Transport coefficients and low energy excitations of a strongly interacting holographic fluid

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On the other hand, one does not yet have a mathematically complete example of a quantum gauge theory in four-dimensional space-time, nor even a precise definition of quantum gauge theory in four dimensions. Will this change in the 21st century? We hope so!

A. Jaffe and E. Witten on Yang–Mills existence and mass gap [1]

1 Introduction

1.1 Preface : What is QFT?

Quantum field theory (QFT) has been an active area of research for several decades by now. By all means, it is one of the greatest accomplishments of theoretical physics from the last century, with applications ranging from high energy physics, such as the standard model, to low energy physics, such as our understanding of metals and semiconductors. We are able to predict the magnetic dipole moment of the electron which agrees with the experimental result to eight decimal places: an unprecedented accuracy in the entire history of science! We are able to predict the existence of anti-particles, $W^{\pm}$, Z bosons, gluons and quarks in the standard model of particle physics. We have a theory of electrons in solids, which enables us to build computers, smartphones, NMR machines, superconductors and many many more applications.

But perhaps we only see the tip of the iceberg. Our achievements from conventional QFT rely on single-particle classical physics intuition: we start from a microscopic Lagrangian description then treat the interaction and the quantum fluctuation as a perturbation. However, we seem to fail miserably when trying to compute physical quantities from the usual QFT formalism when the interactions are strong. One of the most glaring examples is the confinement phenomenon in quantum chromodynamics (QCD), which is responsible for most of the mass of the known matter in the universe. It is known that the interaction strength in QCD (describing physics of quarks and gluons in nuclear matter) is infinitely large at low energy and this dynamically generates a mass gap, resulting in bound states of quarks and gluons, such as protons and neutrons. We know that this is true thanks to the results from lattice simulations, but more evidently, because most of our mass came from protons and neutrons. Still[1], there is no analytic proof why they should form such bound states!

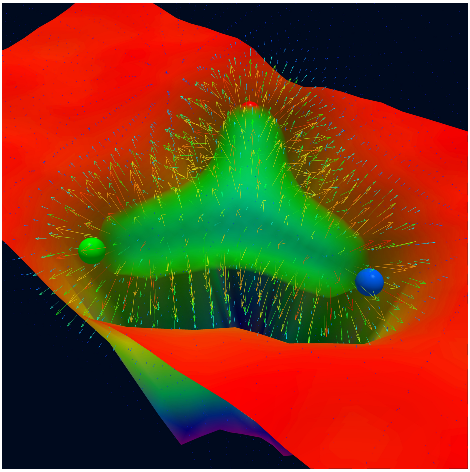

Figure 1.1. A visualisation of a proton in the "vacuum" filled with gluon field. The gluonic field in the proton interior is pushed away by quarks resulting in the flux tube structure confining the quarks together (figure from [2])

Strong interaction are also believed to play a crucial role in materials with high critical temperature superconductivity. There are many intriguing aspects to these materials but I will focus on the strange metallic phase at temperature

[1]See e.g. a Millennium Prize problem [1] by Clay Mathematics Institute involving the existence of this mass gap.

above the, unusually high, superconductor critical temperature[2]. As a name suggests, a strange metal has certain properties that are different from a typical metal. For example, the resistivity of a strange metal grows linearly in contrast to the quadratic growth found in a normal metal. This may not seems like a big deal, but the T^2 dependence is extremely robust. It was shown, from renormlisation group analysis, that any small perturbation around normal metal cannot change this property [5, 6] (see also a review [7]). Thus, one may argue that the “perturbation” has to be very strong in order to obtain the linear resistivity and, again, we enter the regime of strongly interacting quantum field theory. To make matters even worse, the brute-force numerical simulation is categorised as NP-hard in computational complexity [8], thus making the classical simulation of strongly coupled fermions in thermodynamic limits unachievable[3].

