Adapted deformations and the Ekedahl-Oort stratifications of Shimura varieties

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Summary

Let $(\mathbf{G}, \mathbf{X}) \hookrightarrow (\mathbf{GSp}, \mathbf{S}_{\pm})$ be a Shimura datum of Hodge type. Let $p > 2$ be a prime number and assume $\mathbf{G}_{\mathbb{Q}_p}$ has a reductive model $\mathcal{G}$ over $\mathbb{Z}_p$. Let $\mathsf{K} \subset \mathbf{G}(\mathbb{A}_f)$ be an open compact subgroup, hyperspecial at p, and $\mathrm{Sh}_{\mathsf{K}}(\mathbf{G}, \mathbf{X})$ the associated Shimura variety over the reflex field E of $(\mathbf{G}, \mathbf{X})$. Fix a place v of E above p. Denote by $\mathcal{O}_{E,v}$ the ring of integers of the completion of E at v, and by κ the residue field of $\mathcal{O}_{E,v}$. Denote by $\mathcal{S}_{\mathsf{K}}(\mathbf{G}, \mathbf{X})$ the canonical integral model of $\mathrm{Sh}_{\mathsf{K}}(\mathbf{G}, \mathbf{X})$ over $\mathcal{O}_{E,v}$ constructed by Kisin, which is a quasi-projective and smooth scheme over $\mathcal{O}_{E,v}$. Denote by S the special fibre of $\mathcal{S}_{\mathsf{K}}(\mathbf{G}, \mathbf{X})$, over κ.

In her paper *Truncations of level 1 of elements in the loop group of a reductive group* (Annals of Math. 2014), E. Viehmann introduced and studied some new invariants, called "truncations of level 1", for elements in the loop group of G, where G is the special fibre of $\mathcal{G}$. It follows from her results that such invariants can be used to parametrize Ekedahl-Oort strata of S, and to give a criteria for the inclusion relations of these strata. But such a parametrization is quite indirect. In this thesis, using the classification result of p-divisible groups in terms of filtered Breuil-Kisin modules (Breuil-Kisin windows, in our term), we give a more conceptual explanation of Viehmann's results.

Via constructing Breuil-Kisin windows, we obtain a morphism of schemes $\mathrm{I}_+ \to \mathcal{D}_1$, where I_+ is an fppf torsor over S and $\mathcal{D}_1$ is a quotient of a double coset scheme inside the loop group of G. From this morphism we finally obtain a morphism of fpqc sheaves $\eta : S \to \mathcal{D}_1/\mathcal{K}^{\diamond}$, where $\mathcal{D}_1/\mathcal{K}^{\diamond}$ is a quotient sheaf of $\mathcal{D}_1$, whose geometric points correspond to Viehmann's new invariants (truncations of level 1). We show that the fibres of η on geometric points are exactly the Ekedahl-Oort strata of S as defined by C. Zhang and D. Wortmann in their theses.