\nu = \frac{3}{2}$. In that case, the Taylor series of the Bessel function terminates, leaving a simple expression for the bulk-boundary propagator

$$K_{\rm bb}(3; z, z_0, k) = e^{-k(z-z_0)}\frac{1 + kz}{1 + kz_0}.$$

The integral (6.62) can be done explicitly and is of the form

$$R_3(k_1, k_2, k_3) = \frac{I_{-3}}{z_0^3} + \frac{I_{-1}}{z_0} + I_0(k_1, k_2, k_3)z_0^0 + O(z_0). \quad (6.63)$$

The first two terms are singular in the limit $z_0 \to 0$. At the same time, these terms are odd in z_0, indicating that they are imaginary contributions if we would do the Wick-rotation from the anti-de Sitter z_0 to conformal time τ_* [281]. Therefore, the leading contribution comes from the regular coefficient multiplying z_0^0. This term gives the well-known result [77, 78, 257]

$$I_0(k_1, k_2, k_3) = -\frac{1}{3a(3)}\left((-1 + \gamma + \log[-k_t\tau_*])\sum_{j=1}^{3} k_j^3 + k_1k_2k_3 - \sum_{j\neq l} k_jk_l^2\right), \quad (6.64)$$

for spectator fields in a de Sitter background, cf. (2.22).

6.B.4 The nearly marginal case

For general Δ, the expression (6.62) is less easy to evaluate. When Δ is nearly marginal, $\Delta \approx 3$, we may approximate the result via a Taylor series,

$$R_\Delta(k_1,k_2,k_3) = R_3(k_1,k_2,k_3) + (\Delta-3)\,\partial_\Delta R_\Delta(k_1,k_2,k_3)|_{\Delta=3} + O(\lambda^2). \tag{6.65}$$

This approach is very similar to the one employed in the appendix of [79], in which higher order corrections in the slow-roll expansion are calculated. The zeroth order term $R_3(k_1,k_2,k_3)$ equals the result of the previous section, which now gets corrections of order $\Delta-3$ proportional to the derivative of R_Δ at $\Delta = 3$.

Using one of the explicit formulae for the Bessel function [290], $K_\nu = \frac{\sqrt{\pi}}{\Gamma(\nu+\frac{1}{2})}\tilde{K}_\nu$, where

$$\tilde{K}_\nu(Z) = \left(\frac{Z}{2}\right)^\nu \int_1^\infty \mathrm{d}x\, e^{-Zx}(x^2-1)^{\nu-\frac{1}{2}},$$

the first order contribution $\partial_\Delta R_\Delta|_{\Delta=3}$ can be written as

$$\begin{aligned}\partial_\Delta R_\Delta|_{\Delta=3} = {}& -R_3(k_1,k_2,k_3)\left(\frac{\partial_\Delta a(\Delta)}{a(\Delta)} + \frac{\partial_\nu \tilde{K}_\nu(k_1 z_0)}{\tilde{K}_\nu(k_1 z_0)} + \frac{\partial_\nu \tilde{K}_\nu(k_2 z_0)}{\tilde{K}_\nu(k_2 z_0)} + \frac{\partial_\nu \tilde{K}_\nu(k_3 z_0)}{\tilde{K}_\nu(k_3 z_0)}\right)\Bigg|_{\Delta=3} \\ & + L(k_1;k_2,k_3) + L(k_2;k_1,k_3) + L(k_3;k_1,k_2), \qquad \text{where} \end{aligned} \tag{6.66a}$$

$$L(k_1;k_2,k_3) = \frac{1}{a(3)}\int_{z_0}^\infty \frac{\mathrm{d}z}{z^4}\partial_\nu \tilde{K}_\nu(k_1 z)\frac{\tilde{K}_\nu(k_2 z)\tilde{K}_\nu(k_3 z)}{\tilde{K}_\nu(k_1 z_0)\tilde{K}_\nu(k_2 z_0)\tilde{K}_\nu(k_3 z_0)}\Bigg|_{\nu=\frac{3}{2}}. \tag{6.66b}$$

The advantage of using $\tilde{K}_\nu$ is that its derivative can be explicitly evaluated,

$$\begin{aligned}\partial_\nu \tilde{K}_\nu(Z)\big|_{\nu=\frac{3}{2}} &= \left(\frac{Z}{2}\right)^{3/2}\int_1^\infty \mathrm{d}x\, e^{-Zx}(x^2-1)\log\left[\frac{Z}{2}(x^2-1)\right] \\ &= 2(2Z)^{-3/2}e^{-Z}\left(3 + Z - \gamma(1+Z) + (-1+Z)e^{2Z}\mathrm{Ei}(-2Z)\right),\end{aligned} \tag{6.67}$$

where $\mathrm{Ei}(Z) = -\int_{-Z}^\infty \mathrm{d}t\,\frac{e^{-t}}{t}$ is the exponential integral function. Hence, the contribution from the first line of (6.66a) can be directly found from an expansion in z_0. Again, since all singular terms are odd, the leading order contribution comes from the z_0^0-term. The result resembles (6.64), but now also includes terms proportional to $\log[-k_j\tau_*]$.

To find $L(k_1;k_2,k_3)$, the derivative of the Bessel function (6.67) needs to be integrated over $z \in (z_0,\infty)$. The total contribution consists of two parts, one part $L^{(1)}$ coming from the integration of $e^{2k_1 z}\mathrm{Ei}(-2k_1 z)$ and the other part $L^{(2)}$ from the integration over the other terms. The latter contribution can be readily done and it gives an answer of the form

$$L^{(2)}(k_1;k_2,k_3) = \frac{L^{(2)}_{-3}}{z_0^3} + \frac{L^{(2)}_{-2}}{z_0^2} + \frac{L^{(2)}_{-1}}{z_0} + L^{(2)}_0 z_0^0 + O(z_0). \tag{6.68}$$

In this case, the leading order contribution comes from the even, divergent, contribution $L^{(2)}_{-2}$, which is given by

$$L^{(2)}_{-2}(k_1;k_2,k_3) = -k_1\pi^{5/2}. \tag{6.69}$$

Interestingly, this contribution is not homogeneous of degree 3 in the momenta. The contribution $L^{(2)}_0$ is

$$\begin{aligned} L^{(2)}_0(k_1;k_2,k_3) = \frac{\pi^{5/2}}{3}\Big(& (k_2^3+k_3^3)((\gamma-1)(\gamma-3)+\gamma\log[-k_t\tau_*]) \\ & + k_1^3(\gamma(\gamma-1)-3+\gamma\log[-k_t\tau_*]) \\ & - \gamma(k_1^2k_2+k_1^2k_3) + (k_1k_2^2+k_1k_3^2)(3-4\gamma-3\log[-k_t\tau_*]) \\ & + \gamma k_1k_2k_3 + (k_2^2k_3+k_2k_3^2)(\gamma-3)(\gamma-2)\Big). \end{aligned} \tag{6.70}$$

It is again a term that looks very much like (6.64), except for its coefficients.

In order to calculate the integral $L^{(1)}$ over the exponential integral function, we consider the integral of each term of its Taylor series expansion separately,

$$e^Z\mathrm{Ei}(-Z) = \sum_{n=0}^{\infty}\left[\frac{1}{n!}(\gamma+\log[Z])\right]Z^n + \sum_{n=1}^{\infty}\left[\sum_{j=1}^{n}\left(\frac{(-1)^j}{jj!(n-j)!}\right)\right]Z^n.$$

The integral over each of the summands can be straightforwardly computed. Furthermore the restrictions $L^{(1)}_{-2}$ and $L^{(1)}_0$ of $L^{(1)}$ to the z_0^{-2}- and z_0^0-contributions can be resummed into closed expressions. Both expressions are too long to include in this appendix. Most important is the generic momentum dependence, which can already be appreciated by inspection. The contribution $L^{(1)}_{-2}(k_1;k_2,k_3)$ is a polynomial of degree 1 in the momenta, with logarithmic dependence on each of the k_j's. The contribution $L^{(1)}_0(k_1;k_2,k_3)$ is a sixth degree polynomial multiplied by $\frac{1}{k_t^3}$ with polylogarithmic dependence on k_t, but also on $k_1-k_2-k_3$. These expressions show the type of mixing that can occur between the different momenta. After a decent rewriting they may unveil the typical $1/k_t$-momentum dependence of the slow-roll result, although at the moment it seems that there is a much more exotic dependence on the momenta.

7

Conclusions

In this thesis we have collected the findings of three separate studies in inflationary string cosmology. The initial motivation for each of these studies is the important role inflation might play in our understanding of the microscopic structure of nature. This thesis helps to reveal important aspects of the theory, that we need to understand better before we can let inflation fulfill its intended role.

7.1 Microscopic sensitivity

One of the lessons of this thesis is that observations from the early universe might indeed be capable of unveiling hints of the universe at the tiniest scales, but that the sensitivity of inflation to microscopic physics requires a detailed and complete understanding of its underlying microscopic description before we can reap the benefits of the cosmological approach to quantum gravity. Both in the context of supergravity approaches to inflation as well as in the newly developed worldsheet approach, the sensitivity of inflation to the details of the full theory proves to be more restrictive than one would initially imagine.

In chapter 4 we have argued this by studying the η-problem in supergravity, which we have shown cannot be solved without knowledge of the hidden sectors that are gravitationally coupled to the inflaton. If the hidden sector breaks supersymmetry independently, its fields cannot be stabilized during the cosmological evolution of the inflaton. It is shown that both the subsequent dynamical mixing between sectors as well as the mass of the lightest field in the hidden sector are set by the scale of supersymmetry breaking in the hidden sector. The true cosmological η-parameter arises from a linear combination of the lightest mode of the hidden sector with the inflaton. Generically, either the true η deviates considerably from the naïve η implied by the inflaton sector alone, or one has to consider a multi-field model. Only if the lightest mass in the hidden sector is much larger than the inflaton mass and if the inflaton mass is much larger than the scale of hidden sector supersymmetry breaking, is the effect of the hidden sector on the slow-roll dynamics of the inflaton negligible.

Therefore, we argue that the η-problem is even more severe than what is usually understood from the literature. Typical models for inflation in supergravity do not include anything else but inflationary dynamics. The assumption that other physics, such as the standard model, does not contribute, imposes implicit constraints on this other physics that are in many cases unrealistic. One could therefore say that inflation does not only probe the microscopics of its own underlying theory, but also that of other fields that are initially assumed not even to partake in inflation.

For this reason we have turned our attention in chapter 5 to a description of inflation that explicitly includes all known and unknown physics. We have tried to make full use of the constraints imposed by slow-roll inflation on the string worldsheet, by using a general gravity-matter set-up in which the worldsheet consists of an abstract conformal field theory coupled to a 3+1-dimensional nonlinear σ model. The empirical slow-roll parameters are expressed in terms of the β functions of operators in the matter/internal conformal field theory and the β function of the dilaton. The result confirms that inflation is only sensitive to coarse properties of the matter sector, but that in string theory inflation is a non-perturbative (in g_s) phenomenon and one must go beyond tree-level string theory.

