Constraining $f(R)$ gravity with Sunyaev-Zel'dovich clusters detected by the Planck satellite

Simone Peirone,[1] Marco Raveri,[2,3,4,5] Matteo Viel,[3,4] Stefano Borgani,[1,3,4] and Stefano Ansoldi[6,3,7]

[1]*Astronomy Unit, Department of Physics, University of Trieste, Via Tiepolo 11, I-34131 Trieste, Italy*
[2]*SISSA–International School for Advanced Studies, Via Bonomea 265, I-34136 Trieste, Italy*
[3]*INFN, Sezione di Trieste, Via Valerio 2, I-34127 Trieste, Italy*
[4]*INAF-Osservatorio Astronomico di Trieste, Via Gian Battista Tiepolo 11, I-34131 Trieste, Italy*
[5]*Institute Lorentz, Leiden University, P.O. Box 9506, Leiden 2300 RA, The Netherlands*
[6]*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
[7]*Department of Mathematics, Computer Science and Physics, Udine University, 33100 Udine, Italy*

Clusters of galaxies have the potential of providing powerful constraints on possible deviations from General Relativity. We use the catalog of Sunyaev-Zel'dovich (SZ) sources detected by Planck and consider a correction to the halo mass function for a $f(R)$ class of modified gravity models, which has been recently found to reproduce well results from N-body simulations, to place constraints on the scalaron field amplitude at the present time, f_R^0. We find that applying this correction to different calibrations of the halo mass function produces upper bounds on f_R^0 tighter by more than an order of magnitude, ranging from $\log_{10}(-f_R^0) < -5.81$ to $\log_{10}(-f_R^0) < -4.40$ (95% confidence level). This sensitivity is due to the different shape of the halo mass function, which is degenerate with the parameters used to calibrate the scaling relations between SZ observables and cluster masses. Any claim of constraints more stringent that the weaker limit above, based on cluster number counts, appears to be premature and must be supported by a careful calibration of the halo mass function and by a robust calibration of the mass scaling relations.

I. INTRODUCTION

We discuss viability of $f(R)$ gravity [1] by comparing redshift number counts predictions for galaxy clusters with the recently released [2] all-sky, full-mission, Planck catalog of Sunyaev-Zel'dovich (SZ) sources (PSZ2). In particular, we discuss substantial improvements on existing constraints, their stability, and their dependence from the choice of the halo *mass function* (MF) of galaxy clusters, the most massive gravitationally bound structures in the Universe [3,4]. The MF, $n(M,z)$, i.e. the number density of halos in the mass range $[M, M + dM]$ at redshift z, is a sensitive cosmological probe of the late time Universe, and it can provide unique constraints on cosmological parameters and other fundamental physical quantities, like neutrino masses [5,6].

Here we are interested in constraining $f(R)$ gravity [7], which is characterized by a Lagrangian density of the form $R + f(R)$, where f depends on the Ricci scalar, R. Fundamental quantities in the theory are $f_R = df/(dR)$, the *scalaron*, and its Compton wavelength $B = d^2f/(dR)^2$, of which f_R^0 and B_0 represent the values at the current epoch. Deviations from General Relativity (GR) affect gravitational collapse and structure formation, resulting in a dependence of $n(M,z)$ on f_R^0. We describe modified gravity effects on the cluster MF following [8,9], where N-body simulations are used to fit departure from GR predictions for the critical density contrast for the collapse

of a top-hat spherical perturbation, δ_c, in models with $f(R) \sim R^{-n}$, with n a positive integer [10,11].

By taking into account cosmic microwave background (CMB) lensing, constraints from primary CMB temperature anisotropies result in $B_0 < 0.1$ [12] [all results in the text are at 95% confidence level (C.L.), unless otherwise stated]. Adding small scale information from redshift space distortions and weak lensing [13,14] further tightens this constraint to $B_0 < 0.8 \times 10^{-4}$. Similar results are obtained combining CMB and large scale structure (e.g., galaxy clustering) data [15–18]. Constraints coming from cluster number counts [15,19,20] have provided upper limits on f_R^0 in the range $[1.3–4.8] \times 10^{-4}$ by using different data sets and making somewhat different assumptions. Recently a stronger upper limit, $|f_R^0| \lesssim 7 \times 10^{-5}$, was obtained from peak statistics in weak lensing maps [21].

To derive constraints on cosmological models using clusters, a precise calibration of the halo MF is necessary. Significant progress in this direction has been made over the past decade in the context of GR, but only in a few cases modified gravity theories have been considered. In the context of $f(R)$ theory, the good agreement down to nonlinear scales of recent numerical approaches, which compare theoretical models for the MF [22–26] with the results of different implementations of N-body simulations, motivates the use of an updated calibration of the MF to improve existing constraints on modified gravity theories [19,20].

