Latency, energy, and schedulability of real-time embedded systems
Liu, D.; Liu D.

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Chapter 2

Background

Predictability, not speed, is the foremost goal in real-time system design.

John Stankovic [Sta88]

In this chapter, to better understand this dissertation, we introduce some common preliminaries which we use in the subsequent chapters, such as the cyclo-static dataflow (CSDF) model, the real-time theories, and the hard-real-time scheduling of CSDF.

2.1 Cyclo-Static Dataflow (CSDF) Model

In this dissertation, we use the cyclo-static dataflow (CSDF) model to model streaming applications. In this section, we introduce this model and its properties.

In [BELP96], Bilsen et al. proposed the cyclo-static dataflow (CSDF) model to model signal processing applications. CSDF generalizes the well-known synchronous dataflow (SDF) model [LM87]. A CSDF graph is defined as a directed graph $G = (A, E)$, where $A$ is a set of actors and $E$ is a set of edges. Actor $\tau_j \in A$ represents a piece of computation in an application and edge $e_i \in E$ represents the communication between two actors, where an atomic data object that is transferred via an edge is called a token. In a CSDF graph, every actor $\tau_i \in A$ has an execution sequence $[F_i(1), F_i(2), \cdots, F_i(N_i)]$ of length $N_i$, meaning that the $n$th execution/firing of actor $\tau_i$ executes the code of function $F_i(((n - 1) \mod N_i) + 1)$. Similarly, each CSDF actor may produce/consume a variable but predefined number of data tokens in consecutive executions, called production/consumption sequence. The production/consumption sequence has the same length of $N_i$ as the execution sequence. An edge $e_u \in E$ is a first-in, first-out (FIFO) queue defined as pair $e_u = (\tau_i, \tau_j)$, denot-
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ing that actor \( \tau_i \) produces data tokens on edge \( e_u \) and actor \( \tau_j \) consumes data tokens from edge \( e_u \). Let \([x_i^u(1), x_i^u(2), \ldots, x_i^u(N_i)]\) denote the production sequence of actor \( \tau_i \) on edge \( e_u \), meaning that at the \( n \)th execution actor \( \tau_i \) produces \( x_i^u(((n - 1) \mod N_i) + 1) \) data tokens on edge \( e_u \). Let \([x_i^u(1), x_i^u(2), \ldots, x_i^u(N_i)]\) denote the production sequence of actor \( \tau_i \) on edge \( e_u \). Let \([x_i^u(1), x_i^u(2), \ldots, x_i^u(N_i)]\) denote the consumption sequence of actor \( \tau_j \) on edge \( e_u \). Similarly, at the \( n \)th execution actor \( \tau_j \) consumes \( y_j^u(((n - 1) \mod N_j) + 1) \) data tokens from edge \( e_u \) and \( X_{u}^i(n) = \sum_{l=1}^{n} x_i^u(l) \) denotes the total amount of data tokens which actor \( \tau_i \) produces on edge \( e_u \) after its first \( n \) executions. Let \([y_j^u(1), y_j^u(2), \ldots, y_j^u(N_j)]\) denote the consumption sequence of actor \( \tau_j \) on edge \( e_u \). Similarly, at the \( n \)th execution actor \( \tau_j \) consumes \( y_j^u(((n - 1) \mod N_j) + 1) \) data tokens from edge \( e_u \) and \( Y_{u}^j(n) = \sum_{l=1}^{n} y_j^u(l) \) denotes the total amount of data tokens which actor \( \tau_j \) consumes from edge \( e_u \) after its first \( n \) executions.

A compelling and important property of CSDF is its decidability, i.e., a schedule for all actors in a CSDF graph \( G \) can be derived at design time. To derive a valid static schedule of a CSDF graph at design-time, the graph needs to be consistent and live.

**Definition 2.1.1 ([BELP96])**. A CSDF graph \( G \) is said to be consistent if a non-trivial solution exists for a repetition vector \( \vec{q} = [q_1, q_2, \ldots, q_{|A|}]^T \).

