Boethius and the Importance of Basic Logic and Mathematics for Philosophy

Wang, Q.

Author: Wang, Qi
Title: Boethius and the importance of basic logic and mathematics for philosophy
Issue Date: 2014-03-06

II. Logic and Mathematics in Boethius' Curriculum

As I introduced in the first chapter, in the Latin speaking world, logic and mathematics were not well taught,[59] but Boethius regards both of them as elementary disciplines in his curriculum, which are important to philosophy. Therefore, he dedicated most of his time to these two disciplines. He began his curriculum plan with four mathematical disciplines. Boethius believes that quadrivium is the real beginning of learning philosophy.[60] At the same time, he is familiar with Aristotle's logic. I will not focus on the applications of all Boethius' logical knowledge in my dissertation. The theories, such as rules of syllogism and other topics, are useful for proper arguments which permeate Boethius' discussions, but it is not necessary to point out this kind of application of logic in detail. Instead, I will focus on what I call basic logic, which includes knowledge such as division, definition, and categories. This kind of basic logic can already be found in Boethius' works on the quadrivium. Again, in his logical works there are some applications of mathematics, but most of them are just mathematical examples. In other words, as two elementary disciplines, mathematics and logic are not totally independent, but they influence each other, which can be seen in the works on logic and mathematics. Together they become basic knowledge for Boethius' philosophy, which will be introduced in Chapter III and Chapter IV.

In this chapter, I will introduce Boethius' works on logic and mathematics, and I focus on some important ideas[61] which will be employed in his theological treatises and his *Consolatio*. I will also demonstrate the mutual influence of mathematics and logic, that is to say, how mathematical theories and examples are used in his logical works, and how basic logical theories are applied in his mathematical works. It is easy

[59] Cf. Section I.2.1.
[60] Cf. Section I.2.2.
[61] Cf. Section II.1.2, Section II.2.1.3, and Section II.2.2.3.

to understand applications of mathematics to logic, since most of the applications are the use of mathematical examples to support or demonstrate logical notions. Unlike these, the applications of basic logic to Boethius' mathematical works make use of logical knowledge to make the abstract mathematical concepts clearer. In order to understand applications of basic logic in Boethius' mathematical works more easily, I will introduce Boethius' logic first.

II.1. Boethius' Logic

In the second commentary on Porphyry's *Isagoge*, Boethius defines the role of logic as both the part and the instrument of philosophy, which implies the essential role of logic in philosophy. Logic is one of the elementary disciplines necessary to learn philosophy. Boethius' extant logical works can be divided into two large parts. The first part consists of the commentaries on works written by Aristotle, Porphyry, and Cicero and the second part comprises his monographs on the subjects of division, syllogism, and topics.[62]

As two elementary disciplines, mathematics and logic are relatively independent. Still, there are applications of mathematics in Boethius' logical works, though they are very few. In Section II.1.1, I will introduce Boethius' logical works in general and applications of mathematics (especially as mathematical examples) in these works. At the end of this section, I will focus on the basic logical knowledge — including knowledge of categories, division and definition — which will be employed in Boethius' other philosophical works.

II.1.1. Boethius' Logical Works and Applications of Mathematics in Them

In order to discuss the applications of mathematics in Boethius' logical works, I will discuss his logical works[63] in three parts concerning: (1)

[62] Cf. Section I.2.3.
[63] On the discussion on Boethius' logical works and their influence, cf. Casey (2012),

Boethius' commentaries on works in the Aristotelian corpus (Section II.1.1.1); (2) his monographs on division and syllogism (Section II.1.1.2); (3) his works on topics (Section II.1.1.3). I will introduce these three parts of Boethius' logical works and applications of mathematics in them one by one.

II.1.1.1. Boethius' Commentaries on Works in the Aristotelian Corpus

In Boethius' extant logical works, there are commentaries on works in the Aristotelian corpus,[64] which are two commentaries on Porphyry's *Isagoge*, one commentary on Aristotle's *Categories* and two commentaries on Aristotle's *On Interpretation*. In these works, most applications of mathematics follow the original works of Aristotle and Porphyry, and Boethius only adds a few mathematical examples or ideas.

II.1.1.1.a. Commentaries on Porphyry's *Isagoge*[65]

Boethius' first logical work is his commentary on Porphyry's *Isagoge*.[66] From this commentary, Boethius started his great plan of translating all of Aristotle's works.[67] Boethius writes two commentaries on the *Isagoge*. The first one is shorter with two books, written in the form of a dialogue between Boethius as a teacher and Fabius as a student. Boethius' first commentary is based on Victorinus' translation, but he thought that Victorinus' translation was not satisfactory. Boethius points out the errors in Victorinus' translation more than once, saying that Victorinus seems to

"Boethius's Works on Logic in the Middle Ages," in Kaylor and Phillips (eds.), pp. 193-220; Martin (2009), "The Logical Textbooks and Their Influence," in Marenbon (ed.), pp. 56-84; Lewry (1981), "Boethian Logic in the Medieval West," in Gibson (ed.), pp. 90-134. Kneale claims that in the Middle Ages Boethius' logical writings were "better known than those of Aristotle and his reputation as high." Cf. Kneale (1984), pp. 189-198.

[64] Cf. Cameron (2009), "Boethius on Utterances, Understanding and Reality," and Ebbesen (2009), "The Aristotelian Commentator," in Marenbon (ed.), pp. 85-102 and pp. 34-55; Chadwick (1981b), pp. 133-141; Marenbon (2003), pp. 19-42; Reiss (1982), pp. 37-54; Shiel (1958); Solmsen (1944).

[65] The translations of the first book of *₂InIsag.* I cite are McKeon's (1957-1958). The translations of other books are mine from the Latin version. On Porphyry's *Isagoge* and Boethius' commentaries on it, cf. Gracia (1981); Evangeliou (1985).

[66] Cf. Chadwick (1981b), pp. 131-133; Marenbon (2003), pp. 23-32; Reiss (1982), pp. 28-37.

[67] On Boethius' project, cf. Marenbon (2003), pp. 17-18. Also cf. Section I.2.1.

understand Porphyry less clearly, and that his translation is obscure (*$_1$InIsag.*, 53B). The whole style of the first commentary is the same as that of Porphyry's, and there is no application of mathematics in it.

Later, Boethius wrote the second commentary in five books, based on his own translation. In the second commentary, before dealing with Porphyry's five predicables which include genus, species, difference, property, and accident, Boethius devotes most of the first book to the discussion of the utility of studying logic, and the remaining parts are concerned with a penetrating discussion of the problem of the universal. Applications of mathematics to his second commentary on *Isagoge* are not many. At the beginning of this commentary, Boethius clarifies his programme of translation. He says that in order to accomplish the purpose of seeking knowledge, he should translate the Greek books of philosophy into the Latin language without missing anything. In Boethius' point of view, "the most excellent good of philosophy has been related with human souls" (*$_2$InIsag.*, 71A). He believes that if he wants to translate the Greek books of philosophy in which the uncorrupted truth is expressed into the Latin language, then he must begin his exposition with the powers of human soul. Therefore, he begins his second commentary on *Isagoge* with the discussion on the triple power of the soul.

> "There is a triple power of the soul to be found in animated bodies. Of these, one power supports the life for the body that it may arise by birth and subsist by nourishment; another lends judgment to perception; the third is the foundation for the strength of the mind and for reason." (*$_2$InIsag.*, 71B)

In *Musica*, Boethius warns people not to grant all judgment to the senses, which is the second power of the soul. For the sense is just something obedient like a servant. The ultimate perfection should be composed by reason, because reason "holds itself to fixed rules" and "does not falter by any error". This shows that reason, the highest power of soul, "is a judge and carries authority" (*Musica*, I.9.196). Not all judgment ought to be given to the senses, but reason ought to be trusted more, so we should pursue the use of reason. Just like mathematics can help people to arrive at abstract concepts from visual images, the powers of the soul can give people not only the life to exist or the capacity to feel but also the ability to ascend from the known to the unknown. This is why Boethius begins his commentary on *Isagoge* with the statement of powers of the soul.

The influential discussion in Boethius' second commentary on *Isagoge* is the solution to three questions Porphyry raises at the beginning of his *Isagoge*.

> "For example, about genera and species — whether they subsist, whether they actually depend on bare thoughts alone, whether if they actually subsist they are bodies or incorporeal and whether they are separable or are in perceptible items and subsist about them — these matters I shall decline to discuss, such a subject being very deep and demanding another and a larger investigation."[68]

These three questions regard the possible existence of genera and species outside our mind; and also concerning the nature of genera and species, corporeal or incorporeal; and their relations to sensible objects. Porphyry refuses to discuss them, but Boethius gives solutions to them.[69]

In order to answer those three questions, Boethius gives two examples from mathematics, among which one is concerned with the nature of numbers in arithmetic, and the other is concerned with the nature of a line in geometry. First, as for numbers, when any number comes out in computing the digits, there must be no doubt that it eventuates in sensible objects itself. And then in the case of a line, human beings can grasp a line using their mind, and make this line an object of thinking, which seems that a line subsists outside our mind. However, it is just the concept of a line that subsists outside sensible objects. No one can perceive a line which is separable from its form. In agreement with these two mathematical examples, Boethius says,

> "For genera and species subsist in one manner, but are understood in another; and they are incorporeal, but they subsist in sensible

[68] Porphyry, *Isagoge*, 1.10-15; Barnes (2003).
[69] Porphyry's three unanswered questions are about universals. On Boethius' discussions on universals, cf. Cross (2012); Spade (1994).

> things joined to sensible things. They are understood, to be sure, as subsisting through themselves and not as having their being in others." ($_2$*InIsag*., 86A)

Mathematical examples thus help Boethius solve the three important questions, and his statements on genera and species lead to the discussion of Nominalism and Realism[70] in the Middle Ages.

II.1.1.1.b. Commentaries on Aristotle's *Categories*[71]

Porphyry's *Isagoge* is the introduction to Aristotle's *Categories*, and the five predicables discussed in *Isagoge* are the more general types of predicate. After the introductory work, Porphyry also wrote two commentaries on *Categories*. Proposing to follow Porphyry's commentary on the *Categories*, Boethius writes a commentary in four books on Aristotle's *Categories*. He opens the commentary with the intention of Aristotle's work on the categories. The intention of Aristotle's *Categories*, Boethius says, is to examine the words, not significations (*In Categorias*, 159A-D). In *Categories*, Aristotle uses some mathematical examples, and as a commentator, Boethius follows Aristotle and employs mathematical examples in his commentary.

It was regarded as unclear what the purpose is of the first chapter of the *Categories*. Porphyry, in his extant commentary, points out that there are five possible ways to connect names, definitions, and things:

> "When things share the same name but have entirely different accounts, they are called homonyms. When they share both an account and a name, they are referred to as synonyms, since together with (*sun*-) the name they also have the same account. When things share the same account but not the same name, they

[70] Nominalism arose in reaction to the problem of universals. Nominalism is a metaphysical view in philosophy according to which general or abstract terms and predicates exist, while universals or abstract objects, which are sometimes thought to correspond to these terms, do not exist. Opposed to nominalism, the view of realism is that universals do exist over and above particulars. Cf. Armstrong (1978 and 1989).

