Boethius and the Importance of Basic Logic and Mathematics for Philosophy

Wang, Q.

Boethius
and the Importance of
Basic Logic and Mathematics for
Philosophy

Qi WANG

Boethius and the Importance of Basic Logic and Mathematics for Philosophy

Doctoral Thesis, Leiden University, The Netherlands

Author: Qi WANG

Printing: Ridderprint BV, Ridderkerk, The Netherlands

Cover: Robert Kanters, Ridderprint BV, Ridderkerk, The Netherlands

Boethius and the Importance of Basic Logic and Mathematics for Philosophy

Proefschrift

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van Rector Magnificus prof.mr. C.J.J.M. Stolker,
volgens besluit van het College voor Promoties
te verdedigen op donderdag 6 maart 2014
klokke 10.00 uur

door
Qi Wang
geboren te Huludao (China)
in 1982

PROMOTIECOMMISSIE

Promotor:

Prof.dr. F.A.J. de Haas

Overige Leden:

Prof.em.dr. E.P. Bos

Prof.dr. P.J.J.M. Bakker

Dr. R.M. van den Berg

Table of Contents

Abbreviations of Boethius' Works

(1) Mathematical Works

Arithmetica — *De Institutione Arithmetica* (Principles of Arithmetic)
Musica — *De Institutione Musica* (Principles of Music)

(2) Logical Works

C.Topica — *In Ciceronis Topica* (Commentary on Cicero's *Topics*)
Divisio — *De Divisione* (On Division)
In Categorias — *In Categorias Aristotelis* (Commentary on Aristotle's *Categories*)
$_1$InInter., $_2$InInter. — *In Aristotelis De Interpretatione* (*editio prima and editio secunda*) (Commentaries on Aristotle's *On Interpretation*, 1 and 2)
$_1$InIsag., $_2$InIsag. — *In Isagogen Porphyrii Commenta* (*editio prima* and *editio secunda*) (Commentaries on *Isagoge*, 1 and 2)
I.S.Categorico — *Introductio ad Syllogismos Categorico* (Introduction to Categorical Syllogisms)
S.Categorico — *De Syllogismo Categorico* (On the Categorical Syllogisms)
S.Hypothetico — *De Syllogismo Hypothetico* (On Hypothetical Syllogisms)
TopicisD. — *De Topicis Differentiis* (On Topical Differentiae)

(3) Theological Works

De trinitate — *De Trinitate* (The Trinity is One God not Three Gods)
Utrum pater — *Utrum Pater et Filius et Spiritus Sanctus de Divinitate Substantialiter Praedicentur* (Whether Father, Son, and Holy Spirit may be Substantially Predicated of the Divinity)
Quomodo substantiae — *Quomodo Substantiae in eo Quod Sint Bonae Sint cum non Sint Substantialia Bona* (How Substances can be Good in Virtue of Their Existence without Being Absolute Goods)
De fide — *De Fide Catholica* (On the Catholic Faith)
Contra Eutychen et Nestorium — *Contra Eutychen et Nestorium* (A Treatise Against Eutyches and Nestorius)

(4) Last Work

Consolatio — *Consolatio Philosophiae* (Consolation of Philosophy)

I. Introduction

Anicius Manlius Severinus Boethius (480-525 A.D.) is regarded as a great thinker of the early Middle Ages. Indeed, Boethius is a man of immense erudition and his works touch upon many philosophical fields, including mathematics, logic and theology. In Boethius' classification system, mathematics and theology belong to speculative philosophy, and logic is both a part of philosophy and an instrument of philosophy.[1] In his short life[2], he wrote some monographs and commentaries, which can be classified into four main groups: his works on mathematical disciplines including arithmetic, music, geometry, and astronomy[3]; his translations, commentaries, and monographs on Aristotelian logic and areas of rhetoric closely related to logic[4]; his five short theological treatises; and his last but most famous work, *Consolatio Philosophiae*[5].

Despite writing these works, Boethius is not regarded as an original thinker. Instead, his mathematical works and most of his logical works are regarded as translations, and even his monographs on logic are also considered to be little more than translation. His theological tractates and *Consolatio Philosophiae* are thought to patch up Neoplatonic materials. It seems that Boethius was nothing less than a transmitter of ancient thoughts to the Middle Ages. However, I will argue a different perspective on the issue of Boethius as an original thinker.

There are many translations and commentaries of Boethius' works

[1] Boethius discusses the division of philosophy both in his theological treatise on the Trinity and his commentary on Porphyry's *Isagoge*. Cf. *De trinitate*, II.5-21 and *2InIsag.*, 74C-D.

[2] On Boethius' life, cf. Coster (1968); Matthews (1981), "Anicius Manlius Severinus Boethius," and Kirkby (1981), "The Scholar and His Public," in Gibson (ed.), pp. 15-43 and pp. 44-69; Kaylor (2012), "Introduction: The Times, Life, and Work of Boethius," in Kaylor and Phillips (eds.), pp. 1-46; Marenbon (2003), pp. 7-16; Moorhead (2009), "Boethius' Life and the World of Late Antique Philosophy," in Marenbon (ed.), pp. 13-33; Morton (1981); Patch (1947); Stewart (1974), pp. 15-54.

[3] Cf. Section I.2.2.

[4] Cf. Section I.2.3.

[5] Cf. Section I.2.4.

(especially *Consolatio Philosophiae*[6]) in different languages, as well as several studies on Boethius. Nonetheless, Boethius could not be studied as a whole[7]. Boethius is learned, grasping knowledge of various subjects. However, scholars from different fields, such as historians, philosophers, theologians, and literary scholars, tend to focus only on the part of Boethius' thoughts that relate to their own disciplines.[8] The result is a lack of understanding Boethius as a whole. However, there are few scholars researching the relationship between Boethius' mathematics and logic or applications of them in his theology and *Consolatio Philosophiae*.[9] My dissertation will focus on such analysis.

In my dissertation, I will reassess Boethius' mathematics and logic, and their roles in philosophy, and on the basis of these, I will explore the

[6] Godden and Irvine (2009) edit two volumes of the old English versions of *Consolatio*. On translations and commentaries of *Consolatio*, cf. Brancato (2012), "Readers and Interpreters of the *Consolatio* in Italy, 1300–1500," and Johnson (2012), "Making the *Consolatio* in Middle English," in Kaylor and Phillips (eds.), pp. 357-412 and pp. 413-446; Cally (1996); Jefferson (1917); Kaylor (1992 and 2007). And Donato (2013a) reinterprets *Consolatio* in a striking new way. Donato investigates how the study of *Consolatio* can profit from the knowledge of Boethius' cultural, political and social background that is available today.

[7] Cf. Marenbon (2009), "Introduction: Reading Boethius Whole," in Marenbon (ed.). In that introduction, Marenbon gives two main reasons why Boethius is usually not read as a whole. The first one has nothing to do with Boethius, but points to the limitations of academic specialization; the second one is that Boethius is seen as an almost entirely unoriginal thinker, or regarded mainly as a sort of a conduit to the Middle Ages.

[8] On Boethius' logic, cf. De Rijk (1964a, 1964b, and 1988); Dürr (1951); Green-Pedersen (1984); Magee (1989, 1998, and 2010); Shiel (1957, 1958, 1974, and 1982); Stump (1974, 1978, 1981a, 1981b, 1987, and 1988). On Boethius' music, cf. Bower (1978, 1981, 1984, and 1989). On Boethius' Liberal Arts, cf. Masi (1979, 1981a, and 1981b). On commentaries on Boethius, cf. Häring (1966 and 1971); Scott (1993). On Boethius' literary, cf. Lerer (1985). On a concordance of Boethius, cf. Cooper (1928). On comparisons of Boethius with other people, cf. Arber (1942); Coster and Patch (1948); Ford (1968); Merlan (1968); Shiel (1957 and 1974). And Barrett (1940), Chadwick (1981b), Gibson (1981), and Marenbon (2003) give a full description of Boethius and his thoughts, but they, too, introduce Boethius according to separate fields without discussing the connections between these fields.

[9] Chamberlain wrote an article on music in Boethius' *Consolatio* from the perspective of literature; cf. Chamberlain (1970). Kijewska points out some connections between Boethius' mathematics and theology, especially *De trinitate* and *Consolatio* without detailed explanations; cf. Kijewska (2003), pp. 632-637. Schrade wrote an article on music in Boethius' philosophy; cf. Schrade (1947).

connections between mathematics and logic in Boethius and their applications to theological or philosophical topics. However, I will not focus on the applications of all his mathematical and logical knowledge but only on his basic ideas of arithmetic and music, and basic logic including knowledge of categories, theories of division and definition.[10]

Scholars have investigated the chronology of Boethius' work according to various methods. Although they have reached different conclusions on the chronology[11], they agree on a general order. Boethius first finished works on mathematics among which only works on arithmetic and music are extant.[12] After the study on mathematics, he started studying logic. Then he applied theories or examples of mathematics and theories of basic logic to his investigations into theological issues and in his *Consolatio Philosophiae*. Boethius studied Aristotle's categories, and using categorical logic, he discussed the Nature and Person of God. He usually defined a concept first and then put it to use in theology. He also focused explicitly on the arguments and reasoning that serve his discussions, which is particularly helpful in the interlocution with Lady Philosophy in the *Consolatio Philosophiae*.

My dissertation has three main parts. The first part (Chapter II) focuses on Boethius' elementary disciplines, logic and mathematics. I will give a short introduction to these two disciplines and their roles in Boethius' whole knowledge; then I will point out applications of each one to the other. The second and third parts (Chapter III and Chapter IV) trace applications of mathematics and basic logic in Boethius' theological treatises and in the *Consolatio Philosophiae* respectively. Before the three main chapters, the background of Boethius will be introduced first. In this chapter, I will give a description of the curriculum before Boethius and an introduction to Boethius' curriculum on mathematics, logic, theology, and his *Consolatio Philosophiae*.

[10] Cf. Section II.1.2, Section II.2.1.3, and Section II.2.2.3.
[11] Cf. Suto (2012), Chart 2; De Rijk (1964a and 1964b); McKinlay (1907); Phillips (2012), "Anicius Manlius Severinus Boethius: A Chronology and Selected Annotated Bibliography," in Kaylor and Phillips (eds.), pp. 551-590.
[12] Cf. Section I.2.2.

I.1. The Curriculum Before Boethius

The first work of Boethius focuses on arithmetic, *De Institutione Arithmetica*. The preface of this works is a letter to Symmachus[13] in which Boethius sets forth his plans. Boethius says that it was at Symmachus' request that he took it upon himself to make Latin readers acquainted with the riches of Greek culture, which may refer to Symmachus' long-term plan to bring over Greek writings to the Roman language.[14]

At the beginning of his work on arithmetic, Boethius says that he plans to finish works on arithmetic, music, geometry, and astronomy. Boethius gives the name "quadrivium" ("the four ways" or "a place where four roads meet") to these four mathematical disciplines. The quadrivium is regarded as a part of the study of Liberal Arts, and the other part is the trivium which was given the name in imitation of the quadrivium in the Carolingian era, including grammar, logic, and rhetoric. As a matter of fact, the disciplines in Liberal Arts changed many times before the Middle Ages.

Artes liberales (Liberal Arts) used by the Romans stemmed from the Greek term *enkuklios paideia* or *enkuklia mathemata*. The Greeks used *enkuklios paideia* or *enkuklia mathemata* for the subjects that should be learnt for three years after the age of fourteen (from this age, the education should be changed from primary to secondary).[15] The Liberal Arts were taken over from Greek ideas of education with very little change. Greek ideas of education were referred to those subjects, which in classical antiquity were considered essential for a free citizen to study. The persons who were most interested in the full span of subjects were philosophers. Therefore, the curriculum of Liberal Arts was taught only by philosophers, which is demonstrated in the educational programs of Greek philosophers

[13] Quintus Aurelius Memmius Symmachus (died 526 A.D.) was a 6th-century Roman aristocrat and a historian. He was a patron of secular learning and became the consul for the year 485. He supported Pope Symmachus (cf. the preface to Chapter III in my dissertation) in the schism over the Popes' election and was executed with his son-in-law Boethius after being charged with treason. Cf. McGeachy (1942).
[14] Cf. *Arithmetica*, preface; Reiss (1982), p. 12.
[15] Cf. Clarke (1971), p. 2.

Pythagoras (fl. 530 B.C.), Isocrates (436-338 B.C.), and Plato (429-347 B.C.).

Among Liberal Arts, four mathematical disciplines had first been taught first by the Pythagoreans[16]. Pythagoras found the mathematical proportion between intervals, which he used to establish the mathematical foundation for music, and he classified music as one of mathematical disciplines. It is Pythagoras, Proclus (about 411-485 A.D.) believes, who "transformed mathematical philosophy into a scheme of liberal education."[17] And a Pythagorean mathematician Archytas (428-347 B.C.) concludes, at the beginning of his *Harmonics*, that "indeed concerning geometry and arithmetic and sphaeric they handed down to us a clear set of distinctions and not least also concerning music. For these sciences seem to be akin."[18] Thus, he believed that these four sciences had been derived from mathematicians, or, in other words, that these four sciences were subcategories of mathematics. Strictly speaking, because of Pythagoras, mathematics became an intellectual study and had its place among the Liberal Arts. Therefore, Pythagoras should be regarded as the founder of the quadrivium. The real founder of the trivium in Liberal Arts should be the Sophists. During about 460 B.C. to 380 B.C., Sophists began the study of dialectic, grammar, and the technical study of language, of the etymology and usage of words.

In Antiquity, the importance of mathematics was believed to rest on its usefulness. This view came from Isocrates, who says that "rather it seems to me both that those who hold, that this training is of no use in practical life are right and that those who speak in praise of it have truth on their side."[19] He regarded mathematics as "a gymnastic of the mind and a preparation for philosophy."[20] Isocrates taught mathematics, physical science, history, and

[16] On Pythagoreans, cf. Kahn (2001); O'Meara (1989); Zhmud (2012). On Iamblichus' Platonic curriculum, cf. O'Meara (2003), pp. 62-65.
[17] Proclus, *A Commentary on the First Book of Euclid's Elements*, II.iv.65; Morrow (1970). Cf. Heath (1956), pp. 65-117 on Pythagorean achievements in arithmetic; and pp. 141-169 on Pythagorean geometry.
[18] Archytas, *Harmonics*; Huffman (2005), p. 109. Cf. Huffman (2005), pp. 126-127 are on Archytas and the sciences.
[19] Isocrates, *Antidosis*, 263-264; Norlin (1929).
[20] Isocrates, *Antidosis*, 266; Norlin (1929).

other disciplines, but all of such disciplines are considered as preliminary elementary knowledge, and the final purpose is the study of rhetoric and the participation in ruling the state.[21] Not unlike Isocrates, Plato advises in his *Republic* that the rulers of the city-state must receive the education of physical exercise and poetry when they are young, and after they grow up to be adults, trainee philosopher kings should learn arithmetic, geometry, solid geometry, astronomy, and harmonics to surpass the sensible world and enter the abstract intelligible world.[22] We can see the connection between the Liberal Arts and his program of education.

However, the content of Liberal Arts was not fixed. Later, Marcus Terentius Varro (116-27 B.C.), an ancient Roman scholar and writer, added medicine and architecture which were expounded in his *Disciplinarum Libri IX*, so Liberal Arts included nine disciplines. The earliest one who had a clear idea about seven Liberal Arts is Martianus Capella (fl. 5th century A.D.), who wrote a work, *De Nuptiis Philologiae et Mercurii* (*On the Marriage of Philology and Mercury*)[23]. As a matter of fact, between Varro and Capella, there is no evidence that any handbook of these seven Liberal Arts was written, until Capella's contemporary Augustine of Hippo (354-430 A.D.) started one. Later in Roman history, the seven Liberal Arts took a more permanent shape, and included grammar, dialectic (or logic), rhetoric, arithmetic, music, geometry, and astronomy. Augustine planned to write works on all these seven Liberal Arts, but in the end he only finished the works on grammar, rhetoric, dialectic, some works on geometry, and six books on music. Augustine believed the number seven is related to the Holy Spirit. Thanks to the authority of Augustine and the mystery of the number seven, "seven Liberal Arts" were inherited and developed. "Seven" became the standard number of Liberal Arts. A century later, in the second part of *Institutiones Divinarum et Saecularium Litterarum* (*Institutes of Divine and Secular Learning*)[24], Cassiodorus (about 485-580 A.D.) introduces grammar, rhetoric, dialectic, arithmetic, music, geometry, and astronomy. He believed the Liberal Arts as secular learning

[21] Cf. Isocrates, *Antidosis*, 261-268; Norlin (1929).
[22] Cf. Plato, *Republic*, 521e-531d; Waterfield (1993).
[23] Cf. Stahl, Johnson, and Burge (1971 and 1971-1977); Sharples (1991).
[24] Cf. Cassiodorus, *An Introduction to Divine and Human Readings*; Jones (1946).

were not opposed to divine learning, and that the disciplines of the Liberal Arts were part of the content of the Bible. Therefore, Cassiodorus encouraged studying the seven Liberal Arts. Because of him, the seven Liberal Arts received a sacred and unshakable authoritative position.