In principle the observed detailed sensitivity of the worldsheet approach to inflation, i.e. the necessity to go beyond tree-level, might be understood simply as the worldsheet variant of the well-known observation that inflation is sensitive to the details of the theory, like we have already seen in the supergravity situation. However, the worldsheet approach sharpens the statement, making exact what inflation is sensitive to. The results in chapters 4 and 5 complement each other, since their regimes of validity differ subtly but fundamentally, cf. figure 7.1. The supergravity approach is a low energy effective approach, i.e. valid for small values of α', which should essentially be enough to describe the classical phenomenon that is inflation. Although the motivation to study such supergravity theories comes from the low energy limit of g_s-perturbative expressions of the string worldsheet, one might hope that one can generalize this to also capture some higher order (in g_s) unknown physics, still within the low energy limit. In this language, our reasoning in the supergravity limit shows that certainly the low energy limit (in α') is insufficient for a full description of inflation. At that stage a possible resolution might be that either one needs to go to higher order in α' or that the assumption that a low energy limit might be sufficient to encode higher order g_s-effects is invalid. Initially one could hope that higher or-

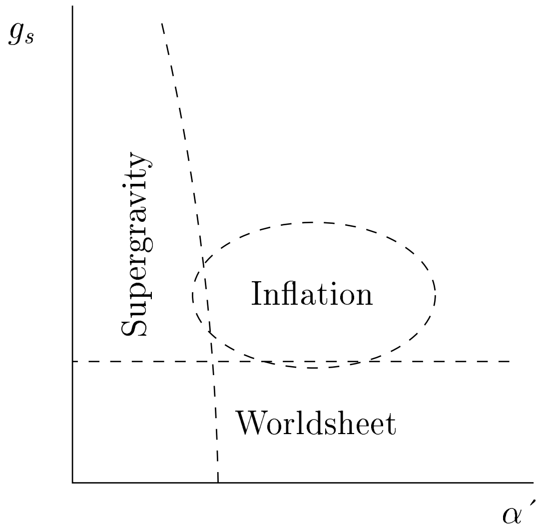


Figure 7.1: *The regimes of applicability for a worldsheet set-up and in a supergravity context. Supergravity is the small α'-regime of string theory and is (without a strong justification) also applied at higher values of g_s. The worldsheet approach of chapter 5 is valid for all α' but certainly not in the large g_s-regime. Inflationary models in supergravity are very sensitive to the physics of hidden sectors, indicating that inflation is strongly coupled in α'. In chapter 5 we have shown that no tree-level (in g_s) string inflationary models can be found. This means that inflation is located in the upper right part of the diagram.*

der effects in g_s are not really relevant, since inflation seems to be not a specifically strongly coupled string phenomenon. In a leap of faith, this would mean that by including all α' corrections to any unknown sector in the theory, inflation can be fully understood. Although the worldsheet approach is only a lowest order expansion in α' for the inflationary part of the theory, the unknown theory is explicitly incorporated to all orders in α'. Hence, while the former should be sufficient to describe the classical evolution of inflation, the latter exactly achieves the sought-for parameterization of the remaining unknown physics. In this language, chapter 5 shows that inflation is also sensitive to strong coupling effects in g_s. As such, the worldsheet approach rules out a whole class of theories that have previously been inaccessible in the approach of the supergravity low energy limit, while at the same time producing a qualitatively very similar conclusion: inflation is more sensitive to the details of the full theory than one would initially imagine. It is both strongly coupled with respect to the α'-as well as with respect to the g_s-corrections of the theory.

7.2 Conformal invariance

The approach taken in chapter 6 could provide a completely new perspective on inflationary cosmology. Guided by holographic principles, it possibly evades the problems from which the more conventional approaches of chapters 4 and 5 suffer. In chapter 6 we have investigated the constraints imposed by symmetry on the three-point correlation function of primordial density fluctuations in slow-roll inflation. It follows from the defining property of slow-roll inflation that primordial correlation functions inherit most of their structure from weakly broken de Sitter symmetries. Using holographic techniques borrowed from the AdS/CFT correspondence, the symmetry constraints on the bispectrum are mapped to a set of stress-tensor Ward identities in a weakly broken three-dimensional Euclidean conformal field theory. The most general solution to these Ward identities can be constructed using conformal perturbation theory. Translating back to the gravity side, our answer illustrates the full underlying symmetry structure of slow-roll non-Gaussianities.

Once again, the approach in chapter 6 is a testimony to the importance of the guiding hand of symmetries in physics. Another conclusion of this thesis is therefore the ubiquity of the applicability of conformal symmetry in our understanding of the universe. The appearance of conformal invariance in any description of quantum gravity is perhaps not very surprising, since it is a core ingredient of the renormalization procedure inherent to any quantum theory. On the other hand, the role conformal symmetry plays in each of the two approaches presented in chapters 5 and 6 is surprisingly different. In chapter 5, conformal invariance of the two-dimensional worldsheet theory is a strict consistency condition for the quantum string description. Formally we have shown that the coarse slow-roll description in terms of ϵ and η can be related to coarse properties of the hidden conformal field theory, like its β functions. Thus, inflation might in principle be described by a two-dimensional conformal field theory and this can be used to observationally constrain the theory. This conformal description happens at the level of the worldsheet, where the gravity and inflaton degrees of freedom appear as background fields. In chapter 6, conformal invariance is used at the level of the three-dimensional asymptotic future of the inflationary gravity theory itself. The conformal symmetry and its spontaneously broken rendition present themselves through constraints on the structure of curvature perturbations, which suggests an underlying holographic transcription of quantum gravity. For now, the relation between both conformal theories eludes any detailed understanding. Such a relation would be highly non-trivial and is perhaps unlikely —for one thing, the dimensions of the theories differ. Nonetheless, it is very interesting to see that conformal field theories play such an important role in inflation at several levels of the string theory description. It puts conformal invariance at the heart of inflationary cosmology, albeit in two variously different guises.

In conclusion, the different viewpoints on string inflation investigated in this thesis shed a brighter light on different aspects of the merits and problems of our current understanding of inflationary cosmology and its underlying microscopic structure. Although —or precisely because— each chapter investigates a different aspect of early universe cosmology, together they sketch a more detailed picture which may prove to be useful for further research. Our studies help crystalizing some of the fundamental problems and overarching guiding principles, which will help to pave the way in the understanding of the primordial universe and, with it, of the microscopic origin of nature.

Samenvatting

Snaarkosmologie

Het wereldoppervlak van de snaar

De mens is een nieuwsgierig wezen, dat al sinds mensenheugenis zijn omgeving aan het beschouwen, verklaren en voorspellen is. Staande op de schouders van onze voorgangers, hebben we onze kennis van de fundamentele bouwstenen van de natuur en hun onderlinge wisselwerkingen weten te verfijnen tot een tweetal theorieën. De kwantumveldentheorie van het *standaard model* beschrijft de natuur op kleine schaal. De *algemene relativiteitstheorie* geeft een meetkundige beschrijving van zwaartekracht, wat weliswaar een zeer zwakke kracht is en daardoor op kleine schaal nagenoeg verwaarloosbaar, maar die met zijn lange dracht dominant is op grote afstandschalen. De volgende stap is om ook deze twee theorieën samen te voegen tot één theorie, een *kwantumzwaartekrachttheorie*. Op dit moment is *snaartheorie* de beste kandidaat voor een kwantumzwaartekrachttheorie.

In de snaartheorie wordt de natuur beschreven in termen van fundamentele ééndimensionale deeltjes, heel anders dan "elementaire" deeltjes zoals elektronen en quarks, welke nul-dimensionale deeltjes zijn. Terwijl een snaar door de ruimte beweegt, genereert hij een twee-dimensionaal oppervlak —één richting langs de lengte van de snaar en één richting die de beweging van de snaar volgt—, het *wereldoppervlak* van de snaar. Snaartheorie is een twee-dimensionale theorie die beschrijft hoe dit wereldoppervlak is ingebed in de vier-dimensionale ruimte om ons heen.[1] De ruimte waardoor de snaar reist, beïnvloedt de mogelijke bewegingen van de snaar. Deze invloeden komen in de twee-dimensionale wereldoppervlaktheorie voor als parameters van de theorie, net zoals de massa en lading van het elektron als parameters voorkomen in de theorie van elektromagnetisme.

De parameters van snaartheorie zijn echter niet willekeurig, aangezien het voor

[1] Sinds het begin van de 20^{e} eeuw behandelen theoretische natuurkundigen tijd en ruimte op gelijke voet. Voor hen is onze ruimte, met één tijdsrichting en drie ruimtelijke richtingen, daarom vierdimensionaal.

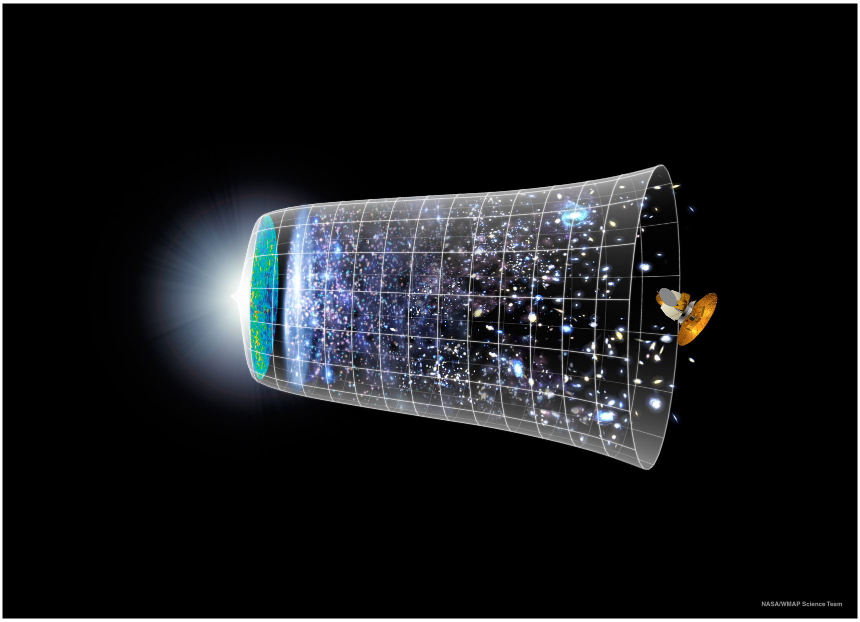
de kwantumconsistentie van snaartheorie cruciaal is dat de wereldoppervlaktheorie invariant is onder lengte veranderende, maar hoekgetrouwe tranformaties. Een theorie die invariant is onder *hoekgetrouwe transformaties*, “conformal transformations”, beschrijft een systeem dat onveranderd blijft bij (plaatselijke) herschalingen, zoals bij het inzoomen op de cirkels op de voorkant van dit proefschrift. Aangezien de wereldoppervlaktheorie hoekgetrouw moet zijn, worden de parameters van snaartheorie en daardoor de mogelijke theorieën sterk beperkt. De eis van hoekgetrouwe invariantie van de twee-dimensionale snaartheorie bepaalt zo de vier-dimensionale achtergrondsruimte van de snaar, oftewel de natuur om ons heen. Het bijzondere aan snaartheorie is dat diepere studie van de theorie onthult dat de snaar zelf deze parameters bepaalt. Hoe dit werkt, begrijpen we alleen voor achtergrondsruimten die statisch zijn, d.w.z. niet veranderen met de tijd. De natuur is echter niet statisch. Alles beweegt en zelfs de kosmos dijt uit met de tijd. Hoe tijdsafhankelijke achtergrondsruimten, en in het bijzonder kosmologische achtergronden, beschreven kunnen worden, is één van de grote puzzels binnen de kwantumzwaartekracht.

Kosmische inflatie

Het heelal is dynamisch. Zo’n 13,8 miljard jaar geleden is het heelal ontstaan in een hete, dichtopeengepakte toestand, welke sindsdien uitdijt. De kosmologie onderzoekt de dynamica van het gehele heelal, waaronder het precieze verloop van het allereerste begin, de *oerknal*. We hebben een goed begrip van reeds luttele momenten na de oerknal, maar onze onzekerheid groeit naarmate we dichter en dichter bij dit eerste begin komen. Het vroegst bekende stadium is *kosmische inflatie*, een korte periode waarin het heelal immens versneld is uitgedijd, zie figuur 1.

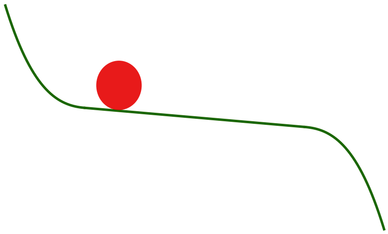
De wiskundige beschrijving van inflatie is verrassend eenvoudig. De dynamica van de versnelde uitdijing van het heelal, is dezelfde als die van een rollende bal in een heuvellandschap, zoals in figuur 2. Het heuvellandschap wordt de *potentiaal* genoemd, de rollende bal het *inflaton*. De uitdijing van het heelal wordt bepaald door de manier waarop het inflaton naar beneden rolt in de potentiaal.

De kosmische dynamica tijdens inflatie, zoals bepaald door de vorm van de potentiaal, kan worden achterhaald aan de hand van de *kosmische microgolf achtergrondstraling*. Deze straling geeft weer hoe het heelal er uitzag tijdens het *recombinatie*-proces waarbij protonen en elektronen stabiele, neutrale atomen vormden, dat zo’n 380 000 jaar na de oerknal heeft plaatsgevonden. Door het licht van de kosmische microgolf achtergrondstraling te detecteren met sattelieten zoals COBE, WMAP en sinds kort PLANCK, krijgen we een afbeelding van de structuur van het jonge heelal, zie figuur 3.