[71] The translations of *In Categorias* are mine from the Latin version. On the discussions of Boethius' commentary on *Categories*, cf. Asztalos (1993 and 2003); Chadwick (1981b), pp. 141-152; Marenbon (2003), pp. 20-23.

> are called polyonyms, and when they have in common neither a name nor an account, they are called heteronyms. ... When certain things come to be from other things, participating in a way in both the name and the account of the things from whence they come, differing however in grammatical form. These are called paronyms."[72]

However, of these five ways Aristotle just mentions three, that is, homonyms, synonyms, and paronyms. Why does Aristotle do this? To answer this question, Porphyry simply attributes it to Aristotle's subsequent discussion: because Aristotle only needs these three ways for his subsequent discussion, and those that he does not need, he does not mention.[73] Boethius answers that question with the help of analogy to geometry in the preface of Book I. Why did Aristotle explain equivocal, univocal, and derivative terms first before discussing the ten categories? Boethius draws analogies with geometry. He says that, "in geometry, the first thing given is the *termini*, and then the *theorematum ordo* can be discussed." (*In Categorias*, 163B-C) This means that before a series of theorems is set out in detail, their principles should be given first. Similarly, in order to discuss the ten categories, first of all, their principles should be explained. This is the relation of the first chapter to what follows.

Among the ten categories, substance, quantity, quality, and relative are closely related to mathematics, so applications of mathematical examples are mainly found in the discussions of these four categories. I will point them out in the following parts of this section.

The category of substance has a number of characteristics, among which one seems to be contrary to what Aristotle says in the early part of the chapter on "substance". This characteristic is "Substance, it seems, does not admit of a more and a less."[74] Earlier in this chapter with regard to the secondary substances, Aristotle says that, "Of the secondary substances the species is more a substance than the genus, since it is nearer to the primary

72 Porphyry, *In Categorias*, 60.26-33; Strange (1992).
73 Porphyry, *In Categorias*, 61.1-5; Strange (1992).
74 Aristotle, *Categories*, 3b33; Ackrill (1963).

substance."[75] Commenting on this apparent contradiction, Porphyry interprets this "not to be understood in an unrestricted sense, but with a distinction"[76]. Following Porphyry, Boethius begins his commentary on this characteristic with similar explanation; that is to say, this is not simply said to be a characteristic but a further distinction is added (*In Categorias*, 197A). Then, unlike Porphyry, Boethius inserts some mathematical examples from other chapters of the *Categories* to explain that this characteristic does not apply to the category of substance solely. For instance, the geometrical figure circle will not be more a circle or less a circle either than itself or than another circle[77]. In addition, he takes some relatives for examples. Double is not more double than another, thus it does not admit "more or less"[78] (*In Categorias*, 197D).

Among the ten categories, "quantity" is most closely related to mathematics, which will be exhibited in Section II.2.1.2.a of this chapter. When introducing quantity, Boethius, following Aristotle, applies more mathematics to his commentary on this part. Quantity can be divided into two kinds, that is, discrete quantity and continuous quantity. And numbers which are studied by arithmetic falls in the discrete quantity, and lines, surfaces, and the like which are the essential objects of geometry falls into the continuous quantity. In his commentary, Boethius enumerates some numbers to show what discrete quantity is. Using lines, surfaces, and bodies as examples, Boethius explains the continuous quantity and how they have a common boundary at which their parts join together (*In Categorias*, 203B-205A).

The second category that has closely relation with mathematics is "relative". One nature of relatives is that usually relatives come into being together, which means that not all relatives are simultaneous by nature. In Aristotle's point of view, "the knowable would seem to be prior to knowledge"[79]. When commenting on this idea, Boethius admits that the first thing related to this idea coming to us is the discipline of mathematics (*In*

[75] Aristotle, *Categories*, 2b7-22; Ackrill (1963).
[76] Porphyry, *In Categorias*, 97.6-10; Strange (1992).
[77] Cf. Aristotle, *Categories*, 11a5-12; Ackrill (1963).
[78] Cf. Aristotle, *Categories*, 6b24-27; Ackrill (1963).
[79] Aristotle, *Categories*, 7b23-24; Ackrill (1963).

Categorias, 229A). Boethius takes triangles for example. We all know that there are three interior angles in a triangle and then these three interior angles equal to two right angles. Hence, it is necessary that the knowable should exist first, and then the knowledge of it may be acquired (*In Categorias*, 229B). In *Categories*, Aristotle takes "the squaring of the circle" as an example to show that "Knowledge of it does not yet exist but the knowable itself exists."[80] Boethius considers this example most obscure (*In Categorias*, 230C), and elaborates a different argument showing that it is possible to draw a triangle equal to a *spatium* with four sides. And similarly, he admits that the knowledge of "the squaring of the circle" also exists, but explains that the reason why Aristotle says it does not exist is that at Aristotle's times the square sought after had not been discovered (*In Categorias*, 230D-231C).

To understand the characteristics of relatives, two terms need to be explained in more detail, namely "simultaneous" and "prior". It is likely that in order to keep the integrity of the discussion on relatives, Aristotle leaves the exposition of these two terms to later chapters. However, in his commentary, Porphyry gives the explanation of "prior" in his comments on the chapter on relatives. Unlike Porphyry, Boethius chooses to follow Aristotle's order. After commenting on the ten categories, Boethius comes to five senses of "prior" in his last book. There are five senses of "prior": (1) whenever we use the term 'prior' in its proper and primary sense, it is time that we have in our minds; (2) 'prior' may be used, when the order of being is fixed and incapable of being reversed; (3) we use the term 'prior' in regard to any order whatever; (4) naturally prior; (5) where in the case of two things the existence of either implies or necessitates that of the other, that thing which is somehow the cause may, in consequence, fairly be considered as naturally prior to the other.[81]

To illustrate the second meaning of prior, Boethius, following Aristotle, takes "one is prior to two" as example, but he gives a more detailed exposition, which could more easily be understood if we refer to his arithmetical work. In Boethius' work on arithmetic, the notion that "unity

[80] Aristotle, *Categories*, 7b30; Ackrill (1963).
[81] Aristotle, *Categories*, 21a34-22a13; Ackrill (1963).

(or one) is the first number" is the starting-point. The definition of "number" is also based on this notion, that is to say, "A number is a collection of unities, or a big mass of quantity issuing from unities." (*Arithmetica*, I.2) In addition, the primary nature of unity is that "it constitutes the primary unit of all numbers which are in the natural order and is rightly recognized as the generator of the total extended plurality of number." (*Arithmetica*, I.7) Thus, if someone proposes the number two, then it definitely follows there is one, for two is the collection of two unities. However, if someone lays down the number one, it is not necessary for it to be multiplied to two (*In Categorias*, 284C-D). As a result, it is clear that between the number one and two, the order is fixed and incapable to be reversed, in other words, one is prior to two, which is the second sense of "prior".

Another characteristic of the relatives is that "if someone knows any relative definitely he will also know definitely that in relation to which it is spoken of."[82] In other words, it is impossible to know that a thing is relative unless its correlative is known. This view is especially related to ratio in arithmetic. Boethius follows Aristotle and takes double as example, but he extends this explanation. As we all know, the number four and the number two have a certain relationship, that is to say "double". It could not be possible to know that the number four is a double without knowing that it is twice the number two. If you definitely know the number four of being "double", then at once will you definitely know to which number the number four stands in relation, that is the number two (*In Categorias*, 235D-236B).

The third category which is close to mathematics is "quality", or more specifically, "the fourth kind of quality". There are four kinds of "quality", including (1) habits and dispositions, (2) capacities, (3) affective qualities and affections, (4) shape, figure and so on. Among these qualities, the fourth kind of quality is close to mathematics, and especially related to geometry, for shape, figure and the like are the subjects focused on by geometry. Boethius lists some geometrical figures, such as a triangle and a square, which belong to the fourth kind of quality. However, there may be some confusion between the fourth kind of quality and continuous quantity, for in

[82] Aristotle, *Categories*, 8a35; Ackrill (1963).

the *Categories* geometrical objects are listed under both headings. Aristotle does not explain this, but following Porphyry, Boethius introduces the following distinction in his commentary. He says that the surface itself is a quantity; however, the shaping of the surface belongs to a quality. Geometrically, the surface bounded by lines is defined by its length and breadth, which is a quantity. For example, a triangle is a particular area produced by three lines which are placed in a certain way to form three angles, so a triangle is called a quantity. However, it is named after a quality in virtue of its certain sort of shaping. It is the same with a line: "it is said to be a quantity due to its length without breadth; and insofar as it is straight, a straight line belongs to quality." (*In Categorias*, 251A-B)

Two characteristics of quality are that most qualities have contraries and admit of degrees. It is helpful to take triangles and other figures that belong to the fourth kind of quality as examples to show that some qualities have no contraries and do not admit of degree. Here, as Aristotle already saw, the introduction of mathematical examples will make arguments easier to be grasped, and more persuasive.

II.1.1.1.c. Commentaries on Aristotle's *On Interpretation*[83]

The purpose of *Categories* is stated again in Boethius' commentaries on Aristotle's *On Interpretation*. The number of significant spoken sounds is divided into ten categories by Aristotle, and the spoken sounds, signifying thought, can also be divided into two parts, name and verb, which are two primary parts to the communication. The former division is discussed in *Categories*, and the latter one is the central topic in *On Interpretation*. Boethius writes two commentaries on Aristotle's *On Interpretation*: the first one is shorter than the second one. Both Boethius' commentaries on Aristotle's *On Interpretation* contain 12 chapters. The difference between two editions is that the first and short one comprises only two books, while the second and larger one comprises six books. There are only a few applications of mathematics in them. In this section, I focus on applications of mathematics to the second commentary.

[83] The translations of *$_2$InInter.* I cite in my dissertation are Smith's (2010 and 2011). On Boethius' commentary on Aristotle's *On Interpretation*, cf. Chadwick (1981b), pp. 152-163; De Rijk (2003); Magee (1989); Marenbon (2003), pp. 32-41; Suto (2012).

At the beginning of Chapter 1, Book I of *₂InInter.*, Boethius first discusses the definition of "spoken sound (*vox*)". The reason why he begins with it is that, as Boethius says, spoken sound is "obviously and clearly the theme of this whole book."[84] Then he gives two definitions of *vox*. The first definition is that "Spoken sound is the striking of the air by the tongue, produced by an animal by means of certain parts of the throat called windpipes." The second possible definition is that "spoken sound is a sound which appears to signify." (*₂InInter.*, 4.17-28)[85] Strictly speaking, the first definition is the meaning of simply sound (*sonus*), thus Boethius gives a second possible definition. The second definition differs from the use of *vox* in Boethius works on music[86], but it is the one he needs in his commentary on *On Interpretation*.

