



Universiteit
Leiden
The Netherlands

Grade-related differences in strategy use in multidigit division in two instructional settings

Hickendorff, M.; Torbeyns, J.; Verschaffel, L.

Citation

Hickendorff, M., Torbeyns, J., & Verschaffel, L. (2018). Grade-related differences in strategy use in multidigit division in two instructional settings. *British Journal Of Developmental Psychology*, 36(2), 169-187.
doi:10.1111/bjdp.12223

Version: Not Applicable (or Unknown)
License: [Leiden University Non-exclusive license](#)
Downloaded from: <https://hdl.handle.net/1887/64876>

Note: To cite this publication please use the final published version (if applicable).



Special issue paper

Grade-related differences in strategy use in multidigit division in two instructional settings

Marian Hickendorff¹* , Joke Torbeyns² and Lieven Verschaffel²

¹Education and Child Studies, Leiden University, the Netherlands

²Center for Instructional Psychology and Technology, KU Leuven, Belgium

We aimed to investigate upper elementary children's strategy use in the domain of multidigit division in two instructional settings: the Netherlands and Flanders (Belgium). A cross-sectional sample of 119 Dutch and 122 Flemish fourth to sixth graders solved a varied set of multidigit division problems. With latent class analysis, three distinct strategy profiles were identified: children consistently using number-based strategies, children combining the use of column-based and number-based strategies, and children combining the use of digit-based and number-based strategies. The relation between children's strategy profiles and their instructional setting (country) and grade were generally in line with instructional differences, but large individual differences remained. Furthermore, Dutch children more frequently made adaptive strategy choices and realistic solutions than their Flemish peers. These results complement and refine previous findings on children's strategy use in relation to mathematics instruction.

Statement of contribution

What is already known?

- Mathematics education reform emphasizes variety, adaptivity, and insight in arithmetic strategies.
- Countries have different instructional trajectories for multidigit division.
- Mixed results on the impact of instruction on children's strategy use in multidigit division.

What does this study add?

- Latent class analysis identified three meaningful strategy profiles in children from grades 4–6.
- These strategy profiles substantially differed between children.
- Dutch and Flemish children's strategy use is related to their instructional trajectory.

Elementary mathematics education has experienced a worldwide reform since the end of the previous century (e.g., Kilpatrick, Swafford, & Findell, 2001; Verschaffel, Greer, & De Corte, 2007). One key characteristic of this reform is the focus on fostering *adaptive expertise* (Hatano, 2003): the competence to solve mathematics problems efficiently, creatively, and adaptively. This contrasts with *routine expertise*: the mastery and efficient application of school-taught standard strategies. In reform-oriented mathematics curricula, the focus on adaptive expertise is reflected by an early and prolonged instruction in

This is an open access article under the terms of the Creative Commons Attribution-NonCommercial License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited and is not used for commercial purposes.

*Correspondence should be addressed to Marian Hickendorff, Educational Science, Education and Child Studies, Leiden University, PO Box 9555, 2300RB Leiden, the Netherlands (email: hickendorff@fsw.leidenuniv.nl).

number-based computation strategies as a stepping stone towards the insightful acquisition of the digit-based algorithms for multidigit arithmetic. After the acquisition of these algorithms, these number-based computation strategies may serve as a valuable computational alternative for the algorithms. There are, however, important differences between curricula in both the importance of adaptive expertise as a goal and the instructional pathways (i.e., content and timing of instruction) to reach it. The aim of this study was to examine elementary school children's strategy use in solving multidigit division problems in two different instructional settings: the Netherlands and Flanders (Belgium). These two countries have similar educational settings and perform at comparable levels in international mathematics assessments (Mullis, Martin, Foy, & Hooper, 2016), but they differ substantially in multidigit arithmetic instruction, particularly in multidigit division. Therefore, comparing Dutch and Flemish children's strategy use in this domain may shed further light on the impact of instruction on children's mathematical competence and adaptive expertise.

Multidigit division strategies and instruction

There are two major types of strategies to solve multidigit arithmetic problems: number-based and digit-based strategies (i.e., the standard algorithms) (for reviews see Fuson, 2003; Kilpatrick *et al.*, 2001; Verschaffel *et al.*, 2007). Number-based strategies operate on numbers, respecting their place value (e.g., van den Heuvel-Panhuizen, Robitzsch, Treffers, & Köller, 2009). These strategies can be applied either entirely in the head (mental computation) or on paper – ranging from writing down only intermediate answers to the entire solution process (cf. Fagginger Auer, Hickendorff, Van Putten, Béguin, & Heiser, 2016; Hickendorff, van Putten, Verhelst, & Heiser, 2010; Selter, 2001; van den Heuvel-Panhuizen *et al.*, 2009). For multidigit division problems such as $168:12 =$, these number-based strategies include repeated addition, repeated subtraction, partitioning, and compensation (Anghileri, Beishuizen, & van Putten, 2002; Hickendorff, 2013a; Hickendorff *et al.*, 2010). *Repeated addition* proceeds by repeatedly adding (multiples of) the divisor, until the dividend is reached. For instance, one can repeatedly add single divisors via $12 + 12 = 24$; $24 + 12 = 36$, ... until the dividend 168 is reached, or one can add multiples of the divisor (e.g., via $120 (=10 \times 12) + 48 (=4 \times 12)$) until the dividend 168 is reached. *Repeated subtraction* proceeds in the opposite direction: repeatedly subtracting (multiples of) the divisor from the dividend until a number less than the divisor is reached. In *partitioning*, the dividend is split decimally and each part is divided by the divisor: for example, via $100:12 =$ and $68:12 =$. Finally, in *compensation*, the dividend is rounded up to a round multiple of the divisor: for example, solving $490:5 =$ via $500:5 = 100$ and $10:5 = 2$, so the answer is $100-2 = 98$. In contrast to these number-based strategies, the *digit-based strategy* (i.e., *the standard algorithm*) operates on the digits of the two given numbers, ignoring the place value of these digits. For division, this so-called long-division algorithm is illustrated in Figures 1A and B.

