Latency, energy, and schedulability of real-time embedded systems

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Appendix

Appendix I

Lemma 1. The minimum value of piece-wise function (6.22) given in Section 6.4 is obtained when $b = b_0$.

$$s(b) = \begin{cases} \frac{(\alpha\lambda^2-\alpha\lambda)b^2+b-1}{(\alpha\lambda-\alpha+1)b-1} & 0 < b \leq b_0 \\ \frac{(1-\alpha)b^2+(\alpha\lambda+\alpha-1)b-\alpha}{(\alpha\lambda-\alpha+1)b-1} & b_0 < b \leq 1 \end{cases} \tag{1}$$

Proof. For case of $0 < b \leq b_0$, its derivative is

$$s'(b) = \frac{\alpha(\lambda-1)(\lambda(\alpha y - \alpha + 1)b^2 - 2\lambda b + 1)}{((\alpha\lambda - \alpha + 1)b - 1)^2}$$

The denominator is obviously positive. For the numerator, since the discriminant of $\lambda(\alpha\lambda - \alpha + 1)b^2 - 2\lambda b + 1 = 0$ is $(2\lambda)^2 - 4\lambda(\alpha\lambda - \lambda + 1)$, which is negative since $0 < \lambda < 1$, so we know $\lambda(\alpha\lambda - \alpha + 1)b^2 - 2\lambda b + 1 > 0$. Moreover, we have $\lambda - 1 < 0$, so putting them together we know the numerator is negative. In summary, $s'(b)$ is negative and thus $s(b)$ is monotonically decreasing with respect to b in the range $b \in (0, b_0]$.

For case of $b_0 < b \leq 1$, we can compute the derivative of $s(b)$ by

$$s'(b) = \frac{(1-\lambda)((\lambda y - x + 1)b^2 - 2b - (\lambda y - x - 1))}{((\lambda y - x + 1)b - 1)^2}$$

The denominator is obviously positive. For the numerator, we focus on $(x\lambda - x + 1)b^2 - 2b - (x\lambda - x - 1)$ part. The following equation

$$(x\lambda - x + 1)b^2 - 2b - (x\lambda - x - 1) = 0$$

has two roots $b_1 = 1$ and $b_2 = \frac{1+(x-x\lambda)}{1-(x-x\lambda)}$, which is greater than 1, so we know $(x\lambda - x + 1)b^2 - 2b - (x\lambda - x - 1)$ is either always positive or always negative in the range of $b \in (b_0, 1)$. Since we can construct $(x\lambda - x + 1)b^2 - 2b - (x\lambda - x - 1) > 0$ with $x = \lambda = b = 0.5$, so we know $(x\lambda - x + 1)b^2 - 2b - (x\lambda - x - 1)$ is always positive. Moreover, since $1 - x > 0$, the numerator of $s'(b)$ is positive, so overall $s'(b)$ is positive, and thus $s(b)$ is monotonically increasing with respect to b in the range of $b \in (b_0, 1]$.

In summary, we have proved $s(b)$ is monotonically decreasing in $(0, b_0]$, and monotonically increasing in $(b_0, 1]$, both with respect to b, so the smallest value of $s(b)$ must occur at b_0. □

Lemma 2. If $0 < \alpha < 1$ and $0 \leq \lambda < 1$, then

$$b_0^1 = \frac{(2 - \alpha\lambda - \alpha) + (1 - \lambda)\sqrt{-3\alpha^2 + 4\alpha}}{2(-\alpha\lambda^2 + \alpha\lambda - \alpha + 1)} > 1 \tag{2}$$

$$b_0^2 = \frac{(2 - \alpha\lambda - \alpha) - (1 - \lambda)\sqrt{-3\alpha^2 + 4\alpha}}{2(-\alpha\lambda^2 + \alpha\lambda - \alpha + 1)} \in [0, 1] \tag{3}$$

Proof. We start with proving $b_0^1 > 1$. We first prove $b_0^1 \geq 0$ by showing both the numerator and dominator are positive. For simplicity, we use N_1 and M_1 to denote the numerator and denominator of b_0^1 in (2), and N_2 and M_2 the numerator and denominator of b_0^2 in (3). Note that the following reasoning relies on that $\alpha \in (0, 1), \lambda \in [0, 1)$.

1. $N_1 > 0$. First, we have

$$\begin{aligned}
&N_1 \times N_2 \\
&= (2 - \alpha\lambda - \alpha)^2 - (1 - \lambda)^2(-3\alpha^2 + 4\alpha) \\
&= 4\alpha\lambda(1 - \lambda)(1 - \alpha) + 4(1 - \alpha)^2 \\
&> 0
\end{aligned}$$

   Moreover, it is easy to see $N_2 > 0$. Therefore, we can conclude that N_1 is also positive.

2. $M_1 > 0$. $2(-\alpha\lambda^2 + \alpha\lambda - \alpha + 1) = 2(\alpha\lambda(1 - \lambda) + (1 - \alpha))$, which is positive.