So in the Middle Ages, "Liberal Arts" referred to seven arts, i.e. arithmetic, geometry, music, and astronomy, which are called the quadrivium; and Grammar, Rhetoric, and Logic, which are called the trivium. The initial medieval university curriculum was constituted of the trivium which dealt with language, and the quadrivium which dealt with mathematical disciplines. In the Middle Ages, these seven Liberal Arts became the preparatory curriculum and the necessary way to the higher wisdom (philosophy and theology) and were taught in the schools. Before the Renaissance, most Greek works were seldom known by people, and Western Europe could get only limited knowledge about ancient civilization. Hence the seven Liberal Arts were regarded as the sum total of all human secular knowledge.

I.2. Boethius' Curriculum

In Boethius' curriculum[25], the seven Liberal Arts are all involved. For example, he wrote works on four mathematical disciplines, and he also wrote, interpreted, or commented on logic, also in its relation to rhetoric. However, Boethius devoted most of his time on logic and four mathematical disciplines. The intellectual environment in which Boethius lived influenced the subjects on which he focused.

I.2.1. Intellectual Environment in Boethius' Times

During the centuries before Theodoric the Great[26], the number of disciplines in Liberal Arts was fixed at seven, including grammar, rhetoric, dialectic, arithmetic, music, geometry, and astronomy. Among them, the

[25] Cf. Reiss (1982, pp. 9-14) discusses a program of education about Boethius.

[26] Theodoric (454-526 A.D.) was king of the Germanic Ostrogoths (471-526 A.D.), ruler of Italy (493-526 A.D.), regent of the Visigoths (511-526A.D.), and a viceroy of the Eastern Roman Empire.

principal one was rhetoric. In Athens and in Rome, rhetoric was essential to men who wanted to pursue political careers. The needs of law and government dominated largely in the cultural tradition of Rome, and rhetoric could provide men with the skills relevant to speaking in law courts and in political assemblies. Thus, it was necessary for those men who wanted a political career to learn rhetoric. Subordinate to rhetoric was grammar, which was preparatory to rhetoric. Other disciplines in Liberal Arts tended to be pushed out.

Although in the Greek-speaking world the Liberal Arts continued to play their role in education, the situation in the Latin-speaking West was different.[27] The Latin world possessed good guides in grammar and in rhetoric but was less well served in the other Liberal Arts. A typical example is Augustine. There were many books on grammar and rhetoric, but no books on other Liberal Arts. The absence of textbooks suggests an absence of teacher; as a result, Augustine had to acquire his knowledge of other Liberal Arts, such as logic and mathematics without the help of any teacher.[28] The example of Augustine suggests that among these seven Liberal Arts, rhetoric and grammar were paid most attention, and were taught by teachers. It is true that the other Liberal Arts, i.e. logic and mathematics, passed to Rome from the Hellenistic world and were still recognized as part of education. However, they were almost neglected in practice.

As a philosopher, Boethius regarded it as part of his vocation to share the richness of Greek thoughts with the Latin speakers. And the weakness of the mathematical disciplines and logic in the Latin world stimulated Boethius' interest, so he dedicated himself to arithmetic, music, geometry, astronomy and the higher studies of philosophical logic.

There is an order in Boethius' curriculum on mathematics and logic, which is affected by the philosophical situation at his time. In other words, the study order of Boethius is in line with that of the Neoplatonic schools.

Ammonius Saccas (3rd century A.D.) tried to harmonize the doctrines of

[27] Cf. Clarke (1971), p. 7.
[28] Augustine, *Confessions*, IV.16.30; Schaff (1886).

Plato and Aristotle, and from his time, the Platonic school tended to absorb Aristotelian doctrine, and it also absorbed the Pythagoreans. Later, a significant development of philosophy happened with the pioneers, Plotinus (about 205-270 A.D.) and his pupil Porphyry (234-305 A.D.). To modern scholars, this movement was known as Neoplatonism, although people of the time referred to its exponents simply as Platonists. In the Neoplatonic tradition, thinkers focused on reworking Plato's ideas, but they began their curriculum of teaching with explanations of some of Aristotle's works.[29] Influenced by Porphyry[30] and other Neoplatonists, Boethius made a great plan to harmonize Plato and Aristotle. At the beginning of Book 2 of the second commentary on Aristotle's *On Interpretation*, Boethius gives a clear exposition of his purpose.

> "It is my fixed purpose, if the favour of the godhead should approve more firmly, to translate into the Roman style and write commentaries on the whole Aristotelian corpus that has come into my hands. ... And so anything written by Aristotle which results from his subtlety in logic, serious mindedness in moral experience and sharp-wittedness in natural truth, all of this I will translate in proper order and illuminate, too, with what light a commentary affords. And by translating all the dialogues of Plato and also commenting on them I will bring them into a Latin form. When this is achieved I would not shrink from somehow bringing the ideas of Plato and Aristotle into a single harmony and proving that they do not disagree in everything as most think but that they agree in most things and in the most important philosophical issues. ... When this is achieved I would not shrink from somehow bringing the ideas of Plato and Aristotle into a single harmony and proving that they do not disagree in everything as most think but that they agree in most things and in the most important philosophical issues." (*2InInter.*, 58)

Although Boethius did not finish his great plan, he carried on the ideas of

[29] Cf. Moorhead (2009), "Boethius' Life and the World of Late Antique Philosophy," in Marenbon (ed.), p. 25.
[30] Cf. Section I.2.3.

his plan in his works. Boethius chose to follow Porphyry and accepted his approach to the interpretation of Aristotelian logic. In his point of view, the subject matters of Aristotle and Plato are different, and there is no conflict between them.[31] For example, in his theological treatise against Eutyches and Nestorius, his treatment of the word "person" varies according to the degree to which Aristotelian language about primary or individual substance is enfolded within a Platonic metaphysic of universals.[32] In his consolation, he also tries to harmonize Aristotle and Plato, which is demonstrated in his definition of man as the combination of the definitions of Aristotle and Plato.[33]

In Plotinus' point of view, the philosopher "must be given mathematical studies to train him in philosophical thought and accustom him to firm confidence in the existence of the immaterial," which means that the first stage of education should be the mathematical disciplines. And after mathematics, the mind must acquire skill in dialectical methods.[34] Accordingly, Boethius started his project with four mathematical disciplines, i.e. arithmetic, music, geometry, and astronomy, and then the study on logic.

I.2.2. Boethius on Mathematics

Among four works on mathematics, only two works on arithmetic and music are extant,[35] which are *De Institutione Arithmetica* (*Principles of*

[31] Cf. Marenbon (2003), p. 41.
[32] Cf. Section III.2.1.2.
[33] Cf. Section IV.2.2.2.
[34] Cf. Plotinus, *Enneads*, I.3; Armstrong (1966).
[35] According to Cassiodorus, Boethius completed his work on geometry, which is the translation of Euclid' work; cf. Cassiodorus, *An Introduction to Divine and Human Readings*, II.vi.3; Jones (1946). From the letter of "Theoderic to the Illustrious Patrician Boethius", Cassiodorus suggests that Boethius finished his works on quadrivium. "For it is in your translations that Pythagoras the musician and Ptolemy the astronomer are read as Italians; that Nicomachus on arithmetic and Euclid on geometry are heard as Ausonians [Italians]; that Plato debates on metaphysics and Aristotle on logic in the Roman tongue; you have rendered Archimedes the engineer to his native Sicilians in Latin dress. (Cassiodorus, *Variae*, I.45.4; Barnish, 1992/2006)" Cf. Chadwick (1981b), p. 103; Stahl (1962), pp. 196-197. On Boethius' works on quadrivium and their influence, cf. Chadwick (1981b), pp. 102-107; Caldwell (1981), "The *De Institutione Arithmetica* and the *De Institutione Musica*," and Pingree (1981), "Boethius' Geometry and Astronomy," and White (1981), "Boethius in the Medieval Quadrivium," in Gibson (ed.), pp. 135-154, and pp.

Arithmetic)[36] and *De Institutione Musica* (*Principles of Music*)[37]. The principal person related to these two works is Nicomachus of Gerasa (60-120 A.D.).

Nicomachus is regarded as a Pythagorean, and Pythagoreanism to him is not simply a mathematically based philosophy. He believes that there is an apparent harmony in the cosmos, and he synthesizes mathematical theories with this belief, which he considers as binding together human soul and human body.[38] Only two works of Nicomachus are extant, viz. *Introduction to Arithmetic* and *Manual of Harmonics*, which have important influences on Boethius' thoughts.

Introduction to Arithmetic is Nicomachus' most famous work, which is of much use to introducing the discipline of arithmetic, as its name suggests. This work is significant for arithmetic to become an independent discipline. Kline gives a proper evaluation of this work.

> "The *Introductio* had value because it was a systematic, orderly, clear, and comprehensive presentation of the arithmetic of integers and ratios of integers freed of geometry. It was not original as far as ideas were concerned, but was a very useful compilation. It also incorporated speculative, aesthetic, mystical, and moral properties of numbers, but no practical applications. The *Introductio* was the standard text in arithmetic for a thousand years. At Alexandria, from the time of Nicomachus, arithmetic rather than geometry became the favorite study."[39]

155-161, and pp. 162-205; Evans (1975 and 1981); Guillaumin (2012), "Boethius's *De institutione arithmetica* and its Influence on Posterity," and McCluskey (2012), "Boethius's Astronomy and Cosmology," and Moyer (2012), "The Quadrivium and the Decline of Boethian Influence," and Rimple (2012), "The Enduring Legacy of Boethian Harmony," in Kaylor and Phillips (eds.), pp. 135-162, and pp. 47-74, and pp. 479-518, and pp. 447-478; Masi (1979, 1981a, and 1981b); Reiss (1982), pp. 14-27.

[36] In the following chapters, I will use *Arithmetica* as the short name of Boethius' work on arithmetic. The translations of *Arithmetica* I cite in my dissertation are Masi's (1983).

[37] In the following chapters, I will use *Musica* as the short name of Boethius' work on music. The translations of *Musica* I cite in my dissertation are Bower's (1989).

[38] To Nicomachus, this is a synthesis of science and religion which has merit. Cf. Chadwick (1981b), p. 72.

[39] Kline (1972), p. 138.

Manual of Harmonics is a brief work that can be considered as a minimal introduction to Pythagorean musical thoughts. In this work, he promises that when he has leisure time and a rest from his journey, he would compose "a longer and more detailed" musical treatise, *Introduction to Music*.[40] As it is, we do not have the *Introduction to Music*, either because Nicomachus did not finish it, or because it was not preserved.

At the beginning of his *Introduction to Arithmetic*, Nicomachus states the standard Neopythagorean order of four mathematical disciplines — arithmetic, music, geometry, and astronomy — which he called *methodoi,* meaning paths or methods for proceeding upwards in a steady progress towards higher knowledge and wisdom. At the beginning of his *Arithmetica* Boethius gives an exposition about why the first discipline to be learned is arithmetic, next music, then geometry, and last astronomy — following Nicomachus. The word "quadrivium[41]" that Boethius has coined, has a similar meaning to *methodoi*, and throughout the Middle Ages, "quadrivium" was the name for the four mathematical disciplines.

It is easy for people to understand that arithmetic and geometry are mathematical disciplines, which is also the modern idea on mathematics. In Antiquity, arithmetic was a discipline that dealt with the theory of number, which means it studies multitude in itself. Geometry, after the day of Euclid (fl. 300 B.C.), would follow in Euclid's footsteps: it began with definitions, then set out certain postulates and axioms, and finally proceeded to prove a series of theorems; geometry studies immovable magnitude. Unlike arithmetic and geometry, astronomy might seem to be different, but essentially they are similar, for astronomy studies movable magnitude. It is not hard to include these three disciplines in mathematics, but it is a little harder to understand that music belongs to mathematical disciplines. The learning of music included two aspects: theoretical and practical. Aristotle stressed musical education which is concerned with the practical aspect of music. When regarding music as one of Liberal Arts, however, one considers theoretical aspects of music. Music, according to the ancient theorists, had two sides. One side of music is rhythm (including metric),

[40] Nicomachus, *Manual of Harmonics*, I; Levin (1994).
[41] Cf. Section I.1.

which was taken over for teaching aims by grammarians in later antiquity. The second side of music is melody, and in later antiquity, this side was left to the musicians who discussed notes, intervals, scales, and the characteristics of the different modes.[42] And all of these subjects in music were studied together with an introduction in arithmetic, especially the proportional principles which were the essential for understanding musical theories. This relation shows that the study of music in Antiquity depended on number theory, or in other words, music became one of mathematical disciplines.

The mathematical disciplines play a vital role in Boethius' philosophy, which derives from the Platonic tradition on mathematics. Plato believes that pure numbers can elevate the soul to understand its own nature. The four mathematical disciplines which all involve the study of number can train the mind to understand truth and reality and then make the mind transcend the physical world of sense-perception.[43] In this way Plato regards mathematics as preparatory to, or forming part of philosophy.

In Boethius' point of view, the quadrivium is an indispensable preliminary to both the investigation of the physical world and the purification of the human soul that is necessary to comprehend the divine. Therefore, by the quadrivium a superior mind is brought from knowledge of things in the world offered by the senses, to the more certain things of the intellect, and is prepared for abstract reasoning. He also summarizes Nicomachus' quotation from *Republic*, 527d-e[44] that "There are various steps and certain dimensions of progressing by which the mind is able to ascend so that by means of the eye of the mind, which (as Plato says) is composed of many corporeal eyes and is of higher dignity than they, truth

[42] Cf. Clarke (1971), p. 54.

[43] Plato, *Republic*, 524e-531d; Waterfield (1993).

[44] Cf. Nicomachus, *Introduction to Arithmetic*, I.III.7; D'Ooge (1938). The original passage of Plato reads: "You amuse me: You're like someone who's afraid that the majority will think he is prescribing useless subjects. It is no easy task — indeed it is very difficult — to realize that in every soul there is an instrument that is purified and rekindled by such subjects when it has been blinded and destroyed by other ways of life, an instrument that it is more important to preserve than ten thousand eyes, since only with it can the truth be seen. (Plato, *Republic*, 527d-e; Cooper, 1997)" Cf. Theon of Smyrna quotes the same passage in *Mathematics Useful for Understanding Plato*, I.1; Lawlor (1979).

can be investigated and beheld. This eye, I say, submerged and surrounded by the corporal senses, is in turn illuminated by the disciplines of the *quadrivium*." (*Arithmetica*, I.1) Boethius clearly expresses that if anyone wants to reach the highest perfection of the disciplines of philosophy, he must dwell on the nobility of such wisdom in the quadrivium, which will hardly be hidden from those properly respectful of expertise. If someone lacks knowledge of the quadrivium, he cannot find the truth, so on this conception of the quadrivium, truth cannot be rightly known without it. Boethius believes the quadrivium is the path to wisdom; philosophy is the love of wisdom, so he who spurns the quadrivium shows contempt for philosophy (*Arithmetica*, I.1).

An illustration of the importance of mathematics to man comes from Boethius work on division, in which the relationship between man and mathematics is taken as an example to explain what the meaning of "conceptually inseparable" is.[45] Boethius shows that mathematics is not inherent in man and does not belong to the class of differentiae; for it is not the ability to do mathematics, but still it cannot be separated from man. Man can use mathematics, and "if this capacity should be removed from man, then man himself no longer remains" (*Divisio*, 881c). Therefore, we can see how vital mathematics is to man.

Mathematical works constitute the first of Boethius' works. His works on the quadrivium focus on "numbers", "propositions", and "harmony" which are close to the research of philosophy and theology. Of these mathematical works, only the *Arithmetica* and an incomplete *Musica* survive.[46] *Arithmetica* deals with pure mathematics; *Musica* and the works on geometry and astronomy touch upon the applied theory of numbers. By the study on numbers, Boethius learned the intrinsic harmony, which served a purpose in philosophy and theology, that is to say, what he focused on is not the mathematical meaning of numbers, but their symbolical or philosophical meaning.

[45] Cf. Section II.1.1.2.a.

[46] Because only these two survive, in my dissertation when I discuss applications of mathematics, I refer to applications of arithmetic and music, and sometimes I refer to geometrical methods.

I.2.3. Boethius on Logic

As I said in Section I.2.1, Boethius devoted himself to two disciplines in his curriculum: mathematics and logic. Logic, in the Platonic tradition, is originally called "dialectic", and the difference between "logical" and "dialectical" reasoning was not marked by the Neoplatonists of Athens and Alexandria, nor by Boethius.[47] The modern sense of "logic" derives from the Peripatetic Alexander of Aphrodisias (fl. 200 A.D.).[48]

Concerning the question what logic itself should be, there are two main opinions: according to one logic is an instrument of philosophy; according to the other logic is a part of philosophy.[49] The important persons holding the first opinion are the Aristotelians, who follow Aristotle's *Topics,* 163b9-11[50], in regarding logic as a practical instrument for the discovery of fallacies in argument on any subject, and as an indispensable tool for every department of human inquiry. Hence they called logic an instrument (*organon*). Alexander of Aphrodisias denies the view that logic is a part of philosophy by saying that logic differs from theoretical philosophy and practical philosophy in subject-matter, and in its end and purpose. Alexander draws an analogy between logic and a hammer and anvil to show that logic is an instrument.[51] Important representatives of the second opinion are the Stoics. In their point of view, logic was a part and an independent branch of philosophy.[52]

Boethius states these two kinds of view in his second commentary on Porphyry's *Isagoge*. And after listing these two opinions, Boethius comes to

[47] Cf. Chadwick (1981b), p. 108.
[48] Cf. Kneale (1984), p. 7.
[49] Cf. Chadwick (1981b), pp. 108-111; Sorabji (2004b), pp. 32-36. Also cf. Ierodiakonou (1997); Mueller (1969).
[50] Aristotle says that "to take and to have taken in at a glance the results of each of two hypotheses is no mean instrument for the cultivation of knowledge and philosophic wisdom." Cf. Aristotle, *Topica*, 163b9-11; Forster (1938).
[51] Cf. Alexander of Aphrodisias, *On Aristotle Prior Analytics I.1-7*, 1.7-3.29; Barnes [etc.] (1991). Also see in Olympiodorus' commentary on Aristotle's *Categories*: like Alexander, Olympiodorus claims that logic has neither the same subject matter nor the same purpose as philosophy, so logic is not a part of philosophy, but an instrument; cf. Sorabji (2004b), p. 35.
[52] Cf. Sorabji (2004b), p. 32.

his conclusion about the role of logic in philosophy.