Across reform-based curricula, number-based strategies are usually instructed before the digit-based strategy. However, the *timing of* and the *pathway towards* the introduction of the digit-based division strategy differ between instructional contexts, with the Netherlands and Flanders as a salient example. In the Netherlands, a particular form of the mathematics education reform – Realistic Mathematics Education (RME; e.g., Freudenthal, 1973) – has heavily influenced mathematics instruction. RME aims at a more gradual and insightful instructional pathway towards the digit-based strategy. To that end, a transitory strategy is explicitly and systematically introduced: the so-called column-

<p>(A)</p> $\begin{array}{r} 12/168 \setminus 14 \\ \underline{12} \\ 48 \\ \underline{48} \\ 0 \end{array}$	<p>(B)</p> $\begin{array}{r} 168 \mid \underline{12} \\ \underline{-12} \mid 14 \\ 48 \mid \\ \underline{-48} \mid \\ 0 \mid \end{array}$	<p>(C)</p> $\begin{array}{r} 168 \\ \underline{60-} \quad 5x \\ 108 \\ \underline{60-} \quad 5x \\ 48 \\ \underline{48-} \quad \underline{4x} \\ 0 \quad 14x \end{array}$	<p>(D)</p> $\begin{array}{r} 168 \\ \underline{120-} \quad 10x \\ 48 \\ \underline{48-} \quad \underline{4x} \\ 0 \quad 14x \end{array}$
---	--	--	---

Figure 1. Examples of strategies for solving the problem $168 : 12 = \underline{\quad}$: the digit-based strategy in the Dutch notation (A) and in the Flemish notation (B), and the column-based strategy with different numbers of solution steps (C and D).

based strategy (van den Heuvel-Panhuizen, 2008; Treffers, 1987). This strategy operates on whole numbers and can therefore be considered a number-based strategy. However, it follows a clear step-by-step procedure supported by structured vertical notation and could therefore also be considered a kind of standard algorithm (Fagginger Auer, Hickendorff, van Putten, Béguin, et al. 2016). For multidigit division, the column-based strategy is a strongly schematized version of repeated subtraction, using a structured vertical notation, see Figures 1C and D. Note that the most abbreviated form of the column-based strategy closely resembles the digit-based strategy. The most common instructional pathway for multidigit division in the Netherlands is as follows: only number-based strategies until grade 4, introduction of the column-based strategy in grade 5, and introduction of the digit-based strategy only in grade 6. These strategies are explicitly linked to each other in the sequence of progressive schematization of the number-based strategy repeated subtraction (Treffers, 1987). It is noteworthy that a substantial proportion of Dutch mathematics educators consider the column-based strategy a valuable computational alternative for the digit-based strategy in division (van den Heuvel-Panhuizen, 2008). Therefore, not all children receive instruction in the digit-based division algorithm: according to the most recent national mathematics assessment, 66% of Dutch sixth grade teachers instruct only the column-based strategy (Scheltens, Hemker, & Vermeulen, 2013).

In Flanders, until the end of grade 3, children are taught only number-based strategies. After the digit-based strategy is introduced at the end of grade 3, children intensively practice it, in addition to continued practice in number-based strategies (Katholiek Onderwijs Vlaanderen, 2002). The two types of strategies are taught separately and, thus, hardly linked to each other. Also, little instructional attention is paid to learning to choose adaptively between the two types of strategies.

Another difference relates to the adaptive use of shortcut strategies in problems with certain number characteristics, such as the compensation strategy when the dividend is slightly below a round multiple of the divisor (e.g., 297 : 3). In the Netherlands, there is a continuous focus on strategy variety and on the adaptive use of such shortcut strategies, whereas in Flanders, it receives much less instructional attention.

A final difference is in the link between mathematics problem-solving and reality. Whereas in Dutch RME many problems are presented in the form of verbally and/or

pictorially presented contextual problems that require children to apply their computational strategies in realistic situations (e.g., Gravemeijer & Doorman, 1999; Hickendorff, 2013a,b), Flemish arithmetic instruction focuses more on solving symbolic problems.

Previous studies

Compared to the other three computational operations, multidigit division has received limited research attention. Recent Dutch studies showed that in the period 1997 to 2011, sixth graders in national assessments had an increasing tendency to solve multidigit division problems mentally without making written notes – most likely with a number-based strategy; that mental strategies were less accurate but faster than written ones; that a substantial number of children did not adaptively choose between written or mental strategies; that the format of the problem (contextual vs. symbolic) did not affect children's strategy use; that children with lower mathematical abilities were more likely to use mental strategies than their higher achieving peers; and that girls were more inclined to use a written strategy than boys (Fagginger Auer, Hickendorff, & van Putten, 2016; Fagginger Auer, Hickendorff, van Putten, Béguin, et al. 2016; Hickendorff, 2013a; Hickendorff, Heiser, Van Putten, & Verhelst, 2009; Hickendorff *et al.*, 2010).

Regarding the impact of instruction on children's strategy use, Anghileri *et al.* (2002); (see also Van Putten, van den Brom-Snijders, & Beishuizen, 2005) found that English fourth graders had more difficulties than their Dutch peers in progressing from number-based to digit-based strategies, reflecting the instructional differences: a gradual transition in the Dutch curriculum based on RME's principles of progressive schematization versus a discontinuous transition in the English curriculum. Fagginger Auer, Hickendorff, & van Putten (2016) found that Dutch sixth graders' use of the written digit-based strategy was related to whether or not teachers instructed this strategy. So, the few studies that have examined the relation between strategy instruction and children's strategy competence in division suggest that instruction impacts strategy use, in particular the use of the digit-based strategy. This is in line with most German and Flemish findings in the domain of multidigit subtraction, where children almost exclusively relied on the digit-based strategy once it had been introduced (Heinze, Marschick, & Lipowsky, 2009; Selter, 2001; Torbeyns & Verschaffel, 2013, 2016), although a recent cross-national study's finding diverged from this dominance of digit-based strategies (Torbeyns, Hickendorff, & Verschaffel, 2017).

Current study

As stated before, there are large differences in instruction in multidigit arithmetic between educational contexts, particularly in multidigit division, with the Netherlands and Flanders (Belgium) as salient examples. Knowing how instruction impacts children's learning is of imminent importance in optimizing (mathematics) education (e.g., National Mathematics Advisory Panel, 2008; Royal Dutch Society of Arts and Sciences, 2009). However, especially in the domain of multidigit division, our understanding of the teaching–learning relationship is limited. This study aimed to investigate children's strategy use in the domain of multidigit division in grades 4–6, in two different instructional settings: the Netherlands and Flanders. Dutch and Flemish fourth to sixth graders solved a varied set of multidigit division problems. We focused on three aspects of children's solutions for which the instruction differed most clearly: (1) the use of number-based, column-based, and digit-based strategies, (2) the adaptive use of the shortcut

strategy compensation, and (3) the proper use of realistic considerations in the solution of division-with-remainder (DWR) contextual problems.

We formulated three hypotheses. First, as strategy use is characterized by variability both between and within individuals (e.g., Siegler, 2007), and this variability can likely be captured by a limited number of distinctive patterns or profiles of strategy use across a set of problems (e.g., Hickendorff, Edelsbrunner, McMullen, Schneider, & Trezise, 2017; Hickendorff *et al.*, 2009), we expected to find different classes of children with similar strategy use profiles across the problems. We used a model-based clustering technique, specifically latent class analysis (e.g., Collins & Lanza, 2010) to identify these profiles. We expected children's strategy profiles to be related to children's instruction, and, therefore, we hypothesized that Dutch children would use only number-based strategies in fourth grade, a mix of number-based and column-based strategies in fifth grade, and a mix of number-based and digit-based strategies in sixth grade (Hypothesis 1a). Conversely, we anticipated that, as a result of their more algorithmically oriented instruction, Flemish children would use a mix of number-based and digit-based strategies, with the digit-based strategies being dominant, in each of the three grades (Hypothesis 1b). In relation to Hypothesis 1, we also asked the question to what extent children's strategy use was related to their intelligence and/or mathematics achievement level, and gender. Due to the absence of clear theoretical arguments and the scarcity of empirical findings, we did not formulate specific hypotheses concerning the impact of these child characteristics on their strategy use in general and in the Dutch and Flemish sample in particular. Second, we anticipated that, due to the stronger emphasis on strategy variety and adaptivity in the Dutch curriculum, Dutch children would show more strategy adaptivity than their Flemish peers (Hypothesis 2), and, third, that Dutch children would solve DWR contextual problems more realistically than their Flemish peers, because of the stronger link between mathematics problem-solving and reality in the Dutch curriculum (Hypothesis 3).