In summary, both the numerator and the denominator of b_0^1 in (2) are positive, so $b_0^1 \geq 0$. Next we prove $b_0^1 \leq 1$ by showing $N_1 - M_1 \leq 0$:

$$\begin{aligned}
&N_1 - M_1 \\
&= (\lambda - 1)(\sqrt{-3\alpha^2 + 4\alpha} + \alpha(2\lambda - 1))
\end{aligned}$$

which is negative if $\lambda \geq 0.5$ (since $\lambda - 1 < 0$ and $\sqrt{-3\alpha^2 + 4\alpha} + \alpha(2\lambda - 1) \geq 0$). So in the following we focus on the case of $\lambda < 0.5$. Since $\lambda < 0.5$, we know $\alpha(2\lambda - 1)$

is negative, so we define two positive number A and B as follows

$$A = \sqrt{-3\alpha^2 + 4\alpha} \tag{4}$$
$$B = \alpha(1 - 2\lambda) \tag{5}$$

so $N_1 - M_1 = (\lambda - 1)(A - B)$. Since $\lambda - 1 < 0$, we only need to prove $A - B > 0$, which is equivalent to proving $A^2 - B^2 > 0$ (as both A and B are positive): $A^2 - B^2 > 0$, which is done as follows:

$$\begin{aligned} A^2 - B^2 =& -3\alpha^2 + 4\alpha - \alpha^2(2\lambda - 1)^2 \\ =& 4\alpha(1 - \alpha) + 4\alpha^2\lambda(1 - \lambda) \\ >& 0 \end{aligned}$$

so we have $A - B > 0$ and thus $N_1 - M_1 = (\lambda - 1)(A - B) < 0$. In summary, we have proved $N_1 - M_1 < 0$ for the cases of both $\lambda \geq 0.5$ and $\lambda < 0.5$, so we know $b_0^1 \in [0, 1]$.

Next we prove $b_0^2 > 1$, by showing $N_2 - M_2 > 0$

$$\begin{aligned} & N_2 - M_2 \\ = & (1 - \lambda)(\sqrt{-3\alpha^2 + 4\alpha} - \alpha(2\lambda - 1)) \end{aligned}$$

If $\lambda \leq 0.5$, then $\sqrt{-3\alpha^2 + 4\alpha} - \alpha(2\lambda - 1) > 0$, and since $1 - \lambda > 0$ we have $N_2 - M_2 > 0$. If $\lambda > 0.5$, we let $C = \alpha(2\lambda - 1) > 0$ and also use A as defined above, $N_2 - M_2 = (1 - \lambda)(A - C)$. To prove $A - C > 0$, it suffices to prove $A^2 - C^2 > 0$, as shown in the following:

$$\begin{aligned} A^2 - C^2 = & -3\alpha^2 + 4\alpha - \alpha^2(2\lambda - 1)^2 \\ = & \; 4\alpha - (3 + (2\lambda - 1)^2)\alpha^2 \\ > & \; 4\alpha - 4\alpha^2 \;\; (\lambda < 1 \text{ ,so } 2\lambda - 1 < 1) \\ > & \; 0 \end{aligned}$$

By now we have proved $N_2 - M_2$ for both cases of $\lambda \leq 0.5$ and $\lambda > 0.5$, so we known $b_0^2 > 1$.

□

Appendix II

Experimental results between EDF-VD and AMC are depicted in Figure 1 - 3, where pCriticality$= 0.3$.

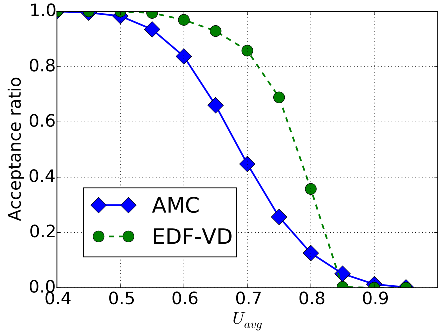

Figure 1: $\lambda = 0.3$

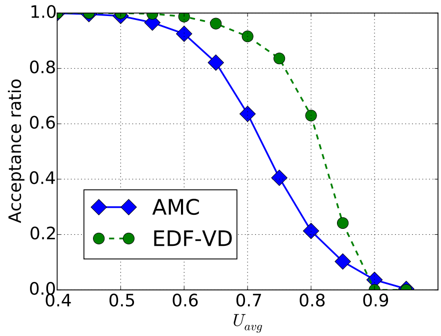

Figure 2: $\lambda = 0.5$

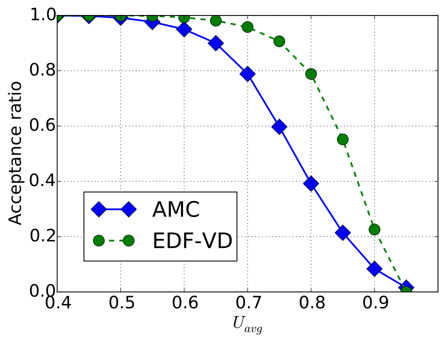

Figure 3: $\lambda = 0.7$