And in *On Interpretation*, Aristotle says "spoken sounds are symbols of affection in the soul"[87]. When commenting on this view, Boethius claims: "these affections in souls are produced from the similarity of the things [to the affections]." (*₂InInter.*, 35.1) Geometrical figures serve to explain this. Boethius takes sphere, square, or other geometrical figures as examples. He points out that, when a person sees a geometrical figure in things, he considers its likeness in the intelligence of his mind, and "when his soul has been affected by the image, he knows the thing by whose image he has been affected". Therefore, the similarity of the figure in things to the affections causes the affection in souls to occur. (*₂InInter.*, 35.1-10)

When commenting on the relation of actuality and potentiality, Boethius says there is something in which there is only potentiality and never actuality by taking numbers as an example.

> "For number can increase to infinity and whatever number has been mentioned, a hundred, a thousand, ten thousand and the rest, must

[84] "This whole book" refers to Aristotle's *On Interpretation.*
[85] The paragraphs of Boethius' second commentary on *On Interpretation* 53ff are closely paralleled in Ammonius 30.1-16, and the discussion of Boethius 4.18-6.5 overlaps with Ammonius 30.2-7. Cf. Ammonius, *On Aristotle On Interpretation 1-8*, 30.1-16; Blank (1996).
[86] In Boethius' work on music, *vox* has a wide spectrum of meaning, for it can mean the human voice, sound in general, or musical pitch.
[87] Aristotle, *On Interpretation*, 16a3; Ackrill (1963).

> be finite. Thus an actual number is never infinite because it can increase to infinity. And for this reason infinite number is only potential." (*₂InInter.*, 463.8-17)

The last example of applications of mathematics that I want to point out is in the last chapter, Book 6 of *On Interpretation*. At 23b33-24a3 of *On Interpretation*, when speaking of the contrary of a statement, Aristotle uses four propositions to argue that every statement has a contrary. The four propositions are: PI. About the good that it is good; PII. About the good that it is not good; PIII. About the not good that it is not good; PIV. About the not good that it is good. When commenting on this, Boethius compares the argument of Aristotle with a ratio, for he believes, "Ratio is in fact the mutual similarity of things to each other." (*₂InInter.*, 490.15-19) Boethius chooses the ratio 2:4=6:12, and according to the numerical relations of this ratio it is also true that 2:6=4:12. Then Boethius transfers the numerical ratio to the force and nature of propositions.[88] He puts PI and PII first, of which PI precedes and PII follows, and then puts PIII and PIV, of which PIII precedes and PIV follows in the same way, and makes there be a similarity.

PI	PII
About the good that it is good	About the good that it is not good
PIII	PIV
About the not good that it is not good	About the not good that it is good

The similar ratio between these four propositions is PI:PII=PIII:PIV, which means that just as PI is true, but PII is false, so too PIII is true, but PIV is false. In other words, the left of the ratio is "a true proposition : a false proposition" and the right of the ratio is the same, so the left equals the right. If we change the places of these four propositions in the way of 2:4=6:12, that is 2:6=4:12, then we can get PI:PIII=PII:PIV. In this new ratio, the left is that PI is true and PIII is true, and the right is that PII is false and PIV is false, so the force of these propositions can also be explained by the ratio. All in all, any statement A, B, C, or D, if A:B=C:D is true, then A:C=B:D is also true, and this correct rule for identifying contraries is the same for all types of statements. Boethius' application of the numerical relation of ratios

[88] Cf. Chadwick (1981b), p. 154.

makes for an easier and more concise way to understand the argument.

II.1.1.2. Boethius' Monographs on Division and Syllogism[89]

II.1.1.2.a. Division[90]

In his work on division, Boethius studies different kinds of division, distinguishes one from another, and points out the logical relations between whatever is being divided (or analysed, or classified) and its dividing elements. There are some uses of mathematics in discussions on division.

The central part of *Divisio* is on the division of genus into species, which is one of the four kinds of division. During the operation of this division style, "differentia" is an essential concept. A differentia is that "in respect of which we indicate that one thing differs from another (*Divisio*, 880b)". There are many kinds of differentiae, and not all of them are suited to the division of genus. "Some differentiae are *per se*, others *per accidens*, and of the latter some are consequent, others regularly departing." Boethius gives some examples to explain which sort of differentiae is suited to the division of genus, one of which is about mathematics and man.

> "Again, there is another thing, which is conceptually inseparable, the separation of which brings destruction of the species, e.g. when we say that it is inherent in man that he alone can use numbers or learn geometry. And if this capacity should be removed from man, then man himself no longer remains; and yet such things do not automatically belong to the class of differentiae that inhere in the substance, for it is not the ability to use numbers and do geometry that accounts for man, but rather being rational and mortal. Hence those differentiae on account of which the species consists are precisely the ones that are placed both in the division of a species and in the division of the genus that contains the species." (*Divisio*, 881b-d)

That Boethius takes this example here also implies that although

89 Cf. Chadwick (1981b), pp. 163-166.

90 The translations of *Divisio* I cite in my dissertation are Magee's (1998). On Boethius' *Divisio*, cf. Marenbon (2003), pp. 44-46.

mathematics (numbers and geometry) does not inhere in man, it still cannot be separated from man. The reason is that if mathematics should be removed from man, "then man himself no longer remains". Therefore, we can see how vital the mathematics is in man.[91]

With regards to affirmation and negation, Boethius takes mathematics as examples. When he discusses the negation used in constructing a species, he gives examples: "Of odd numbers some are prime (e.g. three, five, or seven) others not-prime (e.g. nine);" or again, "Of figures some are rectilinear, others non-rectilinear." (*Divisio*, 882b-c) That is to say when we want to use a simple name to assign a species to something that is not picked out by any word (such as there is no single name applied for not-prime numbers), it is often necessary to use negation in constructing a species (the negation word "not" with the species "prime"). However, it is our need, not nature, that sometimes requires this. Further, when a section is made by negation, the affirmation or simple name should be stated first. So when we divide numbers, first of all, the affirmation "some numbers are prime" is stated, and then the negation "other numbers are not-prime" follows. The relationship between affirmation and negation is that "Affirmation is prior and negation posterior." This can be explained by arithmetical theory, "the equal is prior to the unequal" which will be introduced in Section II.2.2 later. In Chapter 32, Book I of his arithmetical work, Boethius gives a demonstration of "how every inequality proceeds from equality", and in Chapter 1, Book II, Boethius continues to discuss "How every inequality is reduced to equality". The equality is more finite than the inequality. "It is always necessary that finite things be prior to non-finite things." And the equality is prior to the inequality. "All the things that are expressed by a part of speech that is definite or by an affirmation are more finite than a name with a negative particle or a complete negation." (*Divisio*, 882d) Therefore, "affirmation is prior and negation posterior."

The division of one and the same genus occurs in more than one way. To illustrate the multiple divisions, Boethius gives the division of numbers in arithmetic and the division of triangles in geometry as examples. According to different divisions, numbers could be divided into even and

[91] Cf. Section I.2.2.

odd, and alternatively, numbers can also be classified as prime numbers and non-prime numbers. It is the same with triangles. Triangles can be divided into equilateral triangles, triangles with only two equal sides, and triangles with all sides unequal. Another way to divide triangles is that some triangles have a right angle, others have three acute angels, and others have an obtuse one (*Divisio*, 885c). As a result of this, Boethius comes to the conclusion that division of one and the same genus can be made in many ways. In spite of multiple divisions, every division would be split into pairs, if there were names for the species and differentiae. Take geometrical figures with three sides for an example. Three-sided figures can be divided into three species: the figures with equilateral sides, others with two equal sides, and others with unequal sides throughout. Since there are names for the species and differentiae of three-sided figures, the tripartite division of the three-sided figures can change into bipartite division. Those figures with three sides can be divided into figures with equal sides, and figures with unequal sides; of the figures with unequal sides, some have only two equal sides, others have three unequal sides. Therefore, in the same way, "every division would be bipartite if the species and differentiae did not lack names." (*Divisio*, 884a)

The introduction of mathematical examples in *Divisio* makes the points more easily to be understood.

II.1.1.2.b. Syllogism[92]

In Boethius' point of view, syllogism is concerning the statements which are either categorical or hypothetical. Theory of categorical syllogisms is the logic of names and theory of hypothetical syllogisms is propositional logic. Thus his monographs on syllogism are divided into two parts: one is the works on categorical statements including *Introductio ad syllogismos categoricos* and *De syllogismo categorico*[93]; and the other is the work on hypothetical statements called *De hypotheticis syllogismis*[94]. The first concerns categorical statements in which something is predicated of

[92] On Boethius' syllogism, cf. Chadwick (1981b), pp. 166-173; Dürr (1951); Marenbon (2003), pp. 46-56; Speca (2001).
[93] Concerning the relation between the twin monographs on the categorical syllogism, cf. Chadwick (1981b), pp. 165-170.
[94] Cf. Bobzien (2002).

another thing in the form "A is B". As regards the hypothetical statements, Boethius gives four characterizations as follows: (1) hypothetical statements express that something is, if something else is; (2) hypothetical statements consist of categorical statements, while categorical statements are simple; (3) hypothetical statements have their own proper force that differs from the force of categorical statements, in that it rests on a hypothesis rather than on a predication, even with regard to the same terms; (4) hypothetical statements express that something is or is not, if something else is or is not.[95]

All of these monographs refer to how to give proper arguments, and in all Boethius' arguments, he pays attention to the rules of syllogism. Applications of mathematics could not be found in these three monographs, so I will move on to the other logical works.

II.1.1.3. Applications of Mathematics in Boethius' Works on Topics

Boethius wrote two works on topics.[96] One is a commentary on Cicero's *Topics*, and the other is a monograph named *De Topicis Differentiis*. In this section I will introduce both works briefly and list mathematical examples used in both works.

II.1.1.3.a. *In Ciceronis Topica*[97]

In Ciceronis Topica is Boethius' commentary on Cicero's *Topics*, which follows the text in Cicero's work continuously. However, because Cicero's work contained a paragraph 100, and Boethius' commentary ends in the comments on Cicero's paragraph 76, one could say that Boethius' commentary on Cicero's *Topics* is either preserved incompletely or was never finished by him.

According to Boethius, two different sorts of things are Topics: a Topic is both a maximal proposition and the differentia of a maximal proposition. Therefore, "maximal proposition" is a vital concept of Topics. In Book I of

[95] Cf. Speca (2001), pp. 78-80.

[96] On Boethius' theory of Topics and its influence, cf. Marenbon (2003), pp. 56-65; Stump (1981a, 1974, and 1981b).

[97] The translations of *In Ciceronis Topica* I cite in my dissertation are Stump's (1988). On Boethius' commentary on Cicero's *Topics*, cf. Stump (1987).

In Ciceronis Topica, when Boethius states the nature of Topics, he gives the definition of maximal proposition: "We call highest and maximal propositions those propositions that are universal and known and manifest to such an extent that they need no proof but rather themselves provide proof for things that are in doubt, for those propositions that are uncertain." (*C.Topica,* 280/1051) In order to elucidate this concept, he takes examples from mathematical theory. The first example he gives of this sort of propositions is that "Every number is either even or odd". This proposition from arithmetic is universal and there is no need to prove it, for if number is divided from this perspective, it must be that there are two kinds: one is even number and the other is odd number. There is no the third kind of number in this division of number, which suggests that the proposition "Every number is either even or odd" illustrates what it means to be manifest and universal. The second example is "If equals are subtracted from equals, equals remain". This also proves itself by itself which means it is universal and manifest. Therefore, these two propositions from arithmetic are both called "maximal propositions". And from these two arithmetical examples, we can easily understand what "maximal proposition" is and what the characteristics of this proposition are.