In general, the MF can be written as [27,28]

$$\frac{dn(M,z)}{dM} = F(\sigma_M)\frac{\rho_M}{M^2}\frac{d\log\sigma_M^{-1}}{d\log M}, \quad (1)$$

where ρ_M is the comoving density of matter, M the cluster mass, σ_M the variance of the linear matter power spectrum filtered on the mass-scale M, and F the *multiplicity function*. Achitouv *et al.* [8] define a new functional form for $F(\sigma)$ in $f(R)$ gravity. This is done by a reparametrization of δ_c that, contrary to the GR case, becomes a scale dependent function of f_R^0. This derivation of the MF for $f(R)$ models should apply for halo masses computed at the virial radius.

To calibrate the MF parameters, Achitouv *et al.* [8] compared their predictions to the MF results from $f(R)$, N-body simulations in the redshift range $z \in [0, 1.5]$ and for scalaron values in the range $-f_R^0 \in [10^{-4}, 10^{-6}]$ [24], using halos identified by a *Friends-of-Friends* (FoF) algorithm.

On the other hand, in the PSZ2 catalog that we are using here the cluster masses are given as M_{500c}, i.e. the total mass within a radius, R_{500c}, chosen in such a way that the mean enclosed density is $500\rho_c$. To adapt the calibration of the $f(R)$ MF to our case, we implement the Achitouv *et al.* [8] MF as a correction to GR multiplicity functions, computed at R_{500c}, and calibrated from large sets of N-body simulations of standard gravity:

$$F(\sigma) = F_{GR}(\sigma)\frac{F_A^{fR}(\sigma)}{F_A^{GR}(\sigma)}. \quad (2)$$

Here $F_A^{fR}(\sigma)$ and $F_A^{GR}(\sigma)$ are the multiplicity functions defined in Ref. [8]. For the multiplicity function calibrated on N-body simulations in GR, $F_{GR}(\sigma)$, we implement two alternative definitions: the Tinker *et al.* MF [29], and the Watson *et al.* MF [30] (in the following, Tinker and Watson, respectively). We choose to test the Achitouv *et al.* MF in the form of a correction to another MF [see (2)], because its GR limit is markedly different from the Tinker and Watson results. These two MFs have been widely studied thus allowing us to compare our results to past literature. Following this procedure, we are implicitly assuming that the $f(R)$ correction to the MF from Ref. [8] also applies at R_{500c}. This assumption clearly needs to be verified from an extensive calibration of the MF at different overdensities from large $f(R)$ N-body simulations.

Within the PSZ2 catalog, we identify a sample of 429 clusters with a signal-to-noise ratio $q > 6$. These clusters, with masses in the range $M_{500c} \in [1, 10] \times 10^{14} M_\odot$, and redshift $z \in [0, 1]$, are hereafter denoted as the *SZ* data set. The characteristic mass scale of the cluster sample is a critical element in the number counts analysis. The original analysis of the Planck collaboration [2] assumes a calibration of a scaling relation between measured cluster masses

and integrated Compton-y parameter. To parametrize the uncertainty in the calibration of cluster masses [2], a mass bias parameter is introduced, b, the ratio between the masses calibrated through x-ray Multi-Mirror Mission (XMM)-Newton x-ray observations [31] and the true cluster masses. In the following, we assume true cluster masses to be given by the weak lensing results from the *Weighing the Giants* project [32]. This implies for the bias parameter $B_{SZ} = 1 - b$ a Gaussian prior with mean value 0.688 and variance 0.072. This choice is motivated because it provides a better agreement with primary Planck CMB results. It is, thus, a *conservative* choice, since it leaves less freedom for deviations from the standard ΛCDM results. By choosing another prior, the tension between different data sets could result in artificially tighter constraints on f_R^0, when combining CMB and cluster number counts data. Compared to other x-ray selected cluster data sets, like CCCP [33] or REFLEX [34], the Planck sample is biased towards larger masses and higher redshift, and offers a *unique opportunity to test the MF in a complementary regime*. Another key parameter in the likelihood analysis is α_{SZ}, which sets the slope of the scaling relation between Y_{500c}, the strength of the SZ signal in terms of the Compton y-profile integrated within a sphere of radius R_{500c}, and M_{500c}.

We also use Planck measurements of CMB fluctuations in both temperature and polarization [35,36] in the multipoles range $\ell \leq 29$. We account for CMB anisotropies at smaller angular scales by using the Plik likelihood [36] for CMB measurements of the TT, TE and EE power spectra. Finally, we include the Planck 2015 full-sky lensing potential power spectrum [35] in the multipole range $40 \leq \ell \leq 400$.

