Advances in computational methods for Quantum Field Theory calculations

Ruijl, B.J.G.

Author: Ruijl, B.J.G.
Title: Advances in computational methods for Quantum Field Theory calculations
Issue Date: 2017-11-02

Advances in computational methods for Quantum Field Theory calculations

Ben Ruijl

ISBN: 978-94-6233-746-6
Printed by: Gildeprint, Enschede
Cover: thesisexpert.nl

SIKS Dissertation Series No. 2017-31. The research reported in this thesis has been carried out under the auspices of SIKS, the Dutch Research School for Information and Knowledge Systems.
This work is supported by the ERC Advanced Grant no. 320651, "HEPGAME".

Advances in computational methods for Quantum Field Theory calculations

Proefschrift

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van Rector Magnificus prof. mr. C.J.J.M. Stolker,
volgens besluit van het College voor Promoties
te verdedigen op donderdag 2 november 2017
klokke 15.00 uur

door

Ben Jacobus Gertruda Ruijl

geboren te Geleen
in 1989

Promotores:	Prof. dr. H.J. van den Herik
	Prof. dr. A. Plaat
Co-promotor:	dr. J.A.M. Vermaseren (Nikhef)
Promotiecommissie:	Prof. dr. J.N. Kok
	Prof. dr. A. Achúcarro
	Prof. dr. H.H. Hoos
	Prof. dr. J.W. van Holten
	Prof. dr. C. Anastasiou (ETH Zürich)
	Prof. dr. E.L.P.M. Laenen (Universiteit van Amsterdam)
	Prof. dr. A. Vogt (University of Liverpool)
	dr. G. Heinrich (MPP München)

Isn't it a noble, an enlightened way of spending our brief time in the sun, to work at understanding the universe and how we have come to wake up in it?

Richard Dawkins

PREFACE

This PhD thesis is the result of work I performed as part of the HEPGAME (High Energy Physics and Games) project. The HEPGAME project is supported by the ERC Advanced Grant no. 320651 and its primary goal is to apply methods from artificial intelligence to solve problems in high energy physics. The symbolic manipulation toolkit FORM plays a key role in the investigation.

During the PhD I worked in three different locations. The first half year I mostly worked at Tilburg University. The next few years I have worked at Leiden University and Nikhef. At Leiden University, I taught a course, supervised a student, and worked on the computer science aspect of this thesis with my close colleagues Jaap van den Herik, Aske Plaat, and Ali Mirsoleimani. This mainly involved expression simplification, which turned into chapter 2.

At Nikhef, I focused on physics, and worked on designing computer programs with my close colleagues Jos Vermaseren and Takahiro Ueda. This resulted in the FORCER program, described in chapter 3. Later, Andreas Vogt joined to compute new physical quantities with FORCER. The results formed the basis of chapter 4.

Together with Franz Herzog we decided to aim for something we had not thought possible: computing the five-loop beta function. After a year of puzzling, we understood the R^*-method which could help us achieve our goal. The method is explained in chapter 5.

Finally, after combining the FORCER program and the computer code for the R^*-method, we were able to compute the five-loop beta function for Yang-Mills theory with fermions. This formed the basis for chapter 6.

I have had the pleasure to work with experts in the fields of Artificial Intelligence, Computer Science, and Theoretical Physics. This enriching experience has helped shape this thesis, for which I am grateful.

LIST OF ABBREVIATIONS

CSEE Common Subexpression Elimination 11–15, 17, 35, 38

HEP High Energy Physics 15, 16, 18–21, 24, 26–36

IBP Integration by Parts 8, 39, 40, 42, 43, 45, 46, 48, 49, 52–55, 59, 60, 103, 128, 147, 161, 190
IR Infrared 85, 103–110, 113, 118–127, 129–135, 181, 187, 188
IRR Infrared Rearrangement 104, 118

LHC Large Hadron Collider 4–6, 9, 39
LTVG Logarithmic Tensor Vacuum Graph 105, 120, 123, 125, 127, 128, 133, 135