At the introduction of Book II, Boethius refutes various biting censures on the discipline of logic. Everyone wants to appear very skilled at discourse. In order to bring and to refute charges, people would all rush together to the knowledge of the discipline of logic. Then Boethius raises two questions: "But now can anything more absurd be imagined than their trying to argue that the study of dialectic is useless for arguments that are even in their own view readily believable? For what sense does it make to subvert the art of discourse by engaging in discourse, so that you despise the truth of the vary art in which you seek a reputation?" (*C.Topica*, 292/1063) To answer these questions easily, Boethius uses an analogy between a musician and himself. "As that musician directed his disciple to make music for himself and for the Muses, so I too could also have sung for myself and for you, who are not a Muse but a protector of the Muses." (*C.Topica*, 292/1063)

Book II focuses on the nature of related things and their kinds:

conjugates, genus, species, similars, differentiae, contraries, associated things, antecedents, consequents, and incompatibles, cause and effect, comparison of greater, lesser, and equal things. All these are connected to one another and some of them "have not only a linguistic connection to one another but also a certain harmony of nature, although they are not identical with one another (*C.Topica*, 295/1066)". In order to understand the last part of this notion, we need to refer to a conception in Boethius' work on music. In his work on music, Boethius gives a definition of consonance: "a mixture of high and low sound falling pleasantly and uniformly on the ears (*Musica*, I.8.195)". The sounds composing the consonance are not identical with one another, but their mixture can form a pleasant and uniform sound, that is, the consonance. This is the same with the parts of related things. That is why Boethius says, in spite of the difference between the parts, that they could have a certain harmony of nature.

II.1.1.3.b. *De Topicis Differentiis*[98]

At the beginning of Boethius' *De topicis differentiis*, he states the aim of this work. He will show "what the topics are, what their differentiae are and which are suited for which syllogisms." (*TopicisD.*, 1173C.9-10)

The first concept to be discussed is "proposition". Boethius states different ways to divide propositions. The last division is to distinguish some propositions known *per se* from all the rest.

> "Some propositions are known per se, and no proof can be found for these. Others, although the mind of the hearer approves them and assents to them, can nevertheless be proved by other, more fundamental propositions. Those for which there is no proof are called maximal and principal, because it is necessary that these prove those which do not deny that they can be demonstrated." (*TopicisD.,* 1176C.18-24)

Boethius inserts the same mathematical proposition as an example that was also used in *In Ciceronis Topica*: "If you take equals from equals, the remainders are equal." This mathematical proposition produces appropriate

[98] The translations of *De Topicis Differentiis* I cite in my dissertation are Stump's (1978).

belief in itself by nature, so it is known *per se*, and it is indemonstrable, maximal, and principal.

The fourth concept Boethius discusses is "argument" which is a reason producing belief regarding something that is in doubt. "Of all arguments, some are readily believable (*probabilia*) and necessary, some readily believable and not necessary, some necessary but not readily believable, and some neither readily believable nor necessary." (*TopicisD.,* 1180C.24-27) But someone may reckon that things which are necessary only and not also readily believable are not arguments. Boethius gives a refutation of this. This kind of idea, he says, is not based on a correct understanding of "readily believable."

> "Those things are readily believable to which agreement is spontaneously and willingly given, so that they are agreed to as soon as they are heard. However, those things that are necessary and not readily believable are demonstrated before by other things that are necessary and readily believable; and, known and believed, they produce belief regarding something else which is in doubt." (*TopicisD.,* 1181B.33-39)

Boethius continues to illustrate this by the nature of geometrical theory.

> "The theories (theorems) which are considered in geometry are of this sort. For the things presented there are not such that the mind of the student agrees to them spontaneously; but since they are demonstrated by other arguments and so are known and understood, they produce belief regarding other theories. So those things that are not readily believable *per se* but are necessary cannot be arguments to confirm something else for hearers to whom they have not yet been demonstrated. However, to those hearers who by prior reasons have come to believe those things which they [once] did not agree to, they can be invoked as arguments if [the hearers] are in doubt about something." (*TopicisD.,* 1181B.40-1181C.11)

Book III concerns the comparison of divisions between Themistius and Cicero. Boethius believes it is normal that the people of an attentive nature treat the differentiae of Topics variously and in different ways. The reason is that any one thing can often be divided diversely. Here Boethius also shows some divisions in mathematics as examples. "For example, we collect sometimes these differentiae of number: some [numbers] are even and others odd; but sometimes these: some [numbers] are prime and incomposite and others are secondary and composite. The discipline of geometry shows that triangles also may be divided in many ways, though in all cases one should watch that nothing is left out in any form of division and nothing superfluous and beyond what is necessary is added." (*TopicisD.,* 1195B.7-1195C.15)

II.1.2. Boethius' Basic Logic

In logic, the syllogism is used for the statement of knowledge, and topics are used to generate arguments. Syllogisms and topics pervade Boethius' works, so I will neither specially point out where knowledge of the syllogism is used nor how it is used. When I discuss applications of logic in my dissertation, I refer to the basic logic including knowledge of categories and theories of division and definition that will be applied in Boethius' works on mathematics (Section II.2.1.2 and Section II.2.2.2), theology (Chapter III), and *Consolatio* (Section IV.2.1 and Section IV.3). In this section, I will give a short introduction to Boethius' basic logic.

II.1.2.1. Categories

Among the ten categories, Aristotle pays more attention to four of them, that is, substance, quantity, relation, and quality, and he puts other categories in one chapter. Similarly to Aristotle, Boethius also stresses substance, quantity, relation, and quality, which he also applies in his other works. These four main categories are applied in his works on arithmetic and music, which I will discuss in Section II.2.1.2.a and Section II.2.2.2.a. And in his theological treatise on the Trinity, Boethius applies the ten categories to God, and, finally, he finds that only the category of relation can explain the diversity in God, which will be shown in Section III.3.2.

II.1.2.2. Division and Definition

The most important part of the theory of division is the distinction of types of division. In his *Divisio*, Boethius distinguishes two large parts of division, among which there are three types (see Diagram I below).

Diagram I: Boethius' System of Division

Division	
A. Division *per se*	B. Division *per accidens*
Division of a genus into its species	Division of a subject into accidents
Division of a whole into its parts	Division of an accident into its subjects
Division of an utterance into its significations	Division of an accident into accidents

From these six types of division, I will single out the three types of division *per se*[99], because they are important in Boethius' other works.

There are differences between division of a genus into its species and division of a whole into its parts, although these two types are similar. There are three main differences between them. The first difference is "the division of a whole is made in respect of quantity" and "the distribution of a genus is accomplished in respect of quality" (*Divisio*, 879b). The second difference is that "Every genus is by nature prior to its proper species whereas a whole is posterior to its proper parts" (*Divisio*, 879b). The "prior[100]" here is not used in the sense of time, because it means that the destruction of the genus could result in the perishment of the species immediately, but not vice versa. If a species is destroyed, its genus would remain inviolate in its nature. Unlike the relation between the genus and its species, its proper parts are prior to the whole, for if a part of the whole perishes then that of which one part has been destroyed will not be the whole, whereas if the whole perishes parts remain, in separation. The last difference is concerning the similarities and differences between the original one and its divisions. The species is composed of its genus and differentia, in which the genus is the matter of species, and the differentia is the form. The species is always the same as its genus, and only due to adding a

99 Concerning the other three types of division *per accidens*, cf. Section II.2.2.2.b.
100 On five senses of "prior", cf. Section II.1.1.1.b.

differentia, a species differs from its genus. However, it is not the same case with the whole and its parts. It is obvious that a whole consists of parts, and in this case the plurality of parts is the matter, and the composition of those same parts is the form, which is the difference between a whole and its every part. These two types of division will be applied in Boethius' *Consolatio*, which will be discussed below in Section IV.2.2.2 and Section IV.3.1.

Another type of division Boethius employs is the division of an utterance into its signification. When a single spoken sound that signifies many things is opened up and the plurality of its significations is disclosed, we need a division of an utterance into its proper significations. This type of division is used by Boethius in his theological treatise on Christology, *Contra Eutychen et Nestorium*, to give a proper definition of the word "person", which will be introduced in Section III.2.1.2.

There is a piece of vital knowledge related to theory of division, which is knowledge of definition. Boethius talks about "definition" in at least three logical works: one is *Divisio*, another is his commentary on Cicero's *Topics*, and the last one is his commentary on Porphyry's *Isagoge*. I will give a short introduction here. In his monograph on division, Boethius points out that "we may pretty well say that division and definition are in essence concerned with the same thing, since a unified definition is a conglomeration of linked division (*Divisio*, 880c)," which shows the relationship between division and definition. It can be said that division is necessary for full definitions of species, and definition is also necessary for division, for through the use of definition it can be collected together that whatever is equivocal and whatever is univocal. In his commentary on Cicero's *Topica*, Boethius devotes Book III to the discussion of definition, including the nature of definition, kinds of definition, and the method for making definitions. Here I want to introduce two ideas about definition.

The first one is the method for making definitions, which is important in Boethius' other works, such as mathematical works, theological works, and his *Consolatio*, which will be discussed one by one later.[101] For

[101] Cf. Section II.2.1.2.b, Section II.2.2.2.b, Chapter III, and Chapter IV.

division is necessary for full definitions of species, and the method of making definitions rests on division. When defining a species, first of all, this species should be of the sort that it both has a genus and is predicated of subsequent things. For definitions encompass neither higher genera nor the lower things, both of which are excluded from the definition. So it is only "the intermediate things, those that have genera and that are predicated of the others — of genera, of species, or of individuals — that can fall under definition." (*Divisio*, 886a)

The next thing that one should do is to take up the genus of that species, divide the differentiae of that genus, join a differentia to the genus, and see if the differentia joined to the genus is equal to the species which needs to be defined. If so, this is the definition of the species; if not, distribute differentiae under differentiae as often as possible until all of them joined to the genus describe the species in a definition that is equal to it (*Divisio*, 886a).

The second topic I wish to introduce is the division of types of definition. There are four kinds of definition (*C.Topica*, 323/1096).

> (1) When a definition is constructed of genus and differentiae, we unfold substantial parts. This is called "definition" in the strict sense of the name.
>
> (2) There is the sort of definition where accidents are gathered together into one thing and one thing is produced from them; it is a sort of enumeration of parts located not in substance but in a gathering together of accidents. This sort of definition is called a description.
>
> (3) If we are talking not about the accidents of a thing but rather about certain members from which a thing is composed and conjoined, and we attempt to make a definition from such members. This is called a definition by means of enumeration of parts.
>
> (4) If someone makes a definition by presenting species rather than members in the definition, it is called a definition from the division of species.

These four kinds of definition will be applied to Boethius' treatise against Eutyches and Nestorius, and they help him to find a correct definition of nature, which will be discussed in Section III.2.1.1.