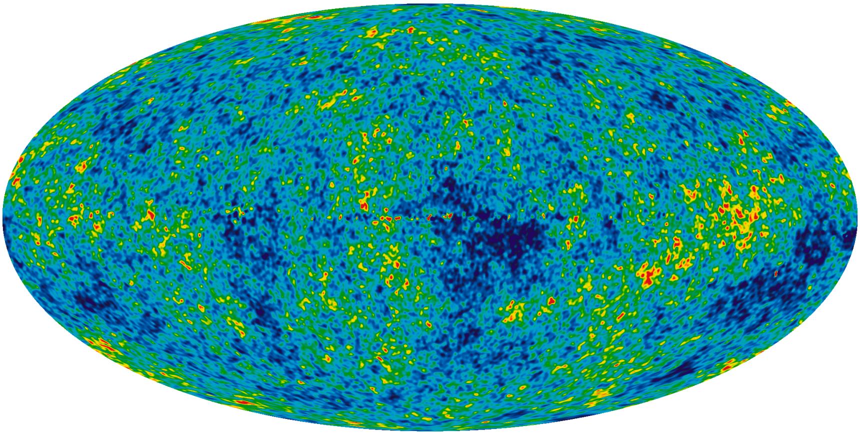


Figuur 1: *Een schematische weergave van de uitdijing van het heelal en structuurvorming gedurende de afgelopen 13,8 miljard jaar. Links in de afbeelding is de oerknal weergegeven. Vrijwel direct erna vindt inflatie plaats, een haast instantane, immense uitdijing van het heelal. Na 380 000 jaar vindt recombinatie plaats, weergegeven met de blauw/groene schijf (zie ook figuur 3), waarvan het licht gedetecteerd kan worden door een satteliet als WMAP (uiterst rechts op de afbeelding). In de periode na recombinatie vindt sterformatie plaats en dijt het heelal geleidelijk verder uit. Recentelijk is de uitdijing weer licht versneld.* Afbeelding afkomstig van NASA/WMAP Science Team.

Hoewel recombinatie veel later heeft plaatsgevonden dan inflatie, weten we hoe we de afdruk van de kosmische microgolf achtergrondstraling, d.w.z figuur 3, wiskundig kunnen terugleiden naar de periode van inflatie. Zo weten we uit het feit dat de temperatuur van de achtergrondstraling maar heel weinig over de ruimte varieert, dat de inflatie-potentiaal zeer vlak moet zijn voor een voldoende lange periode, waarbij inflatie moet hebben plaatsgevonden in een *langzame rol*, "slow-roll inflation". Bij nauwkeurige meting van de microgolfstraling vinden we kleine variaties in de verder homogene temperatuursverdeling. De statistische eigenschappen van de temperatuurinhomogeniteiten kunnen worden vergeleken met de voorspellingen van inflatie en op die manier kunnen we verschillende scenario's voor inflatie testen.



Figuur 2: *Inflatie wordt wiskundig beschreven door een potentiaal waarover het inflaton naar beneden rolt. Om in overeenstemming te zijn met de waarnemingen, moet de potentiaal zeer vlak zijn over een lange afstand.*



Figuur 3: *De kosmische microgolf achtergrondstraling heeft een vrijwel homogene temperatuur over de gehele hemel. Kleine temperatuurschommelingen, maximaal* $0,00003\,^\circ C$*, zijn zichtbaar en geven inzicht in de oorsprong van het heelal.* Afbeelding afkomstig van NASA/WMAP Science Team.

Het heelal als een vergrootglas

Om snaartheorie beter te kunnen begrijpen, moet de theorie experimenteel onderbouwd worden. Echter, snaartheoretische effecten zijn alleen relevant bij zeer hoge energieën, die maar liefst een biljard (1 000 000 000 000 000) keer zo hoog zijn als de energieën in een deeltjesversneller als de Large Hadron Collider van CERN. Dit gaat onze huidige technologische capaciteiten te boven, en eveneens die van de nabije (en verre) toekomst. Inflatie, op haar beurt, lijkt fenomenologisch weliswaar een goede weergave te geven van de omstandigheden vlak na de oerknal, maar de oorsprong ervan blijft een mysterie. Voor een gedegen microscopische onderbouwing is het nodig dat inflatie beschreven kan worden binnen een fundamentele kwantumzwaartekrachttheorie.

In snaarkosmologie vormen deze twee problemen in principe elkaars oplossing. De omstandigheden waarin het heelal verkeerde tijdens inflatie waren dusdanig extreem dat kwantumzwaartekrachteffecten een rol hebben gespeeld. De mogelijke potentialen en ook de kandidaten voor het inflaton worden bepaald door snaartheorie, zoals ook andere aspecten van onze vier-dimensionale natuur daarin hun fundamentele beschrijving vinden. Via de statistische eigenschappen van de kosmische microgolf achtergrondstraling kunnen we achterhalen welke potentialen, d.w.z. welke scenario's voor inflatie, een natuurgetrouwe beschrijving geven. Inflatie en haar implicaties in de achtergrondstraling blijken sterk afhankelijk te zijn van de precieze microscopische, snaartheoretische beschrijving. Mits theoretisch begrip en observaties elkaar volledig aanvullen, vormt inflatie de ideale overgang van de zichtbare, hemeloverspannende structuren van de kosmos naar de kleinste, snaartheoretische schalen van de natuur.

Dit proefschrift

Het onderzoek in dit proefschrift draagt bij aan de zoektocht naar het fundamentele begrip van de natuur en van het begin van ons universum, door de snaartheoretische oorsprong van de oerknal te bestuderen. De titel *Conformal invariance and microscopic sensitivity in cosmic inflation*, wat zich laat vertalen als *Hoekgetrouwe invariantie en microscopische afhankelijkheid in kosmische inflatie*[2], verraadt dat we daarbij met name ingaan op de wijze waarop inflatie afhankelijk is van zijn microscopische, snaartheoretische beschrijving en de mate waarin hoekgetrouwe invariantie daarin een leidende rol speelt. Het onderzoek belicht een drietal onafhankelijke methoden om inflatie te beschrijven binnen de snaartheorie, welke in drie verschillende hoofdstukken gepresenteerd worden.

[2] De afbeelding op de voorkant van dit proefschrift verenigt verschillende belangrijke concepten in het onderzoek. Het stelt een snaar voor die afkomstig is uit de oerknal en die aldoende de hoekgetrouw invariante oorsprong van de oerknal onthult. Tegelijkertijd kan de afbeelding worden opgevat als het hoekgetrouwe wereldoppervlak van de snaartheoretische beschrijving van inflatie. De kleuren van de regenboog verwijzen naar het taalgebruik om microscopische afhankelijkheid in een theorie aan te duiden met "ultraviolet" en macroscopische verschijnselen met "infrarood". De uitdijing van inflatie laat de microscopische details van de snaar uitgroeien tot zichtbare verschijnselen aan de hemel en brengt daarmee de kleinste en grootste schalen van ons universum met elkaar in verband.

Inflatie en het standaard model

De uit de snaartheorie afkomstige beschrijving van onze vier-dimensionale natuur is onderhevig aan *supersymmetrie*, een (nog niet waargenomen) symmetrie tussen de elementaire bouwblokken van de natuur, die de theorie sterk beperkt. Omdat deze vier-dimensionale beschrijving zwaartekracht bevat, is het een voorbeeld van een *superzwaartekracht*theorie. In zo'n theorie heeft de inflatie-potentiaal een specifieke vorm, waarmee expliciete berekeningen mogelijk zijn.

In het algemeen zijn er meerdere kandidaten om als inflaton dienst te doen in een superzwaartekrachttheorie. De superzwaartekrachttheorie die volgt uit snaartheorie bevat bijvoorbeeld een volledige beschrijving van de hele natuur, waaronder zowel inflatie als het standaard model. Hierdoor is het onwaarschijnlijk dat inflatie voldoet aan de simpelste beschrijving van een één-dimensionale potentiaal zoals in figuur 2, waarin de bewegingsrichting van het inflaton maar in één richting is. In superzwaartekrachttheorieën met meerdere sectoren, bijvoorbeeld een inflatie-sector en een sector voor het standaard model, beschrijft de potentiaal een meer-dimensionaal landschap —zoals een sneeuwbal van een alpenhelling— en wordt de dynamica ingewikkelder. Echter, wanneer kosmologen zich richten op inflatie, is het vooralsnog te complex om alle informatie op te nemen in het model en daarom wordt het standaard model (voorlopig) buiten beschouwing gelaten in studies van inflatie. De vraag is of deze vereenvoudiging geoorloofd is en hoe realistisch het is dat de verwaarloosde bewegingsrichtingen niet bijdragen aan de dynamica.

In hoofdstuk 4 bekijken we de interactie tussen het standaard model en de inflatie-sector. We richten ons hierbij op het bestaan van langzame-rol inflatie. Al voor een één-dimensionaal inflatie-scenario in superzwaartekracht blijkt het erg lastig, maar wel mogelijk, om te voldoen aan de fenomenologische eis van langzame rol. Het is daarom allerminst vanzelfsprekend dat de daadwerkelijke, complexere beschrijving waarin het standaard model wél zou worden meegenomen, de langzame rol intact laat. Inderdaad is onze conclusie dat er in een beschrijving van standaard model-loze langzame-rol inflatie impliciete aannames zijn gemaakt over het standaard model. Wanneer niet is voldaan aan deze (sterke) aannames, wordt de daadwerkelijke, meer-dimensionale dynamica verstoord tot een niet-langzame rol. Hiermee tonen we aan dat inflatie zeer gevoelig is voor de microscopische details van de *gehele* snaartheoretische beschrijving, zowel de sector waarin men naïef verwacht dat inflatie plaatsvindt als de (onterecht) als irrelevant aangenomen onderdelen van de theorie zoals het standaard model of andere nog onbekende sectoren. Inflatie biedt hierdoor de mogelijkheid om tot de fundamentele kern van de microscopische beschrijving van de gehele natuur door te dringen.

Inflatie vanuit het wereldoppervlak

Zoals uit het voorgaande blijkt, is inflatie afhankelijk van de microscopische details van de gehele theorie. Aangezien we niet, en waarschijnlijk nooit, de volledige natuur kunnen doorgronden, is het in superzwaartekracht onhandig om afhankelijk te zijn van onbekende sectoren. Aan de andere kant opent inflatie de mogelijkheid om meer te weten te komen van de nu nog onbekende structuur van ons universum. Hiervoor is echter wel een formalisme nodig dat een gehele beschrijving van de natuur mogelijk maakt.

In hoofdstuk 5 gaan we daarom terug naar de basis van de snaartheoretische methode om de natuur te beschrijven, de twee-dimensionale wereldoppervlaktheorie. Dit is een hoekgetrouwe theorie, wiens hoekgetrouwe invariantie, zoals gezegd, aanleiding geeft tot de beschrijving van de vier-dimensionale ruimte waar de snaar doorheen beweegt. Het twee-dimensionale wereldoppervlak kan in principe elke (abstracte) hoekgetrouwe theorie zijn, maar aangezien we inflatie willen beschrijven, zal in ieder geval een deel van de twee-dimensionale theorie hiervoor verantwoordelijk moeten zijn. Het andere deel is onbekend, maar het hoekgetrouwe formalisme van het wereldoppervlak stelt ons in staat om de theorie globaal weer te geven in termen van slechts enkele (abstracte) parameters.

De beide delen van de wereldoppervlaktheorie hoeven afzonderlijk niet hoekgetrouw te zijn, zolang de totale theorie dat maar wel is. De wisselwerking tussen beide onderdelen van de theorie kan geanalyseerd worden door de symmetrie van de onbekende sector licht te breken. Het breken van de hoekgetrouwe invariantie van de onbekende sector heeft een reactie in de inflatie-sector tot gevolg om te voorkomen dat de hoekgetrouwe invariantie van de totale theorie verloren gaat. Deze reactie kan worden herkend als de bijdrage van de onbekende sector op de dynamica van inflatie in de vier-dimensionale ruimte. Door het resultaat te vergelijken met langzame-rol inflatie, kunnen we bepalen welke parameters van de onbekende sector, oftewel welk soort abstracte theorieën, mogelijk zijn voor een snaartheoretische beschrijving van langzame-rol inflatie.