However, it an overstatement to say that we cannot compute anything when the interaction is strong. Numbers of non-perturbative techniques and many toy models where computations are doable even at strong interaction have been developed over the past few years. These non-perturbative results might make one doubt whether the Lagrangian-based formulation of QFT is the best way to understand quantum systems. For example, there are a number of non-perturbative methods which do not rely on microscopic Lagrangian such as conformal bootstrap [13, 14], integrability [15], modern analysis of scattering amplitude [16–18]. Instead they rely heavily on the symmetries of the systems. Many non-perturbative results are also obtained by utilising the property called *duality*. Duality is the phenomenon where by two or more Lagrangians which describe the same physics and, sometimes, enables us to map a difficult problem of strongly interacting QFT to a very simple one. Since this is the most important aspect of this thesis, let me be a little more precise. The strongest form of

[2]See [3] and chapter 2-3 of [4], for a brief history and introduction to high temperature superconductor.

[3]This problem is known as “the sign problem" which, sadly, is largely ignored by most introductions to quantum field theory. However, it is one of the biggest enemies when one tries to directly simulate strongly correlated fermion systems, both in condensed matter and lattice QCD (see e.g. [9, 10]). See also [11] for a discussion of the sign problem in a path integral formalism and the references therein. Solving this problem in polynomial time is equivalent to solving, yet another, Millennium Prize problem by Clay Mathematics Institute [12].

the duality is the following:

> If a theory A is dual to a theory B, then for every physical quantity in A there are corresponding physical quantities in B. The theories A and B may have different microscopic constituents or gauge groups or even live in a different dimension.

If A and B describes the same physics, one may ask a philosophical question: which Lagrangian describe the "true nature" of quantum theory. Are they both physical? Is this just a mathematical trick? Or should we think of theory B as an emergent phenomenon in a certain limit of theory A? At this point, we simply do not know.

So what is QFT? The facts that a conventional Lagrangian formalism seems to hit a brick wall, most non-perturbative results rely so little on the microscopic Lagrangian and, sometimes, more than one Lagrangian describe the same physics, all seem to indicate that we should rethink what we know about it. How should QFT be reformulated? I am afraid I do not know. There seems to be a lot of things we need to understand before answering such a question. *The goal of this thesis* is to make a few small steps toward, hopefully, the answer by exploring a peculiar duality between strongly interacting quantum field theory and the theory of general relativity, known as the gauge/gravity duality.

1.2 Gauge/gravity duality and effective theory

The origin of this duality dates back to the study of black hole entropy. One very peculiar property of black holes is that their entropy is proportional to the area of a surface called black hole horizon, not the enclosed volume. This and many other pieces of evidence prompted 't Hooft and Susskind to conjecture that a black hole can be described by a quantum field theory in one lower dimension [19, 20]. In this picture, the statistical physics interpretation of black hole entropy is nothing but the number of microstates of the corresponding quantum field theory. This statement can be confirmed in the string theory setup by counting the microstates of extended objects called D-brane[21]. Shortly after, it was argued by Maldacena [22] that the large class of quantum field theories constructed in similar manner not only capture the entropy of the black hole but also the dynamics of the spacetime itself! The strongest statement of this duality is:

For a quantum field theory with a holographic dual, there is a theory of (quantum) gravity in higher dimensions in which every physical quantity in the field theory can be mapped into a physical quantity of the dual gravity.

The fact that a certain spacetime in a theory of (super)gravity seems to emerge from quantum field theory is, in itself, a very fascinating phenomenon. In addition, there is a very interesting usage of this duality. It turns out that the dual gravity theory becomes classical when the coupling of the QFT becomes infinitely large. Thus, with this duality, one might hope to use it to compute quantities in the, conventionally, unaccessible regime of quantum field theory.