By looking through Boethius' extant logical works, applications of mathematics can be found in these works. Most of applications of mathematics to logic are as examples to support arguments or notions in logic, which makes logic be understood more easily. In Boethius' mathematical works, there are also applications of logic. In Section II.2, I will introduce Boethius' mathematics and show how logical theories play a role in his mathematical works.

II.2. Boethius' Mathematics

As I introduced in Section I.2.2, there are only two extant mathematical works of Boethius, and both of them are not considered to be original ones with him. As a matter of fact, Boethius composes the works on arithmetic and music according to his purpose of making the Latin-speaking world familiar with the classical Greek knowledge. Thus, in compiling these works Boethius' remains true to his purpose. Here I want to discuss some applications of basic logic in his mathematical works. These are not numerous but can be enough to indicate how Boethius distinguishes himself from his sources.

In the following sections, I will introduce Boethius' arithmetic and music and show how basic logic elements (including division, definition and categories) are applied to them.

II.2.1. Boethius on Arithmetic

II.2.1.1. *De Institutione Arithmetica* and Its Sources

Boethius' *Arithmetica*[102] is the interpretation of Nicomachus' Greek

[102] Cf. Masi (1979) discusses Gerardus Ruffus' commentary on *Arithmetica*; Kibre (1981), "The Boethian *De Institutione Arithmetica* and the Quadrivium in the Thirteen Century University Milieu at Paris," and Masi (1981a), "The Influence of Boethius *De Arithmetica*

Introduction to Arithmetic.[103] The following Table I lists the corresponding chapters between Boethius' *Arithmetica* and Nicomachus' work on arithmetic.

Table I (PS: Nico. is short for Nicomachus, Boe. is short for Boethius)

Nico.	Boe.	Nico.	Boe.
I.1-5	I.1	II.1-2	II.1
I.6	I.2	II.3-4	II.2
I.7	I.3-6	II.5	II.3
I.8	I.7-9	II.6-7	II.4-6
I.9	I.10	II.8	II.7-9
I.10	I.11-12	II.9	II.10-12
I.11-12	I.13-15	II.10	II.13-14
I.13	I.16-18	II.11	II.15-16
I.14-15	I.19	II.12	II.17-19
I.16	I.20	II.13-14	II.20-24
I.17	1.21-23	II.15-16	II.25
I.18	I.23	II.17	II.26-30
I.19	I.24-27	II.18	II.31-32
I.20-21	I.28	II.19	II.33-34
I.22	I.29-30	II.20	II.35-39
I.23	I.31-32	II.21	II.40
		I.22	II.41-42
		II.23	II.43
PS: Nothing in Nicomachus corresponds to II.45		II.24	II.44-46
		II.25-26	II.47-49
		II.28	II.51-53
		II.29	II.54

From the above table we can see that like Nicomachus, Boethius also divides his *Arithmetica* into two books, but Boethius' first book contains 32 chapters (nine chapters longer than Nicomachus' work) and his second book has 54 chapters (twenty-five chapters longer than Nicomachus' work). The

on Late Medieval Mathematics," in Masi (ed.), pp. 67-80 and pp. 81-95; Masi (1981b).

[103] According to Cassiodorus, Apuleius of Madaura also translated Nicomachus' *Introduction to Arithmetic*, but nothing of this translation remains. In the Greek-speaking part of the world, the extant commentaries on Nicomachus' *Introduction to Arithmetic* are those of Iamblichus, Asclepius, and Philoponus. Cf. Tarán (1969), pp. 5-7.

first book focuses on the numbers (the substance of number, different divisions of number and their definitions, productions, and properties, and the relation between numbers); and the second book concerns figure numbers and proportion.

As for the relation between Boethius and his source Nicomachus of Gerasa, Martin Luther D'Ooge gives the following evaluation of Boethius' translation of Nicomachus' works.

> "In the composition of his treatise Boethius more often expands than condenses. His method is to intersperse between sections literally translated, or closely paraphrased, others in which the general principles stated by Nicomachus are furnished with exhaustive explanation and copious numerical examples. Nothing is left to the reader to supply. Almost any chapter, compared with the original one, will prove to be of this character. Boethius also supplies data in tabular form to a far greater extent than did Nicomachus. The order of the original is preserved for the most part, but occasionally a rearrangement is found."[104]

D'Ooge claims that these peculiarities are of minor importance and it is rather the omissions that have to be considered. He regards those omissions as the special difficulty that Boethius had with the logical terminology of Nicomachus.

I do not agree with the evaluation by D'Ooge. Boethius' translation style is due to his purpose, as he says in the preface of his *Arithmetica*: "I do not restrict myself slavishly to traditions of others, but with a well formed rule of translation, having wandered a bit freely, I set upon a different path, not the same footsteps." (*Arithmetica*, preface)

Boethius makes two kinds of changes to Nicomachus' composition. The first one is to add extra exposition or use formulae and diagrams to make some ideas clearer and easier to comprehend. For instance, when introducing the second division of even number, "the even times odd number", Boethius adds more explanation to tell the difference between the

[104] D'Ooge (1938), p. 133.

second division of even number and the first division of even number, "the even times even number". This extra exposition makes the reader understand the division of even number more clearly. In another case, in order to show that "the principle of plane straight-line figures is a triangle", Boethius adds four diagrams[105] to illustrate how a square, a pentagon, hexagon, and even a triangle can be divided into triangles, which gives the readers a visualized picture.

The second kind of changes consists in the reproduction of diffuse discussions in a concise style. One case of this kind is vital to tell the difference between Boethius and Nicomachus. Nicomachus believes that what is true of numbers is also true of the universe. As the discipline studying numbers, arithmetic was preexistent as a cosmic pattern in the mind of God, the creator, and according to this model the material world was formed. This is expressed by Nicomachus in I.4.2 and I.6.1 of his work on arithmetic. Nicomachus refers to the universe more than once. I list two citations from Nicomachus' work, which are completely left out of Boethius' corresponding chapters (see Table I).

> Arithmetic "existed before all the others in the mind of the creating God like some universal and exemplary plan, relying upon which as a design and archetypal example the creator of the universe sets in order his material creations and makes them attain to their proper ends."[106]
>
> "All that has by nature with systematic method been arranged in the universe seems both in part and as a whole to have been determined and ordered in accordance with number, by the forethought and the mind of him that created all things."[107]

In addition, there should be two kinds of numbers, the divine number and the scientific number. From Nicomachus' point of view, the divine number is a wholly conceptual and immaterial number and this kind of number preexisted in God's mind and was the basis of creation; while the scientific

[105] Cf. *Arithmetica*, II.16.
[106] Nicomachus, *Introduction to Arithmetic*, I.4.2; D'Ooge (1938).
[107] Nicomachus, *Introduction to Arithmetic*, I.6.1; D'Ooge (1938).

number is constantly found in connection with material things and measures them, their arrangements and their movements.[108] In I.6 of his work, Nicomachus makes a statement to distinguish the scientific number which he uses in his arithmetic from the divine number. Boethius condenses the contents on the divine number, and he does not discuss the difference between the divine number and the scientific number. Throughout Boethius' work on arithmetic, there is no such term "scientific number". Unlike Nicomachus, Boethius only uses the word "number" instead of the term "scientific number".

Nevertheless, in Boethius' work, when he states the substance of number, he also gives a short introduction saying that "Number was the principal exemplar in the mind of the creator." (*Arithmetica*, I.2) Therefore, Boethius shares the same idea with Nicomachus about the relation between number and universe, but in his *Arithmetica*, Boethius does not intend to give his readers the impression that his arithmetic gives the information about the idea of universe but only leads his reader to the arithmetic *per se* as an elementary discipline in the background of philosophy. In order to emphasize that his work on arithmetic is an elementary introduction for beginners, Boethius confines himself to elementary ideas of arithmetic.

Nicomachus' arithmetic could be seen as his theory of cosmogony, while Boethius regards arithmetic as basic knowledge for the other three mathematical disciplines which together become the preparatory way to the serious study of philosophy.

> "Arithmetic considers that multitude which exists of itself as an integral whole; the measures of musical modulation understand that multitude which exists in relation to some other; geometry offers the notion of stable magnitude; the skill of astronomical discipline explains the science of moveable magnitude." (*Arithmetica*, I.1)

Accordingly, it is not accurate to regard Boethius' work on arithmetic is the translation of Nicomachus', but it should say that Boethius interprets or paraphrases Nicomachus' Greek work on arithmetic in his own way.

[108] Cf. D'Ooge (1938), p. 98.

II.2.1.2. Applications of Basic Logic in *Arithmetica*

In interpreting Nicomachus, Boethius employs basic logical knowledge. The main theories of logic used in arithmetic involve knowledge of categories and definition.

II.2.1.2.a. Categories in *Arithmetica*

Among all logical theories, knowledge of categories relates to arithmetic most closely.[109] Of the ten categories, those that have an obvious relationship with arithmetic are quantities and relations.

The division of mathematics begins with the division of the proper objects of mathematical knowledge (incorporeal *essentiae*), which may be compared to the division of quantities. For in *Categories*, Aristotle divides quantities into two kinds. One is discrete quantity, which includes number; and the other kind is continuous quantity, including lines, surfaces, bodies which are the objects geometry and astronomy study. Similar to this division, of the four mathematical disciplines arithmetic and music concern the genus of discrete *essentiae*, namely multitude; and geometry and astronomy concern the genus of continuous *essentiae*, called magnitude. These two terms, multitude and magnitude, are used here both abstractly and concretely. Abstractly, the two terms refer to the quality; concretely, the two terms refer to the objects of such natures.

One kind of discrete quantity is number, and arithmetic studies number *per se*, therefore, arithmetic focuses on a discrete quantity, which shows how closely the category of quantity and arithmetic are related. Additionally, a characteristic peculiar to a quantity is its being called both equal and unequal, both of which are the vital terms in arithmetical study. Boethius divides I.17 of Nicomachus' work into two chapters: one is concerning a quantity related to another; and the other is relating to the types of major and minor quantity. Boethius believes that, "Any given thing in comparison to another is either equal or unequal with it." (*Arithmetica*, I.21) Thus, the quantity is first divided into equal quantity and unequal quantity. Then of the unequal quantity, some are larger and some are smaller. Finally, the

[109] Cf. Section II.1.1.1.b.

major inequalities and the minor inequality are divided into five types[110] respectively. For example, major inequalities include multiplex, superparticular, superpartient, multiplex superparticular, and multiplex superpartient, and the terms used here are adapted from Boethius' translation, which are important in mathematics.

It is obvious that the category of quantity is intimate to arithmetic. And it also shows another category that is close to arithmetic, which is the category of relation. In *Categories*, Aristotle defines "relatives" as "all such things as are said to be just what they are, *of* or *than* other things, or in some other way *in relation to* something else."[111] This definition of relatives is just the basis of relative quantities in arithmetic. For every relative has a correlative, and in most cases, they come into being together, so Boethius says that, "Any given thing which has a quantity compared to it is not known except by the other term to which it is compared." (*Arithmetica*, I.21) It could be said that without the category of relation, there would be no relative quantities, or there would be no proportion in arithmetic. And what is worse, without the category of relation another mathematical discipline, music, would lose its theoretical basis. Because music in Boethius' quadrivium depends on number theory, it must be built on proportional principles.