MCTS Monte Carlo Tree Search 11, 12, 16–22, 35–37, 61, 161, 189, 190

QCD Quantum Chromodynamics 3, 7–9, 39, 41, 80–82, 85–87, 90, 95, 97, 101, 104, 105, 114, 115, 129, 130, 135, 137, 138, 146, 150, 151, 153–157, 159, 160, 162, 190, 191, 195
QED Quantum Electrodynamics 3, 7, 87, 137, 150–152, 159, 191, 195
QFT Quantum Field Theory 3, 7–9, 39, 95, 103, 104, 129, 162, 189, 192

SA Simulated Annealing 22, 25–27, 32–34
SA-UCT Simulated Annealing - UCT 11, 12, 16, 18, 21–24, 36
SDD Superficial Degree of Divergence 106, 116, 118, 120–123, 126, 129, 133
SHC Stochastic Hill Climbing 22, 25–28, 30–37, 190

UCT Upper Confidence bounds applied to Trees 11, 16, 17, 20–24, 36, 189
UV Ultraviolet 103–109, 111, 113, 115, 118–135, 139, 185, 187, 188

CONTENTS

PREFACE vii

LIST OF ABBREVIATIONS ix

1 INTRODUCTION 3
- 1.1 The Standard Model 3
- 1.2 Precise predictions 5
- 1.3 Computer methods 5
- 1.4 Feynman diagrams 6
- 1.5 Problem statement 7
- 1.6 Three research questions 8
- 1.7 Research methodology and four contributions 9
- 1.8 Structure of the thesis 10

2 EXPRESSION SIMPLIFICATION 11
- 2.1 Horner schemes and common subexpression elimination 12
  - 2.1.1 Horner Schemes 12
  - 2.1.2 Common subexpression elimination 13
  - 2.1.3 The evaluation function 13
  - 2.1.4 Interplay 14
- 2.2 Experimental setup 15
- 2.3 Tree search methods 16
  - 2.3.1 Monte Carlo Tree Search 16
  - 2.3.2 Nested Monte Carlo Search 19
  - 2.3.3 SA-UCT 20
- 2.4 Stochastic Local Search 22
  - 2.4.1 SHC versus SA 22
  - 2.4.2 Neighbourhood structure 27
  - 2.4.3 Two state space properties 31
- 2.5 Performance of SHC vs. MCTS 35
- 2.6 Chapter conclusion 36
  - 2.6.1 Findings and conclusions 37
  - 2.6.2 Future research 37

3 FOUR LOOP FEYNMAN DIAGRAM CALCULATIONS 39
- 3.1 Generalised Feynman diagrams 40
- 3.2 Integration of one-loop two-point functions 42
- 3.3 Carpet rule 43
- 3.4 Integration by parts 45
  - 3.4.1 The rule of the triangle 46
  - 3.4.2 The rule of the diamond 47
  - 3.4.3 Custom solutions 52

3.5 Solving parametric IBP identities by heuristics 53
3.5.1 Heuristics and equation generation 54
3.5.2 Reduction rules beyond S_0 55
3.5.3 Identities for topologies with insertions 59
3.5.4 Solving strategy 60
3.6 The 21 topologies that need custom reductions 61
3.7 The Forcer framework 66
3.7.1 Reduction graph generation 66
3.7.2 Reduction graph execution 69
3.7.3 Example 69
3.8 Usage 72
3.9 Expansions 73
3.10 From physical diagrams to Forcer 74
3.10.1 Self-energy filtering 77
3.10.2 Colour split-off 78
3.10.3 Diagram database 79
3.10.4 Momentum substitutions 79
3.11 Examples and performance 80
3.12 Chapter conclusion 83
3.12.1 Findings and main conclusion 83
3.12.2 Future research 83

4 RESULTS FOR FOUR-LOOP COMPUTATIONS 85
4.1 Group notations 86
4.2 Propagators and vertices 87
4.2.1 Self energies 87
4.2.2 Triple-gluon vertex 88
4.2.3 Ghost-gluon vertex 89
4.2.4 Quark-gluon vertex 90
4.2.5 Renormalisation 90
4.2.6 Anomalous dimensions 92
4.2.7 Computations and checks 93
4.3 Splitting functions and coefficient functions 94
4.3.1 Optical theorem method 96
4.3.2 Operator method 97
4.3.3 Axial vector current 98
4.4 Chapter conclusion 101
4.4.1 Findings and main conclusion 101
4.4.2 Future research 102