Despite such a promising aspect, one has to be careful about the applicability of this duality. With this in mind, let me illustrate this caveat with some technical details. In the prototypical example of gauge/gravity duality, the aforementioned quantum field theoy is a well-known maximally supersymmetric quantum field theory in 3+1 spacetime dimensions, called $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with $SU(N)$ gauge group. It consist of a non-Abelian $SU(N)$ gauge field, four species of fermions and six scalar fields. All of these transform under the adjoint representation of $SU(N)$. Roughly speaking, this theory is a matrix theory where all the fields are $N \times N$ matrices. The coupling between theses fields is extremely constrained by the supersymmetry, leaving only one free parameter in the Lagrangian: the Yang-Mills coupling constant g_{YM}.[4] At first glance, studying this theory or its deformations is very unlikely

[4]In terms of these parameters, the dual gravity theory only becomes tractable when $g^2_{\text{YM}} N \gg 1$ and $N \gg 1$. Otherwise, one needs to include quantum gravity effects in string theory. This issue will be discussed again in chapter 2.

to give us any new insight on the microscopic parameters of a realistic theory (such as hopping parameters, spin orbit coupling etc).

However, it is certainly possible for theories with different microscopic structures to be macroscopically equivalent to one another. This concept, known as *universality*, where many theories with different microscopic constituents become macroscopically indistinguishable, is well-known in the study of critical phenomena. This is the situation we are hoping for: *the infrared physics of the system we are interested in is the same as that of a theory with holographic dual.* If this indeed happens, we should be asking for the macroscopic information rather than the microscopic.

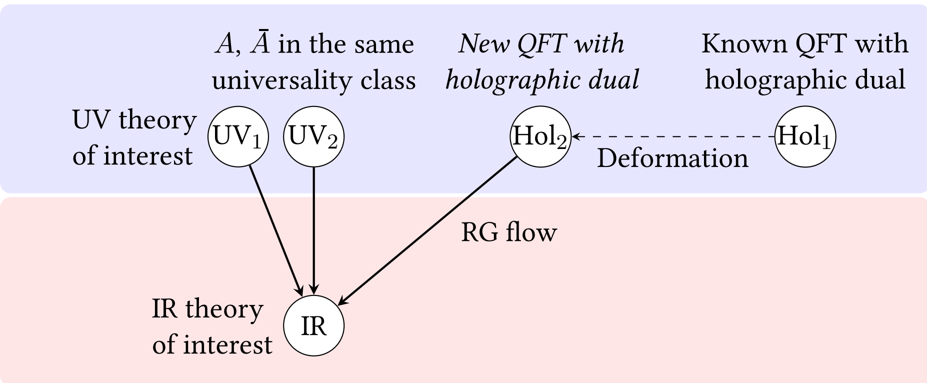

Figure 1.2. A schematic renormalisation group flow from small (UV) to large (IR) length scale where the QFT with holographic dual is in the same universality class as the system we are interested.

Focusing on the macroscopic quantities, such as thermodynamic variables, seems to be a very natural starting point. It is widely known that in strongly interacting quantum systems, thermodynamic quantities exhibit (non-mean field) scaling relations, which are very difficult to achieve using conventional methods. In the contrary, thermodynamic quantities of field theories with holographic dual are almost guaranteed to have such scaling relations. Interestingly, some of these relations are the same as those observed in strongly correlated systems.

Beyond equilibrium analysis, the gauge/gravity duality can be used to study transport phenomena and real-time dynamics of strongly interacting theories at finite temperature. Instead of having a quasi-particle description, a large class of theories with gravity dual exhibit fluid-like excitations. Intuitively, this is due to the fact that a quantum particle lose its individuality because the interaction with its neighbours is infinitely strong. This situation is well-summarised in Fig. 1.3. Such hydrodynamics features have also been observed in the study of the Nernst effect in quantum critical regions [23], graphene near the charge neutrality point [24, 25] and $PdCoO_2$ [26].

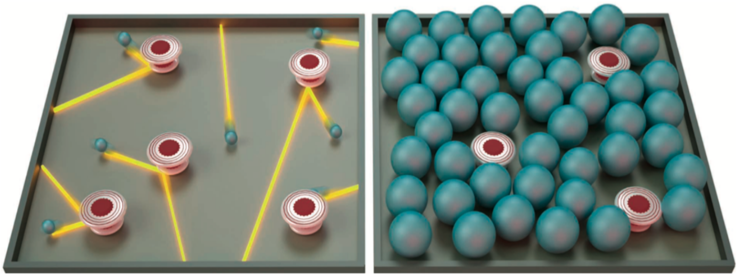

Figure 1.3. (LEFT) The ballistic transport where each quasi-particle barely interacted with each other. (RIGHT) The hydrodynamic transport where quantum particles collide with each other, losing their individuality and becoming a fluid-like entity (figure from [27]).