De conclusie van deze studie is dat, wanneer men alleen wereldoppervlaktheorieën van vrije snaren in de analyse meeneemt, langzame-rol inflatie onmogelijk is, ongeacht welke theorie er in de onbekende sector beschreven wordt. Een consistente beschrijving van inflatie in de wereldoppervlaktheorie vereist dat men ook botsende snaren beschouwt. Dit is verrassend omdat inflatie een beschrijving is binnen de klassieke algemene relativiteitstheorie, welke reeds door vrije snaren beschreven kan worden. Aan de andere kant benadrukt dit resultaat, vanuit een ander oogpunt, iets dat we al wisten: inflatie is zeer afhankelijk van de microscopische details van de kwantumzwaartekrachtbeschrijving.

Holografische inflatie

Omdat inflatie zo afhankelijk is van de microscopische details van de beschrijving, is een alternatieve aanpak nodig. In hoofdstuk 6 bestuderen we zo'n aanpak, op het niveau van de eigenschappen van de statistische variaties van de kosmische microgolf achtergrondstraling. De correlaties tussen koude en hete plekken in de achtergrondstraling kunnen worden berekend vanuit het inflatie-scenario. De gevonden uitdrukking vertoont een uitstekende overeenkomst met de waarnemingen —één van de redenen voor ons vertrouwen in dat inflatie een correcte beschrijving van het vroege heelal is—, maar deze berekening verzuimt te verhelderen waaróm het het specifieke antwoord is. Hierdoor blijft de structuur van inflatie een mysterie.

Eén van de bekende eigenschappen van inflatie is dat er een (licht gebroken) driedimensionale hoekgetrouwe symmetrie is op, voor inflatie, late tijden. In het bijzonder zouden de uitdrukkingen ten tijde van recombinatie, 380 000 jaar later, (vrijwel) invariant moeten zijn onder de hoekgetrouwe symmetrie van inflatie. Aangezien in elke hoekgetrouwe theorie de uitdrukkingen voor correlaties volledig kunnen worden bepaald door de symmetrie, is het interessant om te weten of en op welke manier dit ook geldt voor de correlaties die volgen uit de inflatie-berekening. Deze in hoofdstuk 6 gepresenteerde aanpak gaat in op de structuur van inflatie, op een manier die loodrecht staat op de twee eerder besproken methoden, omdat geen gebruik gemaakt wordt van de onbekende delen/sectoren in de theorie maar alleen de symmetrieën van het systeem als leidraad gebruikt worden.

We kunnen veel leren van de manier waarop de drie-dimensionale hoekgetrouwe symmetrie verpakt zit in de correlaties die volgen uit het vier-dimensionale inflatiemodel. Snaartheorie suggereert dat er een, veel algemenere, relatie moet zijn tussen elke zwaartekrachttheorie en een corresponderende theorie zonder zwaartekracht van één dimensie lager. Deze correspondentie wordt *holografie* genoemd, omdat net als in een hologram de d-dimensionale informatie van het systeem volledig opgeslagen kan worden op een $(d-1)$-dimensionaal "scherm". De holografische dualiteit is een ver reikend inzicht afkomstig uit snaartheorie, dat diep geworteld is in de microscopische, snaartheoretische beschrijving van de natuur. Slechts enkele (potentiële) realisaties van deze correspondentie zijn bekend. De bekendste is er evenwel een waarvan de zwaartekrachttheorie een sterke gelijkenis heeft met inflatie en de corresponderende lager dimensionale theorie hoekgetrouw is. In het licht van deze bekende holografische correspondentie bestuderen we de (licht gebroken) hoekgetrouwe symmetrie van de correlaties uit inflatie.

Uit het onderzoek komt naar voren dat de correlaties, d.w.z. de statistische eigenschappen van de fluctuaties in de achtergrondstraling, inderdaad voor een groot deel bepaald zijn door hoekgetrouwe symmetrie, hoewel het vooralsnog te vroeg is om te zeggen of de symmetrie (en de wijze waarop deze gebroken wordt in inflatie) voldoende is om de correlaties volledig te beschrijven. De variaties lijken de onderliggende hoekgetrouwe structuur van inflatie te onthullen, terwijl tegelijkertijd een beter begrip gecreëerd wordt van de holografische correspondentie, die raakt tot in de kern van snaartheorie.

Conclusie

De gevoeligheid van inflatie voor microscopische effecten is zowel een zegen als een vloek. Het is een zegen omdat we via de kosmische microgolf achtergrondstraling toegang hebben tot het onderzoeken van de kleinste schalen in de natuur. Het is een vloek, omdat de afhankelijkheid aanwezig is in heel de beschrijving van inflatie. Zowel in de superzwaartekracht- als in de wereldoppervlakbeschrijving blijkt het zeer lastig te zijn om een consistente microscopische beschrijving te geven van inflatie. Om de structuur van inflatie beter te begrijpen, hebben we ons daarom laten leiden door de hoekgetrouwe symmetrieën, op een manier die ingaat op de holografische oorsprong van inflatie. Als zodanig biedt dit proefschrift nieuwe inzichten in de oorsprong van ons heelal en de kleinste structuur van de natuur.

List of publications

[151] *The everpresent η-problem: knowledge of all hidden sectors required*
Sjoerd Hardeman, Johannes Oberreuter, Gonzalo Palma, Koenraad Schalm and Ted van der Aalst
JHEP **1104**:009, 2011, arXiv:1012.5966 [Chapter 4]

[152] *Decoupling limits in multi-sector supergravities*
Ana Achúcarro, Sjoerd Hardeman, Johannes Oberreuter, Koenraad Schalm and Ted van der Aalst
Submitted to JCAP, arXiv:1108.2278 [Chapter 4]

[226] *A worldsheet perspective on string inflation*
Koenraad Schalm and Ted van der Aalst
Submitted to Phys. Rev. D., arXiv:1008.5024 [Chapter 5]

[253] *Consistency condition for inflation from (broken) conformal symmetry*
Koenraad Schalm, Gary Shiu and Ted van der Aalst
Submitted to JCAP, arXiv:1211.2157 [Chapter 6]

Curriculum vitae

My name is Ted Adrianus Franciscus van der Aalst. I am born in Eindhoven, the Netherlands, on the 31st of January, 1984. After having completed pre-university education at the Pleincollege Van Maerlant in Eindhoven, I enlisted for the Bachelor's programmes in Physics & Astronomy and in Mathematics at Utrecht University in 2002. In 2005, I graduated cum laude as Bachelor of Science in both programmes with a minor in the History & Philosophy of Science, and continued with the Master's programmes Theoretical Physics and Mathematical Sciences at the same university. This resulted in a cum laude graduation as Master of Science in both programmes in 2008 with a thesis titled *A geometric interpretation of the c-map* under supervision of dr. S. Vandoren and dr. J. Stienstra. My study results were rewarded with a Shell stipend, awarded to the nine best MSc students in Theoretical Physics in the Netherlands. During my study I have been a teaching assistant for several Bachelor and Master courses, among other things "vector calculus" and "advanced quantum mechanics". I have acted as the evaluation manager of the Master's programme in Theoretical Physics and been a student member of the Graduate studies committee and Education advisory committee.

In 2008 I started my PhD research at the Instituut-Lorentz for Theoretical Physics at Leiden University under supervision of prof. dr. A. Achúcarro and dr. K. Schalm. I spent two months as a visiting researcher at the string theory group of the University of Wisconsin in Madison, by invitation of prof. dr. G. Shiu. Teaching activities included answering questions from high school students regarding their "profielwerkstuk" and acting as the teaching assistant for "physics of elementary particles" and "theory of general relativity". During my PhD, I have attended several schools, conferences and workshops in Driebergen, Amsterdam, Paris, Trieste and Princeton and I have presented my work in seminars and talks in Groningen, Veldhoven, Trieste, Chicago, Madison and Ithaca.

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Dit proefschrift is het resultaat van vier jaar onderzoekswerk, waarin ik heb mogen samenwerken met en ben geholpen en bijgestaan door een grote groep familie, vrienden en collega's. Hoewel het proefschrift slechts één naam draagt, is het uiteindelijke resultaat mede tot stand gekomen dankzij hen. Mijn dank gaat als eerste uit naar mijn co-promotor Koenraad Schalm, wiens enthousiasme, positieve instelling, optimisme en passie voor het vakgebied een onuitputtelijke bron voor inspiratie vormt. Zijn, altijd opbouwende, kritieken en gedreven aanmoedigingen hebben mij een beter onderzoeker en professional gemaakt. Ik heb veel waardering voor het feit dat de menselijke maat hierbij nooit uit het oog verloren is geraakt. I feel very privileged to have Ana Achúcarro acting as my promotor. I have benefited greatly from her attention to detail and her sharp thinking and questioning, which set an impressive example of a pure scientist. Her vibrant presence is contagious and her truly caring nature is heart-warming.

Binnen de academische wereld is het niet ongebruikelijk om collega's als in een familie aan te duiden. Ik wil Sjoerd bedanken voor zijn optreden als mijn academische grote broer. Hij heeft mij bekend gemaakt met het instituut, het vakgebied en de wereld, met zijn kennis en brede interesse. Mijn vragen werden altijd beantwoord met zijn liefde voor natuurkunde en het plezier om erover te praten. Zijn invloed is terug te vinden in het hele proefschrift en ik had het gereedkomen dan ook graag met hem gevierd. Zijn initialen staan met eer gegrift in de kosmische microgolf achtergrondstraling. Hij wordt gemist.

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A truly inspiring period was the fall of last year, in which I visited the University

of Wisconsin in Madison. I want to thank Gary Shiu for inviting me and being such an incredible host. His great understanding of physics and the physics community has been a major help in understanding how our research fits in the bigger picture.

My visit also allowed me to intensify discussions about my research with other string cosmologists. I would like to thank the University of Wisconsin, the University of Chicago, New York University, Princeton University, the University of Pennsylvania, Syracuse University and Cornell University for their hospitality and I am especially grateful to Daan & Esther and Riccardo & Paloma for welcoming me as their guest. For interesting discussions and interactions I would like to thank A. Hashimoto, S. Sethi, D. Kutasov, M. Kleban, G. D'Amico, R. Flauger, E. Pajer, J. Khoury, K. Hinterbichler, A. Joyce, C. Armendariz-Picon, L. McAllister, P. McGuirk, P. Creminelli, D. Baumann and S. Patil. I would also like to thank the members of the reading committee and opposition committee for their close look on my work.

My colleagues of the Instituut-Lorentz, the Dutch cosmology group and the DRSTP form a very active and informal network of people with similar thoughts, ideas and interests. It has been an honor to have been part of such a great community with wonderful and intelligent people, who form a pleasant group to work with on a daily basis. My fellow PhD students and postdocs, from the institute, the DRSTP and from Madison, have made the last four years a wonderful experience. Een speciale dank gaat daarbij uit naar Fran, Marianne en Trudy voor de structuur en warmte op het instituut.

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Bibliography

[1] L. McAllister and E. Silverstein. *String cosmology: A review*. General Relativity and Gravitation, **40**:565–605, 2008. `arXiv:0710.2951 [hep-th]`

[2] D. Baumann and L. McAllister. *Advances in inflation in string theory*. Ann.Rev.Nucl.Part.Sci., **59**:67–94, 2009. `arXiv:0901.0265 [hep-th]`

[3] A. Einstein. *The field equations of gravitation*. Sitzungsber.Preuss.Akad.Wiss.Berlin (Math.Phys.), **1915**:844–847, 1915

[4] A. Einstein. *The foundation of the general theory of relativity*. Annalen Phys., **49**:769–822, 1916

[5] A. Friedmann. *On the curvature of space*. Z.Phys., **10**:377–386, 1922

[6] G. Lemaitre. *A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulae*. Mon.Not.Roy.Astron.Soc., **91**:483–490, 1931

[7] H. P. Robertson. *On the foundations of relativistic cosmology*. Proceedings of the National Academy of Science, **15**:822–829, 1929

[8] A. G. Walker. *Distance in an expanding universe*. Monthly Notices of the Royal Astronomical Society, **94**:159, 1933

[9] E. Hubble. *A relation between distance and radial velocity among extra-galactic nebulae*. Proc.Nat.Acad.Sci., **15**:168–173, 1929

[10] E. Komatsu *et al. Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Cosmological interpretation*. Astrophys.J.Suppl., **192**:18, 2011. `arXiv:1001.4538 [astro-ph.CO]`

[11] R. Alpher, H. Bethe and G. Gamow. *The origin of chemical elements*. Phys.Rev., **73**:803–804, 1948

[12] A. A. Penzias and R. W. Wilson. *A measurement of excess antenna temperature at 4080-Mc/s*. Astrophys.J., **142**:419–421, 1965