Method

Participants

Participants were 119 Dutch and 122 Flemish elementary school children from grades 4 to 6 coming from four Dutch and three Flemish schools (35–43 children per grade per country). Only children with parental consent were included in the study, in accordance with the ethical guidelines of the institutes involved. The general mathematical achievement level of the children was assessed via grade-appropriate standardized mathematics tests from, respectively, the Dutch and the Flemish Student Monitoring System (Janssen, Verhelst, Engelen, & Schelkens, 2010; van Dooren, 2000). The Raven Standard Progressive Matrices (Raven, Court, & Raven, 1992) was administered to measure the children's intellectual capacities. Table 1 presents descriptive statistics of the sample. Both the Dutch and the Flemish children performed about average on the mathematics achievement and intelligence tests. Accounting for differences by grade, Dutch and Flemish children did not differ significantly in mathematics achievement percentile score, $F(1, 234) = 3.17$, *n.s.*, but the Dutch children were significantly older.¹ F

¹ These age differences are due to the differences in country-specific regulations for grade inclusion. As we aimed to address children's instruction, we focused on grade instead of age in our analyses.

Table 1. Participants: number, age (in Years), gender, mathematics achievement (Percentile Score) and intelligence (Raven IQ Score) per grade per country

Country	Grade	<i>n</i>	Gender		Age		Mathematics achievement ^a		Intelligence ^b	
			Boy	Girl	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Flanders (Belgium)	Grade 4	43	25	18	9.7	0.30	52.1	28.9	104.9	13.2
	Grade 5	39	19	20	10.7	0.38	51.0	26.1	104.5	9.1
	Grade 6	40	19	21	11.8	0.37	47.2	31.5	103.8	13.5
	All	122	63	59	10.7	0.94	50.1	28.8	104.4	12.1
The Netherlands	Grade 4	42	19	23	10.0	0.52	59.0	24.1	101.5	13.6
	Grade 5	42	20	22	11.0	0.39	57.3	26.2	100.7	10.5
	Grade 6	35	15	20	11.8	0.48	52.9	25.9	97.6	9.9
	All	119	54	65	10.9	0.84	56.6	25.3	100.1	11.6

Note. ^aThe mathematical achievement score of one Dutch sixth grader is missing.

^bThe intelligence score of one Flemish fourth grader is missing.

(1, 235) = 19.76, $p < .001$, and had lower mean intelligence scores, $F(1, 234) = 8.36$, $p = .004$.

Careful analysis of the mathematics textbooks used in these classes, supplemented with a teacher questionnaire, supported the above-mentioned characterization of differences between the Dutch and Flemish instructional pathway and focus.

Materials

We assessed children's strategy use with a multidigit division task consisting of eight divisions. Because children in fourth grade had hardly any instruction or practice in solving multidigit division problems, we created two different task versions: an easier task for grade 4 and a more difficult task for grades 5 and 6 (see Appendix). One random order of these problems was created; a forward and backward version of this order was administered.

All problems had a three-digit dividend, and either a one-digit or two-digit divisor. Each divisor was used only once across the problems in each task version. Four problems were presented in symbolic format and four as contextual problems. Two symbolic and two contextual problems yielded a remainder. The latter two division-with-remainder (DWR) problems required a context-specific interpretation of that remainder: In problem DWR1, the remainder had to be rounded up to yield a contextually meaningful answer, whereas in problem DWR2 it had to be rounded down. Finally, two problems (COM1 and COM2) with the dividend being close to a round multiple of the divisor were designed to diagnose children's adaptive use of the compensation strategy.

Procedure

All children solved the multidigit division task individually in a quiet room at their school. We balanced the two problem orders per class. The task was presented as an A5-sized booklet, with one division problem per page. Children had to solve each problem as accurately and as fast as possible. Furthermore, they were told to write down the answer

to each problem in the answer box that they could use the scrap paper section to make written notes and that they could solve the problems in whatever way they wanted. After they had given an answer, they were asked to verbally report how they had solved the problem. All individual sessions were audiotaped.

Analysis

Strategy coding

Children's strategy use per problem was registered on the basis of their written notes and their verbal strategy reports immediately after solving each problem. The first distinction was into (1) digit-based strategy, (2) column-based strategy, (3) number-based strategies, or (4) unclassifiable solution strategies. First, the *digit-based strategy* involved the long-division algorithm (see Figures 1A and B). Second, in the *column-based strategy* (Figures 1C and D), multiples of the divisor are repeatedly subtracted from the dividend, using a structured vertical notation (Fagginger Auer, Hickendorff, & van Putten, 2016; Treffers, 1987). Note that the number of steps made can differ between solutions. Third, *number-based strategies* involve (1) repeated subtraction without the structured vertical notation, (2) repeated addition, (3) partitioning strategies, (4) compensation strategies, and (5) other number-based strategies, such as trial and error. Fourth and finally, some solutions were unclassifiable because the notations and verbal reports were unclear, a wrong operation was used, or the problem was skipped.

For the two DWR contextual problems, we also classified whether children made realistic considerations when confronted with the remainder, either by giving the contextually correct answer (see Appendix) or by giving an incorrect answer but stating that there is 1 child/rabbit left (problem R1a/R1b) or 20 cm left (problem R2).

To assess the inter-rater reliability, 192 solutions of 24 children (12 Dutch and 12 Flemish) were coded by two independent raters (one Dutch and one Flemish). The agreement on the broad categorization into digit-based, column-based, number-based, and unclassifiable solutions was high with Cohen's $\kappa = .81$. On the more fine-grained categorization into the different types of strategies (see also Table 2), the agreement between raters was substantial, with Cohen's $\kappa = .73$. The classification into whether

Table 2. Overall strategy use frequency in number of solutions and in percentage of solutions

Strategy use	<i>n</i> solutions	Percentage of solutions
Digit-based	500	26
Column-based	201	10
Number-based	1,076	56
Repeated subtraction	485	25
Repeated addition	142	7
Partitioning	328	17
Compensation	92	5
Other number-based	29	2
No strategy classified	151	8
Unclear strategy	90	1
Wrong operation	20	5
Skipped	41	2
Total	1,948	100

the DWR contextual problems were solved realistically had very high inter-rater agreement with Cohen's $\kappa = .94$.