Finally, we complement CMB measurements with the *joint light-curve analysis* "JLA" supernovae sample [37], and with BAO measurements of: the SDSS main galaxy sample at $z_{eff} = 0.15$ [38]; the BOSS DR11 "LOWZ" sample at $z_{eff} = 0.32$ [39]; the BOSS DR11 CMASS at $z_{eff} = 0.57$ [39]; and the 6dFGS survey at $z_{eff} = 0.106$ [40]. We refer to the data combination CMB + BAO + JLA as Planck.

After computing cosmological predictions with EFTCAMB and EFTCosmoMC [41,42] modifications of the CAMB/CosmoMC codes [43,44], we compare these predictions with observations. The EFTCosmoMC code has been modified to account for the $f(R)$ cluster likelihood, a suitable modification of the original likelihood in [2].

II. RESULTS

Table I shows the marginalized constraints obtained from the Planck + SZ data set, with the $f(R)$ correction applied to both Tinker and Watson MFs. Tinker MF results in the tightest constraints on $f(R)$ to date. In particular we improve the bounds in [20] on $\log_{10}(-f_R^0)$ by 1 order of magnitude, and the ones in [21] by almost an order of magnitude. These constraints improve substantially also on

TABLE I. Marginalized constraints obtained from the Planck + SZ data set. Different columns show the two different MFs to which the $f(R)$ correction is being applied, see the discussion following Eq. (2). A prior on B_{SZ} has been applied as in [2].

Parameter	Tinker (95% C.L.)	Watson (95% C.L.)
$\log_{10}(-f_R^0)$	< -5.81	< -4.40
$\log_{10} B_0$	< -5.60	< -4.06
σ_8	$(0.79, 0.83)$	$(0.80, 0.83)$
α_{SZ}	$(1.68, 1.91)$	$(1.57, 1.89)$
B_{SZ}	$(0.55, 0.67)$	$(0.50, 0.63)$

the bounds coming from large scale cosmological observations [14], confirming the leading role of galaxy clusters in constraining modified gravity theories.

At the same time, we find a strong dependence of this upper bound on the choice of the MF, which can affect observational constraints by more than 1 order of magnitude. This strong dependence is clear also from Fig. 1(a): the Tinker MF produces the tightest bounds, while the Watson MF is less constraining. To better understand this result, we first note that SZ cluster measurements break the degeneracy between σ_8 and $\log_{10}(-f_R^0)$ that Planck CMB measurements clearly display. We then considered also a run with SZ clusters without Planck data, adding the previously described BAO constraints, including a prior

