Investigations of radiation pressure : optical side-band cooling of a trampoline resonator and the effect of superconductivity on the Casimir force

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Increased Read-Out Sensitivity with an Optomechanical Cavity

Interest in the Casimir force has increased significantly since technological progress allowed for more sensitive measurements of the force. The Casimir force has been measured between different materials, such as metals [11, 47, 50, 52, 54, 135, 142, 143, 146], semiconductors [51, 175–178], conductive oxides [144], ferromagnets [107] and even superconductors [179].

Also theoretically, methods have been developed for accurate calculations of the force between real materials in different geometries [113, 114, 180, 181]. These calculations in general show good agreement with experiment within several percent. But for a more accurate comparison improvements are required in the force sensitivity of the measurements. One question that is still not answered with satisfaction is what model to use to extrapolate the measured dielectric permittivity of the surfaces to lower frequencies. Two models are proposed, one that includes Ohmic dissipation (Drude model) and one that does not account for dissipation (plasma model).

As discussed in chapter 5 and Refs. [49, 55] differentiating between the two models would be possible by measurements of the Casimir force between superconductors. Since we know that dissipation is not present in superconductors, we can compare these experiments with measurements above the critical temperature. From the presence or absence of notable differences we can deduce the role of dissipation in normal conductors. This experimental investigation of the Casimir force between gold and a superconductor is discussed in chapter 8. We found that the measurement precision was of the same order of magnitude as the maximally expected effect of superconductivity on the Casimir force. To get a more definite insight into this effect, it is necessary to increase our force sensitivity.

In this chapter we will discuss our current sensitivity and remark on some possible changes. One major change would be to combine the Casimir set-up with an optomechanical cavity to read out the motion of the resonator acting as the force sensor. With this set-up it would in principle be possible to increase the force sensitivity by an order of magnitude.

9.1 Considerations for Further Improvements

Measurements of the Casimir force gradient as a function of temperature, see Figure 8.5, show a force sensitivity that is of the same order of magnitude as the expected influence of superconductivity. This influence is negligible near the critical temperature, and only approaches the expected maximum value near $0\,\mathrm{K}$. It is therefore unfortunate that the lowest temperature in our measurements was only a few Kelvin below the critical temperature. We could lower the base temperature of our cryostat, or double the effect by measuring the Casimir force between two superconductors. But even then we need to improve on the force sensitivity of our set-up.

To investigate the force sensitivity of our set-up and its limitations, we first look at the frequency read-out of our cantilever. The cantilever frequency is determined via interferometric read-out of the motion, with the signal sent to a frequency counter (Agilent 53131A). A self-oscillating circuit drives the cantilever at a constant amplitude [161]. In Figure 9.1(a) we show the detected frequency as a function of time with a time interval of one second. During this measurement, the cantilever was in vacuum ($p \approx 10^{-4}\,\mathrm{mbar}$) and at room temperature. The cantilever was withdrawn several millimeters from the plate. Over the measurement time of several hours, no drifts were present in the cantilever frequency, interferometric read-out and frequency counter. From a fit to the histogram of the data, see Figure 9.1(b), we determine that the read-out sensitivity of the cantilever frequency is $13\,\mathrm{mHz}$. Via the system parameter $\mu = 2.22 \times 10^{-12}\,\mathrm{Hz\,m^2\,V^{-2}}$ we calculate that the sensitivity of the normalized force gradient is equal to $0.16\,\mathrm{N/m^2}$, based solely on the read-out of the cantilever motion. This corresponds to a parallel-plate pressure of $0.025\,\mathrm{N/m^2}$. The actual measurement precision based on our Casimir force measurements shown in chapter 8 is an order of magnitude higher. But these measurements were obtained while the distance between the plate and the sphere was fixed. Since the Casimir force depends strongly on the distance, any deviation in the distance results in a