> "The same logic form serves at the same time the function of part and of instrument. For since it retains its own end, and this end is considered by philosophy only, it must be asserted to be a part of philosophy, but since that end of logic, which philosophy alone contemplates, promises its aid to the other parts of philosophy, we do not deny that it is the instrument of philosophy; but the end of logic is the discovery and judgment of reasons." (*2InIsag.*, 74C-D)[53]

Boethius regards logic as a significant discipline in his philosophy. In his second commentary of Porphyry's *Isagoge*, Boethius stresses that reason can show the right way according to which the incorrupt truth of reality cannot be found, and logic is the discipline which can discern truth from falsity, therefore, those who reject logic are bound to make mistakes (*2InIsag.*, 73A). This means that logic is so important that it should not be neglected.

Boethius gives priority to logic and tries to write double commentaries on Aristotle's logical works while also writing monographs on logic.[54] The principal source of his logical works is the Neoplatonist Porphyry.[55] The most well-known work of Porphyry is the *Isagoge* which discusses five key terms: genus, species, differentia, property and accident. Porphyry intended it to throw light on Aristotle's categories and on definition, division and demonstration. His *Isagoge* is designed as an introduction to Aristotle's *Categories*, and the five predicables discussed in *Isagoge* are the more general types of predicate. After the introductory work, Porphyry also wrote two commentaries on *Categories.* One of Porphyry's commentaries is a short one, which survives in part, and the other is a long one, which survives only in fragments. The influence of Porphyry penetrates in Boethius' works.

[53] In his commentary on Aristotle's *Prior Analytics*, Ammonius states that Plato uses logic as both a part and an instrument; cf. Sorabji (2004b), p. 35. And Plotinus maintains that dialectic is "the precious part of philosophy." In *The Enneads*, Plotinus says that "We must not think of it as the mere tool of the metaphysician: Dialectic does not consist of bare theories and rules: it deals with verities." Cf. Plotinus, *The Enneads*, I.3.5; MacKenna (1991).

[54] Cf. Barnes (1981), "Boethius and the Study of Logic," in Gibson (ed.), pp. 73-89.

[55] Cf. Chadwick (1981b), pp. 120-127.

The most complete one is Boethius' commentaries on Porphyry's *Isagoge*. In Boethius' other logical works, he refers to Porphyry more than once. At the beginning of his commentary on Aristotle's *Categories*, Boethius points out that he proposes to follow Porphyry's commentary on the *Categories*. And in his second commentary on Aristotle's *On Interpretation*, Boethius again says that "We have arranged our explication of this work in Latin following Porphyry as far as possible, although we have included material from others too. For Porphyry, our guide, seems pre-eminent in intellectual sharpness and ability to marshal in his ideas." (*2InInter.*, 7.5-8)

In the trivium, Boethius focuses on the study of logic. He regards logic as the leader of the procession of the trivium. He believes that logic is more basic than grammar and rhetoric. The untenable things in logic are far from grammar and rhetoric. However, it does not mean that he thinks little of grammar and rhetoric; on the contrary, people generally admit that Boethius' works are more literary than Cicero's. Boethius' logical works can be divided into three parts: the first part consists of the translations of the works of Aristotle and Porphyry, which are not preserved; the second part consists of the commentaries on logical works, including *In Isagogen Porphyrii Commenta* (*editio prima* and *editio secunda*), *In Categorias Aristotelis*, *In Aristotelis De Interpretatione* (*editio prima* and *editio secunda*), and *In Ciceronis Topica*; the third part consists of his independent works in which he adopts and develops the logic of his predecessors. These works include *De Divisione*, *Introductio ad Syllogismos Categorico*, *De Syllogismo Categorico*, *De Syllogismo Hypothetico*, and *De Topicis Differentiis*.[56]

Logic and mathematics are regarded as elementary disciplines for Boethius, which are preparatory for the study of philosophy. Thus, among

[56] The English titles of these logical works are: *Commentaries on Isagoge* (1 and 2) ("*1InIsag.*" and "*2InIsag.*" for short); *Commentary on Airstotle's Categories* ("*In Categorias*" for short); *Commentaries on On Interpretation* (1 and 2) ("*1InInter.*" and "*2InInter.*" for short); *Commentary on Cicero's Topics* ("*C.Topica*" for short); *On Division* ("*Divisio*" for short); *Introduction to Categorical Syllogisms* ("*I.S.Categorico*" for short); *On the Categorical Syllogism* ("*S.Categorico*" for short); *On Hypothetical Syllogisms* ("*S.Hypothetico*" for short); *On Topical Differentiae* ("*TopicisD.*" for short). On the translations of these works I cite in my dissertation, cf. Section II.1.

Boethius' curriculum, I will first introduce his logic and mathematics, which will be discussed in Chapter II.

I.2.4. Boethius' Theological Treatises and *Consolatio Philosophiae*[57]

In addition to works on mathematics and logic, Boethius also wrote five theological treatises, viz. *De trinitate*, *Utrum pater et filius et spiritus sanctus de divinitate substantialiter praedicentur*, *Quomodo substantiae in eo quod sint bonae sint cum non sint substantialia bona*, *De fide catholica*, and *Contra Eutychen et Nestorium*. Nevertheless, he is not a theologian[58], and the reason he devotes himself to Christian doctrine is related to the political situation in his day, which will be discussed in Chapter III. As a philosopher, Boethius composed his theological tractates, like his mathematical and logical works, from the perspective of philosophy. However, since mathematics and logic are regarded as elementary disciplines, we may expect there to be applications of elementary mathematics and basic logic in his theological treatises, into which I will investigate in Chapter III. Applications of elementary mathematics and basic logic can also be found in his last and most famous work, the *Consolatio Philosophiae*, which I will discuss in Chapter IV.

[57] The English titles of these five treatises are: *The Trinity is One God not Three Gods*; *Whether Father, Son, and Holy Spirit may be Substantially Predicated of the Divinity*; *How Substances can be Good in Virtue of Their Existence without Being Absolute Goods*; *On the Catholic Faith*; *A Treatise Against Eutyches and Nestorius*. In later chapters, I will use short Latin names for each work, viz. "*De trinitate*", "*Utrum pater*", "*Quomodo substantiae*", "*De fide*", and "*Contra Eutychen et Nestorium*". And in later chapters, I will use *Consolatio* for short of *Consolatio Philosophiae*. The translations of the five treatises and the *Consolatio* I cite in my dissertation are Stewart's (1926).

[58] On the reason why Boethius is not a theologian, see Section III.5.

II. Logic and Mathematics in Boethius' Curriculum

As I introduced in the first chapter, in the Latin speaking world, logic and mathematics were not well taught,[59] but Boethius regards both of them as elementary disciplines in his curriculum, which are important to philosophy. Therefore, he dedicated most of his time to these two disciplines. He began his curriculum plan with four mathematical disciplines. Boethius believes that quadrivium is the real beginning of learning philosophy.[60] At the same time, he is familiar with Aristotle's logic. I will not focus on the applications of all Boethius' logical knowledge in my dissertation. The theories, such as rules of syllogism and other topics, are useful for proper arguments which permeate Boethius' discussions, but it is not necessary to point out this kind of application of logic in detail. Instead, I will focus on what I call basic logic, which includes knowledge such as division, definition, and categories. This kind of basic logic can already be found in Boethius' works on the quadrivium. Again, in his logical works there are some applications of mathematics, but most of them are just mathematical examples. In other words, as two elementary disciplines, mathematics and logic are not totally independent, but they influence each other, which can be seen in the works on logic and mathematics. Together they become basic knowledge for Boethius' philosophy, which will be introduced in Chapter III and Chapter IV.

In this chapter, I will introduce Boethius' works on logic and mathematics, and I focus on some important ideas[61] which will be employed in his theological treatises and his *Consolatio*. I will also demonstrate the mutual influence of mathematics and logic, that is to say, how mathematical theories and examples are used in his logical works, and how basic logical theories are applied in his mathematical works. It is easy

[59] Cf. Section I.2.1.
[60] Cf. Section I.2.2.
[61] Cf. Section II.1.2, Section II.2.1.3, and Section II.2.2.3.

to understand applications of mathematics to logic, since most of the applications are the use of mathematical examples to support or demonstrate logical notions. Unlike these, the applications of basic logic to Boethius' mathematical works make use of logical knowledge to make the abstract mathematical concepts clearer. In order to understand applications of basic logic in Boethius' mathematical works more easily, I will introduce Boethius' logic first.

II.1. Boethius' Logic

In the second commentary on Porphyry's *Isagoge*, Boethius defines the role of logic as both the part and the instrument of philosophy, which implies the essential role of logic in philosophy. Logic is one of the elementary disciplines necessary to learn philosophy. Boethius' extant logical works can be divided into two large parts. The first part consists of the commentaries on works written by Aristotle, Porphyry, and Cicero and the second part comprises his monographs on the subjects of division, syllogism, and topics.[62]

As two elementary disciplines, mathematics and logic are relatively independent. Still, there are applications of mathematics in Boethius' logical works, though they are very few. In Section II.1.1, I will introduce Boethius' logical works in general and applications of mathematics (especially as mathematical examples) in these works. At the end of this section, I will focus on the basic logical knowledge — including knowledge of categories, division and definition — which will be employed in Boethius' other philosophical works.

II.1.1. Boethius' Logical Works and Applications of Mathematics in Them

In order to discuss the applications of mathematics in Boethius' logical works, I will discuss his logical works[63] in three parts concerning: (1)

[62] Cf. Section I.2.3.

[63] On the discussion on Boethius' logical works and their influence, cf. Casey (2012),

Boethius' commentaries on works in the Aristotelian corpus (Section II.1.1.1); (2) his monographs on division and syllogism (Section II.1.1.2); (3) his works on topics (Section II.1.1.3). I will introduce these three parts of Boethius' logical works and applications of mathematics in them one by one.

II.1.1.1. Boethius' Commentaries on Works in the Aristotelian Corpus

In Boethius' extant logical works, there are commentaries on works in the Aristotelian corpus,[64] which are two commentaries on Porphyry's *Isagoge*, one commentary on Aristotle's *Categories* and two commentaries on Aristotle's *On Interpretation*. In these works, most applications of mathematics follow the original works of Aristotle and Porphyry, and Boethius only adds a few mathematical examples or ideas.

II.1.1.1.a. Commentaries on Porphyry's *Isagoge*[65]

Boethius' first logical work is his commentary on Porphyry's *Isagoge*.[66] From this commentary, Boethius started his great plan of translating all of Aristotle's works.[67] Boethius writes two commentaries on the *Isagoge*. The first one is shorter with two books, written in the form of a dialogue between Boethius as a teacher and Fabius as a student. Boethius' first commentary is based on Victorinus' translation, but he thought that Victorinus' translation was not satisfactory. Boethius points out the errors in Victorinus' translation more than once, saying that Victorinus seems to

"Boethius's Works on Logic in the Middle Ages," in Kaylor and Phillips (eds.), pp. 193-220; Martin (2009), "The Logical Textbooks and Their Influence," in Marenbon (ed.), pp. 56-84; Lewry (1981), "Boethian Logic in the Medieval West," in Gibson (ed.), pp. 90-134. Kneale claims that in the Middle Ages Boethius' logical writings were "better known than those of Aristotle and his reputation as high." Cf. Kneale (1984), pp. 189-198.

[64] Cf. Cameron (2009), "Boethius on Utterances, Understanding and Reality," and Ebbesen (2009), "The Aristotelian Commentator," in Marenbon (ed.), pp. 85-102 and pp. 34-55; Chadwick (1981b), pp. 133-141; Marenbon (2003), pp. 19-42; Reiss (1982), pp. 37-54; Shiel (1958); Solmsen (1944).

[65] The translations of the first book of *2InIsag*. I cite are McKeon's (1957-1958). The translations of other books are mine from the Latin version. On Porphyry's *Isagoge* and Boethius' commentaries on it, cf. Gracia (1981); Evangeliou (1985).

[66] Cf. Chadwick (1981b), pp. 131-133; Marenbon (2003), pp. 23-32; Reiss (1982), pp. 28-37.

[67] On Boethius' project, cf. Marenbon (2003), pp. 17-18. Also cf. Section I.2.1.

understand Porphyry less clearly, and that his translation is obscure (*1InIsag.*, 53B). The whole style of the first commentary is the same as that of Porphyry's, and there is no application of mathematics in it.

Later, Boethius wrote the second commentary in five books, based on his own translation. In the second commentary, before dealing with Porphyry's five predicables which include genus, species, difference, property, and accident, Boethius devotes most of the first book to the discussion of the utility of studying logic, and the remaining parts are concerned with a penetrating discussion of the problem of the universal. Applications of mathematics to his second commentary on *Isagoge* are not many. At the beginning of this commentary, Boethius clarifies his programme of translation. He says that in order to accomplish the purpose of seeking knowledge, he should translate the Greek books of philosophy into the Latin language without missing anything. In Boethius' point of view, "the most excellent good of philosophy has been related with human souls" (*2InIsag.*, 71A). He believes that if he wants to translate the Greek books of philosophy in which the uncorrupted truth is expressed into the Latin language, then he must begin his exposition with the powers of human soul. Therefore, he begins his second commentary on *Isagoge* with the discussion on the triple power of the soul.

> "There is a triple power of the soul to be found in animated bodies. Of these, one power supports the life for the body that it may arise by birth and subsist by nourishment; another lends judgment to perception; the third is the foundation for the strength of the mind and for reason." (*2InIsag.*, 71B)

In *Musica*, Boethius warns people not to grant all judgment to the senses, which is the second power of the soul. For the sense is just something obedient like a servant. The ultimate perfection should be composed by reason, because reason "holds itself to fixed rules" and "does not falter by any error". This shows that reason, the highest power of soul, "is a judge and carries authority" (*Musica*, I.9.196). Not all judgment ought to be given to the senses, but reason ought to be trusted more, so we should pursue the use of reason. Just like mathematics can help people to arrive at abstract concepts from visual images, the powers of the soul can give people not

only the life to exist or the capacity to feel but also the ability to ascend from the known to the unknown. This is why Boethius begins his commentary on *Isagoge* with the statement of powers of the soul.

The influential discussion in Boethius' second commentary on *Isagoge* is the solution to three questions Porphyry raises at the beginning of his *Isagoge*.

> "For example, about genera and species — whether they subsist, whether they actually depend on bare thoughts alone, whether if they actually subsist they are bodies or incorporeal and whether they are separable or are in perceptible items and subsist about them — these matters I shall decline to discuss, such a subject being very deep and demanding another and a larger investigation."[68]

These three questions regard the possible existence of genera and species outside our mind; and also concerning the nature of genera and species, corporeal or incorporeal; and their relations to sensible objects. Porphyry refuses to discuss them, but Boethius gives solutions to them.[69]

In order to answer those three questions, Boethius gives two examples from mathematics, among which one is concerned with the nature of numbers in arithmetic, and the other is concerned with the nature of a line in geometry. First, as for numbers, when any number comes out in computing the digits, there must be no doubt that it eventuates in sensible objects itself. And then in the case of a line, human beings can grasp a line using their mind, and make this line an object of thinking, which seems that a line subsists outside our mind. However, it is just the concept of a line that subsists outside sensible objects. No one can perceive a line which is separable from its form. In agreement with these two mathematical examples, Boethius says,

> "For genera and species subsist in one manner, but are understood in another; and they are incorporeal, but they subsist in sensible

[68] Porphyry, *Isagoge*, 1.10-15; Barnes (2003).
[69] Porphyry's three unanswered questions are about universals. On Boethius' discussions on universals, cf. Cross (2012); Spade (1994).

> things joined to sensible things. They are understood, to be sure, as subsisting through themselves and not as having their being in others." (*InIsag.*, 86A)

Mathematical examples thus help Boethius solve the three important questions, and his statements on genera and species lead to the discussion of Nominalism and Realism[70] in the Middle Ages.

II.1.1.1.b. Commentaries on Aristotle's *Categories*[71]

Porphyry's *Isagoge* is the introduction to Aristotle's *Categories*, and the five predicables discussed in *Isagoge* are the more general types of predicate. After the introductory work, Porphyry also wrote two commentaries on *Categories*. Proposing to follow Porphyry's commentary on the *Categories*, Boethius writes a commentary in four books on Aristotle's *Categories*. He opens the commentary with the intention of Aristotle's work on the categories. The intention of Aristotle's *Categories*, Boethius says, is to examine the words, not significations (*In Categorias*, 159A-D). In *Categories*, Aristotle uses some mathematical examples, and as a commentator, Boethius follows Aristotle and employs mathematical examples in his commentary.