[13] R. Dicke, P. Peebles, P. Roll and D. Wilkinson. *Cosmic black-body radiation*. Astrophys.J., **142**:414–419, 1965

[14] G. Gamow. *The evolution of the universe*. Nature, **162**:680–682, 1948

[15] R. A. Alpher and R. C. Herman. *On the relative abundance of the elements*. Phys.Rev., **74**:1737–1742, 1948

[16] D. Fixsen. *The temperature of the cosmic microwave background*. Astrophys.J., **707**:916–920, 2009. arXiv:0911.1955 [astro-ph.CO]
[17] V. Springel, S. D. White, A. Jenkins, C. S. Frenk, N. Yoshida *et al.* *Simulating the joint evolution of quasars, galaxies and their large-scale distribution*. Nature, **435**:629–636, 2005. arXiv:astro-ph/0504097 [astro-ph]
[18] W. J. Percival *et al.* *Baryon acoustic oscillations in the Sloan Digital Sky Survey data release 7 galaxy sample*. Mon.Not.Roy.Astron.Soc., **401**:2148–2168, 2010. arXiv:0907.1660 [astro-ph.CO]
[19] A. G. Riess *et al.* *Observational evidence from supernovae for an accelerating universe and a cosmological constant*. Astron.J., **116**:1009–1038, 1998. arXiv:astro-ph/9805201 [astro-ph]
[20] S. Perlmutter *et al.* *Measurements of Omega and Lambda from 42 high redshift supernovae*. Astrophys.J., **517**:565–586, 1999. arXiv:astro-ph/9812133 [astro-ph]
[21] A. Refregier. *Weak gravitational lensing by large scale structure*. Ann.Rev.Astron.Astrophys., **41**:645–668, 2003. arXiv:astro-ph/0307212 [astro-ph]
[22] F. Zwicky. *Spectral displacement of extra galactic nebulae*. Helv.Phys.Acta, **6**:110–127, 1933
[23] V. C. Rubin and J. Ford, W. Kent. *Rotation of the Andromeda nebula from a spectroscopic survey of emission regions*. Astrophys.J., **159**:379–403, 1970
[24] J. Beringer *et al.* (Particle Data Group). *2012 Review of particle physics*. Phys. Rev., **D86**:010001, 2012
[25] A. H. Guth. *The inflationary universe: A possible solution to the horizon and flatness Problems*. Physical Review, **D23**:347–356, 1981
[26] A. D. Linde. *A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems*. Physics Letters, **B108**:389–393, 1982
[27] A. Albrecht and P. J. Steinhardt. *Cosmology for grand unified theories with radiatively induced symmetry breaking*. Physical Review Letters, **48**:1220–1223, 1982
[28] S. Dodelson. *Modern cosmology*. 2003
[29] V. Mukhanov. *Physical foundations of cosmology*. 2005
[30] D. H. Lyth and A. R. Liddle. *The primordial density perturbation: Cosmology, inflation and the origin of structure* (Cambridge University Press, Cambridge, 2009)
[31] D. Baumann. *TASI Lectures on inflation*. 2009. arXiv:0907.5424 [hep-th]

[32] E. W. Kolb and M. Turner. *The early universe* (Westview Press, 1990)

[33] A. R. Liddle and D. H. Lyth. *Cosmological inflation and large-scale structure* (Cambridge University Press, Cambridge, 2000)

[34] A. R. Liddle, P. Parsons and J. D. Barrow. *Formalizing the slow roll approximation in inflation*. Phys.Rev., **D50**:7222–7232, 1994. `arXiv:astro-ph/9408015 [astro-ph]`

[35] D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. Nolta *et al.* *Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Power spectra and WMAP-derived parameters*. Astrophys.J.Suppl., **192**:16, 2011. `arXiv:1001.4635 [astro-ph.CO]`

[36] G. F. Smoot, C. Bennett, A. Kogut, E. Wright, J. Aymon *et al.* *Structure in the COBE differential microwave radiometer first year maps*. Astrophys.J., **396**:L1–L5, 1992

[37] V. F. Mukhanov and G. Chibisov. *Quantum fluctuation and nonsingular universe. (In Russian)*. JETP Lett., **33**:532–535, 1981

[38] V. F. Mukhanov and G. Chibisov. *The vacuum energy and large scale structure of the universe*. Sov.Phys.JETP, **56**:258–265, 1982

[39] A. A. Starobinsky. *Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations*. Phys.Lett., **B117**:175–178, 1982

[40] S. Hawking. *The development of irregularities in a single bubble inflationary universe*. Phys.Lett., **B115**:295, 1982

[41] A. H. Guth and S. Pi. *Fluctuations in the new inflationary universe*. Phys.Rev.Lett., **49**:1110–1113, 1982

[42] J. M. Bardeen, P. J. Steinhardt and M. S. Turner. *Spontaneous creation of almost scale-free density perturbations in an inflationary universe*. Phys.Rev., **D28**:679, 1983

[43] V. F. Mukhanov, H. Feldman and R. H. Brandenberger. *Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions*. Phys.Rept., **215**:203–333, 1992

[44] N. Bartolo, E. Komatsu, S. Matarrese and A. Riotto. *Non-Gaussianity from inflation: Theory and observations*. Phys.Rept., **402**:103–266, 2004. `arXiv:astro-ph/0406398 [astro-ph]`

[45] J. Fergusson and E. Shellard. *The shape of primordial non-Gaussianity and the CMB bispectrum*. Phys.Rev., **D80**:043510, 2009. `arXiv:0812.3413 [astro-ph]`

[46] E. Komatsu, N. Afshordi, N. Bartolo, D. Baumann, J. Bond *et al.* *Non-Gaussianity as a probe of the physics of the primordial universe and*

the astrophysics of the low redshift universe. 2009. arXiv:0902.4759 [astro-ph.CO]

[47] U. Seljak and M. Zaldarriaga. *A line of sight integration approach to cosmic microwave background anisotropies*. Astrophys.J., **469**:437–444, 1996. arXiv:astro-ph/9603033 [astro-ph]

[48] E. R. Harrison. *Fluctuations at the threshold of classical cosmology*. Phys.Rev., **D1**:2726–2730, 1970

[49] Y. B. Zel'dovich. *A hypothesis, unifying the structure and entropy of the universe*. MNRAS, **160**:1P, 1972

[50] S. Dodelson. *Coherent phase argument for inflation*. AIP Conf.Proc., **689**:184–196, 2003. arXiv:hep-ph/0309057 [hep-ph]

[51] M. G. Jackson and K. Schalm. *Model-independent signatures of new physics in slow-roll inflation*. 2011. arXiv:1104.0887 [hep-th]

[52] A. Achúcarro, J.-O. Gong, S. Hardeman, G. A. Palma and S. P. Patil. *Features of heavy physics in the CMB power spectrum*. JCAP, **1101**:030, 2011. arXiv:1010.3693 [hep-ph]

[53] E. Komatsu and D. N. Spergel. *Acoustic signatures in the primary microwave background bispectrum*. Phys.Rev., **D63**:063002, 2001. arXiv:astro-ph/0005036 [astro-ph]

[54] D. Babich, P. Creminelli and M. Zaldarriaga. *The shape of non-Gaussianities*. JCAP, **0408**:009, 2004. arXiv:astro-ph/0405356 [astro-ph]

[55] E. Komatsu and D. N. Spergel. *The cosmic microwave background bispectrum as a test of the physics of inflation and probe of the astrophysics of the low-redshift universe*. 2000. arXiv:astro-ph/0012197 [astro-ph]

[56] V. Acquaviva, N. Bartolo, S. Matarrese and A. Riotto. *Second order cosmological perturbations from inflation*. Nucl.Phys., **B667**:119–148, 2003. arXiv:astro-ph/0209156 [astro-ph]

[57] J. M. Maldacena. *Non-Gaussian features of primordial fluctuations in single field inflationary models*. JHEP, **0305**:013, 2003. arXiv:astro-ph/0210603 [astro-ph]

[58] N. Bartolo, S. Matarrese and A. Riotto. *Nongaussianity from inflation*. Phys.Rev., **D65**:103505, 2002. arXiv:hep-ph/0112261 [hep-ph]

[59] D. Seery and J. E. Lidsey. *Primordial non-Gaussianities from multiple-field inflation*. JCAP, **0509**:011, 2005. arXiv:astro-ph/0506056 [astro-ph]

[60] E. Silverstein and D. Tong. *Scalar speed limits and cosmology: Acceleration from D-cceleration*. Phys.Rev., **D70**:103505, 2004. arXiv:hep-th/0310221 [hep-th]

[61] D. Seery and J. E. Lidsey. *Primordial non-Gaussianities in single field infla-*

tion. JCAP, **0506**:003, 2005. arXiv:astro-ph/0503692 [astro-ph]

[62] X. Chen, M.-x. Huang, S. Kachru and G. Shiu. *Observational signatures and non-Gaussianities of general single field inflation*. JCAP, **0701**:002, 2007. arXiv:hep-th/0605045 [hep-th]

[63] R. Holman and A. J. Tolley. *Enhanced non-Gaussianity from excited initial states*. JCAP, **0805**:001, 2008. arXiv:0710.1302 [hep-th]

[64] P. D. Meerburg, J. P. van der Schaar and P. S. Corasaniti. *Signatures of initial state modifications on bispectrum statistics*. JCAP, **0905**:018, 2009. arXiv:0901.4044 [hep-th]

[65] P. D. Meerburg, J. P. van der Schaar and M. G. Jackson. *Bispectrum signatures of a modified vacuum in single field inflation with a small speed of sound*. JCAP, **1002**:001, 2010. arXiv:0910.4986 [hep-th]

[66] M. G. Jackson and K. Schalm. *Model-independent signatures of new physics in non-Gaussianity*. 2012. arXiv:1202.0604 [hep-th]

[67] A. D. Linde and V. F. Mukhanov. *Nongaussian isocurvature perturbations from inflation*. Phys.Rev., **D56**:535–539, 1997. arXiv:astro-ph/9610219 [astro-ph]

[68] P. Creminelli and L. Senatore. *A smooth bouncing cosmology with scale invariant spectrum*. JCAP, **0711**:010, 2007. arXiv:hep-th/0702165 [hep-th]

[69] Planck. *The scientific programme of Planck*. 2006. arXiv:astro-ph/0604069 [astro-ph]

[70] N. D. Birrell and P. C. W. Davies. *Quantum fields in curved space* (Cambridge University Press, Cambridge, 1982)

[71] R. L. Arnowitt, S. Deser and C. W. Misner. *The dynamics of general relativity*. 1962. arXiv:gr-qc/0405109 [gr-qc]

[72] M. E. Peskin and D. V. Schroeder. *An introduction to quantum field theory* (Westview Press, Boulder, 1995)

[73] J. S. Schwinger. *Brownian motion of a quantum oscillator*. J.Math.Phys., **2**:407–432, 1961

[74] E. Calzetta and B. Hu. *Closed time path functional formalism in curved space-time: Application to cosmological back reaction problems*. Phys.Rev., **D35**:495, 1987

[75] S. Weinberg. *Quantum contributions to cosmological correlations*. Phys.Rev., **D72**:043514, 2005. arXiv:hep-th/0506236 [hep-th]

[76] D. Seery, K. A. Malik and D. H. Lyth. *Non-Gaussianity of inflationary field perturbations from the field equation*. JCAP, **0803**:014, 2008. arXiv:0802.0588 [astro-ph]

[77] T. Falk, R. Rangarajan and M. Srednicki. *The angular dependence of the*

three point correlation function of the cosmic microwave background radiation as predicted by inflationary cosmologies. Astrophys.J., **403**:L1, 1993. arXiv:astro-ph/9208001 [astro-ph]

[78] M. Zaldarriaga. *Non-Gaussianities in models with a varying inflaton decay rate*. Phys.Rev., **D69**:043508, 2004. arXiv:astro-ph/0306006 [astro-ph]

[79] C. Burrage, R. H. Ribeiro and D. Seery. *Large slow-roll corrections to the bispectrum of noncanonical inflation*. JCAP, **1107**:032, 2011. arXiv:1103.4126 [astro-ph.CO]

[80] B. De Wit and J. Smith. *Field theory in particle physics, Vol. 1* (North Holland, 1986)

[81] S. Weinberg. *The quantum theory of fields. Vol. 1: Foundations* (Cambridge University Press, Cambridge, 1995)

[82] S. Weinberg. *The quantum theory of fields. Vol. 2: Modern applications* (Cambridge University Press, Cambridge, 1996)

