Intense automorphisms of finite groups
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Stellingen

behorende bij het proefschrift
Intense automorphisms of finite groups
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Let G be a group. Then $\mathrm{Int}(G)$ denotes the subgroup of $\mathrm{Aut}(G)$ consisting of the automorphisms of G that send each subgroup of G to a conjugate in G.

1. Let $G = \mathbb{F}_{2017^2} \rtimes \mu_9(\mathbb{F}^*_{2017^2})$. Then there are no intense automorphisms of G of order 5.

Let p be a prime number. A p-group G is said to be *extraspecial* if it is finite with $[G, G]$ contained in $\mathrm{Z}(G)$ and $\mathrm{Z}(G)$ of order p.

2. Let p be a prime number. If G is an extraspecial p-group, then $\mathrm{Int}(G) = \mathrm{Inn}(G)$ or $\mathrm{Int}(G)$ is isomorphic to a semidirect product $\mathrm{Inn}(G) \rtimes \mathbb{F}_p^*$.

3. Let p be an odd prime number and let G be a pro-p-group. Assume that G is the absolute Galois group of a field. Then the following conditions are equivalent.

   i If H is a discrete quotient of G and $\mathrm{Int}(H)$ is a p-group, then $H = \{1\}$.

   ii The group G is abelian.

4. Let G denote the group $\mathrm{MC}(3)$, as defined in Section 9.2. Let α be an automorphism of G of order 2 such that, for each $x \in G$, one has $\alpha(x) \equiv x^{-1} \bmod [G, G]$. Then $\mathrm{Aut}(G) = \mathrm{Inn}(G)\,\mathrm{C}_{\mathrm{Aut}(G)}(\alpha)$.

5. Let p be a prime number and let G be a finite group. Let R denote the group algebra $\mathbb{F}_p[G]$ and write $\mathbb{F}_p^* G = \{a\sigma : a \in \mathbb{F}_p^*, \sigma \in G\}$. Then $R^* = \mathbb{F}_p^* G$ if and only if R has cardinality 4, 8, 9, or p.

Let G be a topological group. A *set of topological normal generators* of G is a subset X of G such that $\{gxg^{-1} : x \in X, g \in G\}$ is a set of topological generators of G.

6. Let p be a prime number and let G be a pro-p-group. Let moreover r be a positive integer. Then the following conditions are equivalent.

   i For every open normal subgroup N of G, a set of topological normal generators of N of smallest possible size has cardinality r.

   ii The group G is isomorphic to $\mathbb{Z}_p^r$.

7. Let G be a finite group of odd order and let A be a subgroup of $\mathrm{Aut}(G)$ of order dividing 2. Let moreover p be a prime number and let H be a p-subgroup of G that is A-stable. Then H is contained in a Sylow p-subgroup of G that is A-stable.

8. Let $p > 3$ be a prime number and let G be a p-group of class at least 4. Let $(G_i)_{i \geq 1}$ denote the lower central series of G. Assume that $G_3 = G^p$ and that $|G : G_2| = p^2$. Then the number of maximal subgroups of G that cannot be generated by 2 elements is 0 or 2.

9. Let p be a prime number and let X be a finite set. Let A and B be p-subgroups of $\mathrm{Sym}(X)$. Let X^A and X^B be the collection of fixed points of X under the action of A and of B, respectively. Assume that X^A has cardinality at most $p - 1$. Then, for each $a \in X^A$, there exist $b \in X^B$ and $f \in \langle A, B \rangle$ such that $f(a) = b$.