The repetition vector \( \vec{q} \) is defined as follows:

**Definition 2.1.2 ([BELP96])**. Given a connected CSDF graph \( G \), a vector \( \vec{q} = [q_1, q_2, \ldots, q_{|A|}]^T \) representing the number of invocations of the actors of \( G \) in a valid static schedule is called a repetition vector of \( G \).

And the repetition vector \( \vec{q} \) is computed by using the following theorem,

**Theorem 2.1.1 ([BELP96])**. For a connected CSDF graph \( G \), a repetition vector \( \vec{q} = [q_1, q_2, \ldots, q_{|A|}]^T \) is given by

\[
\vec{q} = \Theta \cdot \vec{r}, \quad \text{with} \quad \Theta_{ik} = \begin{cases} N_i & \text{if } i = k \\ 0 & \text{otherwise} \end{cases} \tag{2.1}
\]

where \( \vec{r} = [r_1, r_2, \ldots, r_{|A|}]^T \) is a positive integer solution of the balance equation

\[
O \cdot \vec{r} = \vec{0} \tag{2.2}
\]

and where the topology matrix \( O \in \mathbb{Z}^{|E| \times |A|} \) is defined by

\[
O_{uj} = \begin{cases} X_j^u(N_j) & \text{if } \tau_j \text{ produces on channel } e_u \\ -Y_j^u(N_j) & \text{if } \tau_j \text{ consumes from channel } e_u \\ 0 & \text{Otherwise.} \end{cases} \tag{2.3}
\]

**Definition 2.1.3 ([BELP96])**. A CSDF graph \( G \) is said to be live if a deadlock-free schedule can be found.


**Definition 2.1.4.** For a consistent and live CSDF graph, the graph completes one iteration, if every actor $\tau_i \in A$ executes for $q_i$ times.

Below, we use an illustrative example to facilitate the understanding of the theories and definitions of CSDF presented above.

**Example 2.1.1.** Consider the CSDF depicted in Figure 2.1. There are four actors $\{\tau_1, \tau_2, \tau_3, \tau_4\}$ and four edges $\{e_1, e_2, e_3, e_4\}$. Each actor has different production/consumption sequences on different edges. For example, actor $\tau_1$ has a production sequence of $[1,1,0]$ on edge $e_1$ and actor $\tau_2$ has a consumption sequence of $[1]$ on edge $e_1$ and a production sequence of $[1]$ on edge $e_3$. Then, according to Equation (2.1), (2.2), (2.3) in Theorem 2.1.1, we obtain

\[
O = \begin{bmatrix}
2 & -1 & 0 & 0 \\
1 & 0 & -1 & 0 \\
0 & 1 & 0 & -2 \\
0 & 0 & 1 & -1
\end{bmatrix}, \quad \vec{r} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \quad \Theta = \begin{bmatrix} 3 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 3 \end{bmatrix}, \quad \vec{q} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 3 \end{bmatrix}
\]

In this dissertation, we consider streaming applications which are modeled as acyclic CSDF graphs. From empirical studies in [TA10], Thies and Amarasinghe showed that 90% of streaming applications can be modeled as acyclic SDF graphs, where SDF graphs are a subset of CSDF graphs. For acyclic CSDF graphs, we have the following lemma,

**Lemma 2.1.1 ([BS11]).** Any acyclic consistent CSDF graph is live.

## 2.2 Real-Time Theories

This section introduces the real-time task models, real-time scheduling algorithms, the schedulability analysis techniques and the multiprocessor real-time scheduling
2.2.1 Real-Time Task Models

A real-time system is comprised of a collection of real-time applications. To ensure the system’s timing correctness, each application in the application set is modeled as a real-time task $\tau_i$ and all tasks form a real-time task set $\Gamma$. A real-time task $\tau_i$ that might generate an infinite sequence of task instances, also called jobs, is usually specified by a parameter tuple \(\{S_i, C_i, D_i, T_i\}\) and the interpretations of the parameters are as follows:

- $S_i$ denotes the start time of $\tau_i$;
- $C_i$ denotes the estimated Worst-Case Execution Time (WCET) of $\tau_i$;
- $D_i$ denotes the relative deadline of $\tau_i$; and
- $T_i$ denotes the period of $\tau_i$ or the minimal arrival interval of two subsequent jobs generated by $\tau_i$.