on n_s [45], $n_s = 0.9624 \pm 0.014$, and adopting big bang nucleosynthesis constraints [46], $\Omega_b = 0.022 \pm 0.002$ (SZ + BAO data set). The results are shown in Fig. 1(a), where we report both the Tinker (yellow) and Watson (orange) contour plots. We can notice that, at least for the Tinker run, the addition of CMB data improves the constraints on f_R^0 by more than 2 orders of magnitude. We especially emphasize that, in the case of SZ + BAO, we do not get the strong dependence on the GR calibration of the MF that we obtain for the SZ + Planck runs. We can then expect that, in the latter case the constraints obtained for the choice of Watson MF are weaker because the shape of this MF is different from Tinker MF exactly in the range of mass and redshift probed by SZ Planck clusters. More precisely, as shown in Fig. 1(b), $N(z)$ falls off at high redshift for the Watson MF more slowly compared to the Tinker case: when combined with CMB Planck data, in order to fit the tail at high redshift in GR, a lower B_{SZ} is required; a lower α_{SZ} is instead preferred in order to fit the low-redshift trend for $N(z)$. When we, instead, consider $f(R)$ models for the Watson MF, there is a more effective way to change the slope of $N(z)$ with this parameter (Fig. 1) than by using α_{SZ}, which is now fairly unconstrained and degenerate with f_R^0. The same is not true for the Tinker case: this degeneracy is not present, and this results in tighter constraints on f_R^0. This would explain the
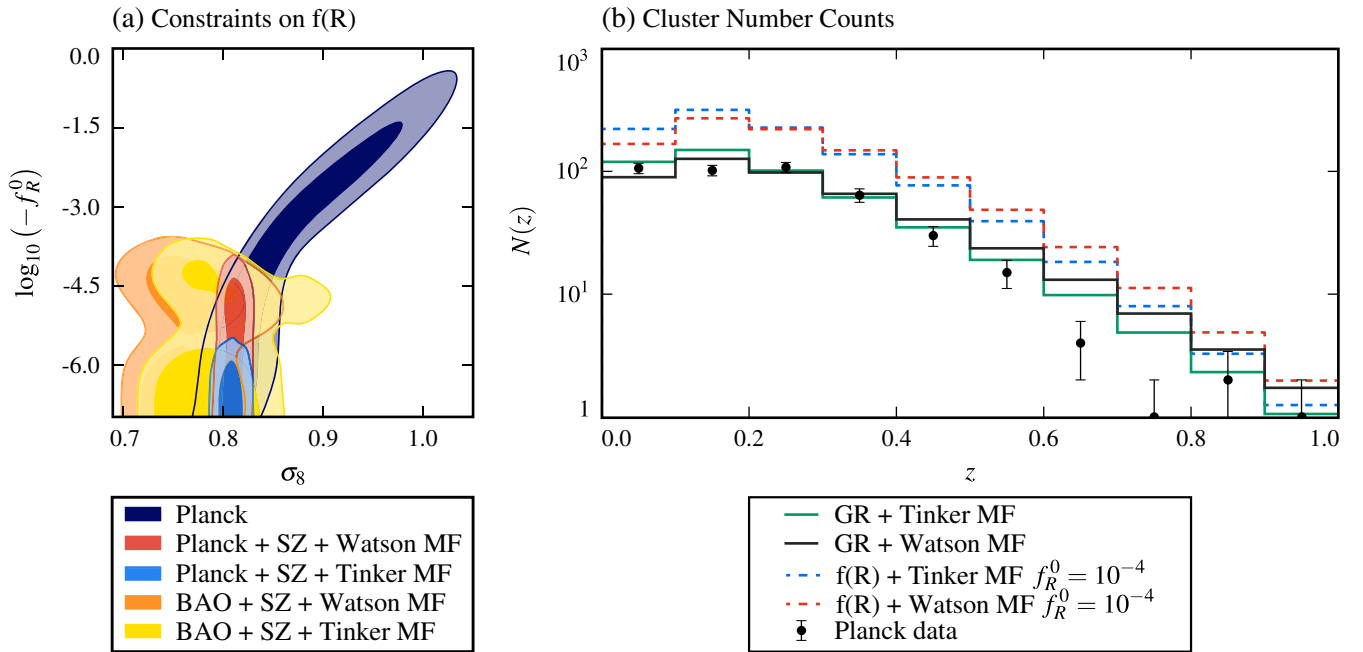

(a) Constraints on f(R)   (b) Cluster Number Counts

FIG. 1. (a) The joint marginalized posterior of $\log_{10}(-f_R^0)$ and σ_8. Different colors correspond to different data set combinations, as shown in the legend. Constraints that do not include Planck have been obtained by using weak priors on n_s and Ω_b. The darker and lighter shades correspond to the 68% C.L. and the 95% C.L. regions, respectively. (b) Comparison between the Planck measurements and the model predictions for the cluster number counts, as a function of redshift. Different colors correspond to different models and different mass functions, as shown in the legend. The black data points are samples from the PSZ2 catalog. The continuous lines represent the best fit prediction of the Planck and Planck cluster GR posterior. The dashed lines correspond to the same values of the parameters, but with $\log_{10}(-f_R^0) = -4$.
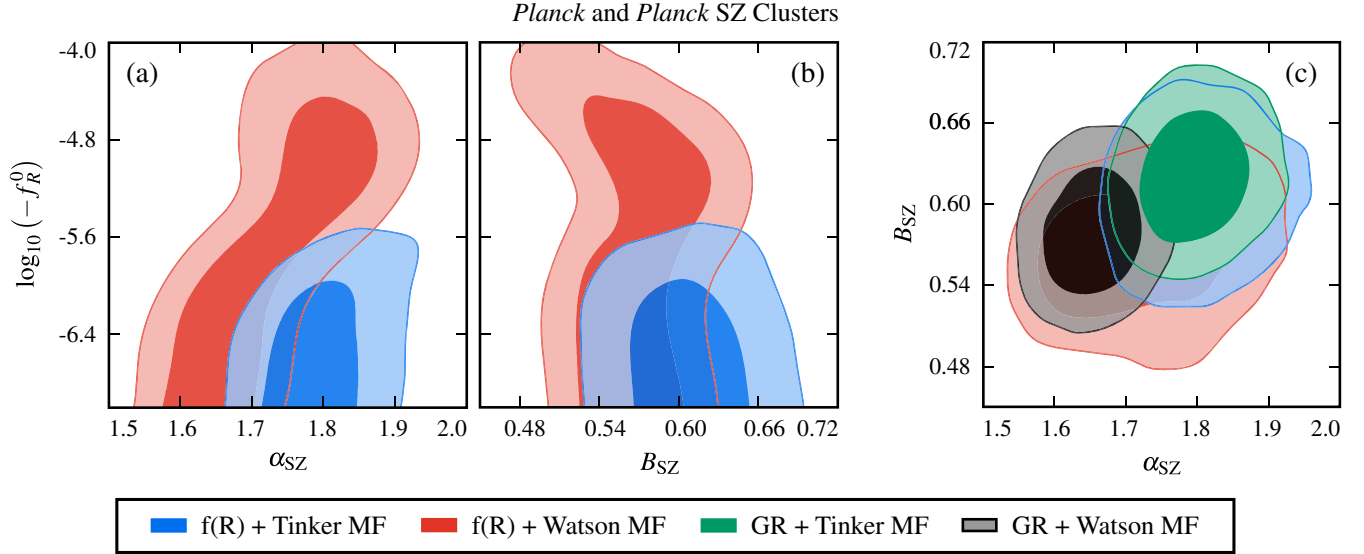
FIG. 2.   The joint marginalized posterior of $\log_{10}(-f_R^0)$, α_{SZ} and B_{SZ} for the Planck and Planck SZ clusters data sets. Different colors correspond to different models, as shown in the legend. The darker and lighter shades correspond to the 68% C.L. and the 95% C.L. regions, respectively.