Of course, the relationship between categories and mathematics is neither original in Boethius nor in Nicomachus, for its history traces back to Aristotle and the Pythagoreans, as we have seen. However, from the arguments in Boethius' work on arithmetic, we find that this relationship is clear in his mind.

II.2.1.2.b. Theory of Definition in *Arithmetica*

Theory of definition[112] is so important in Boethius' logic that he states it in more than one of his logical works, especially in his monograph on division and in his commentary on Cicero's *Topics*. In Boethius' point of

[110] If "m", "n", and "k" are integers, then "multiples" are as mn:n; "superparticulars" are as (n+1):n; "superpartients" are as (n+k):n, k>1; "multiple superparticulars" are as (mn+1):n, m>1; and "multiple superpartients" are as (mn+k):n, both m and k being greater than 1.

[111] Aristotle, *Categories*, 6a36; Ackrill (1963).

[112] Cf. Section II.1.2.2.

view, "the definition shows what the thing defined is; that is, it shows its substance. A definition that consists in a genus and differentiae, however, does lay out a substance." (*C.Topica*, 319/1091) Application of definition is in line with Boethius' goal that his arithmetic is written for the beginners. Therefore, when he discusses one subject, he must, as Nicomachus does, give the definitions of the main terms of that subject, and he also adds a definition which is not found in Nicomachus' work on arithmetic, such as the definition of cyclical or spherical numbers.

II.2.1.3. Basic Ideas of Arithmetic Used in Boethius' Other Works

Applications of logic to Boethius' arithmetic are not many, but they are sufficient to show what Boethius' own approach. Arithmetic is so elementary that in Boethius' other works he employs many ideas of arithmetic. The basis ideas of arithmetic which will be used by Boethius include theory of equality, unity, number and the divine, and number and politics.

II.2.1.3.a. Theory of Equality

At the end of the first book and the beginning of the second book Boethius introduces an important theory that equality is prior to inequality, and inequality can be reduced to equality. In his work on arithmetic, Boethius emphasizes the importance of equality. He states that equality is like a matrix and takes the force of a root, so "it gives depth to the types and orders of inequality" (*Arithmetica*, I.32). Then Boethius follows Nicomachus in presenting the theory of the "three rules", according to which other sets of three in different ratios may be derived from three equal terms (*Arithmetica*, I.32), and by the reversal of which any proportion in three terms may be reduced to the original equality (*Arithmetica*, II.1). These rules are: (1) to make the first number equal to the first; (2) to put down a number equal to the first and the second; (3) to do the sum of one equal to the first, twice the second, and the third. (*Arithmetica*, I.32) Unlike Nicomachus, Boethius draws some diagrams to illustrate the process of generating, from three equal terms 1:1:1, another set of three in a different ratio 1:2:4. According to Boethius' descriptions and his diagrams, I draw a diagram to give a picture of how he uses three rules (see Diagram II).

Diagram II: How to Use three Rules

Three equal terms[113]:	1^a	1^b	1^c
Rule (1)------------------- 1^a			
Rule (2)--------------------------------------1^a+1^b			
Rule (3)--$1^a+2\times1^b+1^c$			
Therefore:	1	2	4
The same three rules:	1	3 (3=1+2)	9 (9=1+2×2+4)

From the illustration, it is obvious that "equality is the principle of all inequalities, and then from inequality all other things are derived." (*Arithmetica*, I.32) Every species of inequality can be resolved into equality. In other words, equality is the mother of any quantity related to any other things and from it comes the first procreation of a relationship and to it again is its final resolution. The notion on the nature of equality becomes one basis for his argument in Boethius' *Consolatio*, which will be discussed in Chapter IV, especially in Section IV.2.

As a Pythagorean, Nicomachus believes that what is true of numbers is also true of the universe, and the significance of "equality" is equal to that of "sameness", that is to say, they are elements and principles of the universe. "Sameness" and "otherness" are the principles of the universe, as Nicomachus says in *Introduction to Arithmetic*.[114] For example, Nicomachus states that "The physical philosopher, however, and those that take their start with mathematics, call 'the same' and 'the other' the principles of the universe."[115] When "sameness" enters into the composition of things, it makes things persist in the same fashion, preserving their identity, while when "otherness" goes into the composition of things, it causes things to change from their original forms and assumes others.[116] Unlike Nicomachus, Boethius regards his work on arithmetic as the elementary discipline, thus he tries to avoid referring "equality" to universe directly, and he does not mention "sameness" at all. Nevertheless, he has the same ideas in mind and employs them to his *Consolatio* and his theological

[113] In order to distinguish "1" in different position, I add "a", "b", "c" to "1".
[114] Cf. Nicomachus, *Introduction to Arithmetic*, II.17.1, 18.1 and 4, 19.1 and 20.2; D'Ooge (1938).
[115] Nicomachus, *Introduction to Arithmetic*, II.18.1; D'Ooge (1938).
[116] Cf. D'Ooge (1938), p. 99.

treatises, especially *De Trinitate*. When Boethius discusses the unity of the Trinity, he begins with a brief analysis of sameness and otherness, and then he invokes Aristotle's point of view on sameness, which will be given in detail in Chapter III.

Nicomachus' equality and inequality are also related to virtue and vice. When discussing division of even numbers into perfect, imperfect, and superabundant, Nicomachus makes a comparison of inequality to vices, ill health and the like.[117] While in Chapter 19, Book I of Boethius' arithmetic, he omits Nicomachus' comparison, but adds analogies to vividly describe the extreme kinds of numbers.

> "And so this number whose parts added together exceed the sum of the same number is called superfluous."... "That number is called diminished whose parts, when put together in the same way, are exceeded by the multitude of the whole term."... The perfect number is that number the sum of whose parts is "not more than the total nor does it suffer from a lack in comparison with the total." (*Arithmetica*, I.19)[118]

Boethius thinks highly of perfect number. He regards the superfluous and diminished numbers as two elements unequal and intemperate, and between them, holding the middle place between the extremes like one who seeks virtue,[119] is the perfect number. In addition, in Chapter 32 of Book I, Boethius omits Nicomachus' I.23.5 which is a reference to the ethical

[117] Nicomachus, *Introduction to Arithmetic*, I.14.2; D'Ooge (1938).
[118] For example, the number 12 is a superfluous number, because half of 12 is 6, a third part is 4, a fourth part is 3, and a sixth part is 2, a twelfth part is 1, and the total sum [6+4+3+2+1] amounts to 16 which surpassed the total of the entire body, that is 12. In the same way, the number 14 is a diminished number, because the total sum [7+2+1] amounts to 10 which is smaller than the original term, that is 14. Similarly, the number 28 is a perfect number, because the total sum [14+7+4+2+1] is 28 which is equal to the original number that is 28.
[119] Cf. Aristotle, *Nicomachean Ethics*, 1108b11-14; Barnes (1984b): "There are three kinds of disposition, then, two of them vices, involving excess and deficiency respectively, and one a virtue, viz. mean." The word "mean" used by Aristotle does not refer to the mathematical mean, and in this sense the middle place Boethius uses is not a point exactly in the middle, but a stretch of the continuum around the middle. This meaning is also used by Boethius in his treatise against Eutyches and Nestorius; cf. Section III.2.3.

virtues. Though they both mention virtue and number, in Chapter 20 of Boethius' Book I which corresponds to Chapter 16 of Nicomachus' Book I, Boethius just uses one sentence to mention this relationship generally, "there is in these a great similarity to the virtues and vices." This similarity is also applied to the theological discussions. In his theological tractate called *Contra Eutychen et Nestorium*, Boethius believes the Catholic faith is the middle way, or just like virtue, it is the perfect way, and the other two faiths of Eutyches and Nestorius are the extremes like vice. I will discuss this in more detail in Section III.2.

II.2.1.3.b. Unity

Another important aspect of arithmetic is the knowledge of "unity". In the natural arrangement of numbers, every number, except unity, has next to it two terms and half of these two terms which come before and after it. Take the number 7 for example. Two terms next to it can be 6 and 8, or 5 and 9, or 4 and 10, and 7 is just half of 6 and 8, or 5 and 9, or 4 and 10. However, unity only has the number 2 next to it and it is half of 2. For this reason, unity has a special role. "It is rightly recognized as the generator of the total extended plurality of numbers." (*Arithmetica*, I.7) And Boethius stresses the elementary role of unity, that is to say, "unity is the substance and principle of any constant quantity" (*Arithmetica*, II.1). The theory of "unity" also becomes one basis for his argument in Boethius' *Consolatio*, which will be discussed in Chapter III and Chapter IV.

II.2.1.3.c. Number and Divinity

Boethius mentions number and divinity several times, which is vital to understand the *Consolatio* of Boethius. When discussing how to produce the even-times even number, he says that "the basic ordering of numbers has come about through careful consideration and through the great constancy of divinity." (*Arithmetica*, I.9) In Chapter 27 of Book I, he also says that "Such is the divine nature of things in this disposition that all the angles are tetragons."[120] In Chapter 2 of Book II, when he discusses the discovery in each number of how many terms of the same proportions are able to precede

[120] Boethius here apparently limits the term "tetragon" to indicate square numbers; cf. Masi (1983), p. 107.

it, Boethius stresses "This must always occur as by a certain divine, not human devising". The relationship between number and divinity will be helpful to understand Boethius' *Consolatio*, which will be shown in Section IV.2.2.

II.2.1.3.d. Number and Politics

The last relevant point of arithmetic I want to point out concerns politics, which is nowhere to be found in Nicomachus' work. In Chapter 45 of Book II, Boethius discusses "which medial proportions are compared to what things in the state of public affairs". The whole chapter has only one paragraph which contains much information.

Boethius compares three types of medial proportions to kinds of state. These three ones are the arithmetic proportion, the musical or harmonic proportion, and the geometric proportion. In the arithmetic proportion of three or any number of stated terms the same and equal difference is found between all terms.[121] For example, in the disposition of 1, 2, 3, there is an equal difference of terms according to an arithmetic interval, that is, 2-1=3-2. There are four properties of the arithmetic proportion, one of them is important, that is, "there is a major proportion found in the smaller terms and a minor proportion in the larger terms." (*Arithmetica*, II.43) This means that in the arithmetic proportion 1, 2, 3, the smaller terms are 1 and 2 of which two to one is a duplex, and the larger terms are 2 and 3 of which three to two is a sesquialter. For 2>2/3, the proportion of a duplex is larger than a sesquialter, in other words, the proportion found in the smaller terms 2 and 1 is a major proportion, but the proportion found in the larger terms 3 to 2 is a minor one. According to this property, the arithmetic proportion may be compared to a kind of state known as Authoritarian Government now, which is ruled by a small group with a greater power. The second proportion is the musical or harmonic proportion[122], "in which as the highest term is when compared to the smallest term, so the difference of the larger two is when compared to the difference of the middle term and the smallest (*Arithmetica*,

[121] If there are three numbers a, b, c (c>b>a), and c-b=b-a, then they form an arithmetic proportion.

