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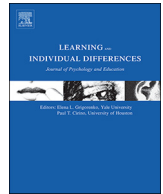
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# Latent variable mixture models in research on learning and individual differences

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## ABSTRACT

Herein, we provide an introduction to the special issue “Latent variable mixture models in research on learning and individual differences”. Latent variable mixture models are argued to be a powerful tool for capturing non-linear and qualitative individual differences in learners' knowledge, characteristics, and development. The current special issue provides an overview of the use of these analytical tools in investigations of learning and individual differences by presenting a wide-range of empirical studies utilizing the methods. A practical non-technical introduction and discussion are also included in the special issue.

## 1. Justification for this special issue

The major aim of a large portion of the studies published in *Learning and Individual Differences* is to examine the nature of learners' knowledge, characteristics, and development. Often this involves probing the relations between individual differences in these features, the influences of instruction on them, and their relations to educational outcomes. A large part of this scholarly work focuses on the relation between measured quantitative variables aggregated using summary statistics, such as means and standard deviations (Snow, 1986). However, it is often the case that creating sum scores and using mean values in analyses may gloss over important non-linear and/or qualitative differences that exist within and across individuals (Hickendorff, Edelsbrunner, McMullen, Schneider, & Trezise, this issue). Indeed, the structure of knowledge, relations between different aspects of learning and cognition, and development does not always map easily onto a linear space.

At the heart of data analysis is information reduction, which aims to answer the question: How can we more concisely, yet accurately, portray this data in a form that is useful for interpretation? In the case of research on learning and individual differences, we are particularly interested in examining how learners' responses to a varied array of questions, tests, or tasks differ across or within individuals and how these responses are related to other relevant indicators. Often times, we are interested in identifying a certain group of individuals (Bergman & Magnusson, 1997), for example those who have a low level of knowledge and may need extra support. However, it is also possible that these individuals may not be reasonably identifiable using an overall score on

a test. For example, a student with a strongly held misconception about the shape of the planet earth and a student who is starting to form a more accurate concept may both perform relatively poorly on the same test, when assessing for correctness (e.g. Straatemeier, van der Maas, & Jansen, 2008). Yet, these two students may need different instructional interventions in order to progress towards a more scientifically correct concept. In this case, using sum scores could lead to overlooking these important differences in the nature of students' knowledge. Giving both students the same type of instructional support may waste valuable resources and even cause negative effects for some students.

When such non-linear differences appear, those of us interested in examining individual differences in learning need other means for examining the structure, relations, and development of learners' knowledge. The present special issue presents the case that latent variable mixture models, such as Latent Profile Analysis (LPA), Latent Class Analysis (LCA), and Latent Transition Analysis (LTA), are powerful and effective tools for capturing the non-linear, qualitative differences that exist in some aspects of learners' knowledge, characteristics, and development. With these tools it is possible to uncover hidden, or unobservable, groups of individuals within a sample that differ on key relevant characteristics (Bergman & Magnusson, 1997). This allows for condensing data into a more manageable and interpretable form, without glossing over important distinctions in the nature and relations between variables.

The current special issue aims to provide an overview to researchers in the field of learning and individual differences of the scope and value provided by latent variable mixture models in examining individual differences in learners' knowledge, characteristics, and development.

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There are two main goals of the present special issue. The first goal is to introduce the methods and their advantages, so that readers may find a new tool for their own research on learning. The second goal is to educate readers on the methods in order to facilitate their own understanding when encountering these methods in the literature. To these ends, we not only present a wide range of empirical examples utilizing latent variable mixture models, but also include a detailed, non-technical introduction to their use in characterizing individual differences in learning.

## 2. Structure of the special issue

This special issue aims to provide readers with a clear picture of the large variation in research questions, designs, and foci that can profit from such statistical approaches, providing an overview of the possible applications and implementations of these techniques in a diverse group of studies. The broad range of possible applications of these models can be seen in the wide range of ages, domains, and research designs employed in the included studies. This breadth of coverage highlights the broad relevance of this special issue for education researchers. Moreover, this special issue provides testament to the necessity for educational research to move beyond the far-too-simple approach of placing all aspects of student knowledge, attitudes and beliefs, and skills along a unidimensional, low-to-high scale. Instead, readers will come away with an understanding of the potential value of looking more closely at the qualitative divides in learners' knowledge and development that may yield a more appropriate and nuanced understanding of student learning and offer key insights into potential instructional improvements.