The fact that hydrodynamics is able to capture the dynamics of such systems may not be very surprising. After all, hydrodynamics is an almost universal effective theory that describes systems with excitation wavelength, λ_{excite} much larger than the mean free path, ℓ_{mfp}, of the constitute particles in the system (which is known to be vanishing as the interaction increases). What sets the strongly interacting quantum fluid apart from a classical fluid, however, is its viscosities. It was found that, in classes of strongly interacting quantum systems, the viscosity is governed by the ratio of Planck's constant and Boltzmann's constant $\hbar/k_B$ where the appearance of $\hbar$ signifies its quantum nature [28] (see also [4, 29] for more stories from the condensed matter viewpoints). In this aspect, gauge/gravity duality not only confirms this observation but also gives the value of the viscosity explicitly. By examining a large class of models with gravity duals, Kovtun, Son and Starinets (KSS) [30] found an interesting pattern and postulated that the ratio of the shear viscosity η and the entropy density s is bounded from below i.e.

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}, \tag{1.1}$$

where the inequality is saturated when the coupling is infinitely large. Remarkably, this ratio can also be extracted from experimental data in known strongly interacting quantum systems, see Fig. 1.4. Interestingly, the viscosity/entropy density ratio gets closer the value $\hbar/4\pi k_B$ as the interaction becomes stronger near the critical point and so far the bound has yet to be violated experimentally. This also strongly suggests that many strongly interacting quantum field theories may belong to the same universality class with the universal diffusion constant $D := \eta/sT = \hbar/4\pi k_B T$ at infinite coupling.

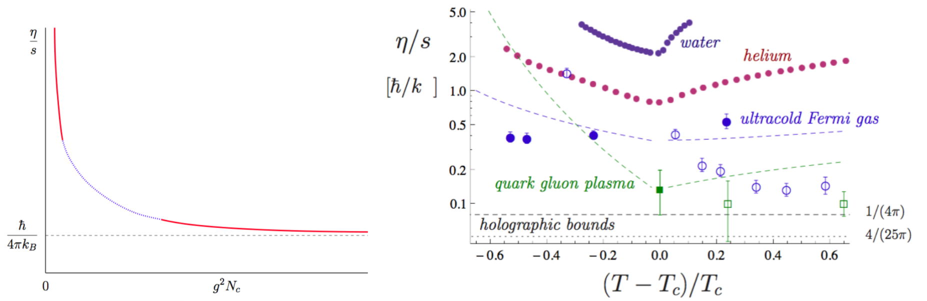

Figure 1.4. (LEFT) The interpolation between value of η/s in $\mathcal{N} = 4$ supersymmetric Yang-Mills between known results at weak coupling $g^2_{\mathrm{YM}}N \ll 1$ and strong coupling $g^2_{\mathrm{YM}}N \gg 1$. (RIGHT) The experimental data in various materials near the critical point compared to the KSS bound (1.1) (figures from [30, 31]).

I would like to emphasise that, although thinking about quantum theory from hydrodynamic point of view may not give us a straightforward knowledge of microscopic information, it nevertheless allows us to make a link between dissipative nature and quantum nature of many body quantum systems. From this perspective, gauge/gravity duality is a playground where computations involving these aspects can be performed explicitly. Moreover, the lower bound on shear viscosity/entropy density can, in principle, be experimentally tested.A result such as the KSS bound inspired searches for similar universal bounds e.g.[32–42] and hopefully these will help us better understand the nature of quantum field theory at strong coupling.