5 INFRARED REARRANGEMENT 103
5.1 Divergences in Euclidean non-exceptional Feynman graphs . . 105
5.1.1 UV divergences in Feynman graphs 106
5.1.2 IR divergences in Feynman graphs 107

5.2 The R-operation in the MS-scheme . . . 110
5.2.1 Definition of the R-operation in the MS-scheme . . . 111
5.2.2 Examples of R-operations . . . 113
5.3 The R-operation for generic Feynman graphs in MS . . . 114
5.3.1 Contraction anomalies and tensor reduction . . . 116
5.3.2 Contraction anomalies and counterterm factorisation . . . 117
5.4 The R*-operation . . . 118
5.4.1 Definition of the R^*-operation . . . 119
5.4.2 The infrared counterterm operation . . . 122
5.4.3 Examples of R^* for generic Feynman graphs . . . 123
5.4.4 Properties of logarithmic vacuum graphs . . . 127
5.5 Applications of R* . . . 128
5.6 Discussion of the literature . . . 132
5.7 Chapter conclusion . . . 134
5.7.1 Findings and main conclusion . . . 135
5.7.2 Future research . . . 135

6 THE FIVE-LOOP BETA FUNCTION 137
6.1 Optimisations . . . 138
6.1.1 Treatment of propagator insertions . . . 138
6.1.2 Delayed Feynman rule substitution . . . 140
6.1.3 Rules to make Feynman diagrams logarithmic . . . 140
6.1.4 Canonical forms for Feynman diagrams . . . 142
6.1.5 Efficient tensor reduction . . . 143
6.2 The background field . . . 145
6.3 Diagram computations and analysis . . . 147
6.4 Results and discussion . . . 149
6.4.1 Analysis of n_f-dependence in QCD . . . 153
6.4.2 Analysis of N-dependence in SU(N) . . . 154
6.4.3 Cumulative effects of the QCD beta function corrections 154
6.5 QCD beta function in the MiniMOM scheme . . . 154
6.6 Chapter conclusions . . . 159
6.6.1 Findings and main conclusion . . . 160
6.6.2 Future work . . . 160

7 CONCLUSIONS 161
7.1 Answers to the research questions . . . 161
7.2 Answer to the problem statement . . . 162
7.3 Future research . . . 162

REFERENCES 165

APPENDICES 181

A CUTVERTEX RULE FOR SCALAR DIAGRAMS 183

B CUTVERTEX RULE FOR TENSOR DIAGRAMS 185
C IR SUBGRAPH SEARCH 187
SUMMARY 189
SAMENVATTING 193
CURRICULUM VITAE 197
ACKNOWLEDGEMENTS 199
SIKS DISSERTATION SERIES 201

The material presented in this thesis is based on the following works.

1. B. Ruijl, T. Ueda and J.A.M. Vermaseren, *Forcer, a FORM program for the parametric reduction of four-loop massless propagator diagrams*, arXiv:1704.06650

2. B. Ruijl, T. Ueda, J.A.M. Vermaseren and A. Vogt, *Four-loop QCD propagators and vertices with one vanishing external momentum*, *JHEP* **2017** (2017) 1, http://dx.doi.org/10.1007/JHEP06(2017)040

3. F. Herzog and B. Ruijl, *The R^*-operation for Feynman graphs with generic numerators*, *JHEP* **2017** (2017) 37, http://dx.doi.org/10.1007/JHEP05(2017)037

4. F. Herzog, B. Ruijl, T. Ueda, J.A.M. Vermaseren and A. Vogt, *The five-loop beta function of Yang-Mills theory with fermions*, *JHEP* **02** (2017) 090 [arXiv:1701.01404]

5. J. Davies, A. Vogt, B. Ruijl, T. Ueda and J.A.M. Vermaseren, *Large-n_f contributions to the four-loop splitting functions in QCD*, *Nucl. Phys.* **B915** (2017) 335

6. F. Herzog, B. Ruijl, T. Ueda, J.A.M. Vermaseren and A. Vogt, *FORM, Diagrams and Topologies*, *PoS* **LL2016** (2016) 073 [arXiv:1608.01834]