The present issue is divided into three sections: an introduction, original empirical papers, and a discussion. The first section is an introduction to the use of latent variable mixture models in examining individual differences in learning as a whole. This section includes the present paper, and also includes an article aimed at providing readers with a basic background in what latent variable mixture models are and what they offer for researchers aiming to examine individual differences that may have non-linear and qualitative features or relations. Readers who are unfamiliar with latent variable mixture models may find this article a useful place to begin. Even those who are familiar with the analytical tools presented in the current issue may still wish to dive deeper into how these tools are used and may find this manuscript useful for further exploration of how latent variable mixture models may be useful for research on learning and individual differences.

The second part of the special issue presents a series of empirical studies that utilize different latent variable mixture models of individual differences between learners. They present studies from various domains (mathematics to psychology), ages (young children to adults), study designs (experimental, longitudinal), and type of latent variable mixture model(s) employed. Table 1 provides an overview of the empirical contributions along these characteristics.

Given the diversity of approaches, target ages, domains, and analyses the simplest structuring was to order the empirical contributions by age. Nonetheless, there remain some commonalities and notable features that can be highlighted across the different contributions.

These include the use of covariates in longitudinal designs, a conceptual change approach, and measures of non-cognitive learner characteristics.

Latent variable mixture models are particularly powerful in measuring unseen developmental patterns (e.g. Van Hoof, Degrande, Ceulemans, Verschaffel, & Van Dooren, 2018), and additionally useful in testing the specific effects of educational interventions on learning (e.g. Schneider & Hardy, 2013). As can be seen in Table 1, the majority of the empirical studies included in the special issue use a longitudinal design. Even more specifically, three of the studies use some form of instructional intervention and assess the effects of this treatment on the development of learners' knowledge (Edelsbrunner, Schalk, Schumacher, & Stern, *this issue*; Schulz & Leuders, *this issue*; Stevenson & Hickendorff, *this issue*). In all three cases, the expectation was that there are non-linear, discontinuous developmental patterns that could not be captured by traditional summary statistical approaches, and that the treatments would move learners through these phases. The use of LTA models to capture these developmental patterns proved useful. In the two experimental studies, the effects of the interventions on discontinuous knowledge development could be targeted by testing the effect of the covariate 'experimental group' on the transition probabilities.

A surprising four papers in the present study refer to or use a conceptual change approach in their theoretical framing. In fact, this may not be entirely surprising, given the discontinuous, slow, arduous process that has been described when moving from a naïve or synthetic concept, which is grounded in a strongly held misconception, towards a scientifically or mathematically correct concept (Vosniadou & Brewer, 1992). It is apparent that the strength of latent variable mixture models in capturing qualitative differences and discontinuous development is particularly useful for modeling conceptual change processes (e.g. Schneider & Hardy, 2013). That this occurs across the domains of mathematics (McMullen, Van Hoof, Degrande, Verschaffel, & Van Dooren, *this issue*; Schulz & Leuders, *this issue*), science (Edelsbrunner et al., *this issue*), and psychology (Flaig et al., *this issue*), and including ages from young children to adults suggests that the marriage of conceptual change theories and latent variable mixture models is a fruitful one.

Finally, this set of empirical studies is particularly interesting as it captures individual differences in not only learners' knowledge, but also domain-general cognitive features, such as short-term memory (Koppenol-Gonzalez, Bouwmeester, & Vermunt, *this issue*), and affective dimensions of learning, such as math anxiety (Trezise & Reeve, *this issue*). This diversity in the types of individual differences captured in the included studies highlights the flexibility of latent variable mixture models and their effectiveness in explaining a variety of cognitive features and correlates of learning.

The final section is a discussion offered by Bethany Bray and John Dziak (*this issue*) which aims to synthesize the empirical contributions and provide a path forward for researchers aiming to use these models in their work on individual differences in learning and development.

In total, we hope that readers of *Learning and Individual Differences* find this special issue to be informative and useful in their understanding and potential use of latent variable mixture models in

**Table 1**  
Overview of empirical contributions to special issue.

Authors	Domain	Age(s) of participants	Study design(s)	Model type
Stevenson & Hickendorff	Analogical reasoning	Grade 1–Grade 3	Experimental	LTA, with covariates
Koppenol-Gonzalez et al.	Cognition (short term memory)	8 year olds	Longitudinal	LTA
Edelsbrunner et al.	Science	Grade 1–Grade 6	Experimental	LTA, with covariates
Schulz & Leuders	Mathematics (arithmetic)	Grade 4	Quasi-experimental/longitudinal	LTA, with covariates
McMullen et al.	Mathematics (rational numbers)	Grade 4–Grade 6	Cross-sectional, comparison	Multi-group LCA
Trezise & Reeve	Mathematics (Algebra)	14–16 year olds	Cross-sectional	LPA, LCA
Flaig et al.	Psychology	Post-secondary students	Longitudinal	LTA

examining learners' knowledge characteristics, and development.

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