It was regarded as unclear what the purpose is of the first chapter of the *Categories*. Porphyry, in his extant commentary, points out that there are five possible ways to connect names, definitions, and things:

> "When things share the same name but have entirely different accounts, they are called homonyms. When they share both an account and a name, they are referred to as synonyms, since together with (*sun*-) the name they also have the same account. When things share the same account but not the same name, they

[70] Nominalism arose in reaction to the problem of universals. Nominalism is a metaphysical view in philosophy according to which general or abstract terms and predicates exist, while universals or abstract objects, which are sometimes thought to correspond to these terms, do not exist. Opposed to nominalism, the view of realism is that universals do exist over and above particulars. Cf. Armstrong (1978 and 1989).

[71] The translations of *In Categorias* are mine from the Latin version. On the discussions of Boethius' commentary on *Categories*, cf. Asztalos (1993 and 2003); Chadwick (1981b), pp. 141-152; Marenbon (2003), pp. 20-23.

> are called polyonyms, and when they have in common neither a name nor an account, they are called heteronyms. ... When certain things come to be from other things, participating in a way in both the name and the account of the things from whence they come, differing however in grammatical form. These are called paronyms."[72]

However, of these five ways Aristotle just mentions three, that is, homonyms, synonyms, and paronyms. Why does Aristotle do this? To answer this question, Porphyry simply attributes it to Aristotle's subsequent discussion: because Aristotle only needs these three ways for his subsequent discussion, and those that he does not need, he does not mention.[73] Boethius answers that question with the help of analogy to geometry in the preface of Book I. Why did Aristotle explain equivocal, univocal, and derivative terms first before discussing the ten categories? Boethius draws analogies with geometry. He says that, "in geometry, the first thing given is the *termini*, and then the *theorematum ordo* can be discussed." (*In Categorias*, 163B-C) This means that before a series of theorems is set out in detail, their principles should be given first. Similarly, in order to discuss the ten categories, first of all, their principles should be explained. This is the relation of the first chapter to what follows.

Among the ten categories, substance, quantity, quality, and relative are closely related to mathematics, so applications of mathematical examples are mainly found in the discussions of these four categories. I will point them out in the following parts of this section.

The category of substance has a number of characteristics, among which one seems to be contrary to what Aristotle says in the early part of the chapter on "substance". This characteristic is "Substance, it seems, does not admit of a more and a less."[74] Earlier in this chapter with regard to the secondary substances, Aristotle says that, "Of the secondary substances the species is more a substance than the genus, since it is nearer to the primary

[72] Porphyry, *In Categorias*, 60.26-33; Strange (1992).
[73] Porphyry, *In Categorias*, 61.1-5; Strange (1992).
[74] Aristotle, *Categories*, 3b33; Ackrill (1963).

substance."[75] Commenting on this apparent contradiction, Porphyry interprets this "not to be understood in an unrestricted sense, but with a distinction"[76]. Following Porphyry, Boethius begins his commentary on this characteristic with similar explanation; that is to say, this is not simply said to be a characteristic but a further distinction is added (*In Categorias*, 197A). Then, unlike Porphyry, Boethius inserts some mathematical examples from other chapters of the *Categories* to explain that this characteristic does not apply to the category of substance solely. For instance, the geometrical figure circle will not be more a circle or less a circle either than itself or than another circle[77]. In addition, he takes some relatives for examples. Double is not more double than another, thus it does not admit "more or less"[78] (*In Categorias*, 197D).

Among the ten categories, "quantity" is most closely related to mathematics, which will be exhibited in Section II.2.1.2.a of this chapter. When introducing quantity, Boethius, following Aristotle, applies more mathematics to his commentary on this part. Quantity can be divided into two kinds, that is, discrete quantity and continuous quantity. And numbers which are studied by arithmetic falls in the discrete quantity, and lines, surfaces, and the like which are the essential objects of geometry falls into the continuous quantity. In his commentary, Boethius enumerates some numbers to show what discrete quantity is. Using lines, surfaces, and bodies as examples, Boethius explains the continuous quantity and how they have a common boundary at which their parts join together (*In Categorias*, 203B-205A).

The second category that has closely relation with mathematics is "relative". One nature of relatives is that usually relatives come into being together, which means that not all relatives are simultaneous by nature. In Aristotle's point of view, "the knowable would seem to be prior to knowledge"[79]. When commenting on this idea, Boethius admits that the first thing related to this idea coming to us is the discipline of mathematics (*In*

[75] Aristotle, *Categories*, 2b7-22; Ackrill (1963).
[76] Porphyry, *In Categorias*, 97.6-10; Strange (1992).
[77] Cf. Aristotle, *Categories*, 11a5-12; Ackrill (1963).
[78] Cf. Aristotle, *Categories*, 6b24-27; Ackrill (1963).
[79] Aristotle, *Categories*, 7b23-24; Ackrill (1963).

Categorias, 229A). Boethius takes triangles for example. We all know that there are three interior angles in a triangle and then these three interior angles equal to two right angles. Hence, it is necessary that the knowable should exist first, and then the knowledge of it may be acquired (*In Categorias*, 229B). In *Categories*, Aristotle takes "the squaring of the circle" as an example to show that "Knowledge of it does not yet exist but the knowable itself exists."[80] Boethius considers this example most obscure (*In Categorias*, 230C), and elaborates a different argument showing that it is possible to draw a triangle equal to a *spatium* with four sides. And similarly, he admits that the knowledge of "the squaring of the circle" also exists, but explains that the reason why Aristotle says it does not exist is that at Aristotle's times the square sought after had not been discovered (*In Categorias*, 230D-231C).

To understand the characteristics of relatives, two terms need to be explained in more detail, namely "simultaneous" and "prior". It is likely that in order to keep the integrity of the discussion on relatives, Aristotle leaves the exposition of these two terms to later chapters. However, in his commentary, Porphyry gives the explanation of "prior" in his comments on the chapter on relatives. Unlike Porphyry, Boethius chooses to follow Aristotle's order. After commenting on the ten categories, Boethius comes to five senses of "prior" in his last book. There are five senses of "prior": (1) whenever we use the term 'prior' in its proper and primary sense, it is time that we have in our minds; (2) 'prior' may be used, when the order of being is fixed and incapable of being reversed; (3) we use the term 'prior' in regard to any order whatever; (4) naturally prior; (5) where in the case of two things the existence of either implies or necessitates that of the other, that thing which is somehow the cause may, in consequence, fairly be considered as naturally prior to the other.[81]

To illustrate the second meaning of prior, Boethius, following Aristotle, takes "one is prior to two" as example, but he gives a more detailed exposition, which could more easily be understood if we refer to his arithmetical work. In Boethius' work on arithmetic, the notion that "unity

[80] Aristotle, *Categories*, 7b30; Ackrill (1963).
[81] Aristotle, *Categories*, 21a34-22a13; Ackrill (1963).

(or one) is the first number" is the starting-point. The definition of "number" is also based on this notion, that is to say, "A number is a collection of unities, or a big mass of quantity issuing from unities." (*Arithmetica*, I.2) In addition, the primary nature of unity is that "it constitutes the primary unit of all numbers which are in the natural order and is rightly recognized as the generator of the total extended plurality of number." (*Arithmetica*, I.7) Thus, if someone proposes the number two, then it definitely follows there is one, for two is the collection of two unities. However, if someone lays down the number one, it is not necessary for it to be multiplied to two (*In Categorias*, 284C-D). As a result, it is clear that between the number one and two, the order is fixed and incapable to be reversed, in other words, one is prior to two, which is the second sense of "prior".

Another characteristic of the relatives is that "if someone knows any relative definitely he will also know definitely that in relation to which it is spoken of."[82] In other words, it is impossible to know that a thing is relative unless its correlative is known. This view is especially related to ratio in arithmetic. Boethius follows Aristotle and takes double as example, but he extends this explanation. As we all know, the number four and the number two have a certain relationship, that is to say "double". It could not be possible to know that the number four is a double without knowing that it is twice the number two. If you definitely know the number four of being "double", then at once will you definitely know to which number the number four stands in relation, that is the number two (*In Categorias*, 235D-236B).

The third category which is close to mathematics is "quality", or more specifically, "the fourth kind of quality". There are four kinds of "quality", including (1) habits and dispositions, (2) capacities, (3) affective qualities and affections, (4) shape, figure and so on. Among these qualities, the fourth kind of quality is close to mathematics, and especially related to geometry, for shape, figure and the like are the subjects focused on by geometry. Boethius lists some geometrical figures, such as a triangle and a square, which belong to the fourth kind of quality. However, there may be some confusion between the fourth kind of quality and continuous quantity, for in

[82] Aristotle, *Categories*, 8a35; Ackrill (1963).

the *Categories* geometrical objects are listed under both headings. Aristotle does not explain this, but following Porphyry, Boethius introduces the following distinction in his commentary. He says that the surface itself is a quantity; however, the shaping of the surface belongs to a quality. Geometrically, the surface bounded by lines is defined by its length and breadth, which is a quantity. For example, a triangle is a particular area produced by three lines which are placed in a certain way to form three angles, so a triangle is called a quantity. However, it is named after a quality in virtue of its certain sort of shaping. It is the same with a line: "it is said to be a quantity due to its length without breadth; and insofar as it is straight, a straight line belongs to quality." (*In Categorias*, 251A-B)

Two characteristics of quality are that most qualities have contraries and admit of degrees. It is helpful to take triangles and other figures that belong to the fourth kind of quality as examples to show that some qualities have no contraries and do not admit of degree. Here, as Aristotle already saw, the introduction of mathematical examples will make arguments easier to be grasped, and more persuasive.

II.1.1.1.c. Commentaries on Aristotle's *On Interpretation*[83]

The purpose of *Categories* is stated again in Boethius' commentaries on Aristotle's *On Interpretation*. The number of significant spoken sounds is divided into ten categories by Aristotle, and the spoken sounds, signifying thought, can also be divided into two parts, name and verb, which are two primary parts to the communication. The former division is discussed in *Categories*, and the latter one is the central topic in *On Interpretation*. Boethius writes two commentaries on Aristotle's *On Interpretation*: the first one is shorter than the second one. Both Boethius' commentaries on Aristotle's *On Interpretation* contain 12 chapters. The difference between two editions is that the first and short one comprises only two books, while the second and larger one comprises six books. There are only a few applications of mathematics in them. In this section, I focus on applications of mathematics to the second commentary.

[83] The translations of *2InInter.* I cite in my dissertation are Smith's (2010 and 2011). On Boethius' commentary on Aristotle's *On Interpretation*, cf. Chadwick (1981b), pp. 152-163; De Rijk (2003); Magee (1989); Marenbon (2003), pp. 32-41; Suto (2012).

At the beginning of Chapter 1, Book I of *2InInter.*, Boethius first discusses the definition of "spoken sound (*vox*)". The reason why he begins with it is that, as Boethius says, spoken sound is "obviously and clearly the theme of this whole book."[84] Then he gives two definitions of *vox*. The first definition is that "Spoken sound is the striking of the air by the tongue, produced by an animal by means of certain parts of the throat called windpipes." The second possible definition is that "spoken sound is a sound which appears to signify." (*2InInter.*, 4.17-28)[85] Strictly speaking, the first definition is the meaning of simply sound (*sonus*), thus Boethius gives a second possible definition. The second definition differs from the use of *vox* in Boethius works on music[86], but it is the one he needs in his commentary on *On Interpretation*.

And in *On Interpretation*, Aristotle says "spoken sounds are symbols of affection in the soul"[87]. When commenting on this view, Boethius claims: "these affections in souls are produced from the similarity of the things [to the affections]." (*2InInter.*, 35.1) Geometrical figures serve to explain this. Boethius takes sphere, square, or other geometrical figures as examples. He points out that, when a person sees a geometrical figure in things, he considers its likeness in the intelligence of his mind, and "when his soul has been affected by the image, he knows the thing by whose image he has been affected". Therefore, the similarity of the figure in things to the affections causes the affection in souls to occur. (*2InInter.*, 35.1-10)

When commenting on the relation of actuality and potentiality, Boethius says there is something in which there is only potentiality and never actuality by taking numbers as an example.

> "For number can increase to infinity and whatever number has been mentioned, a hundred, a thousand, ten thousand and the rest, must

[84] "This whole book" refers to Aristotle's *On Interpretation*.
[85] The paragraphs of Boethius' second commentary on *On Interpretation* 53ff are closely paralleled in Ammonius 30.1-16, and the discussion of Boethius 4.18-6.5 overlaps with Ammonius 30.2-7. Cf. Ammonius, *On Aristotle On Interpretation 1-8*, 30.1-16; Blank (1996).
[86] In Boethius' work on music, *vox* has a wide spectrum of meaning, for it can mean the human voice, sound in general, or musical pitch.
[87] Aristotle, *On Interpretation*, 16a3; Ackrill (1963).

> be finite. Thus an actual number is never infinite because it can increase to infinity. And for this reason infinite number is only potential." (*2InInter.*, 463.8-17)

The last example of applications of mathematics that I want to point out is in the last chapter, Book 6 of *On Interpretation*. At 23b33-24a3 of *On Interpretation*, when speaking of the contrary of a statement, Aristotle uses four propositions to argue that every statement has a contrary. The four propositions are: PI. About the good that it is good; PII. About the good that it is not good; PIII. About the not good that it is not good; PIV. About the not good that it is good. When commenting on this, Boethius compares the argument of Aristotle with a ratio, for he believes, "Ratio is in fact the mutual similarity of things to each other." (*2InInter.*, 490.15-19) Boethius chooses the ratio 2:4=6:12, and according to the numerical relations of this ratio it is also true that 2:6=4:12. Then Boethius transfers the numerical ratio to the force and nature of propositions.[88] He puts PI and PII first, of which PI precedes and PII follows, and then puts PIII and PIV, of which PIII precedes and PIV follows in the same way, and makes there be a similarity.

PI	PII
About the good that it is good	About the good that it is not good
PIII	PIV
About the not good that it is not good	About the not good that it is good

The similar ratio between these four propositions is PI:PII=PIII:PIV, which means that just as PI is true, but PII is false, so too PIII is true, but PIV is false. In other words, the left of the ratio is "a true proposition : a false proposition" and the right of the ratio is the same, so the left equals the right. If we change the places of these four propositions in the way of 2:4=6:12, that is 2:6=4:12, then we can get PI:PIII=PII:PIV. In this new ratio, the left is that PI is true and PIII is true, and the right is that PII is false and PIV is false, so the force of these propositions can also be explained by the ratio. All in all, any statement A, B, C, or D, if A:B=C:D is true, then A:C=B:D is also true, and this correct rule for identifying contraries is the same for all types of statements. Boethius' application of the numerical relation of ratios

[88] Cf. Chadwick (1981b), p. 154.

makes for an easier and more concise way to understand the argument.

II.1.1.2. Boethius' Monographs on Division and Syllogism[89]

II.1.1.2.a. Division[90]

In his work on division, Boethius studies different kinds of division, distinguishes one from another, and points out the logical relations between whatever is being divided (or analysed, or classified) and its dividing elements. There are some uses of mathematics in discussions on division.

The central part of *Divisio* is on the division of genus into species, which is one of the four kinds of division. During the operation of this division style, "differentia" is an essential concept. A differentia is that "in respect of which we indicate that one thing differs from another (*Divisio*, 880b)". There are many kinds of differentiae, and not all of them are suited to the division of genus. "Some differentiae are *per se*, others *per accidens*, and of the latter some are consequent, others regularly departing." Boethius gives some examples to explain which sort of differentiae is suited to the division of genus, one of which is about mathematics and man.

> "Again, there is another thing, which is conceptually inseparable, the separation of which brings destruction of the species, e.g. when we say that it is inherent in man that he alone can use numbers or learn geometry. And if this capacity should be removed from man, then man himself no longer remains; and yet such things do not automatically belong to the class of differentiae that inhere in the substance, for it is not the ability to use numbers and do geometry that accounts for man, but rather being rational and mortal. Hence those differentiae on account of which the species consists are precisely the ones that are placed both in the division of a species and in the division of the genus that contains the species." (*Divisio*, 881b-d)

That Boethius takes this example here also implies that although

[89] Cf. Chadwick (1981b), pp. 163-166.
[90] The translations of *Divisio* I cite in my dissertation are Magee's (1998). On Boethius' *Divisio*, cf. Marenbon (2003), pp. 44-46.

mathematics (numbers and geometry) does not inhere in man, it still cannot be separated from man. The reason is that if mathematics should be removed from man, "then man himself no longer remains". Therefore, we can see how vital the mathematics is in man.[91]

With regards to affirmation and negation, Boethius takes mathematics as examples. When he discusses the negation used in constructing a species, he gives examples: "Of odd numbers some are prime (e.g. three, five, or seven) others not-prime (e.g. nine);" or again, "Of figures some are rectilinear, others non-rectilinear." (*Divisio*, 882b-c) That is to say when we want to use a simple name to assign a species to something that is not picked out by any word (such as there is no single name applied for not-prime numbers), it is often necessary to use negation in constructing a species (the negation word "not" with the species "prime"). However, it is our need, not nature, that sometimes requires this. Further, when a section is made by negation, the affirmation or simple name should be stated first. So when we divide numbers, first of all, the affirmation "some numbers are prime" is stated, and then the negation "other numbers are not-prime" follows. The relationship between affirmation and negation is that "Affirmation is prior and negation posterior." This can be explained by arithmetical theory, "the equal is prior to the unequal" which will be introduced in Section II.2.2 later. In Chapter 32, Book I of his arithmetical work, Boethius gives a demonstration of "how every inequality proceeds from equality", and in Chapter 1, Book II, Boethius continues to discuss "How every inequality is reduced to equality". The equality is more finite than the inequality. "It is always necessary that finite things be prior to non-finite things." And the equality is prior to the inequality. "All the things that are expressed by a part of speech that is definite or by an affirmation are more finite than a name with a negative particle or a complete negation." (*Divisio*, 882d) Therefore, "affirmation is prior and negation posterior."

