A monodromy criterion for existence of Neron models and a result on semi-factoriality

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Stellingen

behorende bij het proefschrift

"A monodromy criterion for existence of Néron models and a result on semi-factoriality"

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1. Let A/k be an abelian variety over a field of characteristic zero, S a regular k-scheme, and $\pi\colon S' \to S$ a proper birational k-morphism. Every k-morphism $S' \to A$ factors via π.

2. Let S be a reduced, locally noetherian, geometrically unibranch $\mathbb{Q}$-scheme, and let $U \subset S$ be an open dense. Let G/S be a smooth group scheme of finite type, with semi-abelian identity component G^0. Let H/U be a smooth subgroup scheme of G_U. Then the schematic closure $\overline{H} \subset G$ is an S-smooth subgroup scheme of G.

3. Let S be a regular, locally noetherian $\mathbb{Q}$-scheme, $D \subset S$ a regular divisor, $U = S \setminus D$ and A/U an abelian scheme with a semiabelian model $\mathcal{A}/S$. The abelian scheme A admits a Néron model over S.

4. Let S be a regular, locally noetherian scheme, $D \subset S$ a regular divisor, $U = S \setminus D$ and C/U a smooth curve with a nodal model $\mathcal{C}/S$. The jacobian $Jac_{C/U}$ admits a Néron model over S.

5. Let S be a regular, locally noetherian, excellent $\mathbb{Q}$-scheme, D a normal crossing divisor on S, $\mathcal{C} \to S$ and $\mathcal{D} \to S$ two nodal curves, smooth over $U = S \setminus D$.

   Assume that over the generic point $\eta \in S$, there exists an isogeny
   $$\mathrm{Pic}^0_{\mathcal{C}_\eta/\eta} \to \mathrm{Pic}^0_{\mathcal{D}_\eta/\eta}.$$
   Then $\mathrm{Pic}^0_{\mathcal{C}_U/U}$ admits a Néron model over S if and only if $\mathrm{Pic}^0_{\mathcal{D}_U/U}$ does.

6. Let S be a regular, locally noetherian scheme, C/S an aligned curve with regular total space C, admitting a section $\sigma\colon S \to C$. Let $U \subset S$ be an open dense, such that C_U is smooth over U, and let $\mathcal{N}/S$ be the Néron model of $\mathrm{Pic}^0_{C_U/U}$. Let $\mathcal{E}$ be a line bundle on C and $J^\sigma_{\mathcal{E}}/S$ be the compactified jacobian of C/S constructed by Esteves. Denote by $J \subset J^\sigma_{\mathcal{E}}$ the open subspace parametrizing line bundles (equivalently, the S-smooth locus of $J^\sigma_{\mathcal{E}}$). Then the natural S-morphism $J \to \mathcal{N}$ is an isomorphism.

7. Let S be the spectrum of a discrete valuation ring, with generic point η. Let $\mathcal{C}/S$ be a nodal curve, and $\mathcal{C}' \to \mathcal{C}$ the blow-up at a closed point of $\mathcal{C}$. Then:

   i) the induced morphism $\mathrm{Pic}_{\mathcal{C}/S} \to \mathrm{Pic}_{\mathcal{C}'/S}$ is étale.

   Let $e\colon \operatorname{Spec} k(\eta) \to \mathrm{Pic}_{\mathcal{C}_\eta/k(\eta)}$ be the unit section. Denote by $\mathrm{cl}(e)$ (resp. $\mathrm{cl}'(e)$) its schematic closure inside $\mathrm{Pic}_{\mathcal{C}/S}$ (resp. inside $\mathrm{Pic}_{\mathcal{C}'/S}$).

   ii) the induced morphism between the fppf-quotients $\frac{\mathrm{Pic}_{\mathcal{C}/S}}{\mathrm{cl}(e)} \to \frac{\mathrm{Pic}_{\mathcal{C}'/S}}{\mathrm{cl}'(e)}$ is an open immersion.

8. Let S be the spectrum of a discrete valuation ring, $\mathcal{C}/S$ a nodal curve, and $\mathcal{C}' \to \mathcal{C}$ the blowing-up of $\mathcal{C}$ at a closed point. Let $\mathcal{L}$ be a line bundle on $\mathcal{C}'$ such that its restriction to every irreducible component of the exceptional locus of π has degree zero. Then $\pi_*\mathcal{L}$ is a line bundle on $\mathcal{C}$ and the natural morphism of $\mathcal{O}_{\mathcal{C}'}$-modules $\pi^*\pi_*\mathcal{L} \to \mathcal{L}$ is an isomorphism.