[83] P. Di Francesco, P. Mathieu and D. Senechal. *Conformal field theory* (Springer-Verlag, Inc., New York, 1997)

[84] P. H. Ginsparg. *Applied conformal field theory*. 1988. arXiv:hep-th/9108028 [hep-th]

[85] M. B. Green, J. H. Schwarz and E. Witten. *Superstring theory, 2 volumes* (Cambridge University Press, Cambridge, 1987)

[86] J. Polchinski. *String theory, 2 volumes* (Cambridge University Press, Cambridge, 1998)

[87] D. Tong. *String theory*. 2009

[88] H. P. Nilles. *Supersymmetry, supergravity and particle physics*. Phys.Rept., **110**:1–162, 1984

[89] J. Wess and J. Bagger. *Supersymmetry and supergravity* (Princeton University Press, Princeton, New Jersey, 1992)

[90] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz. *Large N field theories, string theory and gravity*. Phys.Rept., **323**:183–386, 2000. arXiv:hep-th/9905111 [hep-th]

[91] H. Nastase. *Introduction to AdS-CFT*. 2007. arXiv:0712.0689 [hep-th]

[92] B. Delamotte. *A hint of renormalization*. Am.J.Phys., **72**:170–184, 2004. arXiv:hep-th/0212049 [hep-th]

[93] K. G. Wilson. *Renormalization group and critical phenomena. 1. Renormalization group and the Kadanoff scaling picture*. Phys.Rev., **B4**:3174–3183, 1971

[94] K. G. Wilson. *Renormalization group and critical phenomena. 2. Phase space*

cell analysis of critical behavior. Phys.Rev., **B4**:3184–3205, 1971

[95] K. G. Wilson and J. B. Kogut. *The renormalization group and the epsilon expansion*. Phys.Rept., **12**:75–200, 1974

[96] K. G. Wilson. *The renormalization group: Critical phenomena and the Kondo problem*. Rev.Mod.Phys., **47**:773, 1975

[97] D. Tong. *Quantum field theory*. 2006

[98] C. G. Callan, Jr., S. R. Coleman and R. Jackiw. *A new improved energy-momentum tensor*. Ann. Phys., **59**:42–73, 1970

[99] C. G. Callan, Jr. *Broken scale invariance in scalar field theory*. Phys.Rev., **D2**:1541–1547, 1970

[100] K. Symanzik. *Small distance behavior in field theory and power counting*. Commun.Math.Phys., **18**:227–246, 1970

[101] K. Symanzik. *Small distance behavior analysis and Wilson expansion*. Commun.Math.Phys., **23**:49–86, 1971

[102] B. de Wit, D. J. Smit and N. D. Hari Dass. *Residual supersymmetry of compactified D=10 supergravity*. Nuclear Physics, **B283**:165, 1987

[103] C. M. Hull and P. K. Townsend. *Finiteness and conformal invariance in non-linear sigma models*. Nuclear Physics, **B274**:349, 1986

[104] A. A. Tseytlin. *Conformal anomaly in two-dimensional sigma model on curved background and strings*. Physics Letters, **178B**:34, 1986

[105] A. A. Tseytlin. *Sigma model Weyl invariance conditions and string equations of motion*. Nucl.Phys., **B294**:383, 1987

[106] G. Shore. *A local renormalization group equation, diffeomorphisms, and conformal invariance in sigma models*. Nucl.Phys., **B286**:349, 1987

[107] C. G. Callan, Jr., I. R. Klebanov and M. J. Perry. *String theory effective actions*. Nuclear Physics, **B278**:78, 1986

[108] G. Curci and G. Paffuti. *Consistency between the string background field equation of motion and the vanishing of the conformal anomaly*. Nuclear Physics, **B286**:399, 1987

[109] C. M. Hull and P. K. Townsend. *String effective actions from sigma model conformal anomalies*. Nuclear Physics, **B301**:197, 1988

[110] A. B. Zamolodchikov. *Renormalization group and perturbation theory near fixed points in two-dimensional field theory*. Soviets Journal of Nuclear Physics, **46**:1090, 1987

[111] D. Z. Freedman, M. Headrick and A. Lawrence. *On closed string tachyon dynamics*. Physical Review, **D73**:066015, 2006. `arXiv:hep-th/0510126`

[112] J. L. Cardy. *Conformal invariance and statistical mechanics*. Champs, Cordes et Phénomènes Critiques, Les Houches, Session XLIX, 1989

[113] J. Scherk and J. H. Schwarz. *Dual models for nonhadrons*. Nucl.Phys., **B81**:118–144, 1974

[114] T. Yoneya. *Geometry, gravity and dual strings*. Prog.Theor.Phys., **56**:1310, 1976

[115] D. Friedan. *Nonlinear models in two epsilon dimensions*. Physical Review Letters, **45**:1057, 1980

[116] A. M. Polyakov. *Quantum geometry of bosonic strings*. Physics Letters, **B103**:207–210, 1981

[117] C. G. Callan, Jr., E. J. Martinec, M. J. Perry and D. Friedan. *Strings in background fields*. Nuclear Physics, **B262**:593, 1985

[118] D. Friedan, E. J. Martinec and S. H. Shenker. *Conformal invariance, supersymmetry and string theory*. Nuclear Physics, **B271**:93, 1986

[119] E. Fradkin and A. A. Tseytlin. *Quantum string theory effective action*. Nucl.Phys., **B261**:1–27, 1985

[120] C. G. Callan and L. Thorlacius. *Sigma models and string theory*. Particles, Strings, and Supernovae, TASI 1988, pages 795–878, 1989

[121] R. Metsaev and A. A. Tseytlin. *Order alpha-prime (two loop) equivalence of the string equations of motion and the sigma model Weyl invariance conditions: Dependence on the dilaton and the antisymmetric tensor*. Nucl.Phys., **B293**:385, 1987

[122] A. A. Tseytlin. *Conditions of Weyl invariance of the two-dimensional sigma model from equations of stationarity of the 'central charge' action*. Phys.Lett., **B194**:63, 1987

[123] P. Candelas, G. T. Horowitz, A. Strominger and E. Witten. *Vacuum configurations for superstrings*. Nucl.Phys., **B258**:46–74, 1985

[124] Y. A. Gol'fand and E. P. Likhtman. *Extension of the algebra of Poincaré group generators and violation of P invariance*. JETP Lett., **13**:323–326, 1971

[125] D. Volkov and V. Akulov. *Is the neutrino a Goldstone particle?* Phys.Lett., **B46**:109–110, 1973

[126] J. Wess and B. Zumino. *Supergauge transformations in four-dimensions*. Nucl.Phys., **B70**:39–50, 1974

[127] E. Witten. *Dynamical breaking of supersymmetry*. Nucl.Phys., **B188**:513, 1981

[128] D. Z. Freedman, P. van Nieuwenhuizen and S. Ferrara. *Progress toward a theory of supergravity*. Phys.Rev., **D13**:3214–3218, 1976

[129] G. 't Hooft. *Dimensional reduction in quantum gravity*. 1993. `arXiv:gr-qc/9310026 [gr-qc]`

[130] L. Susskind. *The world as a hologram*. J.Math.Phys., **36**:6377–6396, 1995.

arXiv:hep-th/9409089 [hep-th]

[131] J. M. Maldacena and A. Strominger. *Black hole grey body factors and D-brane spectroscopy*. Phys.Rev., **D55**:861–870, 1997. arXiv:hep-th/9609026 [hep-th]

[132] I. R. Klebanov. *World volume approach to absorption by nondilatonic branes*. Nucl.Phys., **B496**:231–242, 1997. arXiv:hep-th/9702076 [hep-th]

[133] S. S. Gubser, I. R. Klebanov and A. A. Tseytlin. *String theory and classical absorption by three-branes*. Nucl.Phys., **B499**:217–240, 1997. arXiv:hep-th/9703040 [hep-th]

[134] S. S. Gubser and I. R. Klebanov. *Absorption by branes and Schwinger terms in the world volume theory*. Phys.Lett., **B413**:41–48, 1997. arXiv:hep-th/9708005 [hep-th]

[135] J. M. Maldacena and A. Strominger. *Semiclassical decay of near extremal five-branes*. JHEP, **9712**:008, 1997. arXiv:hep-th/9710014 [hep-th]

[136] J. M. Maldacena. *The large N limit of superconformal field theories and supergravity*. Adv.Theor.Math.Phys., **2**:231–252, 1998. arXiv:hep-th/9711200 [hep-th]

[137] K. Becker, M. Becker and J. Schwarz. *String theory and M-theory, a modern introduction* (Cambridge University Press, Cambridge, 2007)

[138] K. Skenderis. *Lecture notes on holographic renormalization*. Class.Quant.Grav., **19**:5849–5876, 2002. arXiv:hep-th/0209067 [hep-th]

[139] J. de Boer, E. P. Verlinde and H. L. Verlinde. *On the holographic renormalization group*. JHEP, **0008**:003, 2000. arXiv:hep-th/9912012 [hep-th]

[140] J. de Boer. *The holographic renormalization group*. Fortsch.Phys., **49**:339–358, 2001. arXiv:hep-th/0101026 [hep-th]

[141] M. Bianchi, D. Z. Freedman and K. Skenderis. *How to go with an RG flow*. JHEP, **0108**:041, 2001. arXiv:hep-th/0105276 [hep-th]

[142] M. Bianchi, D. Z. Freedman and K. Skenderis. *Holographic renormalization*. Nucl.Phys., **B631**:159–194, 2002. arXiv:hep-th/0112119 [hep-th]

[143] J. McGreevy. *Holographic duality with a view toward many-body physics*. Adv.High Energy Phys., **2010**:723105, 2010. arXiv:0909.0518 [hep-th]

[144] E. Witten. *Quantum gravity in de Sitter space*. 2001. arXiv:hep-th/0106109 [hep-th]

[145] A. Strominger. *The dS/CFT correspondence*. JHEP, **0110**:034, 2001. arXiv:hep-th/0106113 [hep-th]

[146] A. Strominger. *Inflation and the dS/CFT correspondence*. JHEP, **0111**:049, 2001. arXiv:hep-th/0110087 [hep-th]

[147] S. Gubser, I. R. Klebanov and A. M. Polyakov. *Gauge theory correlators from noncritical string theory*. Phys.Lett., **B428**:105–114, 1998. `arXiv:hep-th/9802109 [hep-th]`

[148] E. Witten. *Anti-de Sitter space and holography*. Adv.Theor.Math.Phys., **2**:253–291, 1998. `arXiv:hep-th/9802150 [hep-th]`

[149] D. Z. Freedman, S. D. Mathur, A. Matusis and L. Rastelli. *Correlation functions in the CFT(d) / AdS(d+1) correspondence*. Nucl.Phys., **B546**:96–118, 1999. `arXiv:hep-th/9804058 [hep-th]`

[150] W. Mück and K. Viswanathan. *Conformal field theory correlators from classical scalar field theory on AdS(d+1)*. Phys.Rev., **D58**:041901, 1998. `arXiv:hep-th/9804035 [hep-th]`

[151] S. Hardeman, J. M. Oberreuter, G. A. Palma, K. Schalm and T. van der Aalst. *The everpresent eta-problem: knowledge of all hidden sectors required*. JHEP, **1104**:009, 2011. `arXiv:1012.5966 [hep-ph]`

[152] A. Achúcarro, S. Hardeman, J. M. Oberreuter, K. Schalm and T. van der Aalst. *Decoupling limits in multi-sector supergravities*. 2011. `arXiv:1108.2278 [hep-th]`

[153] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands. *False vacuum inflation with Einstein gravity*. Phys. Rev., **D49**:6410–6433, 1994. `arXiv:astro-ph/9401011`

[154] M. Berg, D. Marsh, L. McAllister and E. Pajer. *Sequestering in string compactifications*. 2010. `arXiv:1012.1858 [hep-th]`

[155] R. Kallosh and A. Linde. *New models of chaotic inflation in supergravity*. JCAP, **1011**:011, 2010. `arXiv:1008.3375 [hep-th]`

[156] R. Kallosh, A. Linde and T. Rube. *General inflaton potentials in supergravity*. Phys. Rev., **D83**:043507, 2011. `arXiv:1011.5945 [hep-th]`

[157] T. Banks, D. B. Kaplan and A. E. Nelson. *Cosmological implications of dynamical supersymmetry breaking*. Phys. Rev., **D49**:779–787, 1994. `arXiv:hep-ph/9308292`