Statistical analysis

In the first research question, we aimed to identify classes of children with similar strategy use profiles across the set of problems. We used latent class analysis (LCA, e.g., Collins & Lanza, 2010; Hagenaars & McCutcheon, 2002), a classification analysis technique for categorical response variables (for similar applications see Fagginger Auer, Hickendorff, & van Putten, 2016; Hickendorff *et al.*, 2010). We conducted the LCA with the strategy used on each of the eight problems categorized into (1) digit-based, (2) column-based, and (3) number-based strategies; non-classifiable strategies were excluded. All LCAs were carried out in version 5.0 of the statistical program Latent Gold (Vermunt & Magidson, 2013). To select the optimal number of latent classes, we used the Bayesian information criterion (BIC) – which is a trade-off between model fit (log-likelihood) and model complexity (the number of estimated parameters) – combined with conceptual appeal (interpretability) of the resulting solution (e.g., Collins & Lanza, 2010). We next analysed the relation between children's strategy profile and instruction (grade and country), and also the relation between children's strategy profile and their gender, intelligence, and mathematics achievement level with logistic regression analyses.

Results

Preliminary analyses

Table 2 shows the overall strategy frequency of the fine-grained level of categorization: 26% of all problems were solved with a digit-based strategy, 10% with a column-based strategy; most solutions (56%) involved number-based strategies. Within the number-based strategies, repeated subtraction (without the structured notational procedure) was the most frequent strategy, followed by partitioning.

Table 3 presents strategy frequency classified in the four main categories and performance (per strategy as well as overall), by country and by grade. Because fourth graders completed the easy task version and fifth and sixth graders the difficult task

Table 3. Overall strategy use frequency in percentages (with proportion correct per strategy between brackets) and overall performance per country, by grade

Country	Grade	Strategy use			Overall performance		
		Number-based	Column-based	Digit-based	Not classifiable	Proportion correct	Rasch score
Flanders (Belgium)	Grade 4	54 (0.35)	0 (n.a.)	38 (0.50)	8 (0.00)	0.38	-0.96
	Grade 5	41 (0.42)	0 (n.a.)	53 (0.48)	6 (0.05)	0.43	-0.25
	Grade 6	63 (0.50)	0 (n.a.)	33 (0.60)	3 (0.00)	0.52	0.23
	All	41 (0.43)	0 (n.a.)	53 (0.52)	6 (0.02)	0.44	-0.34
The Netherlands	Grade 4	85 (0.37)	0 (n.a.)	0 (n.a.)	15 (0.00)	0.32	-1.37
	Grade 5	49 (0.44)	40 (0.59)	0 (n.a.)	11 (0.08)	0.46	-0.10
	Grade 6	38 (0.59)	24 (0.68)	35 (0.57)	3 (0.00)	0.59	0.61
	All	53 (0.43)	21 (0.62)	10 (0.57)	10 (0.03)	0.45	-0.34

version, the proportions correct are not directly comparable across grades. We therefore estimated a Rasch model (e.g., Embretson & Reise, 2000) linking the two versions by the four problems they had in common. An ANCOVA on Rasch scores (also in Table 3) with grade and country as factors and mathematics achievement level and intelligence as covariates showed that performance significantly differed by grade, $F(2, 231) = 41.63$, $p < .001$, but not by country or the interaction between grade and country, $F(1, 231) = 0.17$, $p > .05$ and $F(2, 231) = 2.59$, $p > .05$, respectively. A *post-hoc* test with Bonferroni correction showed that all pairwise differences between grades were significant ($ps < .001$). Flemish children never used the column-based strategy while Dutch children in grades 5 and 6 used it; Flemish children used the digit-based strategy in each grade quite frequently (overall 53%) while Dutch children did not use it before grade 6; and Dutch fourth graders only used number-based strategies (and unclassifiable strategies).

Strategy use profiles

Such analysis of overall strategy frequencies has several disadvantages: results are aggregated over problems obscuring information about strategy use *patterns* or *profiles* across problems, and possibilities for statistical testing are limited. To overcome these issues, we conducted latent class analyses (LCAs) which allows identifying classes of children characterized by a particular profile of strategy use across all problems. Children's strategy use (digit-based, column-based, or number-based) on all eight division problems entered the model as the observed variables. Models with a range of one through six latent classes were estimated, showing that the model with three classes had the lowest BIC value. This model had an entropy R-squared of .98 (values between 0 and 1, higher values indicate more certainty of classification) and a classification error of .006, indicating that the three classes differentiated between the children. The lowest posterior class membership probability across individual children was .93, implying that all children could be assigned to only one strategy profile with very high certainty. Figure 2 shows the estimated probability to use each of the strategies per problem in each of the three classes.

The largest class with a prevalence of 50% is characterized by a very high likelihood of using number-based strategies across all problems and is therefore labelled *number-based profile*. Children in the smallest class (16%) were likely to use the column-based strategy on some problems, while they switched between column-based or number-based strategies on other problems; this class is labelled *combined column-based and number-based profile*. Finally, children in the *combined digit-based and number-based profile* class (34%) had a large tendency to use the digit-based strategy, but this also depended on the problem. It is noteworthy that the digit-based and the column-based strategy were very unlikely to be combined in any profile.

Strategy profiles related to instruction

To test Hypotheses 1a and 1b, we investigated whether children's strategy profile reflected their instructional focus, by analysing the differences by country and by grade in the prevalence of each strategy profile. To that end, first, each child was assigned to the class for which (s)he had the highest posterior probability (modal assignment). Next, this class membership variable was related to country and grade, as shown in Table 4. Overall, the strategy profile prevalences differed significantly by country (Fisher Exact Test $p < .001$). The main differences were (1) in the prevalence of the 'combined column-based and number-based profile', which did not occur in Flanders but applied to 31% of