The different interpretations of $T_i$ define two widely-used real-time task models: periodic task model and sporadic task model. In the periodic task model, $T_i$ denotes the period of task $\tau_i$, i.e., task $\tau_i$ releases its jobs strictly periodically at time instants \(t = \{t | S_i + kT_i, k \in \mathbb{Z}^+\}\). In the sporadic task model, $T_i$ denotes the minimal arrival interval of two subsequent jobs generated by $\tau_i$, i.e., task $\tau_i$ can release its next job anytime once the minimal arrival interval $T_i$ elapses.

The task model also features another characteristic related to the tasks’ start times. If all tasks in a task set $\Gamma$ have the same start time, i.e., $S_1 = S_2 = \cdots = S_n$, task set $\Gamma$ is said to be a synchronous task set. Otherwise, it is called an asynchronous task set. Moreover, considering the relation between deadline $D_i$ and period $T_i$, a periodic/sporadic task set can be defined as either implicit deadline task set or constrained deadline task set. The implicit deadline task set indicates that every task $\tau_i \in \Gamma$ has $D_i = T_i$, whereas the constrained deadline task set means that every task $\tau_i \in \Gamma$ has $D_i \leq T_i$. In real-time systems, the utilization is used to denote the ratio of WCET over a period. For a task set $\Gamma$, the utilization $u_i$ of task $\tau_i \in \Gamma$ is $u_i = C_i/T_i$ and thus the total utilization $U_\Gamma$ of task set $\Gamma$ is the sum of the utilizations of all tasks, i.e., $U_\Gamma = \sum_{i=1}^n u_i$. In addition, we have another term, called density, which denotes the ratio of task’s WCET over the lesser of a task’s relative deadline and period, i.e., the density $\delta_i$ of task $\tau_i \in \Gamma$ is $\delta_i = C_i / \min(D_i, T_i)$ and the total density $\delta_\Gamma$ of task set $\Gamma$ is computed by $\delta_\Gamma = \sum_{i=1}^n \delta_i$.

Table 2.1 lists the corresponding task models used in different chapters. Note that in this section we only introduce the basic task model. The exact task model used
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<table>
<thead>
<tr>
<th>Task Model</th>
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<th>Chapter 4</th>
<th>Chapter 5</th>
<th>Chapter 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>synchronous/</td>
<td>asynchronous</td>
<td>asynchronous</td>
<td>synchronous</td>
<td>synchronous</td>
</tr>
<tr>
<td>constrained/implicit</td>
<td>constrained deadline</td>
<td>implicit deadline</td>
<td>constrained deadline</td>
<td>implicit deadline</td>
</tr>
<tr>
<td>periodic/sporadic</td>
<td>periodic</td>
<td>periodic</td>
<td>periodic</td>
<td>sporadic</td>
</tr>
</tbody>
</table>

Table 2.1: Task models considered in the dissertation’ chapters

in each chapter might be slightly different from the basic task model, and we will elaborate the difference in each chapter.

### 2.2.2 Real-Time Scheduling

When a system and a set of real-time tasks are given, a real-time scheduling algorithm is required to schedule the task set on the specified system. A scheduling algorithm schedules tasks based on their priority and there are three ways to assign priority to each task [DB11].

