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Intense automorphisms of finite groups

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Stellingen

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Intense automorphisms of finite groups

van Mima Stanojkovski

Let G be a group. Then $\text{Int}(G)$ denotes the subgroup of $\text{Aut}(G)$ consisting of the automorphisms of G that send each subgroup of G to a conjugate in G .

1. Let $G = \mathbb{F}_{2017^2} \rtimes \mu_9(\mathbb{F}_{2017^2}^*)$. Then there are no intense automorphisms of G of order 5.

Let p be a prime number. A p -group G is said to be *extraspecial* if it is finite with $[G, G]$ contained in $Z(G)$ and $Z(G)$ of order p .

2. Let p be a prime number. If G is an extraspecial p -group, then $\text{Int}(G) = \text{Inn}(G)$ or $\text{Int}(G)$ is isomorphic to a semidirect product $\text{Inn}(G) \rtimes \mathbb{F}_p^*$.
3. Let p be an odd prime number and let G be a pro- p -group. Assume that G is the absolute Galois group of a field. Then the following conditions are equivalent.

i If H is a discrete quotient of G and $\text{Int}(H)$ is a p -group, then $H = \{1\}$.

ii The group G is abelian.

4. Let G denote the group $\text{MC}(3)$, as defined in Section 9.2. Let α be an automorphism of G of order 2 such that, for each $x \in G$, one has $\alpha(x) \equiv x^{-1} \pmod{[G, G]}$. Then $\text{Aut}(G) = \text{Inn}(G) \text{C}_{\text{Aut}(G)}(\alpha)$.
5. Let p be a prime number and let G be a finite group. Let R denote the group algebra $\mathbb{F}_p[G]$ and write $\mathbb{F}_p^*G = \{a\sigma : a \in \mathbb{F}_p^*, \sigma \in G\}$. Then $R^* = \mathbb{F}_p^*G$ if and only if R has cardinality 4, 8, 9, or p .

Let G be a topological group. A *set of topological normal generators* of G is a subset X of G such that $\{g x g^{-1} : x \in X, g \in G\}$ is a set of topological generators of G .

6. Let p be a prime number and let G be a pro- p -group. Let moreover r be a positive integer. Then the following conditions are equivalent.
 - i For every open normal subgroup N of G , a set of topological normal generators of N of smallest possible size has cardinality r .
 - ii The group G is isomorphic to $\mathbb{Z}_p^r$.
7. Let G be a finite group of odd order and let A be a subgroup of $\text{Aut}(G)$ of order dividing 2. Let moreover p be a prime number and let H be a p -subgroup of G that is A -stable. Then H is contained in a Sylow p -subgroup of G that is A -stable.
8. Let $p > 3$ be a prime number and let G be a p -group of class at least 4. Let $(G_i)_{i \geq 1}$ denote the lower central series of G . Assume that $G_3 = G^p$ and that $|G : G_2| = p^2$. Then the number of maximal subgroups of G that cannot be generated by 2 elements is 0 or 2.
9. Let p be a prime number and let X be a finite set. Let A and B be p -subgroups of $\text{Sym}(X)$. Let X^A and X^B be the collection of fixed points of X under the action of A and of B , respectively. Assume that X^A has cardinality at most $p - 1$. Then, for each $a \in X^A$, there exist $b \in X^B$ and $f \in \langle A, B \rangle$ such that $f(a) = b$.