[158] R. L. Arnowitt and P. Nath. *Supersymmetry and supergravity: Phenomenology and grand unification*. 1993. `arXiv:hep-ph/9309277`

[159] J. Bagger, E. Poppitz and L. Randall. *The R axion from dynamical supersymmetry breaking*. Nucl. Phys., **B426**:3–18, 1994. `arXiv:hep-ph/9405345`

[160] J. Bagger. *Dynamical supersymmetry breaking in supergravity theories*. 1994. `arXiv:hep-ph/9503245`

[161] C. Muñoz. *Soft supersymmetry breaking terms and the mu problem*. 1995. `arXiv:hep-th/9507108`

[162] M. Dine. *TASI lectures on M theory phenomenology*. 2000.

arXiv:hep-th/0003175

[163] L. Alvarez-Gaume, C. Gomez and R. Jimenez. *Minimal inflation*. Phys.Lett., **B690**:68–72, 2010. arXiv:1001.0010 [hep-th]

[164] M. Dine and L. Pack. *Studies in small field inflation*. JCAP, **1206**:033, 2012. arXiv:1109.2079 [hep-ph]

[165] A. Achúcarro, S. Mooij, P. Ortiz and M. Postma. *Sgoldstino inflation*. 2012. arXiv:1203.1907 [hep-th]

[166] A. Achúcarro and K. Sousa. *F-term uplifting and moduli stabilization consistent with Kähler invariance*. JHEP, **03**:015, 2008. 0712.3460

[167] A. Achúcarro, S. Hardeman and K. Sousa. *Consistent decoupling of heavy scalars and moduli in N=1 supergravity*. Phys. Rev., **D78**:101901, 2008. arXiv:0806.4364 [hep-th]

[168] S. P. de Alwis. *Effective potentials for light moduli*. Phys. Lett., **B626**:223–229, 2005. hep-th/0506266

[169] S. P. de Alwis. *On integrating out heavy fields in SUSY theories*. Phys. Lett., **B628**:183–187, 2005. hep-th/0506267

[170] A. Achúcarro, S. Hardeman and K. Sousa. *F-term uplifting and the supersymmetric integration of heavy moduli*. JHEP, **11**:003, 2008. arXiv:0809.1441 [hep-th]

[171] D. Gallego. *On the effective description of large volume compactifications*. JHEP, **06**:087, 2011. arXiv:1103.5469 [hep-th]

[172] I. Ben-Dayan, R. Brustein and S. P. de Alwis. *Models of modular inflation and their phenomenological consequences*. JCAP, **0807**:011, 2008. arXiv:0802.3160 [hep-th]

[173] S. C. Davis and M. Postma. *Successfully combining SUGRA hybrid inflation and moduli stabilisation*. JCAP, **0804**:022, 2008. arXiv:0801.2116 [hep-th]

[174] A. Achúcarro, J.-O. Gong, S. Hardeman, G. A. Palma and S. P. Patil. *Mass hierarchies and non-decoupling in multi-scalar field dynamics*. Phys.Rev., **D84**:043502, 2011. arXiv:1005.3848 [hep-th]

[175] V. S. Kaplunovsky and J. Louis. *Model independent analysis of soft terms in effective supergravity and in string theory*. Phys.Lett., **B306**:269–275, 1993. arXiv:hep-th/9303040 [hep-th]

[176] A. D. Linde. *Hybrid inflation*. Phys.Rev., **D49**:748–754, 1994. arXiv:astro-ph/9307002 [astro-ph]

[177] E. Cremmer, P. Fayet and L. Girardello. *Gravity induced supersymmetry breaking and low-energy mass spectrum*. Phys. Lett., **B122**:41, 1983

[178] P. Binetruy and M. K. Gaillard. *Temperature corrections, supersymmetric ef-*

fective potentials ans inflation. Nucl. Phys., **B254**:388, 1985

[179] R. Barbieri, E. Cremmer and S. Ferrara. *Flat and positive potentials in N=1 supergravity*. Phys. Lett., **B163**:143, 1985

[180] J. P. Hsu, R. Kallosh and S. Prokushkin. *On brane inflation with volume stabilization*. JCAP, **0312**:009, 2003. `hep-th/0311077`

[181] P. Binetruy, G. Dvali, R. Kallosh and A. Van Proeyen. *Fayet-Iliopoulos terms in supergravity and cosmology*. Class. Quant. Grav., **21**:3137–3170, 2004. `arXiv:hep-th/0402046`

[182] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski. *Stability of flux compactifications and the pattern of supersymmetry breaking*. JHEP, **11**:076, 2004. `arXiv:hep-th/0411066`

[183] G. Dvali, Q. Shafi and R. K. Schaefer. *Large scale structure and supersymmetric inflation without fine tuning*. Phys.Rev.Lett., **73**:1886–1889, 1994. `arXiv:hep-ph/9406319 [hep-ph]`

[184] A. D. Linde and A. Riotto. *Hybrid inflation in supergravity*. Phys.Rev., **D56**:1841–1844, 1997. `arXiv:hep-ph/9703209 [hep-ph]`

[185] S. Ferrara, C. Kounnas and F. Zwirner. *Mass formulae and natural hierarchy in string effective supergravities*. Nucl. Phys., **B429**:589–625, 1994. `arXiv:hep-th/9405188`

[186] M. Gomez-Reino and C. A. Scrucca. *Locally stable non-supersymmetric Minkowski vacua in supergravity*. JHEP, **05**:015, 2006. `hep-th/0602246`

[187] T. W. B. Kibble. *Topology of cosmic domains and strings*. J. Phys., **A9**:1387–1398, 1976

[188] R. Jeannerot, J. Rocher and M. Sakellariadou. *How generic is cosmic string formation in SUSY GUTs*. Phys. Rev., **D68**:103514, 2003. `arXiv:hep-ph/0308134`

[189] F. L. Bezrukov and M. Shaposhnikov. *The standard model Higgs boson as the inflaton*. Phys. Lett., **B659**:703–706, 2008. `arXiv:0710.3755 [hep-th]`

[190] C. Germani and A. Kehagias. *New model of inflation with non-minimal derivative coupling of standard model Higgs boson to gravity*. Phys. Rev. Lett., **105**:011302, 2010. `arXiv:1003.2635 [hep-ph]`

[191] C. P. Burgess, J. M. Cline, H. Stoica and F. Quevedo. *Inflation in realistic D-brane models*. JHEP, **09**:033, 2004. `arXiv:hep-th/0403119`

[192] S. Groot Nibbelink and B. J. W. van Tent. *Density perturbations arising from multiple field slow- roll inflation*. 2000. `arXiv:hep-ph/0011325`

[193] S. Groot Nibbelink and B. J. W. van Tent. *Scalar perturbations during multiple field slow-roll inflation*. Classical and Quantum Gravity, **19**:613–640, 2002. `arXiv:hep-ph/0107272`

[194] L. Covi, M. Gomez-Reino, J. Louis, G. A. Palma and C. A. Srucca. *Constraints on modular inflation in supergravity and string theory*. 2008. arXiv:0805.3290 [hep-th]

[195] L. Covi, M. Gomez-Reino, J. Louis, G. A. Palma and C. A. Srucca. *De Sitter vacua in no-scale supergravities and Calabi-Yau string models*. JHEP, **06**:057, 2008. arXiv:0804.1073 [hep-th]

[196] L. Covi, M. Gomez-Reino, C. Gross, G. A. Palma and C. A. Scrucca. *Constructing de Sitter vacua in no-scale string models without uplifting*. JHEP, **03**:146, 2009. arXiv:0812.3864 [hep-th]

[197] D. Baumann, A. Dymarsky, I. R. Klebanov, L. McAllister and P. J. Steinhardt. *A delicate universe*. Physical Review Letters, **99**:141601, 2007. arXiv:0705.3837 [hep-th]

[198] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi. *De Sitter vacua in string theory*. Physical Review, **D68**:046005, 2003. arXiv:hep-th/0301240

[199] S. B. Giddings, S. Kachru and J. Polchinski. *Hierarchies from fluxes in string compactifications*. Physical Review, **D66**:106006, 2002. arXiv:hep-th/0105097

[200] S. Mollerach. *Isocurvature baryon perturbations and inflation*. Phys. Rev., **D42**:313–325, 1990

[201] D. Polarski and A. A. Starobinsky. *Isocurvature perturbations in multiple inflationary models*. Phys. Rev., **D50**:6123–6129, 1994. arXiv:astro-ph/9404061

[202] J.-c. Hwang and H. Noh. *Cosmological perturbations with multiple fluids and fields*. Class. Quant. Grav., **19**:527–550, 2002. arXiv:astro-ph/0103244

[203] D. Wands, N. Bartolo, S. Matarrese and A. Riotto. *An observational test of two-field inflation*. Phys. Rev., **D66**:043520, 2002. arXiv:astro-ph/0205253

[204] F. Di Marco, F. Finelli and R. Brandenberger. *Adiabatic and isocurvature perturbations for multifield generalized Einstein models*. Phys. Rev., **D67**:063512, 2003. arXiv:astro-ph/0211276

[205] B. J. W. van Tent. *Multiple-field inflation and the CMB*. Class. Quant. Grav., **21**:349–370, 2004. arXiv:astro-ph/0307048

[206] C. T. Byrnes and D. Wands. *Curvature and isocurvature perturbations from two-field inflation in a slow-roll expansion*. Phys. Rev., **D74**:043529, 2006. arXiv:astro-ph/0605679

[207] D. Wands. *Multiple field inflation*. Lect. Notes Phys., **738**:275–304, 2008. arXiv:astro-ph/0702187

[208] Z. Lalak, D. Langlois, S. Pokorski and K. Turzyński. *Curvature and*

isocurvature perturbations in two-field inflation. JCAP, **0707**:014, 2007. arXiv:0704.0212 [hep-th]

[209] K. A. Malik and D. Wands. *Cosmological perturbations*. Phys. Rept., **475**:1–51, 2009. arXiv:0809.4944 [astro-ph]

[210] D. Langlois, F. Vernizzi and D. Wands. *Non-linear isocurvature perturbations and non-Gaussianities*. JCAP, **0812**:004, 2008. arXiv:0809.4646 [astro-ph]

[211] D. Langlois and S. Renaux-Petel. *Perturbations in generalized multi-field inflation*. JCAP, **0804**:017, 2008. arXiv:0801.1085 [hep-th]

[212] D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka. *Primordial fluctuations and non-Gaussianities in multi-field DBI inflation*. Physical Review Letters, **101**:061301, 2008. arXiv:0804.3139 [hep-th]

[213] D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka. *Primordial perturbations and non-Gaussianities in DBI and general multi-field inflation*. Physical Review, **D78**:063523, 2008. arXiv:0806.0336 [hep-th]

[214] G. I. Rigopoulos, E. P. S. Shellard and B. J. W. van Tent. *Non-linear perturbations in multiple-field inflation*. Phys. Rev., **D73**:083521, 2006. arXiv:astro-ph/0504508

[215] C. M. Peterson and M. Tegmark. *Testing two-field inflation*. 2010. arXiv:1005.4056 [astro-ph.CO]

[216] S. Cremonini, Z. Lalak and K. Turzyński. *Strongly coupled perturbations in two-field inflationary models*. 2010. arXiv:1010.3021 [hep-th]

[217] L. Verde, L.-M. Wang, A. Heavens and M. Kamionkowski. *Large-scale structure, the cosmic microwave background, and primordial non-Gaussianity*. Mon. Not. Roy. Astron. Soc., **313**:L141–L147, 2000. arXiv:astro-ph/9906301

[218] F. Bernardeau and J.-P. Uzan. *Non-Gaussianity in multi-field inflation*. Phys. Rev., **D66**:103506, 2002. arXiv:hep-ph/0207295

[219] G. I. Rigopoulos, E. P. S. Shellard and B. J. W. van Tent. *Large non-Gaussianity in multiple-field inflation*. Phys. Rev., **D73**:083522, 2006. arXiv:astro-ph/0506704

[220] G. I. Rigopoulos, E. P. S. Shellard and B. J. W. van Tent. *Quantitative bispectra from multifield inflation*. Phys. Rev., **D76**:083512, 2007. arXiv:astro-ph/0511041

[221] F. Vernizzi and D. Wands. *Non-Gaussianities in two-field inflation*. JCAP, **0605**:019, 2006. arXiv:astro-ph/0603799

[222] A. J. Tolley and M. Wyman. *The gelaton scenario: Equilateral non-Gaussianity from multi-field dynamics*. Phys. Rev., **D81**:043502, 2010.

