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On the random-matrix theory of Majorana fermions in topological superconductors

Marciani, M.

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On the random-matrix theory of Majorana fermions in topological superconductors

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Cover: The illustration symbolically depicts the search for the unpaired Majorana bound state. The spherical shape is a tribute to the compact space of scattering matrices, the plane to the linear spaces of Hamiltonians and time-delay matrices. The central image is a porcelain with an image from the Dutch painter H. W. Mesdag. Reprinted with small adaptations with permission of the owner: Kramer Kunst & Antiek, Amsterdam.

To the “families on mission” in The Netherlands

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