- **Fixed task priority**: all jobs from a task have a single fixed priority. An example of this is the rate-monotonic scheduling [LL73];

- **Task-level priority**: the jobs of a task might have different priorities, but each job has a single static priority. An example of this is the earliest deadline first (EDF) scheduling [LL73];

- **Dynamic priority**: a job of a task might have different priorities at different times. An example of this is the least laxity first (LLF) scheduling [Leu89].

Preemption is another important feature pertaining to scheduling algorithms.

- If tasks can be preempted by a task with higher priority at any time, the scheduling algorithm is said to be a preemptive scheduling algorithm;

- If tasks start to execute and they cannot be suspended at runtime until their completions, then the scheduling algorithm is said to be a non-preemptive scheduling algorithm.

Since the optimal scheduling algorithms for uniprocessor or multiprocessor systems are both preemptive [LL73][BCPV93a], we in this dissertation focus our study on preemptive scheduling algorithms.
Schedulability Tests and Scheduling Algorithms

As we discussed in Chapter 1, the key problem of real-time systems is to decide whether real-time tasks are schedulable on a specific platform under a certain real-time scheduling algorithm, i.e., to guarantee the system’s timing constraint. Schedulability tests are the formal and verying tools to help us check the schedulability. The formal definition of Schedulability tests can be found in Definition 1.2.1 on page 9. In this section, we introduce the schedulability tests used in this dissertation.

Generally, schedulability tests can be classified as follows [DB11]:

- **Sufficient test**: If a real-time task set which is schedulable according to a schedulability test is indeed schedulable, the schedulability test is said to be sufficient.

- **Necessary test**: If a real-time task set which is unschedulable according to a schedulability test is indeed unschedulable, the schedulability test is said to be necessary.

- **Exact test**: If a schedulability test is both sufficient and necessary, the schedulability test is said to be exact.

In some cases, it is highly difficult to derive an exact test, so we would like to derive a sufficient test to ensure the schedulability of a real-time task set.

The preemptive earliest deadline first (EDF) scheduling algorithm [LL73] is the most studied dynamic-priority scheduling algorithm. In this dissertation, we use EDF as the scheduling algorithm in Chapter 5 and 6. The seminal paper [LL73] proved the exact schedulability test for an implicit deadline periodic task set under EDF on a uniprocessor system.

**Theorem 2.2.1** ([LL73]). For an implicit deadline periodic task set \( \Gamma \), it is schedulable by EDF if and only if

\[
U_\Gamma = \frac{C_1}{T_1} + \frac{C_2}{T_2} + \cdots + \frac{C_n}{T_n} = \sum_{i=1}^{n} \frac{C_i}{T_i} \leq 1. \tag{2.4}
\]

On a uniprocessor system, EDF has been proven to be the optimal scheduling algorithm for implicit deadline tasks [Der74].

**Theorem 2.2.2** ([Der74]). If a job set \( J \) is schedulable by an algorithm \( A \), then it is schedulable by EDF.

**Corollary 2.2.1.** EDF is an optimal scheduling algorithm on a uniprocessor system.
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However, for constrained deadline task set, Equation (2.4) only serves as a necessary test. Barauh et al. in [BMR90] proposed an exact schedulability test for constrained deadline task set under EDF on a uniprocessor system, where the exact schedulability test is derived based on the concept of demand bound function (dbf).

\[
dbf(\tau_i, t_0, t_f) = \max \left\{ 0, \left\lfloor \frac{(t_f - t_0) - D_i}{T_i} \right\rfloor + 1 \right\} C_i
\]

where \( t_f - t_0 \) denotes a time interval, \( t_0 \) and \( t_f \) are the start time and the end time of the interval, respectively. dbf computes the maximum cumulative execution time which a task demands within time interval \([t_0, t_f]\). If the total maximum execution time of task set \( \Gamma \) within the time interval does not exceed the time interval, the task set is schedulable. Otherwise, it is unschedulable for the task set.