1.3 This thesis : A hunt for universality beyond standard hydrodynamics

One reason why hydrodynamics is such a universal effective theory is its underlying principle: global symmetry. From a more modern point of view, hydrodynamics is the low energy dynamics of the Noether currents whose time evolutions governed by conservation laws. In the long wavelength limit, the microscopic details are washed out and the effective theory is fully described by the equations of state and a few parameters in the gradient expansion (such as shear and bulk viscosity). For example the relativistic Navier-Stokes equation can be written as

$$\partial_\mu T^{\mu\nu} = 0, \qquad T^{\mu\nu} = (\mathcal{E}+\mathcal{P})u^\mu u^\nu + \mathcal{P}\eta^{\mu\nu} + \sum_{\text{All } \mathcal{O}(\partial^1) \text{ terms}} \alpha_i \, \mathbb{T}^{\mu\nu}_{(i)}, \quad (1.2)$$

where $T^{\mu\nu}$ is the stress-energy tensor, $\mathbb{T}_{(i)}$ are all possible symmetric traceless rank−2 tensors with one derivative. The conservation of $T^{\mu\nu}$ is linked to the spacetime translational symmetry of the system. The system of equations (1.2) can be extended to include dynamics of additional conserved currents if the system also possesses more global symmetries. The coefficients α_i are called *transport coefficients*. The behaviour of these transport coefficients in a strongly interacting theories is the main focus of this thesis, with an ultimate goal to explore universal relations analogous to (1.1).

Chapter 3,4,5 of this thesis are based on my original works with Piyabut Burikham [43] and Sašo Grozdanov [44, 45] where we study the interplay of additional (or broken) global symmetries, lower energy excitations and transport coefficients in strongly interacting quantum field theories with holographic dual. Chapter 2 contains a brief introduction and background material to relativistic hydrodynamics and implementations of gauge/gravity duality in this context. In the rest of this thesis, all the results are expressed in units where $\hbar = c = 1$. The outline of each chapter is the following.

In chapter 3, we construct a consistent hydrodynamic model where translational symmetry is broken. This is achieved by coupling the strongly interacting matter to the external spatial dependent scalar fields. We carefully define what is the shear viscosity in this system and study the validity of the KSS bound (1.1) in a hope to apply this bound to condensed matter systems where translational symmetry is broken by a lattice or disorder. We find that the bound is violated when we include the first order correction from the scalar fields. We are also able to identify how this violation occurs from the hydrodynamic point of view.

In chapter 4, the system now contains two additional global $U(1)$ symmetries and is subjected to a small external non-dynamical magnetic field. One Noether current is conserved but the other is broken by a quantum anomaly. This gives rise to a new transport channel called*anomalous transport*. Although quantum anomaly is famous for their non-renormalisability (or 1-loop exactness in the diagrammatic approach), it is not entirely clear why these new conductivities should inherit the same property. We find that the information regarding these conductivities is encoded on the black hole horizon in the dual gravity theory. With these horizon formulae, we show that, for all theories with a holographic dual at finite temperature, the form of these anomalous conductivities is fixed by quantum anomaly coefficients, unaffected by the value of the coupling constant and details of the renormalisation group flow.

In chapter 5, the matter no longer breaks any existing global symmetry but is now coupled to a *dynamical* external gauge field, a setup one usually finds in plasma physics. A standard treatment of this system is called magnetohydrodynamics (MHD), which is a very successful framework in the plasma physics community. The problem is that conventional MHD is constructed with an underlying assumption that the equation of state of the matter is independent of the magnetic field. This assumptions makes perfect sense in a normal earthly circumstances where the temperature is very high and the magnetic field is very small. However, there are examples in nature where this is not true. We use a novel concept of generalised global symmetry, put forth by Gaiotto, Kapustin, Seiberg and Willet [46], to identify the true global symmetry of this system. Together with the new formalism of hydrodynamics with generalised global symmetry, recently constructed by Grozdanov, Hofman and Iqbal [47], we implement this new concept in gauge/gravity duality. As a result, for the first time, we are able to investigate low energy dynamics of this system at arbitrary strength of magnetic field.

Lastly, in chapter 6, I summarise the results in this thesis, its contribution to our understanding of quantum field theory at strong coupling and possible future research directions.