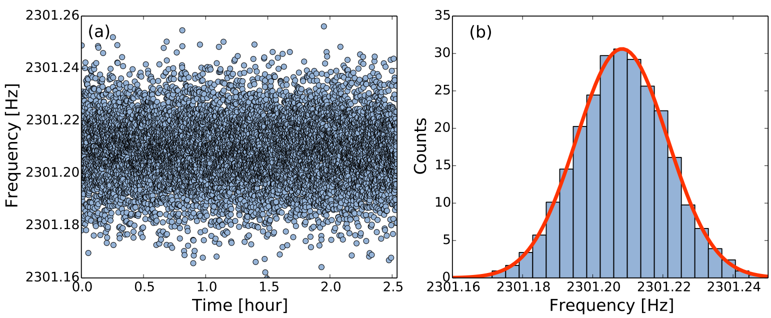

Figure 9.1: Force sensitivity of our set-up: (a) Cantilever frequency measured with fiber interferometry and detected by a frequency counter for several hours. The time interval between data is set at $1\,\mathrm{s}$. The frequency counter, interferometric read-out and cantilever frequency are not sensitive to drifts in a few hours; (b) Histogram of the data in (a). The frequency sensitivity is $13\,\mathrm{mHz}$, which results in a force gradient sensitivity of $0.16\,\mathrm{N/m^2}$.

strong deviation in the measured force. The difference between the predicted and actual force sensitivity can therefore be explained by the fact that the distance lock is not tight enough. This is again a result of our force sensitivity, since small, low frequency fluctuations in the detection lead to small fluctuations in the distance. For the current settings of our force detection, mostly determined by the AC voltage that sets the magnitude of the electrostatic force, the fluctuations in the distance are of the order of a nanometer.

The lock can be improved by setting reasonable feedback parameters. But the distance feedback is based on the electrostatic force and thus on the force read-out. Improving the force sensitivity will therefore also improve the distance lock.

Theoretically, the minimum force gradient that can be detected with the frequency modulation (FM) technique and interferometric read-out is given by [161]

$$\left.\frac{\partial F}{\partial d}\right|_{\min} = \sqrt{\frac{4kBk_BT}{\omega_0 Q\langle x_{\mathrm{osc}}^2\rangle}} \tag{9.1}$$

with k the spring constant, B the measurement bandwidth, k_B Boltzmann's constant, T the temperature and ω_0 and Q the cantilever's angular frequency and quality factor respectively. The mean-square amplitude $\langle x_{\mathrm{osc}}^2\rangle$ is the driven amplitude of the cantilever, which is generally larger than the thermal motion. Via the proximity force approximation (see Eq. 5.31) we can relate this value to the minimal detectable parallel-plate pressure

$$P_{pp}|_{\min} \approx \frac{1}{2\pi R}\sqrt{\frac{4kBk_BT}{\omega_0 Q\langle x_{\mathrm{osc}}^2\rangle}}. \tag{9.2}$$

If we put in our experimental parameters $R = 100\,\mu$m, $k \approx 1\,$N/m, $B = 1\,$Hz, $\omega_0/2\pi = 2.3\,$kHz, $Q = 5000$ and take a driven amplitude of $x_{\mathrm{rms}} = 5\,$nm, we find at room temperature that the minimal detectable pressure is equal to $0.005\,\mathrm{N/m^2}$. The difference between this value and the measured sensitivity in Figure 9.1 is caused either by laser phase noise or by noise in the electronics driving the cantilever at constant amplitude.

At low temperature, the minimal detectable pressure is an order of magnitude lower, $3\times10^{-4}\,\mathrm{N/m^2}$, directly due to the influence of the temperature, but also indirectly because the mechanical quality factor is higher, $Q \approx 30\,000$. The resonance frequency and spring constant shift a few percent up, but this is a negligible effect. We could further lower the detectable force gradient by limiting the bandwidth further, but this will increase our measurement time. Increasing the cantilever amplitude has a positive effect on the detectable pressure. But a large oscillation means a large modulation of the sphere-plate distance. And since the Casimir force, but also the electrostatic force, decreases nonlinearly with distance, this modulation results in an larger average force. If the cantilever amplitude is large compared to the average distance, mixing between the frequency components of our modulation technique can occur. A large cantilever amplitude results in a large average electrostatic force, which causes a large amplitude at the modulation frequency ω_1, such that the Casimir force is also modulated at this frequency. Increasing the cantilever amplitude is therefore limited to only a few nanometers.