**Theorem 2.2.3** ([BMR90]). A task set \( \Gamma \) is schedulable if and only if \( U_\Gamma \leq 1 \) and

\[
\forall t < L_a, \quad dbf(\Gamma, t_0, t_f) = \sum_{i=1}^{n} \max \left\{ 0, \left\lfloor \frac{(t_f - t_0) - D_i}{T_i} \right\rfloor + 1 \right\} C_i
\]

where \( L_a \) is defined as follows:

\[
L_a = \max \left\{ D_1, \ldots, D_n, \max_{1 \leq i \leq n} \left\{ T_i - D_i \right\} \frac{U_\Gamma}{1 - U_\Gamma} \right\}
\]

Theorem 2.2.3 is the exact test to check the schedulability of a task set under EDF scheduling. However, the exact test shown in Theorem 2.2.3 is computationally expensive because this exact test needs to test all absolute deadlines within the time interval and there can be a large number of absolute deadlines which need to be checked. To improve the efficacy of the EDF exact test, Zhang and Burns [ZB09] proposed a new exact test for the EDF scheduling, referred as Quick convergence Processor-demand Analysis (QPA).

**Theorem 2.2.4** ([ZB09]). A task set \( \Gamma \) is schedulable if and only if \( U_\Gamma \leq 1 \) and the result of the QPA iterative algorithm shown in Algorithm 1 is \( dbf(\Gamma, t_0, t_f) \leq d_{min} \), where \( d_{min} = \min \{ D_i \} \).

The extensive experimental results in [ZB09] demonstrate the efficiency of QPA in terms of reducing the time complexity of testing the schedulability. Therefore, in our work, we use QPA to test the schedulability of a task set when the utilization-based test is not applicable.
Algorithm 1: QPA

1. \( t \leftarrow \max\{d_i | d_i < L\} \);
2. while \((dbf(\Gamma, t) \leq t \land dbf(\Gamma, t) > d_{\min})\)
3. \{if \((dbf(\Gamma, t) < t) \) \( t \leftarrow dbf(\Gamma, t) \);
4. else \( t \leftarrow \max\{d_i | d_i < L\} \);
5. \}

2.2.3 Multiprocessor Real-Time Scheduling

Nowadays, the increasing number of real-time systems is implemented on multiprocessor platforms. The prevalence of these real-time multiprocessor systems rises a new problem, namely the assignment problem, i.e., deciding which processor to execute which task. Multiprocessor scheduling algorithms can be classified as follows based on the tasks’ assignment:

- **Partitioned scheduling**: tasks are only permitted to execute on their assigned processors and task migration is prohibited. On each processor, a uniprocessor scheduling algorithm is deployed to schedule the tasks assigned to the processor;

- **Global scheduling**: tasks are permitted to migrate to any processor at anytime and a global scheduling algorithm assigns tasks to proper processors at runtime;

- **Cluster Scheduling**: the system is comprised of several clusters where each cluster consists of a number of processors. Tasks are statically assigned to a fixed cluster and within a cluster tasks are permitted to migrate to the processors in the same cluster, i.e., a global scheduling algorithm is deployed within a cluster, but task migration between clusters is prohibited;

- **Semi-partitioned scheduling/Task-splitting**: the majority of tasks are statically assigned to processors and only a few tasks are permitted to migrate among processors. Usually, the assignment of migrative tasks is also known at design time, i.e., a migrative task executes partially on one processor and then migrates to another processor to complete its execution.

Assignment Algorithms

When partition scheduling is deployed to schedule tasks on a multiprocessor system, a key problem is to efficiently decide how to assign a task to a proper processor so
that a certain metric, e.g., schedulability, energy-efficiency, etc, is satisfied.\footnote{Throughout this dissertation, we may use the term “assign”, “map”, and “partition” interchangeably to denote the procedure that statically decides a processor for a task to complete its execution.}

The assignment problem of real-time tasks on a multiprocessor system is inherently analogous to the well-known bin-packing problem [GJ79]. In the bin-packing problem, objects of different volumes are packed into a finite number of bins with fixed capacity such that the number of bins used is minimized. The bin-packing problem has been proven to be a NP-complete problem [GJ79], so an optimal solution cannot be obtained in a polynominal time unless P=NP. Therefore, many heuristic algorithms are developed to efficiently solve the bin-packing problem and to obtain a suboptimal result in a reasonable time. The close similarity between these two problems allows us to directly utilize the well-established heuristic algorithms for the bin-packing problem to assign real-time tasks on a multiprocessor system. Below, we introduce the most used heuristic algorithms [CGJ97, Joh74].

