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Shtuka cohomology and special values of Goss L-functions

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Citation
Mornev, M. (2018, February 13). *Shtuka cohomology and special values of Goss L-functions*. Retrieved from https://hdl.handle.net/1887/61145

Version:	Not Applicable (or Unknown)
License:	Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden
Downloaded from:	https://hdl.handle.net/1887/61145

Note: To cite this publication please use the final published version (if applicable).

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http://hdl.handle.net/1887/61145

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Issue Date: 2018-02-13

Summary

Let C be a connected smooth projective curve over a finite field $\mathbb{F}_q$. Fix a closed point $\infty \in C$ and set $A = \Gamma(C - \{\infty\}, \mathcal{O}_C)$. We call A the *coefficient ring*. Let F be the fraction field of A. Fix a finite field extension K of F.

Drinfeld A-modules over K behave in a way similar to abelian varieties over number fields. To such a Drinfeld module E and a prime $\mathfrak{p} \subset A$ one can associate the $\mathfrak{p}$-adic Tate module $T_{\mathfrak{p}}E$. It is a finitely generated free module over the completion of A at $\mathfrak{p}$. It carries a natural continuous action of the Galois group of K which is unramified at almost all primes. Given such a prime $\mathfrak{m}$ it makes sense to consider the inverse characteristic polynomial $P_{\mathfrak{m}}(T)$ of the geometric Frobenius element at $\mathfrak{m}$ acting on $T_{\mathfrak{p}}E$. This polynomial has coefficients in F and is independent of the choice of $\mathfrak{p}$.

Assume that the Drinfeld module E has good reduction everywhere. In this case we have a characteristic polynomial $P_{\mathfrak{m}}(T)$ for every prime $\mathfrak{m}$ of K not diving ∞. One can show that the formal product

$$L(E^*, 0) = \prod_{\mathfrak{m}} \frac{1}{P_{\mathfrak{m}}(1)}$$

converges in the local field F_∞ of the curve C at ∞. The construction of $L(E^*, 0)$ resembles the classical construction of an L-function of an abelian variety. Indeed $L(E^*, 0)$ is the value of a certain L-function at $s = 0$, the Goss L-function of the strictly compatible family of Galois representations given by the Tate modules $T_{\mathfrak{p}}E$.

In the case of abelian varieties one expects that the values of L-functions at integral points reflect subtle arithmetic invariants of the varieties in question. The precise relation is given by the celebrated conjecture of Birch and Swinnerton-Dyer and more generally by the equivariant Tamagawa number conjecture. These conjectures are still very far from being solved.

It was a wonderful discovery of Taelman [25] that an analog of the BSD conjecture holds for Goss L-functions of Drinfeld modules with the coefficient ring $A = \mathbb{F}_q[t]$. Building on the work of Taelman, Böckle and Pink [3], Fang [9] and V. Lafforgue [17] we extended the result of Taelman to Drinfeld modules over arbitrary coefficient rings A. Our approach differs substantially from that of Taelman. It is based on a theory of shtukas and their cohomology which we developed for this purpose.