The division of one and the same genus occurs in more than one way. To illustrate the multiple divisions, Boethius gives the division of numbers in arithmetic and the division of triangles in geometry as examples. According to different divisions, numbers could be divided into even and

[91] Cf. Section I.2.2.

odd, and alternatively, numbers can also be classified as prime numbers and non-prime numbers. It is the same with triangles. Triangles can be divided into equilateral triangles, triangles with only two equal sides, and triangles with all sides unequal. Another way to divide triangles is that some triangles have a right angle, others have three acute angels, and others have an obtuse one (*Divisio*, 885c). As a result of this, Boethius comes to the conclusion that division of one and the same genus can be made in many ways. In spite of multiple divisions, every division would be split into pairs, if there were names for the species and differentiae. Take geometrical figures with three sides for an example. Three-sided figures can be divided into three species: the figures with equilateral sides, others with two equal sides, and others with unequal sides throughout. Since there are names for the species and differentiae of three-sided figures, the tripartite division of the three-sided figures can change into bipartite division. Those figures with three sides can be divided into figures with equal sides, and figures with unequal sides; of the figures with unequal sides, some have only two equal sides, others have three unequal sides. Therefore, in the same way, "every division would be bipartite if the species and differentiae did not lack names." (*Divisio*, 884a)

The introduction of mathematical examples in *Divisio* makes the points more easily to be understood.

II.1.1.2.b. Syllogism[92]

In Boethius' point of view, syllogism is concerning the statements which are either categorical or hypothetical. Theory of categorical syllogisms is the logic of names and theory of hypothetical syllogisms is propositional logic. Thus his monographs on syllogism are divided into two parts: one is the works on categorical statements including *Introductio ad syllogismos categoricos* and *De syllogismo categorico*[93]; and the other is the work on hypothetical statements called *De hypotheticis syllogismis*[94]. The first concerns categorical statements in which something is predicated of

[92] On Boethius' syllogism, cf. Chadwick (1981b), pp. 166-173; Dürr (1951); Marenbon (2003), pp. 46-56; Speca (2001).
[93] Concerning the relation between the twin monographs on the categorical syllogism, cf. Chadwick (1981b), pp. 165-170.
[94] Cf. Bobzien (2002).

another thing in the form "A is B". As regards the hypothetical statements, Boethius gives four characterizations as follows: (1) hypothetical statements express that something is, if something else is; (2) hypothetical statements consist of categorical statements, while categorical statements are simple; (3) hypothetical statements have their own proper force that differs from the force of categorical statements, in that it rests on a hypothesis rather than on a predication, even with regard to the same terms; (4) hypothetical statements express that something is or is not, if something else is or is not.[95]

All of these monographs refer to how to give proper arguments, and in all Boethius' arguments, he pays attention to the rules of syllogism. Applications of mathematics could not be found in these three monographs, so I will move on to the other logical works.

II.1.1.3. Applications of Mathematics in Boethius' Works on Topics

Boethius wrote two works on topics.[96] One is a commentary on Cicero's *Topics*, and the other is a monograph named *De Topicis Differentiis*. In this section I will introduce both works briefly and list mathematical examples used in both works.

II.1.1.3.a. *In Ciceronis Topica*[97]

In Ciceronis Topica is Boethius' commentary on Cicero's *Topics*, which follows the text in Cicero's work continuously. However, because Cicero's work contained a paragraph 100, and Boethius' commentary ends in the comments on Cicero's paragraph 76, one could say that Boethius' commentary on Cicero's *Topics* is either preserved incompletely or was never finished by him.

According to Boethius, two different sorts of things are Topics: a Topic is both a maximal proposition and the differentia of a maximal proposition. Therefore, "maximal proposition" is a vital concept of Topics. In Book I of

[95] Cf. Speca (2001), pp. 78-80.
[96] On Boethius' theory of Topics and its influence, cf. Marenbon (2003), pp. 56-65; Stump (1981a, 1974, and 1981b).
[97] The translations of *In Ciceronis Topica* I cite in my dissertation are Stump's (1988). On Boethius' commentary on Cicero's *Topics*, cf. Stump (1987).

In Ciceronis Topica, when Boethius states the nature of Topics, he gives the definition of maximal proposition: "We call highest and maximal propositions those propositions that are universal and known and manifest to such an extent that they need no proof but rather themselves provide proof for things that are in doubt, for those propositions that are uncertain." (*C.Topica,* 280/1051) In order to elucidate this concept, he takes examples from mathematical theory. The first example he gives of this sort of propositions is that "Every number is either even or odd". This proposition from arithmetic is universal and there is no need to prove it, for if number is divided from this perspective, it must be that there are two kinds: one is even number and the other is odd number. There is no the third kind of number in this division of number, which suggests that the proposition "Every number is either even or odd" illustrates what it means to be manifest and universal. The second example is "If equals are subtracted from equals, equals remain". This also proves itself by itself which means it is universal and manifest. Therefore, these two propositions from arithmetic are both called "maximal propositions". And from these two arithmetical examples, we can easily understand what "maximal proposition" is and what the characteristics of this proposition are.

At the introduction of Book II, Boethius refutes various biting censures on the discipline of logic. Everyone wants to appear very skilled at discourse. In order to bring and to refute charges, people would all rush together to the knowledge of the discipline of logic. Then Boethius raises two questions: "But now can anything more absurd be imagined than their trying to argue that the study of dialectic is useless for arguments that are even in their own view readily believable? For what sense does it make to subvert the art of discourse by engaging in discourse, so that you despise the truth of the vary art in which you seek a reputation?" (*C.Topica*, 292/1063) To answer these questions easily, Boethius uses an analogy between a musician and himself. "As that musician directed his disciple to make music for himself and for the Muses, so I too could also have sung for myself and for you, who are not a Muse but a protector of the Muses." (*C.Topica*, 292/1063)

Book II focuses on the nature of related things and their kinds:

conjugates, genus, species, similars, differentiae, contraries, associated things, antecedents, consequents, and incompatibles, cause and effect, comparison of greater, lesser, and equal things. All these are connected to one another and some of them "have not only a linguistic connection to one another but also a certain harmony of nature, although they are not identical with one another (*C.Topica*, 295/1066)". In order to understand the last part of this notion, we need to refer to a conception in Boethius' work on music. In his work on music, Boethius gives a definition of consonance: "a mixture of high and low sound falling pleasantly and uniformly on the ears (*Musica*, I.8.195)". The sounds composing the consonance are not identical with one another, but their mixture can form a pleasant and uniform sound, that is, the consonance. This is the same with the parts of related things. That is why Boethius says, in spite of the difference between the parts, that they could have a certain harmony of nature.

II.1.1.3.b. *De Topicis Differentiis*[98]

At the beginning of Boethius' *De topicis differentiis*, he states the aim of this work. He will show "what the topics are, what their differentiae are and which are suited for which syllogisms." (*TopicisD.*, 1173C.9-10)

The first concept to be discussed is "proposition". Boethius states different ways to divide propositions. The last division is to distinguish some propositions known *per se* from all the rest.

> "Some propositions are known per se, and no proof can be found for these. Others, although the mind of the hearer approves them and assents to them, can nevertheless be proved by other, more fundamental propositions. Those for which there is no proof are called maximal and principal, because it is necessary that these prove those which do not deny that they can be demonstrated." (*TopicisD.,* 1176C.18-24)

Boethius inserts the same mathematical proposition as an example that was also used in *In Ciceronis Topica*: "If you take equals from equals, the remainders are equal." This mathematical proposition produces appropriate

[98] The translations of *De Topicis Differentiis* I cite in my dissertation are Stump's (1978).

belief in itself by nature, so it is known *per se*, and it is indemonstrable, maximal, and principal.

The fourth concept Boethius discusses is "argument" which is a reason producing belief regarding something that is in doubt. "Of all arguments, some are readily believable (*probabilia*) and necessary, some readily believable and not necessary, some necessary but not readily believable, and some neither readily believable nor necessary." (*TopicisD.,* 1180C.24-27) But someone may reckon that things which are necessary only and not also readily believable are not arguments. Boethius gives a refutation of this. This kind of idea, he says, is not based on a correct understanding of "readily believable."

> "Those things are readily believable to which agreement is spontaneously and willingly given, so that they are agreed to as soon as they are heard. However, those things that are necessary and not readily believable are demonstrated before by other things that are necessary and readily believable; and, known and believed, they produce belief regarding something else which is in doubt." (*TopicisD.,* 1181B.33-39)

Boethius continues to illustrate this by the nature of geometrical theory.

> "The theories (theorems) which are considered in geometry are of this sort. For the things presented there are not such that the mind of the student agrees to them spontaneously; but since they are demonstrated by other arguments and so are known and understood, they produce belief regarding other theories. So those things that are not readily believable *per se* but are necessary cannot be arguments to confirm something else for hearers to whom they have not yet been demonstrated. However, to those hearers who by prior reasons have come to believe those things which they [once] did not agree to, they can be invoked as arguments if [the hearers] are in doubt about something." (*TopicisD.,* 1181B.40-1181C.11)

Book III concerns the comparison of divisions between Themistius and Cicero. Boethius believes it is normal that the people of an attentive nature treat the differentiae of Topics variously and in different ways. The reason is

that any one thing can often be divided diversely. Here Boethius also shows some divisions in mathematics as examples. "For example, we collect sometimes these differentiae of number: some [numbers] are even and others odd; but sometimes these: some [numbers] are prime and incomposite and others are secondary and composite. The discipline of geometry shows that triangles also may be divided in many ways, though in all cases one should watch that nothing is left out in any form of division and nothing superfluous and beyond what is necessary is added." (*TopicisD.,* 1195B.7-1195C.15)

II.1.2. Boethius' Basic Logic

In logic, the syllogism is used for the statement of knowledge, and topics are used to generate arguments. Syllogisms and topics pervade Boethius' works, so I will neither specially point out where knowledge of the syllogism is used nor how it is used. When I discuss applications of logic in my dissertation, I refer to the basic logic including knowledge of categories and theories of division and definition that will be applied in Boethius' works on mathematics (Section II.2.1.2 and Section II.2.2.2), theology (Chapter III), and *Consolatio* (Section IV.2.1 and Section IV.3). In this section, I will give a short introduction to Boethius' basic logic.

II.1.2.1. Categories

Among the ten categories, Aristotle pays more attention to four of them, that is, substance, quantity, relation, and quality, and he puts other categories in one chapter. Similarly to Aristotle, Boethius also stresses substance, quantity, relation, and quality, which he also applies in his other works. These four main categories are applied in his works on arithmetic and music, which I will discuss in Section II.2.1.2.a and Section II.2.2.2.a. And in his theological treatise on the Trinity, Boethius applies the ten categories to God, and, finally, he finds that only the category of relation can explain the diversity in God, which will be shown in Section III.3.2.

II.1.2.2. Division and Definition

The most important part of the theory of division is the distinction of types of division. In his *Divisio*, Boethius distinguishes two large parts of division, among which there are three types (see Diagram I below).

Diagram I: Boethius' System of Division

Division	
A. Division *per se*	B. Division *per accidens*
Division of a genus into its species	Division of a subject into accidents
Division of a whole into its parts	Division of an accident into its subjects
Division of an utterance into its significations	Division of an accident into accidents

From these six types of division, I will single out the three types of division *per se*[99], because they are important in Boethius' other works.

There are differences between division of a genus into its species and division of a whole into its parts, although these two types are similar. There are three main differences between them. The first difference is "the division of a whole is made in respect of quantity" and "the distribution of a genus is accomplished in respect of quality" (*Divisio*, 879b). The second difference is that "Every genus is by nature prior to its proper species whereas a whole is posterior to its proper parts" (*Divisio*, 879b). The "prior[100]" here is not used in the sense of time, because it means that the destruction of the genus could result in the perishment of the species immediately, but not vice versa. If a species is destroyed, its genus would remain inviolate in its nature. Unlike the relation between the genus and its species, its proper parts are prior to the whole, for if a part of the whole perishes then that of which one part has been destroyed will not be the whole, whereas if the whole perishes parts remain, in separation. The last difference is concerning the similarities and differences between the original one and its divisions. The species is composed of its genus and differentia, in which the genus is the matter of species, and the differentia is the form. The species is always the same as its genus, and only due to adding a

[99] Concerning the other three types of division *per accidens*, cf. Section II.2.2.2.b.
[100] On five senses of "prior", cf. Section II.1.1.1.b.

differentia, a species differs from its genus. However, it is not the same case with the whole and its parts. It is obvious that a whole consists of parts, and in this case the plurality of parts is the matter, and the composition of those same parts is the form, which is the difference between a whole and its every part. These two types of division will be applied in Boethius' *Consolatio*, which will be discussed below in Section IV.2.2.2 and Section IV.3.1.

Another type of division Boethius employs is the division of an utterance into its signification. When a single spoken sound that signifies many things is opened up and the plurality of its significations is disclosed, we need a division of an utterance into its proper significations. This type of division is used by Boethius in his theological treatise on Christology, *Contra Eutychen et Nestorium*, to give a proper definition of the word "person", which will be introduced in Section III.2.1.2.

There is a piece of vital knowledge related to theory of division, which is knowledge of definition. Boethius talks about "definition" in at least three logical works: one is *Divisio*, another is his commentary on Cicero's *Topics*, and the last one is his commentary on Porphyry's *Isagoge*. I will give a short introduction here. In his monograph on division, Boethius points out that "we may pretty well say that division and definition are in essence concerned with the same thing, since a unified definition is a conglomeration of linked division (*Divisio*, 880c)," which shows the relationship between division and definition. It can be said that division is necessary for full definitions of species, and definition is also necessary for division, for through the use of definition it can be collected together that whatever is equivocal and whatever is univocal. In his commentary on Cicero's *Topica*, Boethius devotes Book III to the discussion of definition, including the nature of definition, kinds of definition, and the method for making definitions. Here I want to introduce two ideas about definition.

The first one is the method for making definitions, which is important in Boethius' other works, such as mathematical works, theological works, and his *Consolatio*, which will be discussed one by one later.[101] For

[101] Cf. Section II.2.1.2.b, Section II.2.2.2.b, Chapter III, and Chapter IV.

division is necessary for full definitions of species, and the method of making definitions rests on division. When defining a species, first of all, this species should be of the sort that it both has a genus and is predicated of subsequent things. For definitions encompass neither higher genera nor the lower things, both of which are excluded from the definition. So it is only "the intermediate things, those that have genera and that are predicated of the others — of genera, of species, or of individuals — that can fall under definition." (*Divisio*, 886a)

The next thing that one should do is to take up the genus of that species, divide the differentiae of that genus, join a differentia to the genus, and see if the differentia joined to the genus is equal to the species which needs to be defined. If so, this is the definition of the species; if not, distribute differentiae under differentiae as often as possible until all of them joined to the genus describe the species in a definition that is equal to it (*Divisio*, 886a).

The second topic I wish to introduce is the division of types of definition. There are four kinds of definition (*C.Topica*, 323/1096).

> (1) When a definition is constructed of genus and differentiae, we unfold substantial parts. This is called "definition" in the strict sense of the name.
>
> (2) There is the sort of definition where accidents are gathered together into one thing and one thing is produced from them; it is a sort of enumeration of parts located not in substance but in a gathering together of accidents. This sort of definition is called a description.
>
> (3) If we are talking not about the accidents of a thing but rather about certain members from which a thing is composed and conjoined, and we attempt to make a definition from such members. This is called a definition by means of enumeration of parts.
>
> (4) If someone makes a definition by presenting species rather than members in the definition, it is called a definition from the division of species.

These four kinds of definition will be applied to Boethius' treatise against Eutyches and Nestorius, and they help him to find a correct definition of nature, which will be discussed in Section III.2.1.1.

By looking through Boethius' extant logical works, applications of mathematics can be found in these works. Most of applications of mathematics to logic are as examples to support arguments or notions in logic, which makes logic be understood more easily. In Boethius' mathematical works, there are also applications of logic. In Section II.2, I will introduce Boethius' mathematics and show how logical theories play a role in his mathematical works.

II.2. Boethius' Mathematics

As I introduced in Section I.2.2, there are only two extant mathematical works of Boethius, and both of them are not considered to be original ones with him. As a matter of fact, Boethius composes the works on arithmetic and music according to his purpose of making the Latin-speaking world familiar with the classical Greek knowledge. Thus, in compiling these works Boethius' remains true to his purpose. Here I want to discuss some applications of basic logic in his mathematical works. These are not numerous but can be enough to indicate how Boethius distinguishes himself from his sources.

In the following sections, I will introduce Boethius' arithmetic and music and show how basic logic elements (including division, definition and categories) are applied to them.

II.2.1. Boethius on Arithmetic

II.2.1.1. *De Institutione Arithmetica* and Its Sources

Boethius' *Arithmetica*[102] is the interpretation of Nicomachus' Greek

[102] Cf. Masi (1979) discusses Gerardus Ruffus' commentary on *Arithmetica*; Kibre (1981), "The Boethian *De Institutione Arithmetica* and the Quadrivium in the Thirteen Century University Milieu at Paris," and Masi (1981a), "The Influence of Boethius *De Arithmetica*

Introduction to Arithmetic.[103] The following Table I lists the corresponding chapters between Boethius' *Arithmetica* and Nicomachus' work on arithmetic.