- **First-Fit (FF)** algorithm: the FF algorithm always tries to place an item $I_i$ to the first bin $B_j$ (i.e., lowest index). That is

\[
j = \min\{k : \text{size}(I_i) + \text{capacity}(B_k) \leq 1\}
\]

If no existing bin can accommodate the item, a new bin is opened and the item is placed in the new bin;

- **Worst-Fit (WF)** algorithm: the WF algorithm always tries to assign an item $I_i$ to the bin $B_j$ which has the most residual capacity after placing item $I_i$. That is

\[
j = \min\{k : \text{size}(I_i) + \text{capacity}(B_k) \text{ minimized}\}
\]

If no existing bin can accommodate the item, a new bin is opened and the item is placed in the new bin;

- **Best-Fit (BF)** algorithm: the BF algorithm always tries to assign an item $I_i$ to the bin $B_j$ which has the least residual capacity after placing item $I_i$. That is

\[
j = \min\{k : \text{size}(I_i) + \text{capacity}(B_k) \text{ maximized & not exceed the bin volume.}\}
\]

If no existing bin can accommodate the item, a new bin is opened and the item is placed in the new bin.

Performance of these heuristic algorithms can be improved by sorting tasks in order of decreasing utilization or density. Then, we have:
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Table 2.2: Scheduling algorithms considered in each chapter

<table>
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<td>cluster scheduling</td>
</tr>
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<td>Chapter 5</td>
<td>task-level</td>
<td>multiprocessor</td>
<td>task-splitting</td>
</tr>
<tr>
<td>Chapter 6</td>
<td>dynamic²</td>
<td>uniprocessor</td>
<td>None</td>
</tr>
</tbody>
</table>

- **First-Fit-Decreasing (FFD):** all tasks are sorted in decreasing order of their utilization or density (see Section 2.2.1) and then tasks are assigned using the FF algorithm;

- **Worst-Fit-Decreasing (WFD):** all tasks are sorted in decreasing order of their utilization or density and then tasks are assigned using the WF algorithm;

Although partitioned scheduling has no migration cost and can directly apply a wealth of well-developed uniprocessor real-time theories, it suffers from low resource utilization due to the capacity loss during the assignment procedure [DB11]. The rest of the multiprocessor real-time scheduling algorithms can achieve better resource utilization and additionally the research on them is an increasingly hot topic in the real-time community. Therefore, in this dissertation, we consider the global scheduling, cluster scheduling, and task-splitting approach.

In the cluster scheduling, the initial step is to assign tasks to a cluster such that a global scheduling can be applied to the tasks assigned to the cluster. Similarly, in the task-splitting approach, a subset of tasks are statically assigned to processors and the rest of the tasks are split among the processors. The assignment procedures in both the cluster scheduling and the task-splitting approach are similar to the assignment procedure in partitioned scheduling. Therefore, all heuristic assignment algorithms introduced above can be applied to the cluster scheduling and the task-splitting approach.

Table 2.2 summarizes the priority assignment schemes, platforms, and multiprocessor scheduling algorithms considered in each chapter.

²Although the original EDF is a task-level priority scheduling algorithm, EDF-VD may change deadlines of some tasks during runtime. As a result, the priority of these tasks will be changed upon their execution. Thus, we here consider EDF-VD as a dynamic priority scheduling.
2.3 Hard-Real-Time (HRT) Scheduling of CSDF graphs

Throughout this dissertation, instead of traditional dataflow scheduling techniques [MB07], we use a new scheduling framework [BS11, BS12, BS13] to schedule CSDF graphs. In this section, we give a brief introduction about this new scheduling framework.

