On the random-matrix theory of Majorana fermions in topological superconductors

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Issue Date: 2017-06-21

Summary

The 2016 Nobel Prize in Physics was awarded for the theoretical discovery in the 1970's and 1980's of topological states of matter. In the last decade, experimental discoveries have placed topological insulators, semimetals, and superconductors at the center of attention. Applications, in the area of spintronics and quantum computing, are still in the future, but at present these materials offer a wealth of fundamentally new effects to explore.

In this thesis we focus on topological superconductors. The topology manifests itself in the presence of gapless excitations bound to edges or surfaces, which cannot be removed by disorder or other perturbations. The excitations are so-called Majorana bound states, charge neutral particles that are their own antiparticles. A pair of Majorana fermions bound to two magnetic vortices can be used to store quantum information in a way that is nonlocal, insensitive to decoherence. Because of this potential application to quantum computing their properties are under intense investigation. The approach taken in this thesis is to study the universal, model-independent properties of Majorana fermions by means of random-matrix theory: a statistical approach in which only fundamental symmetries enter. The model system to which we apply the theory is a superconducting quantum dot coupled to metal leads (a so-called Andreev billiard).

Random-matrix theory was previously applied to condensed matter systems that were governed by the presence or absence of time-reversal symmetry, producing the three Wigner-Dyson symmetry classes. Chiral symmetry, studied mostly in the context of particle physics, doubled this to six symmetry classes. In a superconductor the particle-hole symmetry gives rise to the four Altland-Zirnbauer symmetry classes, for a total of 10 — the celebrated "tenfold way" of random-matrix theory. The central objective of this thesis is to investigate how particle-hole symmetry modifies the statistics of spectral properties and transport properties, in particular in systems where there is an additional chiral symmetry.

In Chapter 2, we investigate how the coupling to metal leads broadens the midgap spectral peak in a superconducting quantum dot. This peak is a key signature of the presence of a Majorana bound state, observed in conduction experiments as a peak in the differential conductance around zero voltage. A surprising result of our calculation is that ballistic

coupling, without a tunnel barrier, completely hides the Majorana peak in the background density of states. The technical ingredient that enables us to arrive at this result is the calculation of the eigenvalue statistic of the so-called time-delay matrix, the energy derivative of the scattering matrix. With that knowledge we can also access thermo-electric properties such as the Seebeck coefficient, and we find that it is similarly insensitive to the presence or absence of a Majorana bound state.

In Chapter 3 we include the effect of chiral symmetry, which is present at the surface of a topological insulator with induced superconductivity. Chiral symmetry stabilizes multiple Majorana bound states, by preventing a splitting of the midgap states. In contrast to the situation without chiral symmetry, we now find that the density of states and the thermo-electric properties do become sensitive to the Majorana bound states. At the technical level this chapter is more demanding than the previous one, where we could directly apply a technique developed for the Wigner-Dyson ensembles. With the chiral symmetry a new "trick" was needed.

In Chapter 4 we turn to an alternative way to restore the sensitivity to the Majorana bound states in the density of states, which is to introduce a tunnel barrier in the metal leads. While indeed the Majorana peak returns when the ballistic coupling is removed, one needs tunnel coupling in all leads attached to the quantum dot. The Majorana signature remains hidden if only a single metal lead has a ballistic coupling.

Finally, the last chapter is devoted to a problem in a different field: the search for adatomic nanomagnets with stable magnetization. These systems provide a benchmark for studies of decoherence effects and may provide the smallest-size logic unit possible in a condensed matter system. Since a few years, it has been recognized that symmetries are important to qualitatively characterize the stability of the magnetization. However, a complete study of the typical substrate symmetries (rotational and mirror ones) combined with time-reversal symmetry was missing, even for the single-adatom case. We have identified all combinations of symmetry groups that allow for a magnetized doubly degenerate ground state, robust under small crystal field-induced transversal anisotropies and first-order scattering from the substrate electrons. We could generalize the classification to arbitrary bipartite multiadatom nanomagnets with Heisenberg couplings. Our results can be seen as an extension of the celebrated Lieb-Mattis theorem on the ordering of energy levels in magnetic systems.