arXiv:0910.1853 [hep-th]
[223] X. Chen and Y. Wang. *Large non-Gaussianities with intermediate shapes from quasi-single field inflation*. Phys. Rev., **D81**:063511, 2010. arXiv:0909.0496 [astro-ph.CO]
[224] X. Chen and Y. Wang. *Quasi-single field inflation and non-Gaussianities*. JCAP, **1004**:027, 2010. arXiv:0911.3380 [hep-th]
[225] X. Dong, B. Horn, E. Silverstein and A. Westphal. *Simple exercises to flatten your potential*. 2010. arXiv:1011.4521 [hep-th]
[226] K. Schalm and T. van der Aalst. *A worldsheet perspective on string inflation*. 2010. arXiv:1008.5024 [hep-th]
[227] S. H. Henry Tye. *Brane inflation: String theory viewed from the cosmos*. Lecture Notes in Physics, **737**:949–974, 2008. arXiv:hep-th/0610221
[228] J. M. Cline. *String cosmology*. 2006. arXiv:hep-th/0612129
[229] R. Kallosh. *On inflation in string theory*. Lecture Notes in Physics, **738**:119–156, 2008. arXiv:hep-th/0702059
[230] C. P. Burgess. *Lectures on cosmic inflation and its potential stringy realizations*. PoS, **P2GC**:008, 2006. arXiv:0708.2865 [hep-th]
[231] J. Martin and R. H. Brandenberger. *A cosmological window on trans-Planckian physics*. 2000. arXiv:astro-ph/0012031
[232] A. Kempf and J. C. Niemeyer. *Perturbation spectrum in inflation with cutoff*. Physical Review, **D64**:103501, 2001. arXiv:astro-ph/0103225
[233] R. Easther, B. R. Greene, W. H. Kinney and G. Shiu. *Inflation as a probe of short distance physics*. Physical Review, **D64**:103502, 2001. arXiv:hep-th/0104102
[234] N. Kaloper, M. Kleban, A. E. Lawrence and S. Shenker. *Signatures of short distance physics in the cosmic microwave background*. Physical Review, **D66**:123510, 2002. arXiv:hep-th/0201158
[235] B. Greene, K. Schalm, J. P. van der Schaar and G. Shiu. *Extracting new physics from the CMB*. 2005. arXiv:astro-ph/0503458
[236] M. G. Jackson and K. Schalm. *Model independent signatures of new physics in the inflationary power spectrum*. 2010. arXiv:1007.0185 [hep-th]
[237] C. Schmidhuber and A. A. Tseytlin. *On string cosmology and the RG flow in 2-d field theory*. Nuclear Physics, **B426**:187–202, 1994. arXiv:hep-th/9404180
[238] S. Hellerman and I. Swanson. *Cosmological solutions of supercritical string theory*. Physical Review, **D77**:126011, 2008. arXiv:hep-th/0611317
[239] G. W. Gibbons. *Aspects of supergravity theories*. 1984. Three lectures given at GIFT Seminar on Theoretical Physics, San Feliu de Guixols, Spain, Jun 4-11,

1984
[240] S. R. Green, E. J. Martinec, C. Quigley and S. Sethi. *Constraints on string cosmology*. 2011. `arXiv:1110.0545 [hep-th]`
[241] W. Fischler and L. Susskind. *Dilaton tadpoles, string condensates and scale invariance*. Physics Letters, **B171**:383, 1986
[242] W. Fischler and L. Susskind. *Dilaton tadpoles, string condensates and scale invariance. 2*. Physics Letters, **B173**:262, 1986
[243] S. R. Das and S.-J. Rey. *Dilaton condensates and loop effects in open and closed bosonic strings*. Physics Letters, **B186**:328, 1987
[244] M. Dine and N. Seiberg. *Couplings and scales in superstring models*. Physical Review Letters, **55**:366, 1985
[245] M. Dine and N. Seiberg. *Is the superstring weakly coupled?* Physics Letters, **B162**:299, 1985
[246] A. B. Zamolodchikov. *"Irreversibility" of the flux of the renormalization group in a 2D field theory*. Journal of Experimental and Theoretical Physics Letters, **43**:730–732, 1986
[247] H. Kodama and M. Sasaki. *Cosmological perturbation theory*. Progress of Theoretical Physics Supplement, **78**:1–166, 1984
[248] A. A. Starobinsky. *Multicomponent de Sitter (inflationary) stages and the generation of perturbations*. Journal of Experimental and Theoretical Physics Letters, **42**:152–155, 1985
[249] K. Dasgupta, G. Rajesh and S. Sethi. *M theory, orientifolds and G-flux*. Journal of High Energy Physics, **08**:023, 1999. `arXiv:hep-th/9908088`
[250] B. R. Greene, K. Schalm and G. Shiu. *Warped compactifications in M and F theory*. Nuclear Physics, **B584**:480–508, 2000. `arXiv:hep-th/0004103`
[251] M. R. Douglas and S. Kachru. *Flux compactification*. Review of Modern Physics, **79**:733–796, 2007. `arXiv:hep-th/0610102`
[252] D. Kutasov. *Geometry on the space of conformal field theories and contact terms*. Physics Letters, **B220**:153, 1989
[253] K. Schalm, G. Shiu and T. van der Aalst. *Consistency condition for inflation from (broken) conformal symmetry*. 2012. `arXiv:1211.2157 [hep-th]`
[254] D. Seery and J. E. Lidsey. *Non-Gaussian inflationary perturbations from the dS/CFT correspondence*. JCAP, **0606**:001, 2006. `arXiv:astro-ph/0604209 [astro-ph]`
[255] I. Antoniadis, P. O. Mazur and E. Mottola. *Conformal invariance, dark energy, and CMB non-Gaussianity*. 2011. `arXiv:1103.4164 [gr-qc]`
[256] J. M. Maldacena and G. L. Pimentel. *On graviton non-Gaussianities during inflation*. JHEP, **1109**:045, 2011. `arXiv:1104.2846 [hep-th]`

[257] P. Creminelli. *Conformal invariance of scalar perturbations in inflation*. 2011. arXiv:1108.0874 [hep-th]

[258] S. A. Hartnoll, P. K. Kovtun, M. Muller and S. Sachdev. *Theory of the Nernst effect near quantum phase transitions in condensed matter, and in dyonic black holes*. Phys.Rev., **B76**:144502, 2007. arXiv:0706.3215 [cond-mat.str-el]

[259] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz. *Building a holographic superconductor*. Phys.Rev.Lett., **101**:031601, 2008. arXiv:0803.3295 [hep-th]

[260] S.-S. Lee. *A non-Fermi liquid from a charged black hole: A critical Fermi ball*. Phys.Rev., **D79**:086006, 2009. arXiv:0809.3402 [hep-th]

[261] H. Liu, J. McGreevy and D. Vegh. *Non-Fermi liquids from holography*. Phys.Rev., **D83**:065029, 2011. arXiv:0903.2477 [hep-th]

[262] M. Čubrović, J. Zaanen and K. Schalm. *String theory, quantum phase transitions and the emergent Fermi-liquid*. Science, **325**:439–444, 2009. arXiv:0904.1993 [hep-th]

[263] F. Larsen, J. P. van der Schaar and R. G. Leigh. *De Sitter holography and the cosmic microwave background*. JHEP, **0204**:047, 2002. arXiv:hep-th/0202127 [hep-th]

[264] F. Larsen and R. McNees. *Inflation and de Sitter holography*. JHEP, **07**:051, 2003. arXiv:hep-th/0307026

[265] J. P. van der Schaar. *Inflationary perturbations from deformed CFT*. JHEP, **0401**:070, 2004. arXiv:hep-th/0307271 [hep-th]

[266] L. Dyson, M. Kleban and L. Susskind. *Disturbing implications of a cosmological constant*. JHEP, **0210**:011, 2002. arXiv:hep-th/0208013 [hep-th]

[267] N. Goheer, M. Kleban and L. Susskind. *The trouble with de Sitter space*. JHEP, **0307**:056, 2003. arXiv:hep-th/0212209 [hep-th]

[268] L. Dyson, J. Lindesay and L. Susskind. *Is there really a de Sitter/CFT duality?* JHEP, **0208**:045, 2002. arXiv:hep-th/0202163 [hep-th]

[269] I. Antoniadis, P. O. Mazur and E. Mottola. *Conformal invariance and cosmic background radiation*. Phys.Rev.Lett., **79**:14–17, 1997. arXiv:astro-ph/9611208 [astro-ph]

[270] P. Creminelli, J. Norena and M. Simonovic. *Conformal consistency relations for single-field inflation*. 2012. arXiv:1203.4595 [hep-th]

[271] V. Assassi, D. Baumann and D. Green. *On soft limits of inflationary correlation functions*. 2012. arXiv:1204.4207 [hep-th]

[272] A. Kehagias and A. Riotto. *Operator product expansion of inflationary correlators and conformal symmetry of de Sitter*. 2012. arXiv:1205.1523 [hep-th]

[273] A. Kehagias and A. Riotto. *The four-point correlator in multifield inflation, the operator product expansion and the symmetries of de Sitter*. 2012. `arXiv:1210.1918 [hep-th]`

[274] V. Assassi, D. Baumann and D. Green. *Symmetries and loops in inflation*. 2012. `arXiv:1210.7792 [hep-th]`

[275] K. Hinterbichler and J. Khoury. *The pseudo-conformal universe: Scale invariance from spontaneous breaking of conformal symmetry*. 2011. `arXiv:1106.1428 [hep-th]`

[276] K. Hinterbichler, A. Joyce and J. Khoury. *Non-linear realizations of conformal symmetry and effective field theory for the pseudo-conformal universe*. JCAP, **1206**:043, 2012. `arXiv:1202.6056 [hep-th]`

[277] K. Hinterbichler, L. Hui and J. Khoury. *Conformal symmetries of adiabatic modes in cosmology*. 2012. `arXiv:1203.6351 [hep-th]`

[278] K. Hinterbichler, A. Joyce, J. Khoury and G. E. Miller. *DBI realizations of the pseudo-conformal universe and Galilean genesis scenarios*. 2012. `arXiv:1209.5742 [hep-th]`

[279] P. Creminelli and M. Zaldarriaga. *Single field consistency relation for the 3-point function*. JCAP, **0410**:006, 2004. `arXiv:astro-ph/0407059 [astro-ph]`

[280] C. Cheung, A. L. Fitzpatrick, J. Kaplan and L. Senatore. *On the consistency relation of the 3-point function in single field inflation*. JCAP, **0802**:021, 2008. `arXiv:0709.0295 [hep-th]`

[281] P. McFadden and K. Skenderis. *Holographic non-Gaussianity*. JCAP, **1105**:013, 2011. `arXiv:1011.0452 [hep-th]`

[282] P. McFadden and K. Skenderis. *Cosmological 3-point correlators from holography*. JCAP, **1106**:030, 2011. `arXiv:1104.3894 [hep-th]`

[283] A. Bzowski, P. McFadden and K. Skenderis. *Holographic predictions for cosmological 3-point functions*. 2011. `arXiv:1112.1967 [hep-th]`

[284] M. Spradlin and A. Volovich. *Vacuum states and the S matrix in dS/CFT*. Phys.Rev., **D65**:104037, 2002. `arXiv:hep-th/0112223 [hep-th]`

[285] A. J. Tolley and N. Turok. *Quantization of the massless minimally coupled scalar field and the dS/CFT correspondence*. 2001. `arXiv:hep-th/0108119 [hep-th]`

[286] M. Bianchi, M. Prisco and W. Mück. *New results on holographic three point functions*. JHEP, **0311**:052, 2003. `arXiv:hep-th/0310129 [hep-th]`

[287] W. Mück. *Running scaling dimensions in holographic renormalization group flows*. JHEP, **1008**:085, 2010. `arXiv:1006.2987 [hep-th]`

[288] W. H. Kinney. *Inflation: Flow, fixed points and observables to arbitrary or-*

der in slow roll. Phys.Rev., **D66**:083508, 2002. arXiv:astro-ph/0206032 [astro-ph]

[289] B. Underwood. *A breathing mode for warped compactifications*. Class.Quant.Grav., **28**:195013, 2011. arXiv:1009.4200 [hep-th]

[290] M. Abramowitz and I. A. Stegun. *Handbook of mathematical functions with formulas, graphs, and mathematical tables* (Dover Publications, New York, 1972)