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Abstract

This thesis is subdivided in two parts.

In the first part, we introduce a new condition, called toric-additivity, on a family of abelian varieties degenerating to a semi-abelian scheme over a normal crossing divisor. The condition depends only on the Tate module $T_l A(K^{sep})$ of the generic fibre, for a prime l invertible on the base. We show that toric-additivity is a sufficient condition for the existence of a Néron model if the base is a \mathbb{Q} -scheme. In the case of the jacobian of a smooth curve with semi-stable reduction, we obtain the same result without assumptions on the base characteristic; and we show that toric-additivity is also necessary for the existence of a Néron model, when the base is a \mathbb{Q} -scheme.

In the second part, we consider the case of a family of nodal curves over a discrete valuation ring, having split singularities. We say that such a family is semi-factorial if every line bundle on the generic fibre extends to a line bundle on the total space. We give a necessary and sufficient condition for semi-factoriality, in terms of combinatorics of the dual graph of the special fibre. In particular, we show that performing one blow-up with center the non-regular closed points yields a semi-factorial model of the generic fibre.

As an application, we extend the result of Raynaud relating Néron models of smooth curves and Picard functors of their regular models to the case of nodal curves having a semi-factorial model.

Samenvatting

Dit proefschrift bestaat uit twee delen.

In het eerste deel introduceren we een nieuwe voorwaarde, torische-additiviteit genaamd, voor een familie van abelse variëteiten die tot een semi-abelse schema degenereren boven een divisor met normale kruisingen. De voorwaarde hangt alleen af van het Tate-moduul $T_l A(K^{sep})$ van de generieke vezel, voor een priemgetal l dat inverteerbaar is op de basis. We laten zien dat torische-additiviteit een voldoende voorwaarde is voor het bestaan van een Néron model, als de basis een \mathbb{Q} -schema is. In het geval van de jacobian van een gladde kromme met semi-stabiele reductie, verkrijgen we hetzelfde resultaat zonder veronderstellingen over de karakteristiek van de basis; bovendien, laten we zien dat torische-additiviteit ook nodig is voor het bestaan van een Néron model, wanneer de basis een \mathbb{Q} -schema is.

In het tweede deel beschouwen we het geval van een familie van semi-stabiele krommen over een discrete valuatie ring, met gespleten singulariteiten. We zeggen dat zo'n familie semi-factorieel is als elke lijnbundel op de generieke vezel de restrictie is van een lijnbundel op de totale ruimte. We geven een noodzakelijke en voldoende voorwaarde voor semi-factorialiteit, in termen van de combinatoriek van de duale graaf van de speciale vezel. In het bijzonder laten we zien dat het uitvoeren van één blow-up met centrum de niet-reguliere gesloten punten een semi-factorieel model oplevert van de generieke vezel.

Als toepassing, breiden we het resultaat van Raynaud met betrekking tot Néron-modellen van gladde krommen en Picard-functoren van hun reguliere modellen uit naar het geval van (mogelijk singuliere) krommen met een semi-factorieel model.

Résumé

Cette thèse est divisée en deux parties. Dans la première partie, nous introduisons une nouvelle condition, appelée additivité torique, sur une famille de variétés abéliennes qui dégénèrent en un schéma semi-abelien au-dessus d'un diviseur à croisements normaux. La condition ne dépend que du module de Tate $T_l A(K^{sep})$ de la fibre générique. Nous montrons que l'additivité torique est une condition suffisante pour l'existence d'un modèle de Néron, si la base est un schéma de caractéristique nulle. Dans le cas de la jacobienne d'une courbe lisse à réduction semi-stable, on obtient le même résultat sans aucune hypothèse sur la caractéristique de base; et nous montrons que l'additivité torique est aussi nécessaire pour l'existence d'un modèle de Néron, si la base est un schéma de caractéristique nulle.

Dans la deuxième partie, on considère le cas d'une famille de courbes nodales sur un anneau de valuation discrète. On donne une condition combinatoire sur le graphe dual de la fibre spéciale, appelée semi-factorialité, qui équivaut au fait que tous les faisceaux inversibles sur la fibre générique s'étendent en des faisceaux inversibles sur l'espace total de la courbe. Il est démontré par la suite que cette condition est automatiquement satisfaite après un éclatement centré aux points fermés non-réguliers de la famille de courbes.

On applique le résultat ci-dessus pour généraliser un théorème de Raynaud sur le modèle de Néron des jacobiniennes de courbes lisses, au cas des courbes nodales.

Curriculum Vitae

Giulio Orecchia was born on January 11, 1990, in Rapallo, Italy. He lived until 2009 in Genova, where he attended high school at Liceo Scientifico Gian Domenico Cassini, which offered him the opportunity to participate in a number of mathematical competitions for high school students.

After finishing high school, he moved to Padova for his undergraduate studies in mathematics. During those years, he was also a student at the Scuola Galileiana di Studi Superiori, from which he obtained a diploma in 2015.

In 2012, Giulio was awarded an Erasmus Mundus Master scholarship to pursue his master degree within the ALGANT program at Concordia University and Universiteit Leiden. The program had a strong focus on Algebra, Geometry and Number theory courses. He graduated in July 2014 with a thesis in algebraic geometry titled “Torsion-free rank one sheaves on a semi-stable curve”, written under the supervision of Prof. Bas Edixhoven.

In September of the same year, he began his Ph.D. in mathematics, again within the ALGANT doctorate program, under the joint supervision of Dr. David Holmes (Universiteit Leiden) and Prof. Qing Liu (Université de Bordeaux). He plans to defend his thesis in February 2018.