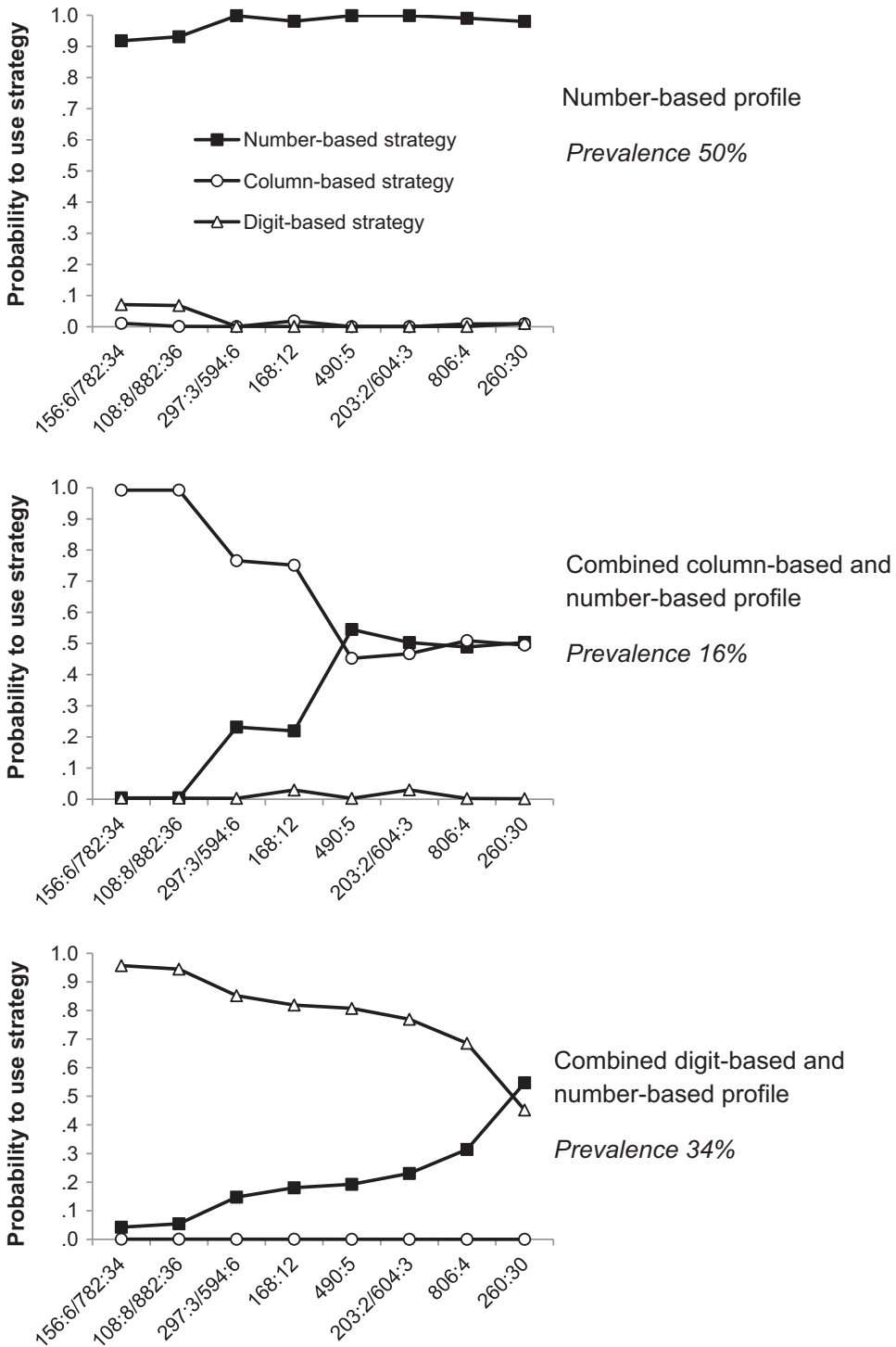


Figure 2. The three strategy profiles from the latent class analysis. Per profile the estimated probability of solving each division problem with a digit-based, column-based, or number-based strategy is graphed; problems are sorted in increasing order of observed frequency of number-based strategy use.

Table 4. Strategy profile frequencies (in percentages) per country, by grade

Country	Grade	Strategy profile		
		Consistent number-based	Combined column-based with number-based	Combined digit-based with number-based
Flanders (Belgium)	Grade 4	47	0	53
	Grade 5	33	0	67
	Grade 6	55	0	45
	All	45	0	55
The Netherlands	Grade 4	100	0	0
	Grade 5	40	60	0
	Grade 6	26	34	40
	All	57	31	12

the Dutch children, and (2) the prevalence of the ‘combined digit-based and number-based profile’ being much higher in Flanders (55%) than in the Netherlands (12%).

Within each country, the differences by grade were significant in the Netherlands (Fisher exact test $p < .001$) but not in Flanders ($p > .05$). For Dutch children, the general grade-related pattern aligned with the instructional focus as stated in Hypothesis 1b: in fourth grade, all children were in the consistent number-based profile; in fifth grade, more than half were in the combined column-based and number-based profile; and in sixth grade, all three strategy profiles occurred. Although these overall grade-related differences are in accordance with the instructional pathways, supporting Hypothesis 1a, noteworthy are the large interindividual differences between children that still remained. For Flemish children, it was an approximate 50/50 split in each grade between the number-based profile and the combined digit-based and number-based profile, which is partly in line with Hypothesis 1b: as expected, the Flemish children quite frequently used the digit-based strategy in each grade, but, unexpectedly, the digit-based strategy was not the dominant strategy.

To gain further insight into the interindividual variability in strategy use related to children’s instructional setting, we analysed the differences between schools. In the Flemish sample, there were differences between schools only in grade 4 (Fisher Exact $p = .001$); there was one school in which relatively a high number of children (86%) were in the consistent number-based profile compared to the other schools (25–31%). In the Dutch sample, children’s school was significantly related to their strategy profile in the Dutch sample in grade 5 (Fisher exact $p = .009$) and grade 6 (Fisher exact $p < .001$); in one school, the percentage of fifth graders in the combined number-based and column-based profile was relatively higher (92%) than in the other schools (41–60%), and all 14 sixth grade children who were in the combined number-based and digit-based profile were from that same school.

Further investigation of the strategy profiles

Next, we analysed the relation between children’s strategy profile and their intelligence, mathematics achievement level, and gender. Table 5 presents the descriptive statistics. Because strategy profile is a categorical variable, we used logistic regression analyses with strategy profile as the dependent variable and intelligence, mathematics achievement level, and gender as predictors. As the number of strategy profiles observed

Table 5. Descriptive statistics of strategy profiles per country, by grade

Country	Grade	Strategy profile	<i>n</i>	Intelligence <i>M</i> (<i>SD</i>)	Math achievement percentile <i>M</i> (<i>SD</i>)	Gender (% girls within profile)
Flanders (Belgium)	Grade 4	Consistent NB	20	108.5 (10.4)	67.0 (23.1)	55
		Combined DB/NB	23	101.8 (14.7)	39.1 (27.6)	30
	Grade 5	Consistent NB	13	107.5 (6.1)	60.2 (27.1)	46
		Combined DB/NB	26	103.0 (10.1)	46.4 (24.8)	54
	Grade 6	Consistent NB	22	107.5 (11.9)	55.6 (26.5)	41
		Combined DB/NB	18	99.3 (14.4)	36.8 (34.7)	67
The Netherlands	Grade 4	Consistent NB	42	101.5 (13.6)	59.0 (24.1)	55
	Grade 5	Consistent NB	17	97.3 (10.0)	45.8 (28.9)	59
		Combined CB/NB	25	103.0 (10.4)	65.0 (21.4)	48
	Grade 6	Consistent NB	9	96.2 (10.7)	67.9 (23.8)	44
		Combined CB/NB	12	94.2 (10.7)	42.6 (25.1)	58
		Combined DB/NB	14	101.4 (8.0)	53.1 (25.2)	64

differed between countries (i.e., two observed profiles in the Flemish sample and three in the Dutch sample), we conducted separate analyses in the Flemish and Dutch samples.