7. T. Ueda, B. Ruijl and J.A.M. Vermaseren, *Forcer: a FORM program for 4-loop massless propagators*, *PoS* **LL2016** (2016) 070 [arXiv:1607.07318]

8. B. Ruijl, T. Ueda, J.A.M. Vermaseren, J. Davies and A. Vogt, *First Forcer results on deep-inelastic scattering and related quantities*, *PoS* **LL2016** (2016) 071, https://arxiv.org/abs/1605.08408

9. T. Ueda, B. Ruijl and J.A.M. Vermaseren, *Calculating four-loop massless propagators with Forcer*, *J. Phys. Conf. Ser.* **762** (2016) 012060 [arXiv:1604.08767]

10. B. Ruijl, T. Ueda and J. Vermaseren, *The diamond rule for multi-loop Feynman diagrams*, *Phys. Lett.* **B746** (2015) 347 [arXiv:1504.08258]

11. B. Ruijl, J. Vermaseren, A. Plaat and H.J. van den Herik, *Why Local Search Excels in Expression Simplification*, http://arxiv.org/abs/1409.5223, 2014, arXiv:1409.5223

12. B. Ruijl, J. Vermaseren, A. Plaat and J. van den Herik, *Problems and New Solutions in the Boolean Domain*, ch. Simplification of Extremely Large Expressions, p. 76. Cambridge Scholars Publishing, 2016

13. B. Ruijl, J. Vermaseren, A. Plaat and H.J. van den Herik, *HEPGAME and the Simplification of Expressions*, *Proceedings of the 11th International Workshop on Boolean Problems* (2014), http://arxiv.org/abs/1405.6369

14. B. Ruijl, J. Vermaseren, A. Plaat and H.J. van den Herik, *Combining Simulated Annealing and Monte Carlo Tree Search for Expression Simplification*, *Proceedings of ICAART Conference 2014* **1** (2014) 724 [arXiv:1312.0841], http://arxiv.org/abs/1312.0841

1 INTRODUCTION

1

Understanding how Nature works on a fundamental level is one of the key goals of physics. Consequently, physicists try to identify what the smallest building blocks of our universe are and how they interact. The ideas about these fundamental building blocks have undergone many revolutions, each radically changing the way we describe Nature.

The ancients reasoned that all objects were composed of the elements fire, earth, water, and air. As technology progressed, scientists discovered cells, molecules, and atoms. Atoms only have a radius of about 30 trillionths of a meter. For a while it was believed that the atom was the smallest component (the Greek name means *indivisible*). This idea lasted until the early 20th century with the discoveries of the electron and proton. Six decades later, it turned out that even protons and neutrons were not fundamental, but consisted of quarks [15, 16].

From the invention of quantum mechanics in the early 1920s, it became clear that these small particles behave differently from everyday experience: particles could be in two places at once, act like waves, or spontaneously emerge from the vacuum, and quickly disappear again [17]. The fact that the fundamental building blocks had both particle-like and wave-like features was later unified by the framework of Quantum Field Theory (QFT) [18]. The new quantum field theoretic description of the electromagnetic interaction, Quantum Electrodynamics (QED), was hugely successful and is still used to this day.

Below we provide a brief introduction to the world of QFT. We describe the Standard Model in section 1.1, the aim for precise predictions in section 1.2, computer methods in section 1.3, and Feynman diagrams in section 1.4. Then we formulate our Problem Statement (PS) in section 1.5, and the Research Questions (RQs) in section 1.6. In section 1.7 we list our contributions and in section 1.8 we outline the structure of the thesis.

1.1 THE STANDARD MODEL

In the early 1960s, the first version of the Standard Model was constructed [19]. The goal was to capture all fundamental particles and interactions in this model. The first version contained several particles, such as quarks and electrons, and the electromagnetic and weak forces. The electromagnetic force governs the interactions of photons (light) with charged particles. The weak forces govern nuclear decay and are mediated by the W and Z bosons. The Higgs boson, which is responsible for giving elementary particles mass, was added to the model shortly after, in 1964 [20–22]. Finally, the strong force, mediated by gluons, was added in 1973 [23, 24]. The theory for the strong interaction, Quantum Chromodynamics (QCD), explained why quarks of opposite charge can stay in a stable configuration in the nucleus of an