Traditionally, CSDF graphs are scheduled by self-timed scheduling [MB07], where an actor starts to execute as soon as it receives enough tokens from its predecessors. However, since the self-timed scheduling normally provides best-effort services, it is difficult to provide a hard-real-time timing guarantee for every task in an application. Bamakhrama and Stefanov in [BS11, BS12, BS13] proposed a Hard-Real-Time (HRT) scheduling framework in which an acyclic CSDF graph is converted into an independent periodic task set. This conversion effectively bridges the gap between data-flow models and real-time theories and enables us to apply a plethora of well-developed real-time theories to CSDF graphs. The advantage of this framework is the direct application of real-time theories on dataflow models, such as schedulability tests and assignment algorithms, and the designers are able to accomplish fast admission control and temporal isolation and provide hard-real-time guarantees. A good example of the application of the HRT scheduling framework is demonstrated in [BZNS12], where the merit of the HRT scheduling framework helps to significantly reduce the complexity of the design space exploration when designing a real-time streaming multiprocessor system.

The HRT scheduling framework takes as an input a CSDF graph where the WCET of each actor in the CSDF graph is known a priori, and finally it outputs a periodic task sets which can be scheduled by a real-time scheduler. Note that the WCETs considered in the HRT scheduling framework also account for the worst-case communication overhead because the HRT scheduling framework strives to ensure the feasibility of this approach regardless of the variance of different task assignments. The basic concept behind this framework is to derive the real-time parameters, i.e., period, start time, and deadline, for each actor according to actors’ WCETs and the CSDF graph properties, e.g., the repetition vector explained in Section 2.1. The procedure goes through the following steps: 1) it computes a period for each actor in the input CSDF graph according to its repetition values and WCETs; 2) it computes the actors’ start times such that the precedence constraints between actors are respected and thus the deadlock in execution is avoided; and 3) it determines the deadline of each actor.

Computing Periods

To compute the period of an actor in a CSDF graph, we first define the workload of an actor and the maximum actor workload of a graph as follows:
Definition 2.3.1. The \textbf{workload} of an actor \( \tau_i \) is \( W_i = q_iC_i \) and the \textbf{maximum actor workload} of the graph is \( \hat{W} = \max_{\tau_i \in G} \{ W_i \} \)

Then, we can use the \textit{maximum actor workload} and the \textit{repetition value} \( q_i \) of actor \( \tau_i \) to compute the minimum period \( \hat{T}_i \) of actor \( \tau_i \) as follows [BS11]:

\[
\hat{T}_i = \frac{\text{lcm}(\vec{q})}{q_i} \left\lfloor \frac{\hat{W}}{\text{lcm}(\vec{q})} \right\rfloor
\]  

(2.8)

where \( \text{lcm}(\vec{q}) \) is the \textit{least common multiple} of the repetition vector \( \vec{q} \) (explained in Section 2.1). The minimum periods deliver the maximal throughput for the CSDF graph under the \textit{HRT} scheduling. Based on this minimum period, a \textit{period scaling technique} can be applied to uniformly scale up the period of each actor under the \textit{HRT} scheduling framework, thereby adjusting the throughput of the CSDF graph [ZBS13, SLS16].