In the Dutch sample, we conducted a multinomial logistic regression analysis, excluding the fourth graders (as all of them were in the same strategy profile). The likelihood-ratio tests of the effects of intelligence, $\chi^2 (df=2) = 1.91, p > .05$, mathematics achievement level, $\chi^2 (df=2) = 0.34, p > .05$, and gender, $\chi^2 (df=2) = 0.41, p > .05$, were not significant. A binary logistic regression analysis in the Flemish sample showed that only the effect of mathematics achievement level was significant, $b = 0.021$, Wald ($df = 1$) = 5.89, $p = .015$, indicating that children in the consistent number-based profile had significantly higher mathematics achievement levels than their peers in the combined digit-based and number-based profile. The effects of intelligence, $b = 0.019$, Wald ($df = 1$) = 0.85, $p > .05$, and gender, $b = 0.066$, Wald ($df = 1$) = 0.03, $p > .05$, were not significant.

Adaptive use of compensation strategy

To test Hypothesis 2, we analysed the frequency of using the compensation strategy on the two problems that were designed to diagnose the adaptive use of this strategy, COM1 and COM2. On these two problems, the compensation strategy was used more often ($M = 14\%$) than on the six remaining problems ($M = 1\%$); $t (273) = 7.81, p < .001$. Furthermore, in line with Hypothesis 3, on problem COM1, the frequency of compensation was significantly higher in Dutch (29%) than in Flemish children (14%), Fisher exact test $p = .008$. Also on problem COM2, the Dutch children compensated more frequently (19%) than their Flemish peers (6%), $p = .003$.

Realistic consideration in solutions to DWR contextual problems

Table 6 presents the percentage of realistic solutions on both DWR contextual problems as a percentage of all solutions (left columns) or only of the incorrect solutions (right

Table 6. Frequency (in Percentages) of realistic nature of division-with-remainder (DWR) contextual problems, within all solutions and within incorrect solutions, by country

	All solutions		Incorrect solutions	
	Problem DWR1 ^a	Problem DWR2	Problem DWR1 ^a	Problem DWR2
Flanders (Belgium)	15% (<i>n</i> = 119)	25% (<i>n</i> = 118)	8% (<i>n</i> = 96)	1% (<i>n</i> = 102)
The Netherlands	32% (<i>n</i> = 119)	39% (<i>n</i> = 118)	16% (<i>n</i> = 86)	15% (<i>n</i> = 95)
Fisher exact test for difference Flanders vs. the Netherlands	<i>p</i> = .003	<i>p</i> = .036	<i>p</i> = .12 n.s.	<i>p</i> < .001

Note. ^aFourth graders solved the easy task and fifth/ sixth graders solved the difficult task, which had different versions of problem DWR (see Appendix).

columns). Results generally supported Hypothesis 3: in three out of the four comparisons, Dutch children were significantly more likely than their Flemish peers to make realistic considerations when confronted with the remainder.

Discussion

The present study aimed to examine upper elementary children's strategy use in solving multidigit division problems in two different instructional settings, the Netherlands and Flanders. These countries have comparable general (mathematics) educational features but salient specific differences in the instruction from number-based strategies to the digit-based strategy, and in the emphasis on adaptive strategy use and realism in the problem-solving process. Focusing on three instructionally important strategies (number-based, column-based, and digit-based), latent class analysis (LCA) identified three distinct strategy profiles: one in which children quite consistently used number-based strategies, one in which children combined the (transitory) column-based strategy with number-based strategies, and one in which children combined the digit-based strategy with number-based strategies. LCA proved a sensitive and informative tool to capture this naturally occurring heterogeneity in children's strategy use. Without the need to aggregate over problems, a limited number of distinct classes characterized children's strategy profiles across the individual problems.

Generally speaking, the prevalence of each of these profiles was in line with children's instruction. Dutch fourth graders were all in the consistent number-based profile, Dutch fifth graders were either in the combined column-based and number-based profile or in the consistent number-based profile, and it was not before sixth grade that some Dutch children were classified in the combined digit-based and number-based profile. Notably, the likelihood that children combined the column-based strategy with the digit-based strategy was almost zero, implying that Dutch children did not switch between the digit-based strategy and column-based approaches, similar to earlier findings (Fagginger Auer, Hickendorff, & van Putten, 2016; Hickendorff *et al.*, 2009). By contrast, Dutch fifth and sixth graders did switch between number-based strategies on the one hand, and either column-based or digit-based on the other. Noteworthy in this respect is that the Dutch sixth graders who used the digit-based strategy originated from only one school, indicating that the way in which the written curriculum is implemented and enacted by the school

and teachers plays an important role (Stein, Remillard, & Smith, 2009). In contrast, approximately half of the Flemish children had a consistent number-based profile while the other half combined the digit-based strategy with number-based strategies. As expected, Flemish children were quite likely to use the digit-based strategy from grade 4 onwards. Contrary to our expectations based on previous studies in multidigit subtraction (Heinze *et al.*, 2009; Selter, 2001; Torbeyns & Verschaffel, 2013, 2016), but more in line with a recent study in multidigit subtraction (Torbeyns *et al.*, 2017), the digit-based strategy did not become dominant after its introduction, and a substantial part of Flemish children was not inclined to use it at all.

The results regarding the relation between strategy profile and mathematics achievement level showed that in the Dutch sample, mathematics achievement level was unrelated to strategy profile, whereas in the Flemish sample there was a relation: children in the consistent number-based profile had higher mathematics achievement scores than their peers combining number-based and digit-based strategies. A possible explanation may be that in Flanders, the children who have difficulties with the number-based strategies are encouraged to make an earlier switch to the digit-based strategy, as that strategy requires less mathematical insight and creativity and puts fewer demands on working memory. This explanation would require further study into the instructional practices in the mathematics classroom. In both samples, intelligence did not add to the prediction of strategy use profile when mathematics achievement level was already in the model. This may imply that mathematical strategy use is more strongly predicted by mathematics achievement level than by more general intellectual functioning. Finally, gender was unrelated to children's strategy profile in both countries.

In line with the second hypothesis, Dutch children were more likely to adaptively use compensation on problems supporting this strategy than their Flemish peers, reflecting the stronger focus on adaptive strategy use in the Netherlands compared to Flanders. However, most solutions (also among the Dutch children) did *not* involve compensation. In line with previous studies on subtraction (e.g., Blöte, van der Burg, & Klein, 2001; Heinze *et al.*, 2009; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009), children's use of compensation was limited, even in the RME-oriented curriculum.

Finally, in line with the third hypothesis, Dutch children were more likely to provide a realistic answer when confronted with a contextual problem involving a division with a remainder than their Flemish peers, reflecting the stronger focus on realistic problem-solving in the Netherlands compared to Flanders. However, again, the majority of the solutions did *not* involve realistic considerations. Children's sense-making in word problem-solving seemed limited (cf. Verschaffel, Greer, & De Corte, 2000), even in the RME-oriented curriculum.