Measuring at low temperatures theoretically lowers the minimal detectable pressure, but in our experiment we see no difference between the room temperature force sensitivity and the sensitivity at low temperature. This means that other noise sources, like laser noise, are still dominant [51]. If we want to increase our force sensitivity, we can gain a lot by improving the feedback parameters of the distance lock, by increasing the electrostatic force set-point and by upgrading the electronics that drive the cantilever. Increasing the read-out sensitivity would also be a great improvement.

9.2 A New Measurement Set-up

The force sensitivity of our current set-up can be increased via the recommendations mentioned above. But even if all the technical noise is cleared up, there is still a limit from the interferometric read-out. In chapter 7 we found that the noise floor of our interferometric read-out is equal to $\sigma = 100\,\mathrm{fm}/\sqrt{\mathrm{Hz}}$. If we take a measurement bandwidth of $1\,\mathrm{Hz}$, spring constant of $1\,\mathrm{N/m}$, a sphere with radius $100\,\mu\mathrm{m}$ and a sphere-plate distance of $100\,\mathrm{nm}$, we find via

$$P_{pp} = \frac{\sigma k \sqrt{B}}{2\pi R d} \tag{9.3}$$

that the detection limit of the parallel-plate pressure is equal to $0.002\,\mathrm{N/m^2}$. To reach the minimal detectable pressure at low temperature as given by Eq. 9.3, we should also improve the detection of the cantilever motion.

In general, an optical cavity provides a more sensitive read-out of the resonator motion than a fiber interferometer, due to a stronger light-resonator interaction. To demonstrate this, with our optomechanical cavity we measure a noise floor of $300\,\mathrm{am}/\sqrt{\mathrm{Hz}}$, which is a factor 300 lower than what we can achieve with our fiber interferometer. Note that this is measured at the resonance frequency of the trampoline resonator, which is two orders of magnitude higher than for the cantilevers with microspheres.

We can use the trampoline resonator as our force sensor, the only adaption to our current optomechanical set-up would be a conductive coating of the resonator, which is required for our calibration and compensation scheme based on the electrostatic force. An advantage of the trampoline resonators is that we can increase the interaction area of the force. Whereas spheres with a large radius generally also have a larger surface roughness [156], trampoline resonators or nanomembranes can be manufactured with an area of the order of $1\,\mathrm{mm^2}$ while maintaining practically flat surfaces. The idea of using a nanomembrane to read out the motion has been proposed before [182–185], but in these proposals it was not combined with a high-finesse cavity for read-out.

At first glance, it would seem imprudent to pair our flat trampoline resonator with a flat conductive plate for the Casimir force measurements, because of the technical challenge to align them perfectly parallel. But having to align a sphere directly opposite the resonator is also not ideal. Fortunately, it was shown [186] that the trampoline resonators are curved due to the tensile stress in the silicon nitride. The radius of curvature is commonly about $R \approx 1\,\mathrm{mm}$. We can therefore pair a curved trampoline resonator with a flat conductive plate.

Another advantage of using trampoline resonators is that its mechanical properties can be tuned. Compared to microspheres attached to cantilevers, the mechanical quality factor, resonance frequency and radius of curvature are higher. The spring constant can be tuned by the design of the arms of the resonator. A low spring constant is better for the force sensitivity, but when it is too low, the resonator may snap under the influence of the large forces. Fortunately, further tunability is possible due to the optical spring effect [187]. In principle, the spring constant can be adapted as the distance between the resonator and the plate changes. A comparison of typical parameters of the two systems is shown in Table 9.1.

	microsphere on cantilever	trampoline resonator
radius of curvature	$100\,\mu\mathrm{m}$	$1\,\mathrm{mm}$
resonance frequency	$2\,\mathrm{kHz}$	$200\,\mathrm{kHz}$
mechanical quality factor	5000 - 30 000	400 000
spring constant	$1\,\mathrm{N/m}$	$0.2\,\mathrm{N/m}$, tunable
read-out	interferometric	high-finesse cavity

Table 9.1: Comparison of typical parameters of two systems for Casimir force measurements; a microshere attached to an atomic force microscope cantilever that was used in this thesis and the proposed improvement based on a trampoline resonator.

