Intense automorphisms of finite groups

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Summary

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Let G be a group. An automorphism α of G is *intense* if, for every subgroup H of G, there exists an element x of G such that $\alpha(H) = xHx^{-1}$. The collection of intense automorphisms of G is a subgroup of $\mathrm{Aut}(G)$, which is denoted by $\mathrm{Int}(G)$.

In this thesis we classify the pairs (p, G), where p is a prime number and G is a finite p-group but $\mathrm{Int}(G)$ is not. To this end, for each finite p-group G, we define the *intensity* $\mathrm{int}(G)$ of G to be the index of any Sylow p-subgroup of $\mathrm{Int}(G)$ in $\mathrm{Int}(G)$. We prove that finite 2-groups have intensity 1. Next, we prove that, for every prime number p, each non-trivial finite abelian p-group has intensity $p-1$. We proceed with the classification by progressively increasing the nilpotency class of the groups we are looking at. Let p be an odd prime number. We show that the finite p-groups of class 2 and intensity greater than 1 are exactly the *extraspecial* p-groups of exponent p. We prove moreover that, if the class is 3, then a finite p-group has intensity greater than 1 if and only if its abelianization has order p^2. The classification process becomes more difficult as the class increases. We prove that there exists a unique 3-group, up to isomorphism, of class at least 4 and intensity greater than 1; that group has order 729. In contrast with the case of 3-groups, we demonstrate that, if $p > 3$, there exists, for each positive integer c, a p-group G of class c for which $\mathrm{Int}(G)$ is not itself a p-group. To do so, we extend the notion of intensity to pro-p-groups and, if $p > 3$, we construct an infinite non-abelian pro-p-group of intensity greater than 1. We later prove that the infinite group we constructed is the unique infinite non-abelian pro-p-group of intensity greater than 1, up to isomorphism. In conclusion, for each prime number $p > 3$, we define a new family of 2-generated finite p-groups, which we call *p-obelisks*, and we show that they have exceptionally pleasant properties. The classification of finite p-groups of intensity greater than 1 is completed, modulo the existence of a special kind of automorphisms of p-obelisks.