strong influence of the choice of the MF on the final constraints.

Coming to the interplay between cosmology and astrophysical parameters of the cluster scaling relations, in Fig. 2 we show the contour plots for f_R^0, and for the SZ parameters α_{SZ} and B_{SZ}. The first two panels show that the degeneracy between f_R^0 and the other two parameters is clear in the Watson case, but absent in the Tinker one. The wider range of α_{SZ} probed by the Watson MF when compared to the Tinker MF in $f(R)$ models is evident, and it explains the weaker constraints obtained in the former case. We comment below about a possibility to reduce this dependence related to the cluster mass bias.

III. STABILITY OF THE RESULTS

To test the dependence of our results on other effects, we first add the contribution of baryons, and implement the baryonic correction to the MF in [47]. In particular, we consider the correction to the MF obtained by inclusion of feedback from active galactic nuclei (AGN) in hydrodynamic simulations. We obtain $\log_{10}(-f_R^0) < -5.84$ when considering the Tinker MF and the SZ + Planck data set. We thus conclude that the presence of baryons does not have a substantial influence on our results unlike the larger effects found in Ref. [5], where, however, cluster data probed smaller masses, which are more affected by feedback effects than those probed by SZ clusters.

We then investigate the dependence from the signal-to-noise ratio of Planck data, by using the most conservative choice $q > 8.5$, which reduces the sample to 40% of the original one. In this case we obtain $\log_{10}(-f_R^0) < -5.54$ using the Tinker MF. Again, we can conclude that our constraints are stable, as a change in q affects them much less than a change in the MF would.

IV. DISCUSSION

We compare our results with a recent work [20], where galaxy clusters have been used in order to get constraints on $f(R)$ gravity theory. In that case the authors got $\log_{10}(-f_R^0) < -4.79$ by considering the Tinker MF. In this sense, with the same choice of the MF and leveraging on the higher constraining power of Planck SZ cluster catalog, our work improves the constraint by 1 order of magnitude and gives $\log_{10}(-f_R^0) < -5.81$, the key ingredients of this improvement being the extended mass and redshift coverage of Planck clusters. We stress that this result should be compared with the one in [20], since both come from the same choice of the MF. However, according to us, the main result of this work is not only the exposition of a tighter constraint. Indeed, we also show that the implementation of a $f(R)$ correction to the MF strongly depends on the calibration of the MF in GR. In this context we show that, by keeping the $f(R)$ correction constant and changing the MF for GR, e.g. by switching from Tinker to Watson, we obtain a change of more than 1 order of magnitude in the f_R^0 constraint. In the case of Tinker we get $\log_{10}(-f_R^0) < -5.81$, while for the Watson $\log_{10}(-f_R^0) < -4.40$. We discussed in detail how this strong dependence on the MF arises from the degeneracy between f_R^0 and the SZ parameters, α_{SZ} and B_{SZ}. Therefore, in order to reduce this dependence it would be effective to further constrain the cluster mass bias; by reducing the distribution of this parameter, and thus of B_{SZ}, one would minimize the region of the parameter space in which the degeneracy occurs.

Thus, we expect that a better determination of the variables describing SZ clusters would directly translate into a more robust estimation of modified gravity parameters with respect to the choice of the GR MF.

In our analysis we also considered stability of the final results. We implemented the corrections on the MF induced by baryons and, more specifically, the effect of stars formation and AGN feedback in hydrodynamic simulations [47]. In particular, we speculated that the baryonic processes would not depend on the model of gravity, i.e. on the value of f_R^0. In principle, since these effects strongly influence the shape of the MF and, consequently, of the cluster number counts, we would expect some change in the final constraints on the scalaron amplitude. However, taking into account these effects did not appreciably influence the result.