Together, these findings show that Dutch and Flemish children's strategy use profiles are generally aligned to their instructional experiences, although large individual differences remain. The correspondence between strategy instruction and children's strategy use is thus less straightforward than previous research findings suggest (e.g., Fagginger Auer, Hickendorff, & van Putten, 2016; Selter, 2001; Torbeyns & Verschaffel, 2013, 2016), as was also concluded based on a recent study in multidigit subtraction (Torbeyns *et al.*, 2017).

Notwithstanding the differences in strategy profiles, overall performance did not differ significantly between countries. At first sight, this may seem undercutting the present findings on the varied nature of children's strategy use: How important is it that

the children solve the problems in different ways if they turn out to reach the same performance level? However, we argue that it *does* matter. Mathematical competence is much more than merely performing accurately, in particular in the framework of the worldwide reform in mathematics education, where children's strategy competence and real-world problem-solving are of major importance (e.g., Verschaffel *et al.*, 2007). The current study's findings thus present a much more complete picture of children's mathematical competence in different instructional settings. Addressing only performance would have obscured the relevant strategy use differences.

An important educational implication relates to the debate about skills versus understanding in elementary mathematics education (e.g., Baroody, 2003). It has been argued that a heavy focus on strategy efficiency may be at odds with stimulating other aspects of mathematical competence, such as insightful and adaptive strategy use (Verschaffel *et al.*, 2007). The current results counter this argument. They show that a carefully constructed instructional pathway that focuses on strategy variety and adaptivity, progressively building towards standard (digit-based) procedures in an insightful way, and focusing on realistic problem-solving, may lead to the same performance level as an instructional approach with a heavier focus on efficiency and less attention for insight, strategy variety and adaptivity, and realistic problem-solving. This adds to the literature arguing that conceptual and procedural knowledge is intertwined and strengthens each other (e.g., Schneider, Rittle-Johnson, & Star, 2011). Conceptual knowledge may aid the construction, selection, and execution of strategies, whereas practice with procedures may help deepen the conceptual understanding. An implication is that instruction focusing on only one kind of knowledge may be undesirable (see also Schneider *et al.*). However, the current study's correlational and cross-sectional design clearly limits the causal interpretation of relations between instructional practice and children's strategy competence.

Other limitations are our method to examine adaptive strategy use, which was restricted to one type of strategy (compensation) and our normative definition for which problems this strategy was adaptive (see for instance Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009; for alternatives). Another methodological issue is that we characterized the instruction in Flanders and the Netherlands primarily based on the written materials (educational goals, curricular guidelines, and mathematics textbooks). This 'written curriculum' may differ from the 'enacted curriculum' – the classroom implementation (Stein *et al.*, 2009). An important avenue for further research is to measure the enacted curriculum by classroom observations and relate that to children's strategy competence.

Conclusion

The present study is the first that investigated children's strategy use across grades in the domain of multidigit division, in two different instructional settings. This cross-national comparison allowed examining the extent to which children's strategy use reflects instruction in an ecological valid setting, without the need for an experiment. Latent class analysis proved very informative to characterize individual differences in the use of instructionally meaningful strategies across a set of varied problems. The findings complement current insights into the potential impact of instruction on children's strategy competence.

Acknowledgements

This research was partially supported by Grant GOA 2012/10 ‘Number sense: analysis and improvement’ from the Research Fund KU Leuven, Belgium.

References

- Anghileri, J., Beishuizen, M., & van Putten, K. (2002). From informal strategies to structured procedures: Mind the gap!. *Educational Studies in Mathematics*, *49*(2), 149–170. <https://doi.org/10.1023/A:1016273328213>
- Baroody, A. J. (2003). The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 1–33). Mahwah, NJ: Lawrence Erlbaum.
- Blöte, A. W., van der Burg, E., & Klein, A. S. (2001). Students’ flexibility in solving two-digit addition and subtraction problems: Instruction effects. *Journal of Educational Psychology*, *93*, 627–638. <https://doi.org/10.1037/0022-0663.93.3.627>
- Collins, L. M., & Lanza, S. T. (2010). *Latent class and latent transition analysis: With applications in the social behavioral, and health sciences*. Hoboken, NJ: Wiley.
- Embretson, S. E., & Reise, S. P. (2000). *Item response theory for psychologists*. Mahwah, NJ: Lawrence Erlbaum.
- Fagginger Auer, M. F., Hickendorff, M., & van Putten, C. M. (2016). Solution strategies and adaptivity in multidigit division in a choice/no-choice experiment: Student and instructional factors. *Learning and Instruction*, *41*, 52–59. <https://doi.org/10.1016/j.learninstruc.2015.09.008>
- Fagginger Auer, M. F., Hickendorff, M., Van Putten, C. M., Béguin, A. A., & Heiser, W. J. (2016). Multilevel latent class analysis for large-scale educational assessment data: Exploring the relation between the curriculum and students’ mathematical strategies. *Applied Measurement in Education*, *29*(2), 144–159. <https://doi.org/10.1080/08957347.2016.1138959>
- Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht, The Netherlands: Reidel.
- Fuson, K. C. (2003). Developing mathematical power in whole number operations. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 68–94). Reston, VA: National Council of Teachers of Mathematics.
- Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics*, *39*(1/3), 111–129. <https://doi.org/10.1023/A:1003749919816>
- Hagenaars, J. A., & McCutcheon, A. L. (2002). *Applied latent class analysis*. Cambridge, UK: Cambridge University Press. <https://doi.org/10.1017/CBO9780511499531>
- Hatano, G. (2003). Foreword. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. xi–xiii). Mahwah, NJ: Lawrence Erlbaum.
- Heinze, A., Marschick, F., & Lipowsky, F. (2009). Addition and subtraction of three-digit numbers: Adaptive strategy use and the influence of instruction in German third grade. *ZDM*, *41*(5), 591–604. <https://doi.org/10.1007/s11858-009-0205-5>
- Hickendorff, M. (2013a). The effects of presenting multidigit mathematics problems in a realistic context on sixth graders’ problem solving. *Cognition and Instruction*, *31*(3), 314–344. <https://doi.org/10.1080/07370008.2013.799167>
- Hickendorff, M. (2013b). The language factor in elementary mathematics assessments: Computational skills and applied problem solving in a multidimensional IRT framework. *Applied Measurement in Education*, *26*(January 2015), 253–278. <https://doi.org/10.1080/08957347.2013.824451>
- Hickendorff, M., Edelsbrunner, P., McMullen, J., Schneider, M., & Trezise, K. (2017). Informative tools for characterizing individual differences in learning: latent class, latent profile, and latent

