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Summary

Let $C$ be a connected smooth projective curve over a finite field $\mathbb{F}_q$. Fix a closed point $\infty \in C$ and set $A = \Gamma(C - \{\infty\}, \mathcal{O}_C)$. We call $A$ the coefficient ring. Let $F$ be the fraction field of $A$. Fix a finite field extension $K$ of $F$.

Drinfeld $A$-modules over $K$ behave in a way similar to abelian varieties over number fields. To such a Drinfeld module $E$ and a prime $\mathfrak{p} \subset A$ one can associate the $\mathfrak{p}$-adic Tate module $T_\mathfrak{p}E$. It is a finitely generated free module over the completion of $A$ at $\mathfrak{p}$. It carries a natural continuous action of the Galois group of $K$ which is unramified at almost all primes. Given such a prime $\mathfrak{m}$ it makes sense to consider the inverse characteristic polynomial $P_\mathfrak{m}(T)$ of the geometric Frobenius element at $\mathfrak{m}$ acting on $T_\mathfrak{p}E$. This polynomial has coefficients in $F$ and is independent of the choice of $\mathfrak{p}$.

Assume that the Drinfeld module $E$ has good reduction everywhere. In this case we have a characteristic polynomial $P_\mathfrak{m}(T)$ for every prime $\mathfrak{m}$ of $K$ not diving $\infty$. One can show that the formal product

$$L(E^*, 0) = \prod_{\mathfrak{m}} \frac{1}{P_\mathfrak{m}(1)}$$

converges in the local field $F_\infty$ of the curve $C$ at $\infty$. The construction of $L(E^*, 0)$ resembles the classical construction of an $L$-function of an abelian variety. Indeed $L(E^*, 0)$ is the value of a certain $L$-function at $s = 0$, the Goss $L$-function of the strictly compatible family of Galois representations given by the Tate modules $T_\mathfrak{p}E$.

In the case of abelian varieties one expects that the values of $L$-functions at integral points reflect subtle arithmetic invariants of the varieties in question. The precise relation is given by the celebrated conjecture of Birch and Swinnerton-Dyer and more generally by the equivariant Tamagawa number conjecture. These conjectures are still very far from being solved.

It was a wonderful discovery of Taelman [25] that an analog of the BSD conjecture holds for Goss $L$-functions of Drinfeld modules with the coefficient ring $A = \mathbb{F}_q[t]$. Building on the work of Taelman, Böckle and Pink [3], Fang [9] and V. Lafforgue [17] we extended the result of Taelman to Drinfeld modules over arbitrary coefficient rings $A$. Our approach differs substantially from that of Taelman. It is based on a theory of shtukas and their cohomology which we developed for this purpose.