We also investigated the effects of the signal-to-noise ratio q for the identification of the clusters in the Planck catalog. Setting this threshold to the most conservative one, $q > 8.5$, we obtain $\log_{10}(-f_R^0) < -5.54$, i.e. a difference of about 5% from the original result for $\log_{10}(-f_R^0)$. Also for this setup, we can then state the stability of our results.

Concluding, we quantitatively investigated the important role that SZ clusters have in constraining theories of modified gravity once cluster physics is properly understood and modeled, by implementing a state-of-the-art conservative analysis, and using the best available data set together with recent results in terms of cluster MF. While studies in GR are already at an advanced stage, modified gravity theories can benefit from additional insight on cluster physics that can be directly translated in tighter constraints on gravitational physics. Here, we obtained the tightest constraints to date on the scalaron amplitude, and, especially, we emphasized and discussed a strong dependence from the choice of the GR mass function. This is relevant to present and future cosmological surveys, like Euclid and CMB-S4, that are expected to deliver unprecedented quality cluster measurements, as it shows that a deep understanding of the physics of clusters is essential to fully exploit the constraining power of these observations [48,49].

ACKNOWLEDGMENTS

[1] L. Pogosian and A. Silvestri, Pattern of growth in viable f(R) cosmologies, Phys. Rev. D **77,** 023503 (2008).

[2] P. A. R. Ade, N. Aghanim, M. Arnaud, M. Ashdown, J. Aumont, C. Baccigalupi, A. J. Banday, R. B. Barreiro, J. G. Bartlett et al. (Planck Collaboration), Planck 2015 results. XXIV. Cosmology from Sunyaev-Zeldovich cluster counts, Astron. Astrophys. **594,** A24 (2015)..

[3] S. W. Allen, A. E. Evrard, and A. B. Mantz, Cosmological parameters from observations of galaxy clusters, Annu. Rev. Astron. Astrophys. **49,** 409 (2011).

[4] A. V. Kravtsov and S. Borgani, Formation of galaxy clusters, Annu. Rev. Astron. Astrophys. **50,** 353 (2012).

[5] M. Costanzi, F. Villaescusa-Navarro, M. Viel, J.-Q. Xia, S. Borgani, E. Castorina, and E. Sefusatti, Cosmology with massive neutrinos III: The halo mass function and an application to galaxy clusters, J. Cosmol. Astropart. Phys. 12 (2013) 012.

[6] A. B. Mantz, A. von der Linden, S. W. Allen, D. E. Applegate, P. L. Kelly, R. G. Morris, D. A. Rapetti, R. W. Schmidt, S. Adhikari, M. T. Allen, P. R. Burchat, D. L. Burke, M. Cataneo, D. Donovan, H. Ebeling, S. Shandera, and A. Wright, Weighing the giants—IV. Cosmology and neutrino mass, Mon. Not. R. Astron. Soc. **446,** 2205 (2015).

[7] F. Schmidt, M. Lima, H. Oyaizu, and W. Hu, Nonlinear evolution of f(R) cosmologies. III. Halo statistics, Phys. Rev. D **79,** 083518 (2009).

[8] I. Achitouv, M. Baldi, E. Puchwein, and J. Weller, Imprint of f (R) gravity on nonlinear structure formation, Phys. Rev. D **93,** 103522 (2016).

[9] M. Kopp, S. A. Appleby, I. Achitouv, and J. Weller, Spherical collapse and halo mass function in f(R) theories, Phys. Rev. D **88,** 084015 (2013).

[10] W. Hu and I. Sawicki, Models of f(R) cosmic acceleration that evade solar system tests, Phys. Rev. D **76,** 064004 (2007).

[11] A. A. Starobinsky, Disappearing cosmological constant in f(R) gravity, JETP Lett. **86,** 157 (2007).

[12] B. Hu, M. Raveri, A. Silvestri, and N. Frusciante, Exploring massive neutrinos in dark cosmologies with EFTCAMB/ EFTCosmoMC, Phys. Rev. D **91,** 063524 (2015).

[13] P. A. R. Ade, N. Aghanim, M. Arnaud, M. Ashdown, J. Aumont, C. Baccigalupi, A. J. Banday, R. B. Barreiro, N. Bartolo et al. (Planck Collaboration), Planck 2015 results.

XIV. Dark energy and modified gravity, Astron. Astrophys. **594,** A14 (2016)..

[14] B. Hu, M. Raveri, M. Rizzato, and A. Silvestri, Testing Hu-Sawicki f(R) gravity with the effective field theory approach, Mon. Not. R. Astron. Soc. **459,** 3880 (2016).