Table I (PS: Nico. is short for Nicomachus, Boe. is short for Boethius)

Nico.	Boe.	Nico.	Boe.
I.1-5	I.1	II.1-2	II.1
I.6	I.2	II.3-4	II.2
I.7	I.3-6	II.5	II.3
I.8	I.7-9	II.6-7	II.4-6
I.9	I.10	II.8	II.7-9
I.10	I.11-12	II.9	II.10-12
I.11-12	I.13-15	II.10	II.13-14
I.13	I.16-18	II.11	II.15-16
I.14-15	I.19	II.12	II.17-19
I.16	I.20	II.13-14	II.20-24
I.17	1.21-23	II.15-16	II.25
I.18	I.23	II.17	II.26-30
I.19	I.24-27	II.18	II.31-32
I.20-21	I.28	II.19	II.33-34
I.22	I.29-30	II.20	II.35-39
I.23	I.31-32	II.21	II.40
PS: Nothing in Nicomachus corresponds to II.45		I.22	II.41-42
		II.23	II.43
		II.24	II.44-46
		II.25-26	II.47-49
		II.28	II.51-53
		II.29	II.54

From the above table we can see that like Nicomachus, Boethius also divides his *Arithmetica* into two books, but Boethius' first book contains 32 chapters (nine chapters longer than Nicomachus' work) and his second book has 54 chapters (twenty-five chapters longer than Nicomachus' work). The

on Late Medieval Mathematics," in Masi (ed.), pp. 67-80 and pp. 81-95; Masi (1981b).

[103] According to Cassiodorus, Apuleius of Madaura also translated Nicomachus' *Introduction to Arithmetic*, but nothing of this translation remains. In the Greek-speaking part of the world, the extant commentaries on Nicomachus' *Introduction to Arithmetic* are those of Iamblichus, Asclepius, and Philoponus. Cf. Tarán (1969), pp. 5-7.

first book focuses on the numbers (the substance of number, different divisions of number and their definitions, productions, and properties, and the relation between numbers); and the second book concerns figure numbers and proportion.

As for the relation between Boethius and his source Nicomachus of Gerasa, Martin Luther D'Ooge gives the following evaluation of Boethius' translation of Nicomachus' works.

> "In the composition of his treatise Boethius more often expands than condenses. His method is to intersperse between sections literally translated, or closely paraphrased, others in which the general principles stated by Nicomachus are furnished with exhaustive explanation and copious numerical examples. Nothing is left to the reader to supply. Almost any chapter, compared with the original one, will prove to be of this character. Boethius also supplies data in tabular form to a far greater extent than did Nicomachus. The order of the original is preserved for the most part, but occasionally a rearrangement is found."[104]

D'Ooge claims that these peculiarities are of minor importance and it is rather the omissions that have to be considered. He regards those omissions as the special difficulty that Boethius had with the logical terminology of Nicomachus.

I do not agree with the evaluation by D'Ooge. Boethius' translation style is due to his purpose, as he says in the preface of his *Arithmetica*: "I do not restrict myself slavishly to traditions of others, but with a well formed rule of translation, having wandered a bit freely, I set upon a different path, not the same footsteps." (*Arithmetica*, preface)

Boethius makes two kinds of changes to Nicomachus' composition. The first one is to add extra exposition or use formulae and diagrams to make some ideas clearer and easier to comprehend. For instance, when introducing the second division of even number, "the even times odd number", Boethius adds more explanation to tell the difference between the

[104] D'Ooge (1938), p. 133.

second division of even number and the first division of even number, "the even times even number". This extra exposition makes the reader understand the division of even number more clearly. In another case, in order to show that "the principle of plane straight-line figures is a triangle", Boethius adds four diagrams[105] to illustrate how a square, a pentagon, hexagon, and even a triangle can be divided into triangles, which gives the readers a visualized picture.

The second kind of changes consists in the reproduction of diffuse discussions in a concise style. One case of this kind is vital to tell the difference between Boethius and Nicomachus. Nicomachus believes that what is true of numbers is also true of the universe. As the discipline studying numbers, arithmetic was preexistent as a cosmic pattern in the mind of God, the creator, and according to this model the material world was formed. This is expressed by Nicomachus in I.4.2 and I.6.1 of his work on arithmetic. Nicomachus refers to the universe more than once. I list two citations from Nicomachus' work, which are completely left out of Boethius' corresponding chapters (see Table I).

> Arithmetic "existed before all the others in the mind of the creating God like some universal and exemplary plan, relying upon which as a design and archetypal example the creator of the universe sets in order his material creations and makes them attain to their proper ends."[106]

> "All that has by nature with systematic method been arranged in the universe seems both in part and as a whole to have been determined and ordered in accordance with number, by the forethought and the mind of him that created all things."[107]

In addition, there should be two kinds of numbers, the divine number and the scientific number. From Nicomachus' point of view, the divine number is a wholly conceptual and immaterial number and this kind of number preexisted in God's mind and was the basis of creation; while the scientific

[105] Cf. *Arithmetica*, II.16.
[106] Nicomachus, *Introduction to Arithmetic*, I.4.2; D'Ooge (1938).
[107] Nicomachus, *Introduction to Arithmetic*, I.6.1; D'Ooge (1938).

number is constantly found in connection with material things and measures them, their arrangements and their movements.[108] In I.6 of his work, Nicomachus makes a statement to distinguish the scientific number which he uses in his arithmetic from the divine number. Boethius condenses the contents on the divine number, and he does not discuss the difference between the divine number and the scientific number. Throughout Boethius' work on arithmetic, there is no such term "scientific number". Unlike Nicomachus, Boethius only uses the word "number" instead of the term "scientific number".

Nevertheless, in Boethius' work, when he states the substance of number, he also gives a short introduction saying that "Number was the principal exemplar in the mind of the creator." (*Arithmetica*, I.2) Therefore, Boethius shares the same idea with Nicomachus about the relation between number and universe, but in his *Arithmetica*, Boethius does not intend to give his readers the impression that his arithmetic gives the information about the idea of universe but only leads his reader to the arithmetic *per se* as an elementary discipline in the background of philosophy. In order to emphasize that his work on arithmetic is an elementary introduction for beginners, Boethius confines himself to elementary ideas of arithmetic.

Nicomachus' arithmetic could be seen as his theory of cosmogony, while Boethius regards arithmetic as basic knowledge for the other three mathematical disciplines which together become the preparatory way to the serious study of philosophy.

> "Arithmetic considers that multitude which exists of itself as an integral whole; the measures of musical modulation understand that multitude which exists in relation to some other; geometry offers the notion of stable magnitude; the skill of astronomical discipline explains the science of moveable magnitude." (*Arithmetica*, I.1)

Accordingly, it is not accurate to regard Boethius' work on arithmetic is the translation of Nicomachus', but it should say that Boethius interprets or paraphrases Nicomachus' Greek work on arithmetic in his own way.

[108] Cf. D'Ooge (1938), p. 98.

II.2.1.2. Applications of Basic Logic in *Arithmetica*

In interpreting Nicomachus, Boethius employs basic logical knowledge. The main theories of logic used in arithmetic involve knowledge of categories and definition.

II.2.1.2.a. Categories in *Arithmetica*

Among all logical theories, knowledge of categories relates to arithmetic most closely.[109] Of the ten categories, those that have an obvious relationship with arithmetic are quantities and relations.

The division of mathematics begins with the division of the proper objects of mathematical knowledge (incorporeal *essentiae*), which may be compared to the division of quantities. For in *Categories*, Aristotle divides quantities into two kinds. One is discrete quantity, which includes number; and the other kind is continuous quantity, including lines, surfaces, bodies which are the objects geometry and astronomy study. Similar to this division, of the four mathematical disciplines arithmetic and music concern the genus of discrete *essentiae*, namely multitude; and geometry and astronomy concern the genus of continuous *essentiae*, called magnitude. These two terms, multitude and magnitude, are used here both abstractly and concretely. Abstractly, the two terms refer to the quality; concretely, the two terms refer to the objects of such natures.

One kind of discrete quantity is number, and arithmetic studies number *per se*, therefore, arithmetic focuses on a discrete quantity, which shows how closely the category of quantity and arithmetic are related. Additionally, a characteristic peculiar to a quantity is its being called both equal and unequal, both of which are the vital terms in arithmetical study. Boethius divides I.17 of Nicomachus' work into two chapters: one is concerning a quantity related to another; and the other is relating to the types of major and minor quantity. Boethius believes that, "Any given thing in comparison to another is either equal or unequal with it." (*Arithmetica*, I.21) Thus, the quantity is first divided into equal quantity and unequal quantity. Then of the unequal quantity, some are larger and some are smaller. Finally, the

[109] Cf. Section II.1.1.1.b.

major inequalities and the minor inequality are divided into five types[110] respectively. For example, major inequalities include multiplex, superparticular, superpartient, multiplex superparticular, and multiplex superpartient, and the terms used here are adapted from Boethius' translation, which are important in mathematics.

It is obvious that the category of quantity is intimate to arithmetic. And it also shows another category that is close to arithmetic, which is the category of relation. In *Categories*, Aristotle defines "relatives" as "all such things as are said to be just what they are, *of* or *than* other things, or in some other way *in relation to* something else."[111] This definition of relatives is just the basis of relative quantities in arithmetic. For every relative has a correlative, and in most cases, they come into being together, so Boethius says that, "Any given thing which has a quantity compared to it is not known except by the other term to which it is compared." (*Arithmetica*, I.21) It could be said that without the category of relation, there would be no relative quantities, or there would be no proportion in arithmetic. And what is worse, without the category of relation another mathematical discipline, music, would lose its theoretical basis. Because music in Boethius' quadrivium depends on number theory, it must be built on proportional principles.

Of course, the relationship between categories and mathematics is neither original in Boethius nor in Nicomachus, for its history traces back to Aristotle and the Pythagoreans, as we have seen. However, from the arguments in Boethius' work on arithmetic, we find that this relationship is clear in his mind.

II.2.1.2.b. Theory of Definition in *Arithmetica*

Theory of definition[112] is so important in Boethius' logic that he states it in more than one of his logical works, especially in his monograph on division and in his commentary on Cicero's *Topics*. In Boethius' point of

[110] If "m", "n", and "k" are integers, then "multiples" are as mn:n; "superparticulars" are as (n+1):n; "superpartients" are as (n+k):n, k>1; "multiple superparticulars" are as (mn+1):n, m>1; and "multiple superpartients" are as (mn+k):n, both m and k being greater than 1.
[111] Aristotle, *Categories*, 6a36; Ackrill (1963).
[112] Cf. Section II.1.2.2.

view, "the definition shows what the thing defined is; that is, it shows its substance. A definition that consists in a genus and differentiae, however, does lay out a substance." (*C.Topica*, 319/1091) Application of definition is in line with Boethius' goal that his arithmetic is written for the beginners. Therefore, when he discusses one subject, he must, as Nicomachus does, give the definitions of the main terms of that subject, and he also adds a definition which is not found in Nicomachus' work on arithmetic, such as the definition of cyclical or spherical numbers.

II.2.1.3. Basic Ideas of Arithmetic Used in Boethius' Other Works

Applications of logic to Boethius' arithmetic are not many, but they are sufficient to show what Boethius' own approach. Arithmetic is so elementary that in Boethius' other works he employs many ideas of arithmetic. The basis ideas of arithmetic which will be used by Boethius include theory of equality, unity, number and the divine, and number and politics.

II.2.1.3.a. Theory of Equality

At the end of the first book and the beginning of the second book Boethius introduces an important theory that equality is prior to inequality, and inequality can be reduced to equality. In his work on arithmetic, Boethius emphasizes the importance of equality. He states that equality is like a matrix and takes the force of a root, so "it gives depth to the types and orders of inequality" (*Arithmetica*, I.32). Then Boethius follows Nicomachus in presenting the theory of the "three rules", according to which other sets of three in different ratios may be derived from three equal terms (*Arithmetica*, I.32), and by the reversal of which any proportion in three terms may be reduced to the original equality (*Arithmetica*, II.1). These rules are: (1) to make the first number equal to the first; (2) to put down a number equal to the first and the second; (3) to do the sum of one equal to the first, twice the second, and the third. (*Arithmetica*, I.32) Unlike Nicomachus, Boethius draws some diagrams to illustrate the process of generating, from three equal terms 1:1:1, another set of three in a different ratio 1:2:4. According to Boethius' descriptions and his diagrams, I draw a diagram to give a picture of how he uses three rules (see Diagram II).

Diagram II: How to Use three Rules

Three equal terms[113]:	1^a	1^b	1^c
Rule (1)----------	1^a		
Rule (2)----------		1^a+1^b	
Rule (3)----------			$1^a+2\times1^b+1^c$
Therefore:	1	2	4
The same three rules:	1	3 (3=1+2)	9 (9=1+2×2+4)

From the illustration, it is obvious that "equality is the principle of all inequalities, and then from inequality all other things are derived." (*Arithmetica*, I.32) Every species of inequality can be resolved into equality. In other words, equality is the mother of any quantity related to any other things and from it comes the first procreation of a relationship and to it again is its final resolution. The notion on the nature of equality becomes one basis for his argument in Boethius' *Consolatio*, which will be discussed in Chapter IV, especially in Section IV.2.

As a Pythagorean, Nicomachus believes that what is true of numbers is also true of the universe, and the significance of "equality" is equal to that of "sameness", that is to say, they are elements and principles of the universe. "Sameness" and "otherness" are the principles of the universe, as Nicomachus says in *Introduction to Arithmetic*.[114] For example, Nicomachus states that "The physical philosopher, however, and those that take their start with mathematics, call 'the same' and 'the other' the principles of the universe."[115] When "sameness" enters into the composition of things, it makes things persist in the same fashion, preserving their identity, while when "otherness" goes into the composition of things, it causes things to change from their original forms and assumes others.[116] Unlike Nicomachus, Boethius regards his work on arithmetic as the elementary discipline, thus he tries to avoid referring "equality" to universe directly, and he does not mention "sameness" at all. Nevertheless, he has the same ideas in mind and employs them to his *Consolatio* and his theological

[113] In order to distinguish "1" in different position, I add "a", "b", "c" to "1".
[114] Cf. Nicomachus, *Introduction to Arithmetic*, II.17.1, 18.1 and 4, 19.1 and 20.2; D'Ooge (1938).
[115] Nicomachus, *Introduction to Arithmetic*, II.18.1; D'Ooge (1938).
[116] Cf. D'Ooge (1938), p. 99.

treatises, especially *De Trinitate*. When Boethius discusses the unity of the Trinity, he begins with a brief analysis of sameness and otherness, and then he invokes Aristotle's point of view on sameness, which will be given in detail in Chapter III.

Nicomachus' equality and inequality are also related to virtue and vice. When discussing division of even numbers into perfect, imperfect, and superabundant, Nicomachus makes a comparison of inequality to vices, ill health and the like.[117] While in Chapter 19, Book I of Boethius' arithmetic, he omits Nicomachus' comparison, but adds analogies to vividly describe the extreme kinds of numbers.

> "And so this number whose parts added together exceed the sum of the same number is called superfluous."... "That number is called diminished whose parts, when put together in the same way, are exceeded by the multitude of the whole term."... The perfect number is that number the sum of whose parts is "not more than the total nor does it suffer from a lack in comparison with the total." (*Arithmetica*, I.19)[118]

Boethius thinks highly of perfect number. He regards the superfluous and diminished numbers as two elements unequal and intemperate, and between them, holding the middle place between the extremes like one who seeks virtue,[119] is the perfect number. In addition, in Chapter 32 of Book I, Boethius omits Nicomachus' I.23.5 which is a reference to the ethical

[117] Nicomachus, *Introduction to Arithmetic*, I.14.2; D'Ooge (1938).

[118] For example, the number 12 is a superfluous number, because half of 12 is 6, a third part is 4, a fourth part is 3, and a sixth part is 2, a twelfth part is 1, and the total sum [6+4+3+2+1] amounts to 16 which surpassed the total of the entire body, that is 12. In the same way, the number 14 is a diminished number, because the total sum [7+2+1] amounts to 10 which is smaller than the original term, that is 14. Similarly, the number 28 is a perfect number, because the total sum [14+7+4+2+1] is 28 which is equal to the original number that is 28.

[119] Cf. Aristotle, *Nicomachean Ethics*, 1108b11-14; Barnes (1984b): "There are three kinds of disposition, then, two of them vices, involving excess and deficiency respectively, and one a virtue, viz. mean." The word "mean" used by Aristotle does not refer to the mathematical mean, and in this sense the middle place Boethius uses is not a point exactly in the middle, but a stretch of the continuum around the middle. This meaning is also used by Boethius in his treatise against Eutyches and Nestorius; cf. Section III.2.3.

virtues. Though they both mention virtue and number, in Chapter 20 of Boethius' Book I which corresponds to Chapter 16 of Nicomachus' Book I, Boethius just uses one sentence to mention this relationship generally, "there is in these a great similarity to the virtues and vices." This similarity is also applied to the theological discussions. In his theological tractate called *Contra Eutychen et Nestorium*, Boethius believes the Catholic faith is the middle way, or just like virtue, it is the perfect way, and the other two faiths of Eutyches and Nestorius are the extremes like vice. I will discuss this in more detail in Section III.2.