[122] Concerning the reason why it is called a harmonic proportion, please see *Arithmetica*, II.48. I will not explain it here.

II.47)".[123] For instance, in the disposition of 3, 4, 6, there is an equation, that is 6/3=(6-4)/(4-3), so 3, 4, 6 forms a harmonic proportion. Contrary to that important property of the arithmetic proportion, in a harmonic proportion, the smaller terms have minor proportions, and the larger terms have a major proportion, such as in 3, 4, 6, the proportion of the smaller terms is 4/3, and the proportion of the larger terms is 6/4, and 4/3<6/4, so in the smaller terms a minor proportion is found, and in the larger terms there is a major proportion. The harmonic proportion is compared to the state which is called Democratic Government today. This kind of state is regarded as the very best, in which there is a proportionality in the larger term, viz. the major power is owned by the larger proportions of the population. The last kind of medial proportion is the geometric proportion, in which "an equal ration is always kept and the quantity and multitude of number is regularly ignored (*Arithmetica*, II.44)".[124] Take 2, 4, 8 for example. In the disposition of 2, 4, 8, in the smaller terms there is 4/2=2 which is duplex, and in the larger terms there is 8/4=2 which is also the duplex. Unlike the property of the arithmetic proportion and the harmonic proportion, the geometric proportion provides the middle position, in which either larger or smaller terms maintains equal quantities of numbers in proportionality. The geometric proportion is compared to the state which is "of the people, as it were, and of a balanced citizenry. For in either larger or in smaller, the whole is put together with an equal proportionality of all, and there is an equality between all; there is a certain equal right balance in preserving proportions." (*Arithmetica*, II.45)

By using this analogy, Boethius makes his readers vividly grasp the characteristics of three kinds of proportions, and have better ideas on the state, such as what is the state in general, what is the state of the very best, and what is the state of the people. It may seem strange in this work to add a paragraph on number and politics, for Boethius tries to write his arithmetical work as a basic one. However, this chapter implies that Boethius is interested in state affairs, which is also shown by his motivation to write

[123] If there are three numbers a, b, c (c>b>a), and c/b=(c-b)/(b-a), then they form an musical or harmonic proportion.

[124] If there are three numbers a, b, c (c>b>a), and c/b=b/a, then they form a geometric proportion.

theological treatises[125].

II.2.2. Boethius on Music

II.2.2.1. *De Institutione Musica* and Its Sources

The extant edition of Boethius' *Musica* includes five books and the whole work ends with Chapter 19 of Book V.[126] Unlike his work on arithmetic, the sources of his *Musica* are complicated. Many citations of authors occur in Boethius' *Musica*. Among these authors, there are two important ones, Nicomachus and Ptolemy (90-168 A.D.).

Nicomachus is the person cited most often in the first four books. As I said in the earlier section about Nicomachus[127], only two of his works are preserved to us in their entirety, namely *Introduction to Arithmetic* and *Manual of Harmonics*. The first four books, especially Book I-III of Boethius' *Musica* show clearly a development from and dependence on Nicomachus' two works. After comparing Boethius' music with that of Nicomachus, Calvin Bower has reached the conclusion that the extant and the more extended musical work promised by Nicomachus served as the principal source of Boethius' first four books of *Musica*.[128]

The first book of Boethius' *Musica* is the general introduction to music. In the prologue of Book I (including Chapter 1 and Chapter 2), Boethius introduces the nature of music, the role of music, and the division of music. In Chapter 33 of Book I, Boethius sums up the first book, "how the things thus far said are to be taken" and promises that "all these will be proved both through mathematical reasoning and aural judgment." Book II and III involve the technical theories on numerical proportions to fulfill the expectation of logical demonstrations of the theories presented as dogma,

---

125 Cf. the preface to Chapter III of my dissertation.

126 Cf. Bower (1981), "The Role of Boethius' *De Institutione Musica* in the Speculative Tradition of Western Musical Thought," and Hoolloway (1981), " 'The Asse to the Harpe': Boethian Music in Chaucer," and Pizzani (1981), "The Influence of the *De Institutione Musica* of Boethius up to Gerbert D'Aurillac: A Tentative Contribution," in Masi (ed.), pp. 157-174, pp. 97-156, and pp. 175-186.

127 Cf. Section I.2.2.

128 Cf. Bower (1978).

which are introduced in Book I with little comment. The content of these two books is related to Nicomachus' work on arithmetic, including the mathematical theories which are applied to the monochord in Book IV. There are two main parts in Book IV: one is the division of the monochord, and the other is the theory of modes, which seems unrelated. From the whole structure, no matter how loose it is, the first four books can be put together as treating instrumental music, throughout which only one dissenting voice is allowed. This voice is from Ptolemy. And later in Book V, Ptolemy assumes the leading role. Book V continues to deal with instrumental music but develops in new directions. Boethius picks out the first chapter of Ptolemy's *Harmonics* as the source for Book V to complete his Latin record of Greek musical thoughts.

It is a pity that Boethius' extant work on music is not preserved in its entirety. On the one hand, it is obvious that his extant work ends abruptly, leaving eleven chapter titles without content. On the other hand, he dedicates one chapter of the first book to the general introduction to three kinds of music, which are cosmic music, human music, and instrumental music, but in his extant five books there is only knowledge of instrumental music without any further exposition on cosmic music[129] and human music. There are two possibilities. The first one is that Boethius did finish translating Ptolemy's work, but those parts have not been preserved. The second one is that he did not finish interpreting Ptolemy's *Harmonics*. No matter which possibility it is, one thing is sure: if the whole *Harmonics* had been interpreted by Boethius, he would have completed his thoughts on the three kinds of music.

II.2.2.2. Applications of Basic Logic to *Musica*

In his extant work, when interpreting his sources, Boethius applies a few basic logical theories to make his expression clearer, viz. the theory of categories, division and definition.

[129] Boethius does say a few words on cosmic music in his extant work on music. In I.27 of *Musica*, Boethius discusses "to what heavenly bodies the strings are compared", but this comparison of strings to the disposition of the heavenly spheres is hardly enough to fulfill his promises to discuss cosmic music "later more studiously"; cf. *Musica*, I.2.188.

II.2.2.2.a. Categories in *Musica*

Music considers the relationship between numbers, so sound should be known through numbers which are related through proportions. That is to say, sound is regarded as quantity. Music studies the multitude, the discrete quantities. Of quantities, some are equal and others are unequal, thus of sounds, "some sounds are also equal, while others stand at an interval from each other by virtue of an inequality (*Musica*, I.3.191)". Except the category of quantity, sound is also related to another category of action. If there is no motion, there will no sound. Sound is "a percussion of air remaining undissolved all the way to the hearing (*Musica*, I.3.189)". The frequency of motion causes different sounds. The slow and less frequent motion of the string will produce low sounds, so by contrast, the fast and more frequent motion of the string will produce high sounds. In other words, high sounds consist of more motions than low sounds. According to the property of discrete quantity, low sounds and high sounds should preserve the nature of consonance which is a vital concept in music. "In those pitches which do not harmonize through any inequality, there is no consonance at all. For consonance is the concord of mutually dissimilar pitches brought together into one." (*Musica*, I.3.191) By the help of theory of categories, sound, which is a basic but abstract term in music, can be understood easily.

II.2.2.2.b. Theories of Division and Definition in *Musica*

Other important applications of logic to music are theories of definition and division.[130] As a matter of fact, division and definition are necessary requisites for each other, in other words, division is necessary for full definitions of species, since a unified definition is made up of divisions joined together; and definition is necessary for division, since by means of definition, it could be determined what is equivocal and what is univocal (*Divisio*, 880c-880d). It could be said that division and definition are close sisters, or more specifically, division of a genus into its species is most intimate with definition. This could be demonstrated from what Boethius describes in *Divisio*.

"When I have been given a species of the sort that both has a genus

[130] Cf. Section II.1.2.2.

> and is predicated of subsequent things, I first take up its genus, I divide the differentiae of that genus, I join a differentia to the genus, and I see whether that differentia joined to the genus can be equal to the species I have undertaken to circumscribe with a definition. .... Finally, we distribute differentiae under differentiae as often as we must until all of them joined to the genus describe the species in a definition that is equal to it." (*Divisio*, 886a)

Just due to this knowledge about division and definition, Boethius makes changes to the conception of some musical terms, which makes himself distinct from his sources.

The most obvious example is the definition of consonant and dissonant sounds. These two terms appear in Boethius' Book IV, the introduction of which derives from parts of Euclid's *Sectio Canonis*. In *Sectio Canonis*, Euclid gives defines the term "consonant sounds" as two sounds which mingle, and similar to this simple way, the term "dissonant sounds" is defined as two sounds which do not mingle. Boethius was not satisfied with definitions of these two terms. Obviously, as the names of two terms show, consonant sounds and dissonant sounds are two species of the genus sound. When defining them, the first thing to be taken up is, of course, their genus, the sound. Then the significant thing is to add the differentiae. And the reason why Boethius changes the definition is likely that he thinks the differentia "which mingle" joined to the genus sounds could not be equal to the species of "consonant sounds", or in other words, could not be equal to the definition of "consonant sounds". And it is the same with the definition of "dissonant sounds". Thus, Boethius joins the differentia "when struck at the same time" to the genus. And it is still not enough, so he adds as the last differentia "pleasant and intermingled". So far, in Boethius' point of view, the description of the species "consonant sounds" is integral and equal to the definition of this term. Accordingly, in the introduction of Book IV of his work, Boethius defines, "Consonant pitches are those which when struck at the same time sound pleasant and intermingled with each other; dissonant pitches are those which when struck at the same time do not yield intermingled sound." (*Musica*, IV.1.302)

The theory of division is also employed by Boethius in his work on music. There are a few divisions in musical theories, and because of Boethius' familiarity with the theory of division, his expositions of musical divisions are clear and easy to be grasped.

The first division I want to discuss is a famous division of people whose jobs are related to music. In Chapter 34 of Book I, Boethius distinguishes musicians from performers and composers. This division is different from the divisions I described in Section II.1.2.2. Division of a genus into its species, division of a whole into its parts, and division of a spoken sound into its significations are divisions *per se*. Except those three divisions, there is another one, division *per accidens*. There are three types of division *per accidens*, which are the division of a subject into its accidents, division of an accident into its subjects, and division of an accident into accidents. Here the division of people whose jobs are related to music belongs to division of a subject into its accidents. The same subject is men who are engaged in the musical art, and its accidents include what they do, and how they deal with music. The first kind of man performs on instruments, but they make no use of reason, acting as slaves, so they are excluded from comprehension of musical knowledge. The second kind of man is like the poet, and when they are composing songs, they take advantage of a certain natural instinct but not so much by thought and reason. Therefore, the first kind of man who is related to music is called performer, and the second is named composer, both of which are not musicians in the proper sense. Unlike the former two classes, the third class of man makes full use of his reason and thought to "exhibit the faculty of forming judgments according to speculation or reason relative and appropriate to music concerning modes and rhythms, the genera of songs, consonances, and all the things which are to be explained subsequently, as well as concerning the songs of the poets (*Musica*, I.34.225)". The last kind of man who is related to music can be esteemed as musician. Thus, by applications of theory of division, Boethius defines what the musician in the proper sense is.

