Adapted deformations and the Ekedahl-Oort stratifications of Shimura varieties

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Stellingen

behorende bij het proefschrift

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(1) Let k be a perfect field of characteristic $p > 2$. The functor of sending a p-divisible group to its associated Breuil-Kisin window induces an equivalence between the category of p-divisible groups over $W(k)$ and the category of Breuil-Kisin windows over $W(k)[[u]]$. [Theorem 1.4.2]

(2) Let k be a field and G a linear algebraic group over k. Let $\mathcal{K}$ be the positive loop group of G, which is the functor associating to a k-algebra A the group $G(A[[u]])$. Given a point $x \in G(k((u)))$, the fpqc sheafification $\mathcal{D}$ of the functor associating to A the double coset $\mathcal{K}(A)x\mathcal{K}(A)$ inside of $G\big(A((u))\big)$ is represented by a k-scheme. [Proposition 3.2.3]

(3) Let $\mathcal{K}_1$ be the kernel of the natural reduction map from $\mathcal{K}$ to G. Let $\mathcal{K}_1$ act on $\mathcal{D}$ by right multiplication, then the fpqc quotient sheaf $\mathcal{D}_1 := \mathcal{D}/\mathcal{K}_1$ is represented by a k-scheme of finite type. [Section 3.4]

As introduced in Section 6.4, let $\mathcal{S}$ be the canonical integral model over $W(\kappa)$ of a Shimura variety $\mathrm{Sh}_{\mathrm{K}}(\mathbf{G}, \mathbf{X})$, and denote by S the special fibre of $\mathcal{S}$. Then S is a smooth quasi-projective scheme over the finite field κ. Let $\mathbb{I}_+$ be the scheme defined in Section 7.2, and I_+ its special fibre, which is an fppf torsor over S.

(4) Let $\mathcal{D}_1$ be as in (2) by setting $k = \kappa$ and $x = \mu(u) \in G(\kappa((u)))$, where $\mu : \mathbb{G}_{m,\kappa} \to G_\kappa$ is the cocharacter defined in Section 1.3. Let l be an algebraically closed field extension of κ. For a point $x \in \mathrm{I}_+(l)$ and a lift $\tilde{x} \in \mathbb{I}_+(W(l))$ of x, the Breuil-Kisin window given by $\tilde{x}$ induces an element in $\mathcal{D}_1(l)$, which turns out to be independent of the lift $\tilde{x}$. This induces a map $\theta(l) : \mathrm{I}_+(l) \to \mathcal{D}_1(l)$. [Section 7.4.2]

(5) There is a morphism of κ-schemes $\theta : \mathrm{I}_+ \to \mathcal{D}_1$ that, on l-valued points, is given by $\theta(l)$. [Section 7.4.1]

(6) Write $\mathcal{K}^\diamond := \mathcal{K}_1 \rtimes_{\mathbb{F}_p} \mathcal{K}$, where $\mathcal{K}$ acts on $\mathcal{K}_1$ on the right by conjugation. Let $\mathcal{K}^\diamond$ act on $\mathcal{D}_1$ on the right as in (3.4.2), Section 3.4. Then the composition $S \xrightarrow{\theta} \mathcal{D}_1 \to \mathcal{D}_1/\mathcal{K}^\diamond$ of fpqc sheaves induces a morphism of fpqc sheaves $\eta : S \to \mathcal{D}_1/\mathcal{K}^\diamond$. [Theorem 7.5.2]

(7) The fibres of η on geometric points give back the Ekedahl-Oort strata of S. [Proposition 8.4.1]

(8) The quotient stack $[\mathcal{D}_1/\mathcal{K}^\diamond]$ and the stack $G\text{-}\mathsf{Zip}^\chi$ of G-zips of type χ (introduced in Section 8.1) have the same topological spaces, but they are not isomorphic as stacks.

(9) What one finds difficult is often easy for others: this is because people learn things in different orders.