- transition analysis. *Learning and Individual Differences*, <https://doi.org/10.1016/j.lindif.2017.11.001>
- Hickendorff, M., Heiser, W. J., Van Putten, C. M., & Verhelst, N. D. (2009). Solution strategies and achievement in dutch complex arithmetic: Latent variable modeling of change. *Psychometrika*, *74*(2), 331–350. <https://doi.org/10.1007/s11336-008-9074-z>
- Hickendorff, M., van Putten, C. M., Verhelst, N. D., & Heiser, W. J. (2010). Individual differences in strategy use on division problems: Mental versus written computation. *Journal of Educational Psychology*, *102*, 438–452. <https://doi.org/10.1037/a0018177>
- Janssen, J., Verhelst, N., Engelen, R., & Scheltens, F. (2010). *Wetenschappelijke verantwoording van de toetsen LOVS rekenen-wiskunde voor groep 3 tot en met groep 8 [Technical report for the student monitoring system mathematics tests for grade 1 to 6]*. Arnhem, the Netherlands: CITO.
- Katholiek Onderwijs Vlaanderen. (2002). *Toelichtingen bij het leerplan wiskunde: bewerkingen* [Notes to the mathematics curriculum: Arithmetic operations]. Brussel, Belgium: Author.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learning mathematics*. Washington, DC: National Academy Press.
- Mullis, I. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 international results in mathematics*. Chestnut Hill, MA: Boston College. Retrieved from Boston College, TIMSS & PIRLS International Study Center website: <http://timssandpirls.bc.edu/timss2015/international-results/>
- National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel. Foundations (Vol. 37)*. <https://doi.org/10.3102/0013189x08329195>
- Raven, J. C., Court, J. H., & Raven, J. (1992). *Standard progressive matrices*. Oxford, UK: Psychologists Press.
- Royal Dutch Society of Arts and Sciences. (2009). *Rekenonderwijs op de basisschool. Analyse en sleutels tot verbetering [Mathematics education in primary school. Analysis and recommendations for improvement]*. Amsterdam, The Netherlands: KNAW.
- Scheltens, F., Hemker, B., & Vermeulen, J. (2013). *Balans van het reken-wiskundeonderwijs aan het eind van de basisschool. Uitkomsten van de vijfde peiling in 2011 [Results of the fifth national mathematics assessment at the end of primary school]*. Arnhem, the Netherlands: CITO.
- Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations among conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology*, *47*, 1525–1538. <https://doi.org/10.1037/a0024997>
- Selter, C. (2001). Addition and subtraction of three-digit numbers: German elementary children's success, methods and strategies. *Educational Studies in Mathematics*, *47*(2), 145–173. <https://doi.org/10.1023/A:1014521221809>
- Siegler, R. S. (2007). Cognitive variability. *Developmental Science*, *10*(1), 104–109. <https://doi.org/10.1111/j.1467-7687.2007.00571.x>
- Stein, M. K., Remillard, J., & Smith, M. S. (2009). How curriculum influences student learning. In J. T. Remillard, B. A. Herbel-Eisenmann & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 319–378). New York and London: Routledge, Taylor & Francis Group.
- Torbeyns, J., De Smedt, B., Ghesquière, P., & Verschaffel, L. (2009). Acquisition and use of shortcut strategies by traditionally schooled children. *Educational Studies in Mathematics*, *71*(1), 1–17. <https://doi.org/10.1007/s10649-008-9155-z>
- Torbeyns, J., Hickendorff, M., & Verschaffel, L. (2017). the use of number-based versus digit-based strategies on multi-digit subtractions: 9-12-year-olds' strategy use profiles and task performances. *Learning and Individual Differences*, *58*, 64–74. <https://doi.org/10.1016/j.lindif.2017.07.004>
- Torbeyns, J., & Verschaffel, L. (2013). Efficient and flexible strategy use on multi-digit sums: A choice/no-choice study. *Research in Mathematics Education*, *15*, 129–140. <https://doi.org/10.1080/14794802.2013.797745>

- Torbeyns, J., & Verschaffel, L. (2016). Mental computation or standard algorithm? Children's strategy choices on multi-digit subtractions. *European Journal of Psychology of Education, 31* (2), 99–116. <https://doi.org/10.1007/s10212-015-0255-8>
- Treffers, A. (1987). Integrated column arithmetic according to progressive schematisation. *Educational Studies in Mathematics, 18*(2), 125–145. <https://doi.org/10.1007/BF00314723>
- van den Heuvel-Panhuizen, M. (2008). *Children learn mathematics*. Rotterdam, the Netherlands: Sense Publishers.
- van den Heuvel-Panhuizen, M., Robitzsch, A., Treffers, A., & Köller, O. (2009). Large-scale assessment of change in student achievement: Dutch primary school students' results on written division in 1997 and 2004 as an example. *Psychometrika, 74*(2), 351–365. <https://doi.org/10.1007/s11336-009-9110-7>
- van Dooren, L. (2000). *Leerlingvolgsysteem. Algemene handleiding [System for following pupils' learning trajectories. General information]*. Leuven, Belgium: Garant.
- Van Putten, C. M., van den Brom-Snijders, P. A., & Beishuizen, M. (2005). Progressive mathematization of long division strategies in dutch primary schools. *Journal for Research in Mathematics Education, 36*(1), 44–73. Retrieved from <http://www.jstor.org/stable/10.2307/30034920>
- Vermunt, J. K., & Magidson, J. (2013). *Latent GOLD 5.0 upgrade manual*. Belmont, MA: Statistical Innovations.
- Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse, the Netherlands: Swets & Zeitlinger.
- Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. K. Lester Jr (Ed.), *Second handbook of research on mathematics teaching and learning pages* (pp. 557–628). Charlotte, NC: Information Age Publishing.
- Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education, 24*(3), 335–359. <https://doi.org/10.1007/BF03174765>

Received 30 November 2016; revised version received 26 October 2017

Appendix

The multidigit division problems (texts translated from Dutch)

label	Format	Remainder	Easy Task (grade 4)	Difficult Task (grade 5 and 6)
COM1	Symbolic	No	$297 : 3 = \dots$ ANSWER: 99	$594 : 6 = \dots$ ANSWER: 99
COM2	Contextual (Partitive)	No	Grandma has €490. She divides that money over her 5 grandchildren. How much does each child get? ANSWER: 98 euros	
DWR1	Contextual (Quotitive)	Yes	There are 203 children at the fairground amusement. Per ride, 2 children can take place. How many rides are needed? ANSWER: 102 rides	A farmer has 604 rabbits that have to be put in cages. In each cage, 3 rabbits fit. How many cages does the farmer need? ANSWER: 202 cages
DWR2	Contextual (Quotitive)	Yes	Father has a piece of wood of 260 cm. He saws planks of 30 cm out of that. How many complete planks can he saw? ANSWER: 8 planks	
A	Contextual (Partitive)	No	Lisa has 156 candies. She divides these candies over 6 bags. How many candies are there in each bag? ANSWER: 26 candies	Lisa has 34 piano lessons per year. Each year she pays €782 for those lessons. How much does a piano lesson cost? ANSWER: 23 euros
B	Symbolic	Yes	$108 : 8 = \dots$ ANSWER: 13 remainder 4; or 13,5	$882 : 36 = \dots$ ANSWER: 24 remainder 18; or 24,5
C	Symbolic	Yes	$806 : 4 = \dots$ ANSWER: 201 remainder 2; or 201,5	
D	Symbolic	No	$168 : 12 = \dots$ ANSWER: 14	