[15] L. Lombriser, A. Slosar, U. Seljak, and W. Hu, Constraints on f(R) gravity from probing the large-scale structure, Phys. Rev. D **85,** 124038 (2012).

[16] J. Dossett, B. Hu, and D. Parkinson, Constraining models of f(R) gravity with Planck and WiggleZ power spectrum data, J. Cosmol. Astropart. Phys. 03 (2014) 046.

[17] J. Bel, P. Brax, C. Marinoni, and P. Valageas, Cosmological tests of modified gravity: Constraints on F(R) theories from the galaxy clustering ratio, Phys. Rev. D **91,** 103503 (2015).

[18] E. Di Valentino, A. Melchiorri, and J. Silk, Cosmological hints of modified gravity?, Phys. Rev. D **93,** 023513 (2016).

[19] F. Schmidt, A. Vikhlinin, and W. Hu, Cluster constraints on f(R) gravity, Phys. Rev. D **80,** 083505 (2009).

[20] M. Cataneo, D. Rapetti, F. Schmidt, A. B. Mantz, S. W. Allen, D. E. Applegate, P. L. Kelly, A. von der Linden, and R. G. Morris, New constraints on f(R) gravity from clusters of galaxies, Phys. Rev. D **92,** 044009 (2015).

[21] X. Liu, B. Li, G.-B. Zhao, M.-C. Chiu, W. Fang, C. Pan, Q. Wang, W. Du, S. Yuan, L. Fu, and Z. Fan, Constraining $f(R)$ Gravity Theory Using CFHTLenS Weak Lensing Peak Statistics, Phys. Rev. Lett. **117,** 051101 (2016).

[22] L. Lombriser, B. Li, K. Koyama, and G.-B. Zhao, Modeling halo mass functions in chameleon f(R) gravity, Phys. Rev. D **87,** 123511 (2013).

[23] J.-h. He, B. Li, and Y. P. Jing, Revisiting the matter power spectra in f(R) gravity, Phys. Rev. D **88,** 103507 (2013).

[24] E. Puchwein, M. Baldi, and V. Springel, Modified-gravity-GADGET: A new code for cosmological hydrodynamical simulations of modified gravity models, Mon. Not. R. Astron. Soc. **436,** 348 (2013).

[25] M. Baldi, F. Villaescusa-Navarro, M. Viel, E. Puchwein, V. Springel, and L. Moscardini, Cosmic degeneracies—I. Joint N-body simulations of modified gravity and massive neutrinos, Mon. Not. R. Astron. Soc. **440,** 75 (2014).

[26] H. A. Winther, F. Schmidt, A. Barreira, C. Arnold, S. Bose, C. Llinares, M. Baldi, B. Falck, W. A. Hellwing, K. Koyama, B. Li, D. F. Mota, E. Puchwein, R. E. Smith, and G.-B. Zhao, Modified gravity N-body code comparison project, Mon. Not. R. Astron. Soc. **454,** 4208 (2015).

[27] J. R. Bond, S. Cole, G. Efstathiou, and Nick Kaiser, Excursion set mass functions for hierarchical Gaussian fluctuations, Astrophys. J. **379,** 440 (1991).

[28] William H. Press and Paul Schechter, Formation of galaxies and clusters of galaxies by self-similar gravitational condensation, Astrophys. J. **187,** 425 (1974).

[29] J. Tinker, A. V. Kravtsov, A. Klypin, K. Abazajian, M. Warren, G. Yepes, S. Gottlöber, and D. E. Holz, Toward a halo mass function for precision cosmology: The limits of universality, Astrophys. J. **688,** 709 (2008).

[30] W. A. Watson, I. T. Iliev, A. D'Aloisio, A. Knebe, P. R. Shapiro, and G. Yepes, The halo mass function through the cosmic ages, Mon. Not. R. Astron. Soc. **433,** 1230 (2013).

[31] M. Arnaud, G. W. Pratt, R. Piffaretti, H. Böhringer, J. H. Croston, and E. Pointecouteau, The universal galaxy cluster pressure profile from a representative sample of nearby

systems (REXCESS) and the Y_{SZ}-M_{500} relation, Astron. Astrophys. **517,** A92 (2010).

[32] A. von der Linden, M. T. Allen, D. E. Applegate, P. L. Kelly, S. W. Allen, H. Ebeling, P. R. Burchat, D. L. Burke, D. Donovan, R. G. Morris, R. Blandford, T. Erben, and A. Mantz, Weighing the giants—I. Weak-lensing masses for 51 massive galaxy clusters: Project overview, data analysis methods and cluster images, Mon. Not. R. Astron. Soc. **439,** 2 (2014).