In Chapter 21 of Book I, Boethius introduces three genera of melodies: diatonic, chromatic, and enharmonic. The heart of Book IV is the division of the monochord, comprising Chapters 5-13. A monochord division is truly exceptional in ancient musical theory, and began from the Pythagorean diatonic genus. These are called the division of genus into its species. Similar to these divisions, division of notes also belongs to division of genus. It is common in all classical Greek musical theory that the notes are classified as either fixed or movable. However, here Boethius adds intermediate ones, "Of all these pitches, some sound completely fixed, some completely movable, whereas others sound neither completely fixed nor completely movable." (*Musica*, IV.13.335) This division is identical with Nicomachus' theory, which is found in no other author of antiquity. Nicomachus gives the definition of the category of sound which is neither completely fixed nor completely movable as one that does not move between the diatonic and chromatic but move in the enharmonic, which is unique to Nicomachus.[131] Boethius' division of notes is a little different from those above, for it is not division of genus into its species, but into to differentiae. "A genus is divided sometimes into species, sometimes into differentiae, if the species by which the genus ought rightly to be divided lack names." (*Divisio*, 880b) "Fixed" and "movable" are not the species but the differentiae. Since there is no single name of the species "fixed notes", the differentia is put in place of the species and connected to the higher genus. "For every differentia produces a species when it comes into conjunction with its proper genus." (*Divisio*, 880b)

The threefold division of notes is the same with twofold division of notes. "That with the imposition of names division is always into two terms is made clear when one on one's own initiative imposes a name for a genus or differentia that has none." (*Divisio*, 883d-884a) This means that the tripartite division, "of notes some are completely fixed, some completely movable, and others are neither completely fixed nor completely movable," could be made bipartite if expressed in this way: "Of notes that some are fixed, others movable; of the movable ones some are completely movable, others partly movable." Therefore, although Boethius chooses to follow Nicomachus' theory, he does not disprove the mainstream of Greek musical theory; on the contrary, he coincides with that.

The last division but the most important one is the three-fold kinds of music. Bower thinks highly of Boethius' dividing the kinds of music:

[131] Cf. Bower (1978), p. 26.

> "Just as Boethius seems to have coined the concept of *quadruvium* in the introduction to *De institutione arithmetica*, a concept not found in Nicomachus' treatise, so Boethius may have conceived the three-fold concept of music placed in certain instruments, music of the human being, and music of the universe in the *proemium* of *De institutione musica*."[132]

As we know, the source of the first four books is Nicomachus' works on arithmetic and music. In the first four books, human music and cosmic music are not dealt with, and in Nicomachus' extant musical work, there is no the division of music and we do not have the longer one which Nicomachus promised to write. Therefore, we may well say that the division of music was not to be part of the original text of Nicomachus. Boethius' threefold division of kinds of music is unlikely to be original with him, because implicit in the thoughts of Plato and Pythagoras there were similar ideas. However, it is sure that the first person who expressed this idea distinctly in Latin is Boethius.

Boethius' clear exposition of dividing music may result from Peripatetics' division of the forms of speech. In the commentary on Aristotle's *On Interpretation*, Boethius says, "Peripatetics who draw from Aristotle were right to posit three forms of speech (*oratio*): one which can be written in letters, a second which can be vocally expressed and a third which can be connected by thought; one contained in thoughts, the second by spoken sound, the third by letters." ($_2$*InInter.*, 29.15-20) The division of speech and the division of music are both divisions of a genus into its species. The differentia of different kinds of speed is their form, which is the same with that of music. There are hierarchies in both divisions. The lowest kind of speech is the speech that can be written in letters. In other words, this kind of speech exists in tangible objects. The highest kind of speech can be connected by thought, the incorporeal form. And the second kind of speech, which is contained by spoken sound, is the connection between the extremes. Similar to the hierarchy of speech, the lowest kind of music, instrumental music, is the music which exists in tangible instruments; and the highest type of music, cosmic music, is contained by incorporeal form.

[132] Bower (1978), p. 44.

Human music, the middle one in the hierarchy, can connect cosmic music and instrumental music. The threefold division of kinds of music is so important in Boethius' thoughts that I will provide a more detailed discussion in Section II.2.2.3.

II.2.2.3. Basic Ideas of Music Used in Boethius' Other Works

The basic idea of music which will be applied to Boethius' other works, especially *Consolatio*, is the view on three kinds of music. The lowest kind of music is instrumental music, which is produced by strings, winds and percussion. It is not hard to grasp instrumental music, for this kind of music is the one which we can listen to directly with our ears. Another reason why instrumental music is understood without effort is that the whole extant work on music of Boethius deals with instrumental music. As for the two higher kinds of music, human music and cosmic music, we only have general introductions to them at the beginning of Boethius' work. After each short introduction, Boethius uses similar sentences: one is "these things ought to be discussed later more studiously", and the other is "I shall also speak about these things later" (*Musica*, I.2.188-189), both of which together show that human music and cosmic music would be explained in detail in the later parts of his work. However, unfortunately, at the end of his extant work on music, Boethius does not come back to the topics of human music and cosmic music.

Boethius begins to choose Ptolemy as his source of his Book V, but his extant work ends abruptly at Chapter 19 of Book V with eleven titles of the remaining chapters, which just finishes the first book of Ptolemy's. Some ideas of Ptolemy's second book are condensed and larded with the corresponding part in first four books. Inferentially, it is said that if Boethius had finished his whole work on music, he would have come to the discussion on human music and cosmic music following the third book of Ptolemy' *Harmonics*. It is possible that Boethius changed some ideas on Ptolemy's human music and cosmic music, but from what Boethius applies in his *Consolatio* the knowledge of human music and cosmic music in Boethius is similar to Ptolemy's. In order to better comprehend applications of the three kinds of music in his later works, I shall here add more of Boethius' ideas on human music and cosmic music with reference to the third book of *Harmonics*.

Human music can be perceived, in Boethius' point of view, by whoever penetrates into his own self. As to how to comprehend human music, Boethius and Ptolemy both resort to a kind of analogy. Low sounds and high sounds can be brought into one to produce one consonance. Similar to this, Boethius believes the incorporeal nature of reason can be united with the body by a certain harmony. And in his third book of *Harmonics*, Ptolemy says that the consonances are in accord with the soul. Before explaining this relationship, a short introduction to the knowledge of the consonances should be given first.

One consonant sound consists of two unequal sounds, which is based on the proportional theory in arithmetic. The consonance of the diapason is that which is made in duple ratio (2:1). The diapente is that which consists of the ratio sesquialter (3:2). The diatessaron is that which occurs in the ratio sesquitertian (4:3). Ptolemy connected these three consonances to three first parts of the soul — the intellectual, aesthetic, and habitual.

> "So that that of the diapason accords with the intellectual, for mostly in each is what is simple, equal, and not different, the diapente to the aesthetic, and the diatessaron to habitual."[133]

For the ratio 3:2 is nearer the ratio 2:1 than the ratio 4:3, which means that the diapente is nearer the diapason than the diatessaron, thus the corresponding parts of the soul have the same relation, that is to say, "the aesthetic is nearer the intellectual than the habitual on account of its sharing some of the same perception." Our soul can be divided in another way, viz. into the rational, emotional, and cupidinous.

> "The rational, for the sake of an equality similar to what we have previously discussed, we equate properly to the diapason, the emotional somehow approaching it, to the diapente, and the cupidinous, arranged below it, to the diatessaron."[134]

By virtue of these three important consonances, Ptolemy discusses the

[133] Ptolemy, *Harmonics*, III.5: 96.1-3; Solomon (2000).
[134] Ptolemy, *Harmonics*, III.5: 96.28-9; Solomon (2000).

nature and species of each part of the soul. In addition, harmonic modulations resemble the circumstantial modulations of souls. For example, the peaceful conditions make the souls of citizens more tranquil and equitable, but by contrast, martial conditions cause the souls to be more rash and contemptuous. In a similar way, the tension in strings or a higher pitch gives a more arousing sense, and the relaxation in strings or the lower pitch produce a more depressing sense. Consequently, it can be said that "our souls evidently experience the same effects as the melody, as if they recognize the kindred relationship of the ratios of each state and are modeled by some movements appropriate to individual musical forms."[135] With the help of instrumental music, human music would be understood, but both music types imitate the highest level of music, cosmic music.

As I discussed above, the motion can cause sounds, so it must be the case that when a heavenly machine moves extremely fast, there is a sound that does not penetrate our ears. This kind of sound belongs to cosmic music. In the Chapter 9-16, Book II of *Harmonics*, Ptolemy makes an analogy to examine musical principles in the heavenly bodies or movements. He discusses how the harmonic consonances and dissonances resemble those in the Zodiac, how the succession in the notes resembles the longitudinal movement of the stars, how the stellar movement in altitude compares with the harmonic genera, that modulations of *tonoi* are like stellar crossings in latitude, on the similarity of the tetrachords and the aspects of the sun, by what first numbers might the fixed notes of the perfect system be compared to the first spheres in the cosmos, and how the combinations of the planets should be compared to those of the notes. From these comparisons, it is remarkable that although we could not hear cosmic music, it does exist. Boethius believes that the failure of the sound of a moving heavenly machine to penetrate our ears happens necessarily for many reasons. Nevertheless, it is impossible that such extremely fast motion of such large bodies should produce absolutely no sound, especially since the courses of the stars are joined by such harmonious union that nothing so perfectly united, nothing so perfectly fitted together, can be realized (*Musica,* I.2.187-188). Cosmic music means cosmic harmony, which holds together

[135] Ptolemy, *Harmonics*, III.7: 99.25; Solomon (2000).

the four elements of earth, air, fire, and water, or the cycle of the four seasons, in consonance and equilibrium. It leads to the change of season and determines the movement of the celestial bodies. Cosmic music is discernible especially in those things that are observed in heaven itself or in the combination of elements or the diversity of seasons.

Of course one discerns musical principles through instrumental music, but Boethius suggests his readers to go beyond the music of instruments to the music of human beings, and even further to the music of the universe. For likeness attracts, and thus the harmony of proportions in human beings could help us be attracted to the harmony in the universe which is the pure proportion or the idea of harmony itself. The three kinds of music and the process from instrumental music to human music and finally to cosmic music are very important in Boethius' thought, which will be fully exhibited in Boethius' *Consolatio.*[136]

II.3. Conclusion

From the discussion of Boethius' logic, we can find that applications of mathematics to logic are mostly examples. Boethius' logical knowledge does have an influence on his mathematical works, as has been discussed above. Although Boethius' mathematical works are not original, his application of logic gives his works a distinct character. Even at this level there is a certain connection between mathematics and logic.

As elementary disciplines, both mathematics and logic play an essential role in Boethius' philosophy. When Boethius discusses theological issues, he employs this elementary knowledge of mathematics and logic, and even when he faces death, he uses his knowledge of mathematics and logic to console himself, which will be discussed in Chapter III and Chapter IV, respectively.

[136] Cf. Section IV.2.2.1.