Advances in computational methods for Quantum Field Theory calculations

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door

Ben Jacobus Gertruda Ruijl

geboren te Geleen
in 1989

Promotores:	Prof. dr. H.J. van den Herik
	Prof. dr. A. Plaat
Co-promotor:	dr. J.A.M. Vermaseren (Nikhef)
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	Prof. dr. H.H. Hoos
	Prof. dr. J.W. van Holten
	Prof. dr. C. Anastasiou (ETH Zürich)
	Prof. dr. E.L.P.M. Laenen (Universiteit van Amsterdam)
	Prof. dr. A. Vogt (University of Liverpool)
	dr. G. Heinrich (MPP München)

Isn't it a noble, an enlightened way of spending our brief time in the sun, to work at understanding the universe and how we have come to wake up in it?

Richard Dawkins

PREFACE

This PhD thesis is the result of work I performed as part of the HEPGAME (High Energy Physics and Games) project. The HEPGAME project is supported by the ERC Advanced Grant no. 320651 and its primary goal is to apply methods from artificial intelligence to solve problems in high energy physics. The symbolic manipulation toolkit Form plays a key role in the investigation.

During the PhD I worked in three different locations. The first half year I mostly worked at Tilburg University. The next few years I have worked at Leiden University and Nikhef. At Leiden University, I taught a course, supervised a student, and worked on the computer science aspect of this thesis with my close colleagues Jaap van den Herik, Aske Plaat, and Ali Mirsoleimani. This mainly involved expression simplification, which turned into chapter 2.

At Nikhef, I focused on physics, and worked on designing computer programs with my close colleagues Jos Vermaseren and Takahiro Ueda. This resulted in the Forcer program, described in chapter 3. Later, Andreas Vogt joined to compute new physical quantities with Forcer. The results formed the basis of chapter 4.

Together with Franz Herzog we decided to aim for something we had not thought possible: computing the five-loop beta function. After a year of puzzling, we understood the R^*-method which could help us achieve our goal. The method is explained in chapter 5.

Finally, after combining the Forcer program and the computer code for the R^*-method, we were able to compute the five-loop beta function for Yang-Mills theory with fermions. This formed the basis for chapter 6.

I have had the pleasure to work with experts in the fields of Artificial Intelligence, Computer Science, and Theoretical Physics. This enriching experience has helped shape this thesis, for which I am grateful.

LIST OF ABBREVIATIONS

CONTENTS

The material presented in this thesis is based on the following works.