II.2.1.3.b. Unity

Another important aspect of arithmetic is the knowledge of "unity". In the natural arrangement of numbers, every number, except unity, has next to it two terms and half of these two terms which come before and after it. Take the number 7 for example. Two terms next to it can be 6 and 8, or 5 and 9, or 4 and 10, and 7 is just half of 6 and 8, or 5 and 9, or 4 and 10. However, unity only has the number 2 next to it and it is half of 2. For this reason, unity has a special role. "It is rightly recognized as the generator of the total extended plurality of numbers." (*Arithmetica*, I.7) And Boethius stresses the elementary role of unity, that is to say, "unity is the substance and principle of any constant quantity" (*Arithmetica*, II.1). The theory of "unity" also becomes one basis for his argument in Boethius' *Consolatio*, which will be discussed in Chapter III and Chapter IV.

II.2.1.3.c. Number and Divinity

Boethius mentions number and divinity several times, which is vital to understand the *Consolatio* of Boethius. When discussing how to produce the even-times even number, he says that "the basic ordering of numbers has come about through careful consideration and through the great constancy of divinity." (*Arithmetica*, I.9) In Chapter 27 of Book I, he also says that "Such is the divine nature of things in this disposition that all the angles are tetragons."[120] In Chapter 2 of Book II, when he discusses the discovery in each number of how many terms of the same proportions are able to precede

[120] Boethius here apparently limits the term "tetragon" to indicate square numbers; cf. Masi (1983), p. 107.

it, Boethius stresses "This must always occur as by a certain divine, not human devising". The relationship between number and divinity will be helpful to understand Boethius' *Consolatio*, which will be shown in Section IV.2.2.

II.2.1.3.d. Number and Politics

The last relevant point of arithmetic I want to point out concerns politics, which is nowhere to be found in Nicomachus' work. In Chapter 45 of Book II, Boethius discusses "which medial proportions are compared to what things in the state of public affairs". The whole chapter has only one paragraph which contains much information.

Boethius compares three types of medial proportions to kinds of state. These three ones are the arithmetic proportion, the musical or harmonic proportion, and the geometric proportion. In the arithmetic proportion of three or any number of stated terms the same and equal difference is found between all terms.[121] For example, in the disposition of 1, 2, 3, there is an equal difference of terms according to an arithmetic interval, that is, 2-1=3-2. There are four properties of the arithmetic proportion, one of them is important, that is, "there is a major proportion found in the smaller terms and a minor proportion in the larger terms." (*Arithmetica*, II.43) This means that in the arithmetic proportion 1, 2, 3, the smaller terms are 1 and 2 of which two to one is a duplex, and the larger terms are 2 and 3 of which three to two is a sesquialter. For 2>2/3, the proportion of a duplex is larger than a sesquialter, in other words, the proportion found in the smaller terms 2 and 1 is a major proportion, but the proportion found in the larger terms 3 to 2 is a minor one. According to this property, the arithmetic proportion may be compared to a kind of state known as Authoritarian Government now, which is ruled by a small group with a greater power. The second proportion is the musical or harmonic proportion[122], "in which as the highest term is when compared to the smallest term, so the difference of the larger two is when compared to the difference of the middle term and the smallest (*Arithmetica*,

[121] If there are three numbers a, b, c (c>b>a), and c-b=b-a, then they form an arithmetic proportion.

[122] Concerning the reason why it is called a harmonic proportion, please see *Arithmetica*, II.48. I will not explain it here.

II.47)".[123] For instance, in the disposition of 3, 4, 6, there is an equation, that is 6/3=(6-4)/(4-3), so 3, 4, 6 forms a harmonic proportion. Contrary to that important property of the arithmetic proportion, in a harmonic proportion, the smaller terms have minor proportions, and the larger terms have a major proportion, such as in 3, 4, 6, the proportion of the smaller terms is 4/3, and the proportion of the larger terms is 6/4, and 4/3<6/4, so in the smaller terms a minor proportion is found, and in the larger terms there is a major proportion. The harmonic proportion is compared to the state which is called Democratic Government today. This kind of state is regarded as the very best, in which there is a proportionality in the larger term, viz. the major power is owned by the larger proportions of the population. The last kind of medial proportion is the geometric proportion, in which "an equal ration is always kept and the quantity and multitude of number is regularly ignored (*Arithmetica*, II.44)".[124] Take 2, 4, 8 for example. In the disposition of 2, 4, 8, in the smaller terms there is 4/2=2 which is duplex, and in the larger terms there is 8/4=2 which is also the duplex. Unlike the property of the arithmetic proportion and the harmonic proportion, the geometric proportion provides the middle position, in which either larger or smaller terms maintains equal quantities of numbers in proportionality. The geometric proportion is compared to the state which is "of the people, as it were, and of a balanced citizenry. For in either larger or in smaller, the whole is put together with an equal proportionality of all, and there is an equality between all; there is a certain equal right balance in preserving proportions." (*Arithmetica*, II.45)

By using this analogy, Boethius makes his readers vividly grasp the characteristics of three kinds of proportions, and have better ideas on the state, such as what is the state in general, what is the state of the very best, and what is the state of the people. It may seem strange in this work to add a paragraph on number and politics, for Boethius tries to write his arithmetical work as a basic one. However, this chapter implies that Boethius is interested in state affairs, which is also shown by his motivation to write

[123] If there are three numbers a, b, c (c>b>a), and c/b=(c-b)/(b-a), then they form an musical or harmonic proportion.

[124] If there are three numbers a, b, c (c>b>a), and c/b=b/a, then they form a geometric proportion.

theological treatises[125].

II.2.2. Boethius on Music

II.2.2.1. *De Institutione Musica* and Its Sources

The extant edition of Boethius' *Musica* includes five books and the whole work ends with Chapter 19 of Book V.[126] Unlike his work on arithmetic, the sources of his *Musica* are complicated. Many citations of authors occur in Boethius' *Musica*. Among these authors, there are two important ones, Nicomachus and Ptolemy (90-168 A.D.).

Nicomachus is the person cited most often in the first four books. As I said in the earlier section about Nicomachus[127], only two of his works are preserved to us in their entirety, namely *Introduction to Arithmetic* and *Manual of Harmonics*. The first four books, especially Book I-III of Boethius' *Musica* show clearly a development from and dependence on Nicomachus' two works. After comparing Boethius' music with that of Nicomachus, Calvin Bower has reached the conclusion that the extant and the more extended musical work promised by Nicomachus served as the principal source of Boethius' first four books of *Musica*.[128]

The first book of Boethius' *Musica* is the general introduction to music. In the prologue of Book I (including Chapter 1 and Chapter 2), Boethius introduces the nature of music, the role of music, and the division of music. In Chapter 33 of Book I, Boethius sums up the first book, "how the things thus far said are to be taken" and promises that "all these will be proved both through mathematical reasoning and aural judgment." Book II and III involve the technical theories on numerical proportions to fulfill the expectation of logical demonstrations of the theories presented as dogma,

[125] Cf. the preface to Chapter III of my dissertation.
[126] Cf. Bower (1981), "The Role of Boethius' *De Institutione Musica* in the Speculative Tradition of Western Musical Thought," and Hoolloway (1981), " 'The Asse to the Harpe': Boethian Music in Chaucer," and Pizzani (1981), "The Influence of the *De Institutione Musica* of Boethius up to Gerbert D'Aurillac: A Tentative Contribution," in Masi (ed.), pp. 157-174, pp. 97-156, and pp. 175-186.
[127] Cf. Section I.2.2.
[128] Cf. Bower (1978).

which are introduced in Book I with little comment. The content of these two books is related to Nicomachus' work on arithmetic, including the mathematical theories which are applied to the monochord in Book IV. There are two main parts in Book IV: one is the division of the monochord, and the other is the theory of modes, which seems unrelated. From the whole structure, no matter how loose it is, the first four books can be put together as treating instrumental music, throughout which only one dissenting voice is allowed. This voice is from Ptolemy. And later in Book V, Ptolemy assumes the leading role. Book V continues to deal with instrumental music but develops in new directions. Boethius picks out the first chapter of Ptolemy's *Harmonics* as the source for Book V to complete his Latin record of Greek musical thoughts.

It is a pity that Boethius' extant work on music is not preserved in its entirety. On the one hand, it is obvious that his extant work ends abruptly, leaving eleven chapter titles without content. On the other hand, he dedicates one chapter of the first book to the general introduction to three kinds of music, which are cosmic music, human music, and instrumental music, but in his extant five books there is only knowledge of instrumental music without any further exposition on cosmic music[129] and human music. There are two possibilities. The first one is that Boethius did finish translating Ptolemy's work, but those parts have not been preserved. The second one is that he did not finish interpreting Ptolemy's *Harmonics*. No matter which possibility it is, one thing is sure: if the whole *Harmonics* had been interpreted by Boethius, he would have completed his thoughts on the three kinds of music.

II.2.2.2. Applications of Basic Logic to *Musica*

In his extant work, when interpreting his sources, Boethius applies a few basic logical theories to make his expression clearer, viz. the theory of categories, division and definition.

[129] Boethius does say a few words on cosmic music in his extant work on music. In I.27 of *Musica*, Boethius discusses "to what heavenly bodies the strings are compared", but this comparison of strings to the disposition of the heavenly spheres is hardly enough to fulfill his promises to discuss cosmic music "later more studiously"; cf. *Musica*, I.2.188.

II.2.2.2.a. Categories in *Musica*

Music considers the relationship between numbers, so sound should be known through numbers which are related through proportions. That is to say, sound is regarded as quantity. Music studies the multitude, the discrete quantities. Of quantities, some are equal and others are unequal, thus of sounds, "some sounds are also equal, while others stand at an interval from each other by virtue of an inequality (*Musica*, I.3.191)". Except the category of quantity, sound is also related to another category of action. If there is no motion, there will no sound. Sound is "a percussion of air remaining undissolved all the way to the hearing (*Musica*, I.3.189)". The frequency of motion causes different sounds. The slow and less frequent motion of the string will produce low sounds, so by contrast, the fast and more frequent motion of the string will produce high sounds. In other words, high sounds consist of more motions than low sounds. According to the property of discrete quantity, low sounds and high sounds should preserve the nature of consonance which is a vital concept in music. "In those pitches which do not harmonize through any inequality, there is no consonance at all. For consonance is the concord of mutually dissimilar pitches brought together into one." (*Musica*, I.3.191) By the help of theory of categories, sound, which is a basic but abstract term in music, can be understood easily.

II.2.2.2.b. Theories of Division and Definition in *Musica*

Other important applications of logic to music are theories of definition and division.[130] As a matter of fact, division and definition are necessary requisites for each other, in other words, division is necessary for full definitions of species, since a unified definition is made up of divisions joined together; and definition is necessary for division, since by means of definition, it could be determined what is equivocal and what is univocal (*Divisio*, 880c-880d). It could be said that division and definition are close sisters, or more specifically, division of a genus into its species is most intimate with definition. This could be demonstrated from what Boethius describes in *Divisio*.

"When I have been given a species of the sort that both has a genus

[130] Cf. Section II.1.2.2.

> and is predicated of subsequent things, I first take up its genus, I divide the differentiae of that genus, I join a differentia to the genus, and I see whether that differentia joined to the genus can be equal to the species I have undertaken to circumscribe with a definition. .... Finally, we distribute differentiae under differentiae as often as we must until all of them joined to the genus describe the species in a definition that is equal to it." (*Divisio*, 886a)

Just due to this knowledge about division and definition, Boethius makes changes to the conception of some musical terms, which makes himself distinct from his sources.

The most obvious example is the definition of consonant and dissonant sounds. These two terms appear in Boethius' Book IV, the introduction of which derives from parts of Euclid's *Sectio Canonis*. In *Sectio Canonis*, Euclid gives defines the term "consonant sounds" as two sounds which mingle, and similar to this simple way, the term "dissonant sounds" is defined as two sounds which do not mingle. Boethius was not satisfied with definitions of these two terms. Obviously, as the names of two terms show, consonant sounds and dissonant sounds are two species of the genus sound. When defining them, the first thing to be taken up is, of course, their genus, the sound. Then the significant thing is to add the differentiae. And the reason why Boethius changes the definition is likely that he thinks the differentia "which mingle" joined to the genus sounds could not be equal to the species of "consonant sounds", or in other words, could not be equal to the definition of "consonant sounds". And it is the same with the definition of "dissonant sounds". Thus, Boethius joins the differentia "when struck at the same time" to the genus. And it is still not enough, so he adds as the last differentia "pleasant and intermingled". So far, in Boethius' point of view, the description of the species "consonant sounds" is integral and equal to the definition of this term. Accordingly, in the introduction of Book IV of his work, Boethius defines, "Consonant pitches are those which when struck at the same time sound pleasant and intermingled with each other; dissonant pitches are those which when struck at the same time do not yield intermingled sound." (*Musica*, IV.1.302)

The theory of division is also employed by Boethius in his work on

music. There are a few divisions in musical theories, and because of Boethius' familiarity with the theory of division, his expositions of musical divisions are clear and easy to be grasped.

The first division I want to discuss is a famous division of people whose jobs are related to music. In Chapter 34 of Book I, Boethius distinguishes musicians from performers and composers. This division is different from the divisions I described in Section II.1.2.2. Division of a genus into its species, division of a whole into its parts, and division of a spoken sound into its significations are divisions *per se*. Except those three divisions, there is another one, division *per accidens*. There are three types of division *per accidens*, which are the division of a subject into its accidents, division of an accident into its subjects, and division of an accident into accidents. Here the division of people whose jobs are related to music belongs to division of a subject into its accidents. The same subject is men who are engaged in the musical art, and its accidents include what they do, and how they deal with music. The first kind of man performs on instruments, but they make no use of reason, acting as slaves, so they are excluded from comprehension of musical knowledge. The second kind of man is like the poet, and when they are composing songs, they take advantage of a certain natural instinct but not so much by thought and reason. Therefore, the first kind of man who is related to music is called performer, and the second is named composer, both of which are not musicians in the proper sense. Unlike the former two classes, the third class of man makes full use of his reason and thought to "exhibit the faculty of forming judgments according to speculation or reason relative and appropriate to music concerning modes and rhythms, the genera of songs, consonances, and all the things which are to be explained subsequently, as well as concerning the songs of the poets (*Musica*, I.34.225)". The last kind of man who is related to music can be esteemed as musician. Thus, by applications of theory of division, Boethius defines what the musician in the proper sense is.

In Chapter 21 of Book I, Boethius introduces three genera of melodies: diatonic, chromatic, and enharmonic. The heart of Book IV is the division of the monochord, comprising Chapters 5-13. A monochord division is truly exceptional in ancient musical theory, and began from the Pythagorean

diatonic genus. These are called the division of genus into its species. Similar to these divisions, division of notes also belongs to division of genus. It is common in all classical Greek musical theory that the notes are classified as either fixed or movable. However, here Boethius adds intermediate ones, "Of all these pitches, some sound completely fixed, some completely movable, whereas others sound neither completely fixed nor completely movable." (*Musica*, IV.13.335) This division is identical with Nicomachus' theory, which is found in no other author of antiquity. Nicomachus gives the definition of the category of sound which is neither completely fixed nor completely movable as one that does not move between the diatonic and chromatic but move in the enharmonic, which is unique to Nicomachus.[131] Boethius' division of notes is a little different from those above, for it is not division of genus into its species, but into to differentiae. "A genus is divided sometimes into species, sometimes into differentiae, if the species by which the genus ought rightly to be divided lack names." (*Divisio*, 880b) "Fixed" and "movable" are not the species but the differentiae. Since there is no single name of the species "fixed notes", the differentia is put in place of the species and connected to the higher genus. "For every differentia produces a species when it comes into conjunction with its proper genus." (*Divisio*, 880b)

The threefold division of notes is the same with twofold division of notes. "That with the imposition of names division is always into two terms is made clear when one on one's own initiative imposes a name for a genus or differentia that has none." (*Divisio*, 883d-884a) This means that the tripartite division, "of notes some are completely fixed, some completely movable, and others are neither completely fixed nor completely movable," could be made bipartite if expressed in this way: "Of notes that some are fixed, others movable; of the movable ones some are completely movable, others partly movable." Therefore, although Boethius chooses to follow Nicomachus' theory, he does not disprove the mainstream of Greek musical theory; on the contrary, he coincides with that.

The last division but the most important one is the three-fold kinds of music. Bower thinks highly of Boethius' dividing the kinds of music:

[131] Cf. Bower (1978), p. 26.

> "Just as Boethius seems to have coined the concept of *quadruvium* in the introduction to *De institutione arithmetica*, a concept not found in Nicomachus' treatise, so Boethius may have conceived the three-fold concept of music placed in certain instruments, music of the human being, and music of the universe in the *proemium* of *De institutione musica*."[132]

As we know, the source of the first four books is Nicomachus' works on arithmetic and music. In the first four books, human music and cosmic music are not dealt with, and in Nicomachus' extant musical work, there is no the division of music and we do not have the longer one which Nicomachus promised to write. Therefore, we may well say that the division of music was not to be part of the original text of Nicomachus. Boethius' threefold division of kinds of music is unlikely to be original with him, because implicit in the thoughts of Plato and Pythagoras there were similar ideas. However, it is sure that the first person who expressed this idea distinctly in Latin is Boethius.