Based on the noise floor of $300\,\mathrm{am}/\sqrt{\mathrm{Hz}}$ of our optomechanical cavity, we find that the optical cavity can read out a pressure of $1 \times 10^{-7}\,\mathrm{N/m^2}$, at a distance of $100\,\mathrm{nm}$ and a bandwidth of $1\,\mathrm{Hz}$. The minimal pressure that can be detected by the trampoline resonator, using the FM technique, is according to Eq. 9.3 equal to $9 \times 10^{-5}\,\mathrm{N/m^2}$ at room temperature based on a thermal resonator RMS amplitude of $144\,\mathrm{pm}$. The trampoline resonator can be driven [80, 188] to an amplitude of around $1\,\mathrm{nm}$, which would result in a minimal detectable pressure of $1.3 \times 10^{-5}\,\mathrm{N/m^2}$, a significant enhancement compared to our current sensitivity.

To set the distance between the trampoline resonator, it is possible to position the plate on a piezo-electric translation stage. The distance can be determined by modulating the electrostatic force between the two surfaces in a similar fashion described in this thesis. This method requires an independent read-out of the plate's distance change d_{pz} during a measurement run. We therefore need to install a fiber interferometer directed towards the plate. Since we are not interested in the absolute direction of the distance change, we can place the fiber on either side of the plate. Behind the plate would be more advantageous for spatial arguments. Another, more challenging, but also more sensitive option would be to align a second beam between the large cavity mirror and the plate to form a second optical cavity that is tilted slightly with respect to the first cavity.

To set the distance more accurately than can be done with a piezo, we can use the nested resonator described in chapter 4. The coated outer resonator and the plate form a capacitor. By setting an adequate voltage between them, the electrostatic force will pull the nested resonator closer to the plate. Since this requires different volt-

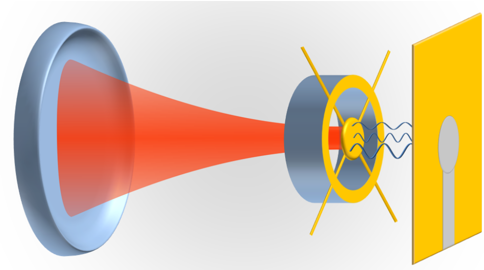

Figure 9.2: Impression of the set-up for Casimir force measurements with an optomechanical cavity for force detection. The large cavity mirror (left), nested resonator (middle) and plate (right) are indicated, as well as the cavity light and vacuum fluctuations. The plate is separated into two areas which can be set at different potentials. The center area (grey) is used for calibration and compensation as described in chapter 7 of this thesis, the outer area (yellow) is used to set the distance between the nested resonator and the plate.

ages than needed for our calibration and compensation technique, the plate needs to be divided into two areas that can be set at different voltages with respect to the nested resonator. The voltages V_{DC} and V_{AC}, required for our modulation scheme, are applied to the inner area, indicated in grey in Figure 9.2. The distance between the outer resonator and plate is set by the potential of the outer area. A large degree of control of the nested resonator motion with the dielectric force was already shown in Ref. [96]. A similar degree of control can be expected with the capacitative force, as the main objection in chapter 4 against this force was the alignment procedure, which is required in this set-up anyway.

Combining the Casimir force measurement method based on modulation of the electrostatic force with an optomechanical cavity to read out the motion of the trampoline resonator serving as a force sensor is a promising way to significantly improve our force sensitivity. Our current force gradient sensitivity of $2\,\mathrm{N/m^2}$ (corresponding to a pressure sensitivity of $0.3\,\mathrm{N/m^2}$) is limited by noise in our electronics. If we can eliminate this noise, we are still limited to a minimal detectable pressure of $0.005\,\mathrm{N/m^2}$ at room temperature based on the read-out sensitivity of our fiber interferometer. We can improve on this further if we switch to optomechanical cavities. The resulting pressure sensitivity of $1.3 \times 10^{-5}\,\mathrm{N/m^2}$ at low temperatures is significantly better than with our current set-up. For comparison, the lowest experimental error reported so far is $2.2 \times 10^{-3}\,\mathrm{N/m^2}$ [11].