1. B. Ruijl, T. Ueda and J.A.M. Vermaseren, *Forcer, a FORM program for the parametric reduction of four-loop massless propagator diagrams*, arXiv:1704.06650
2. B. Ruijl, T. Ueda, J.A.M. Vermaseren and A. Vogt, *Four-loop QCD propagators and vertices with one vanishing external momentum*, *JHEP* **2017** (2017) 1, http://dx.doi.org/10.1007/JHEP06(2017)040
3. F. Herzog and B. Ruijl, *The R^*-operation for Feynman graphs with generic numerators*, *JHEP* **2017** (2017) 37, http://dx.doi.org/10.1007/JHEP05(2017)037
4. F. Herzog, B. Ruijl, T. Ueda, J.A.M. Vermaseren and A. Vogt, *The five-loop beta function of Yang-Mills theory with fermions*, *JHEP* **02** (2017) 090 [arXiv:1701.01404]
5. J. Davies, A. Vogt, B. Ruijl, T. Ueda and J.A.M. Vermaseren, *Large-n_f contributions to the four-loop splitting functions in QCD*, *Nucl. Phys.* **B915** (2017) 335
6. F. Herzog, B. Ruijl, T. Ueda, J.A.M. Vermaseren and A. Vogt, *FORM, Diagrams and Topologies*, *PoS* **LL2016** (2016) 073 [arXiv:1608.01834]
7. T. Ueda, B. Ruijl and J.A.M. Vermaseren, *Forcer: a FORM program for 4-loop massless propagators*, *PoS* **LL2016** (2016) 070 [arXiv:1607.07318]
8. B. Ruijl, T. Ueda, J.A.M. Vermaseren, J. Davies and A. Vogt, *First Forcer results on deep-inelastic scattering and related quantities*, *PoS* **LL2016** (2016) 071, https://arxiv.org/abs/1605.08408
9. T. Ueda, B. Ruijl and J.A.M. Vermaseren, *Calculating four-loop massless propagators with Forcer*, *J. Phys. Conf. Ser.* **762** (2016) 012060 [arXiv:1604.08767]
10. B. Ruijl, T. Ueda and J. Vermaseren, *The diamond rule for multi-loop Feynman diagrams*, *Phys. Lett.* **B746** (2015) 347 [arXiv:1504.08258]
11. B. Ruijl, J. Vermaseren, A. Plaat and H.J. van den Herik, *Why Local Search Excels in Expression Simplification*, http://arxiv.org/abs/1409.5223, 2014, arXiv:1409.5223
12. B. Ruijl, J. Vermaseren, A. Plaat and J. van den Herik, *Problems and New Solutions in the Boolean Domain*, ch. Simplification of Extremely Large Expressions, p. 76. Cambridge Scholars Publishing, 2016
13. B. Ruijl, J. Vermaseren, A. Plaat and H.J. van den Herik, *HEPGAME and the Simplification of Expressions*, *Proceedings of the 11th International Workshop on Boolean Problems* (2014), http://arxiv.org/abs/1405.6369
14. B. Ruijl, J. Vermaseren, A. Plaat and H.J. van den Herik, *Combining Simulated Annealing and Monte Carlo Tree Search for Expression Simplification*, *Proceedings of ICAART Conference 2014* **1** (2014) 724 [arXiv:1312.0841], http://arxiv.org/abs/1312.0841

1 INTRODUCTION

Understanding how Nature works on a fundamental level is one of the key goals of physics. Consequently, physicists try to identify what the smallest building blocks of our universe are and how they interact. The ideas about these fundamental building blocks have undergone many revolutions, each radically changing the way we describe Nature.

The ancients reasoned that all objects were composed of the elements fire, earth, water, and air. As technology progressed, scientists discovered cells, molecules, and atoms. Atoms only have a radius of about 30 trillionths of a meter. For a while it was believed that the atom was the smallest component (the Greek name means *indivisible*). This idea lasted until the early 20th century with the discoveries of the electron and proton. Six decades later, it turned out that even protons and neutrons were not fundamental, but consisted of quarks [15, 16].

From the invention of quantum mechanics in the early 1920s, it became clear that these small particles behave differently from everyday experience: particles could be in two places at once, act like waves, or spontaneously emerge from the vacuum, and quickly disappear again [17]. The fact that the fundamental building blocks had both particle-like and wave-like features was later unified by the framework of Quantum Field Theory (QFT) [18]. The new quantum field theoretic description of the electromagnetic interaction, Quantum Electrodynamics (QED), was hugely successful and is still used to this day.

Below we provide a brief introduction to the world of QFT. We describe the Standard Model in section 1.1, the aim for precise predictions in section 1.2, computer methods in section 1.3, and Feynman diagrams in section 1.4. Then we formulate our Problem Statement (PS) in section 1.5, and the Research Questions (RQs) in section 1.6. In section 1.7 we list our contributions and in section 1.8 we outline the structure of the thesis.

1.1 THE STANDARD MODEL

In the early 1960s, the first version of the Standard Model was constructed [19]. The goal was to capture all fundamental particles and interactions in this model. The first version contained several particles, such as quarks and electrons, and the electromagnetic and weak forces. The electromagnetic force governs the interactions of photons (light) with charged particles. The weak forces govern nuclear decay and are mediated by the W and Z bosons. The Higgs boson, which is responsible for giving elementary particles mass, was added to the model shortly after, in 1964 [20–22]. Finally, the strong force, mediated by gluons, was added in 1973 [23, 24]. The theory for the strong interaction, Quantum Chromodynamics (QCD), explained why quarks of opposite charge can stay in a stable configuration in the nucleus of an