\textbf{Computing Start Times}

Once the periods of all actors are computed, we need to deal with the precedence constraints between actors. The execution semantics of a dataflow model requires an actor to receive sufficient data from its predecessors in order to trigger its execution, thus the existence of precedence constraints prohibits all actors to start their execution at the same time. To resolve the precedence constraints, the converting procedure in the \textit{HRT} scheduling framework offsets the start times of actors such that by its start time every actor is capable of obtaining sufficient data on its input edges and its subsequently periodic executions are free from deadlock. The following lemma is used to compute the earliest start time for each actor in a CSDF graph

\textbf{Lemma 2.3.1} (From [BS11]). \textit{For an acyclic CSDF graph} \( G \), \textit{the earliest start time of an actor} \( \tau_j \in A \), \textit{denoted} \( S_j \), \textit{under HRT scheduling is given by}

\[
S_j = \begin{cases} 
0 & \text{if } \Omega(\tau_j) = \emptyset \\
\max_{\tau_i \in \Omega(\tau_j)} (S_{i \rightarrow j}) & \text{if } \Omega(\tau_j) \neq \emptyset 
\end{cases}
\]  

(2.9)

where \( \Omega(\tau_j) \) is the set of predecessors of \( \tau_j \), and \( S_{i \rightarrow j} \) is given by

\[
S_{i \rightarrow j} = \min_{t \in [0, S_i + \alpha]} \{ t : \text{prd}_{[S_i, \max(S_i, t)+k]}(\tau_i) \geq \text{cns}_{[t, \max(S_i, t)+k]}(\tau_j) \forall k \in [0, \alpha] \}
\]  

(2.10)

where \( \alpha = q_iT_i = q_jT_j \), \( \text{prd}_{[t_s, t_e]}(\tau_i) \) is the number of tokens produced by \( \tau_i \) during the time interval \( [t_s, t_e] \), and \( \text{cns}_{[t_s, t_e]}(\tau_j) \) is the number of tokens consumed by \( \tau_j \) during the time interval \( [t_s, t_e] \).
Determining Deadlines

After the step of computing the start times, we have three parameters to specify an actor $\tau_i$ as a periodic task: the given WCET $C_i$, the period $T_i$, and the start time $S_i$ computed by using Equation (2.8) and Equation (2.9), respectively. Recall that a periodic task is specified by a tuple of 4 parameters $\{S_i, C_i, D_i, T_i\}$, so we need to determine a deadline for each actor. In the HRT scheduling framework, the deadline $D_i$ of actor $\tau_i$ can be selected within a well-defined range $[C_i, T_i]$, i.e., $D_i \in [C_i, T_i]$. The deadline selection is a highly relevant problem for the HRT scheduling framework because the deadline selection is able to influence both the performance (mainly the latency) of the CSDF graph and the number of processors required to schedule the CSDF graph. Later in Chapter 3, we propose a novel approach to optimally select deadlines in the HRT scheduling framework such that the latency requirement is ensured and the number of processors required is minimized.

Latency and Throughput

After the periodic tasks’ parameters are computed, as explained above, the latency and throughput of the CSDF graph scheduled in the HRT scheduling framework can be computed. Equation (2.11) is used to compute the minimum latency of the CSDF graph,

$$L(G) = \max_{w \in W}(S_{\text{out}} + (g_{\text{out}}^C + 1)T_{\text{out}} - (S_{\text{in}} + g_{\text{in}}^P T_{\text{in}})) \quad (2.11)$$

where $w$ is one path of set $W$ which consists of all paths from the input actor to the output actor. Here, $S_{\text{out}}$ and $T_{\text{out}}$ are the start time and period, respectively, of output actor $\tau_{\text{out}}$, while $S_{\text{in}}$ and $T_{\text{in}}$ denote the start time and period, respectively, of input actor $\tau_{\text{in}}$. $g_{\text{out}}^C$ and $g_{\text{in}}^P$ are two constants which denote the number of invocations the actor waits for the non-zero consumption/production of tokens on a path $w \in W$. Note that when $D_i = C_i, \forall i$, the graph can reach the minimum latency achievable by the HRT scheduling framework. The throughput of the CSDF graph is computed as follows:

$$R = 1/T_{\text{out}} \quad (2.12)$$

Note that when all actors have the minimum periods $\tilde{T}_i$, the graph can reach the maximum throughput achievable by the HRT scheduling framework.