[33] H. Hoekstra, R. Herbonnet, A. Muzzin, A. Babul, A. Mahdavi, M. Viola, and M. Cacciato, The Canadian cluster comparison project: Detailed study of systematics and updated weak lensing masses, Mon. Not. R. Astron. Soc. **449,** 685 (2015).

[34] C. A. Collins, H. Bohringer, L. Guzzo, P. Schuecker, D. Neumann, S. Schindler, R. Cruddace, S. Degrandi, G. Chincarini, A. C. Edge, H. T. MacGillivray, P. Shaver, G. Vettolani, and W. Voges, The REFLEX cluster survey, in *19th Texas Symposium on Relativistic Astrophysics and Cosmology*, edited by J. Paul, T. Montmerle, and E. Aubourg (1998).

[35] P. A. R. Ade, N. Aghanim, M. Arnaud, M. Ashdown, J. Aumont, C. Baccigalupi, A. J. Banday, R. B. Barreiro, J. G. Bartlett *et al.* (Planck Collaboration), Planck 2015 results. XIII. Cosmological parameters, Astron. Astrophys. **594,** A13 (2016)..

[36] N. Aghanim, M. Arnaud, M. Ashdown, J. Aumont, C. Baccigalupi, A. J. Banday, R. B. Barreiro, J. G. Bartlett, N. Bartolo *et al.* (Planck Collaboration), Planck 2015 results. XI. CMB power spectra, likelihoods, and robustness of parameters, Astron. Astrophys. **594,** A11 (2016)..

[37] M. Betoule *et al.*, Improved cosmological constraints from a joint analysis of the SDSS-II and SNLS supernova samples, Astron. Astrophys. **568,** A22 (2014).

[38] A. J. Ross, L. Samushia, C. Howlett, W. J. Percival, A. Burden, and M. Manera, The clustering of the SDSS DR7 main galaxy sample—I. A 4 percent distance measure at $z = 0.15$, Mon. Not. R. Astron. Soc. **449,** 835 (2015).

[39] L. Anderson *et al.*, The clustering of galaxies in the SDSS-III baryon oscillation spectroscopic survey: Baryon acoustic oscillations in the data releases 10 and 11 galaxy samples, Mon. Not. R. Astron. Soc. **441,** 24 (2014).

[40] F. Beutler, C. Blake, M. Colless, L. Staveley-Smith, and H. Jones, The 6dF galaxy survey: Baryon acoustic oscillations and the local Hubble constant, in *American Astronomical Society Meeting Abstracts No. 219*, 2012, Vol. 219, p. 402.01.

[41] B. Hu, M. Raveri, N. Frusciante, and A. Silvestri, Effective field theory of cosmic acceleration: An implementation in CAMB, Phys. Rev. D **89,** 103530 (2014).

[42] M. Raveri, B. Hu, N. Frusciante, and A. Silvestri, Effective field theory of cosmic acceleration: Constraining dark energy with CMB data, Phys. Rev. D **90,** 043513 (2014).

[43] A. Lewis, A. Challinor, and A. Lasenby, Efficient computation of cosmic microwave background anisotropies in closed Friedmann-Robertson-Walker models, Astrophys. J. **538,** 473 (2000).

[44] A. Lewis and S. Bridle, Cosmological parameters from CMB and other data: A Monte Carlo approach, Phys. Rev. D **66,** 103511 (2002).

[45] P. A. R. Ade, N. Aghanim, Y. Akrami, P. K. Aluri, M. Arnaud, M. Ashdown, J. Aumont, C. Baccigalupi, A. J. Banday *et al.* (Planck Collaboration), Planck 2015 results. XVI. Isotropy and statistics of the CMB, Astron. Astrophys. **594**, A16 (2016).

[46] G. Steigman, Neutrinos and BBN (and the CMB), arXiv:0807.3004.

[47] W. Cui, S. Borgani, and G. Murante, The effect of active galactic nuclei feedback on the halo mass function, Mon. Not. R. Astron. Soc. **441**, 1769 (2014).

[48] D. S. Y. Mak, E. Pierpaoli, F. Schmidt, and N. Macellari, Constraints on modified gravity from Sunyaev-Zeldovich cluster surveys, Phys. Rev. D **85**, 123513 (2012).

[49] B. Sartoris, A. Biviano, C. Fedeli, J. G. Bartlett, S. Borgani, M. Costanzi, C. Giocoli, L. Moscardini, J. Weller, B. Ascaso, S. Bardelli, S. Maurogordato, and P. T. P. Viana, Next generation cosmology: Constraints from the Euclid galaxy cluster survey, Mon. Not. R. Astron. Soc. **459**, 1764 (2016).