Boethius' clear exposition of dividing music may result from Peripatetics' division of the forms of speech. In the commentary on Aristotle's *On Interpretation*, Boethius says, "Peripatetics who draw from Aristotle were right to posit three forms of speech (*oratio*): one which can be written in letters, a second which can be vocally expressed and a third which can be connected by thought; one contained in thoughts, the second by spoken sound, the third by letters." ($_2$*InInter.*, 29.15-20) The division of speech and the division of music are both divisions of a genus into its species. The differentia of different kinds of speed is their form, which is the same with that of music. There are hierarchies in both divisions. The lowest kind of speech is the speech that can be written in letters. In other words, this kind of speech exists in tangible objects. The highest kind of speech can be connected by thought, the incorporeal form. And the second kind of speech, which is contained by spoken sound, is the connection between the extremes. Similar to the hierarchy of speech, the lowest kind of music, instrumental music, is the music which exists in tangible instruments; and the highest type of music, cosmic music, is contained by incorporeal form.

[132] Bower (1978), p. 44.

Human music, the middle one in the hierarchy, can connect cosmic music and instrumental music. The threefold division of kinds of music is so important in Boethius' thoughts that I will provide a more detailed discussion in Section II.2.2.3.

II.2.2.3. Basic Ideas of Music Used in Boethius' Other Works

The basic idea of music which will be applied to Boethius' other works, especially *Consolatio*, is the view on three kinds of music. The lowest kind of music is instrumental music, which is produced by strings, winds and percussion. It is not hard to grasp instrumental music, for this kind of music is the one which we can listen to directly with our ears. Another reason why instrumental music is understood without effort is that the whole extant work on music of Boethius deals with instrumental music. As for the two higher kinds of music, human music and cosmic music, we only have general introductions to them at the beginning of Boethius' work. After each short introduction, Boethius uses similar sentences: one is "these things ought to be discussed later more studiously", and the other is "I shall also speak about these things later" (*Musica*, I.2.188-189), both of which together show that human music and cosmic music would be explained in detail in the later parts of his work. However, unfortunately, at the end of his extant work on music, Boethius does not come back to the topics of human music and cosmic music.

Boethius begins to choose Ptolemy as his source of his Book V, but his extant work ends abruptly at Chapter 19 of Book V with eleven titles of the remaining chapters, which just finishes the first book of Ptolemy's. Some ideas of Ptolemy's second book are condensed and larded with the corresponding part in first four books. Inferentially, it is said that if Boethius had finished his whole work on music, he would have come to the discussion on human music and cosmic music following the third book of Ptolemy' *Harmonics*. It is possible that Boethius changed some ideas on Ptolemy's human music and cosmic music, but from what Boethius applies in his *Consolatio* the knowledge of human music and cosmic music in Boethius is similar to Ptolemy's. In order to better comprehend applications of the three kinds of music in his later works, I shall here add more of Boethius' ideas on human music and cosmic music with reference to the

third book of *Harmonics*.

Human music can be perceived, in Boethius' point of view, by whoever penetrates into his own self. As to how to comprehend human music, Boethius and Ptolemy both resort to a kind of analogy. Low sounds and high sounds can be brought into one to produce one consonance. Similar to this, Boethius believes the incorporeal nature of reason can be united with the body by a certain harmony. And in his third book of *Harmonics*, Ptolemy says that the consonances are in accord with the soul. Before explaining this relationship, a short introduction to the knowledge of the consonances should be given first.

One consonant sound consists of two unequal sounds, which is based on the proportional theory in arithmetic. The consonance of the diapason is that which is made in duple ratio (2:1). The diapente is that which consists of the ratio sesquialter (3:2). The diatessaron is that which occurs in the ratio sesquitertian (4:3). Ptolemy connected these three consonances to three first parts of the soul — the intellectual, aesthetic, and habitual.

> "So that that of the diapason accords with the intellectual, for mostly in each is what is simple, equal, and not different, the diapente to the aesthetic, and the diatessaron to habitual."[133]

For the ratio 3:2 is nearer the ratio 2:1 than the ratio 4:3, which means that the diapente is nearer the diapason than the diatessaron, thus the corresponding parts of the soul have the same relation, that is to say, "the aesthetic is nearer the intellectual than the habitual on account of its sharing some of the same perception." Our soul can be divided in another way, viz. into the rational, emotional, and cupidinous.

> "The rational, for the sake of an equality similar to what we have previously discussed, we equate properly to the diapason, the emotional somehow approaching it, to the diapente, and the cupidinous, arranged below it, to the diatessaron."[134]

By virtue of these three important consonances, Ptolemy discusses the

[133] Ptolemy, *Harmonics*, III.5: 96.1-3; Solomon (2000).
[134] Ptolemy, *Harmonics*, III.5: 96.28-9; Solomon (2000).

nature and species of each part of the soul. In addition, harmonic modulations resemble the circumstantial modulations of souls. For example, the peaceful conditions make the souls of citizens more tranquil and equitable, but by contrast, martial conditions cause the souls to be more rash and contemptuous. In a similar way, the tension in strings or a higher pitch gives a more arousing sense, and the relaxation in strings or the lower pitch produce a more depressing sense. Consequently, it can be said that "our souls evidently experience the same effects as the melody, as if they recognize the kindred relationship of the ratios of each state and are modeled by some movements appropriate to individual musical forms."[135] With the help of instrumental music, human music would be understood, but both music types imitate the highest level of music, cosmic music.

As I discussed above, the motion can cause sounds, so it must be the case that when a heavenly machine moves extremely fast, there is a sound that does not penetrate our ears. This kind of sound belongs to cosmic music. In the Chapter 9-16, Book II of *Harmonics*, Ptolemy makes an analogy to examine musical principles in the heavenly bodies or movements. He discusses how the harmonic consonances and dissonances resemble those in the Zodiac, how the succession in the notes resembles the longitudinal movement of the stars, how the stellar movement in altitude compares with the harmonic genera, that modulations of *tonoi* are like stellar crossings in latitude, on the similarity of the tetrachords and the aspects of the sun, by what first numbers might the fixed notes of the perfect system be compared to the first spheres in the cosmos, and how the combinations of the planets should be compared to those of the notes. From these comparisons, it is remarkable that although we could not hear cosmic music, it does exist. Boethius believes that the failure of the sound of a moving heavenly machine to penetrate our ears happens necessarily for many reasons. Nevertheless, it is impossible that such extremely fast motion of such large bodies should produce absolutely no sound, especially since the courses of the stars are joined by such harmonious union that nothing so perfectly united, nothing so perfectly fitted together, can be realized (*Musica,* I.2.187-188). Cosmic music means cosmic harmony, which holds together

[135] Ptolemy, *Harmonics*, III.7: 99.25; Solomon (2000).

the four elements of earth, air, fire, and water, or the cycle of the four seasons, in consonance and equilibrium. It leads to the change of season and determines the movement of the celestial bodies. Cosmic music is discernible especially in those things that are observed in heaven itself or in the combination of elements or the diversity of seasons.

Of course one discerns musical principles through instrumental music, but Boethius suggests his readers to go beyond the music of instruments to the music of human beings, and even further to the music of the universe. For likeness attracts, and thus the harmony of proportions in human beings could help us be attracted to the harmony in the universe which is the pure proportion or the idea of harmony itself. The three kinds of music and the process from instrumental music to human music and finally to cosmic music are very important in Boethius' thought, which will be fully exhibited in Boethius' *Consolatio.*[136]

II.3. Conclusion

From the discussion of Boethius' logic, we can find that applications of mathematics to logic are mostly examples. Boethius' logical knowledge does have an influence on his mathematical works, as has been discussed above. Although Boethius' mathematical works are not original, his application of logic gives his works a distinct character. Even at this level there is a certain connection between mathematics and logic.

As elementary disciplines, both mathematics and logic play an essential role in Boethius' philosophy. When Boethius discusses theological issues, he employs this elementary knowledge of mathematics and logic, and even when he faces death, he uses his knowledge of mathematics and logic to console himself, which will be discussed in Chapter III and Chapter IV, respectively.

[136] Cf. Section IV.2.2.1.

III. Applications of Basic Logic and Mathematics in the Theological Tractates

At the beginning of Chapter I, I set out that the fields Boethius mainly wrote about were mathematics, logic and theology, and that Boethius regarded all of them as philosophy. From the above chapters, it has become clear that mathematics and logic are elementary disciplines in Boethius' curriculum. In the chapter on logic and mathematics,[137] I gave a short introduction into some basic ideas of these two disciplines and I promised to show how Boethius employs them in his philosophical writing. From this chapter onwards, I will discuss the applications of basic logic and mathematics. In this chapter, I first investigate their use in Boethius' theology.

Boethius' theological works are usually called *Opuscula Sacra.*[138] The traditional order of these five theological tractates is the order in which the tractates were found in the manuscripts and in which they are generally printed: *De trinitate*, *Utrum pater*, *Quomodo substantiae*, *De fide*, and *Contra Eutychen et Nestorium.* However, this is not the chronological order. I follow Chadwick in regarding *De fide* as the first in the chronological order. Then Boethius wrote the *Contra Eutychen et Nestorium*, followed by *De trinitate* and *Utrum pater*. The relative position of *Quomodo substantiae* remains uncertain.

Why did Boethius see the need to write on Christian theology? In the age of Boethius, religious and political affairs were often inseparably bound

[137] Cf. Section II.1.2, Section II.2.1.3 and Section II.2.2.3.

[138] Cf. Bradshaw (2009), "The *Opuscula Sacra*: Boethius and Theology," in Marenbon (ed.), pp. 105-128; Mair (1981), "The Text of the *Opuscula Sacra*," in Gibson (ed.), pp. 206-213; Reiss (1982), pp. 58-79. — Thomas Aquinas wrote two works on Boethius' *De trinitate*; cf. Aquinas (1953 and 1987). He also wrote a work on Boethius' *Quomodo substantiae*; cf. Aquinas (2001). McInerny has argued that Thomas Aquinas was a reliable interpreter of Boethius' thought (McInerny, 1990 and 1992). On Boethius and Aquinas, cf. Hall (1992); Hankey (1981).

up with each other.[139] As a consul, therefore, Boethius had to closely follow the theological developments of his day.[140] Among his five theological tractates, only the *Contra Eutychen et Nestorium* gives us clear information on the circumstances of its composition.

> "I have been long and anxiously waiting for you to discuss with me the problem which was raised at the meeting. ... You no doubt remember how, when the letter was read in the assembly, it was asserted that the Eutychians confess that Christ is formed from two natures but does not consist of them ..." (*Contra Eutychen et Nestorium*, 1-2; 6-7)

The meeting Boethius refers to took place in the year 512 A.D..[141] In 512 A.D., some Eastern European bishops from the Byzantine Empire were caught between Eutychians and Nestorians, so they appealed to Pope Symmachus[142] for his advice and help. In the same year, an assembly of high clergy and Roman senators was held, and Boethius and John the Deacon, to whom *Contra Eutychen et Nestorium* is addressed, attended this meeting. At this assembly, the letter that Boethius refers to was read; it contained a plea for help from the Eastern bishops.[143] In this letter, a formula was submitted, to the effect that Christ both consisted *of* two natures and subsisted *in* two natures ("*ex* duabus naturis et *in* duabus naturis"). Boethius agrees to this formula, and he says that "among followers of the true Faith He is equally believed to be of two natures and in two nature" (*Contra Eutychen et Nestorium*, 9-12). This formula is different from that of Eutychians, who "confess that Christ is formed from two

[139] Cf. Charanis (1974); Dvornik (1951). — In section II.2.1.3.d above, I noted that Boethius added chapter 45 of Book II to his work on arithmetic, discussing "which medial proportions are compared to what things in the state of public affairs", which shows that even in the context of arithmetic, Boethius' interest in state affairs perspires.

[140] Cf. Reiss (1982), "The Philosopher as Public Administrator", "Theological Controversies", and "Joining East and West", pp. 55-58, pp. 64-66, and pp. 76-79.

[141] Cf. Chadwick (1981b), pp. 180-190; Marenbon (2003), pp. 68-69.

[142] Pope Symmachus (died 19 July 514 A.D.) was the head of the Catholic Church from 22 November 498 to his death in 514. His tenure was marked by a serious schism over who was legitimately elected pope by the citizens of Rome. Cf. Davis (1989), pp. 43-46 and Richards (1979), pp. 69-99 on the Symmachan Schism; see Townsend (1937) on councils held under Pope Symmachus.

[143] For the text of the letter, see Mansi (1901-1927) (V.8), pp. 224-225.

natures but does not consist of them" (*Contra Eutychen et Nestorium*, 7-8). In the letter, the Eastern bishops say that, there is a middle way between Arius[144] and Sabellius[145]. Since the position of Nestorius resembles that of Arius, and the position of Eutyches resembles that of Sabellius, therefore, they in fact asked Pope Symmachus to give them a middle way between Nestorius and Eutyches. Boethius attaches importance to this issue raised by the bishops and thus effectively lends them his aid by means of a tractate against Eutyches and Nestorius. He provides them with the requested middle way, which is the position of the Christian faith. He says that he was struck by the novelty of the assertion in the letter that was read in the assembly. Here we see Boethius' motivation for writing the *Contra Eutychen et Nestorium*, together with its political significance.

After the completion of the treatise *Contra Eutychen et Nestorium*, Boethius turns his interests to the doctrine of the Trinity. He wrote two treatises on the Trinity, in both of which there is no certain information to show what caused Boethius to focus on this issue. From the study of the religious and political events at that time, it is clear that issues concerning the Trinity directly resulted from the Christological debate, especially from an anti-Nestorian slogan.[146] This slogan "*unus ex trinitate passus carne* (one of the Trinity suffered in the flesh)" was offered in 519 A.D. by Scythian monks from the East to protect Christ's divinity by allowing for his suffering as a human, i.e. in his passible nature.[147] The theology of the Scythian monks sounded dangerously close to making an intimate link between Monophysite Christology and a strongly pluralistic doctrine of God, in antithesis to Nestorius' tendency to link a diphysite Christology with an

[144] Arius (250 or 256-336 A.D.) was an ascetic North African Christian presbyter and priest in Alexandria, Egypt, of the church of Baucalis; Arianism originated with him. Arius asserted that the Son of God was a subordinate entity to God the Father. Cf. Anatolios (2011), p. 44; Williams (2001).

[145] Sabellius is a theologian and priest from the 3rd century A.D.; the theological teaching attributed to him is Sabellianism. In Christianity, Sabellianism (also known as modalism, modalistic monarchianism, or modal monarchism) is the nontrinitarian belief that the Heavenly Father, the Resurrected Son and the Holy Spirit are different modes or aspects of one God, as perceived by the believer, rather than three distinct persons in God Himself. Cf. Pelikan (1975), pp. 179-181.

[146] Cf. Chadwick (1981b), p. 211.

[147] Bark (1944), p. 417. Cf. Marenbon (2003), pp. 76-77.

exclusively unitary doctrine of God.[148] This event led to a schism in the Church, beginning in the year 482 A.D.. In 482 A.D., the Emperor Zeno had issued a document known as the *Henotikon* aimed at upholding the Nicene and Constantinopolitan creeds. The original purpose of the *Henotikon* was the reconciliation of the theological differences, so it avoided any definitive statement on whether Christ had one or two natures. This document condemned the teachings of Nestorius and Eutyches, but Zeno's attempt to appease the parties was not successful, and it was followed by the schism of the Eastern and Western churches.[149] Boethius undoubtedly realized the importance of the question for the restoration and maintenance of peace within the Church. Thus, Boethius set down to discuss the doctrine of the Trinity in his next two theological tractates, *De trinitate* and *Utrum pater*.

Besides the three treatises introduced above, Boethius also composed a tractate named *Quomodo substantiae*, in which he does not discuss any point of Christian dogma. Despite this difference in character, these four works (*De trinitate*, *Utrum pater*, *Quomodo substantiae*, and *Contra Eutychen et Nestorium*) adopt a similar method: Boethius uses numerous applications of basic logic, but very few applications of mathematics, to throw further light on the issues at stake. In the *De fide* Boethius makes no use of basic logic and mathematics at all.[150] The *De fide* is by far the least studied work of the theological tractates, but this treatise is of great importance, for it gives a brief sketch of the central doctrines of Christianity and covers the topics which are discussed in other tractates. For this reason, I shall investigate *De fide* first and point out how it prepares for the other theological treatises. Then I shall discuss the three tractates on Christian doctrine in chronological order. The last treatise I shall analyze is *Quomodo substantiae*, for it looks like a philosophical essay, and is therefore closer to his last work, the *Consolatio*, which I shall discuss in Chapter IV.

Mair points out that "the relationship between the logical and

[148] Cf. Chadwick (1981b), p. 212.
[149] Cf. Baker (1973), pp. 17-22; Bark (1944); Bury (1958), pp. 314-347; Dvornik (1951); Matthews (1981), "Anicius Manlius Severinus Boethius," in Gibson (ed.), p. 33.
[150] The authenticity of *De fide* has often been discussed by scholars, but now it is generally accepted as one of Boethius' theological works. Cf. Bark (1946); Chadwick (1980a).

theological works of Boethius himself"[151] still awaits detailed examination, which is what I am trying to do now. In this chapter, I will examine applications of basic logic and mathematics in Boethius' theological tractates, and establish the relationship between Boethius' logical and mathematical works, and his theological works.

III.1. On the Treatise *De Fide Catholica*

The title of *De fide* was lacking from the manuscripts. It is Renatus Vallinus, in the Leyden edition of *Consolatio* and the tractates (1656), who first gave it the title under which it appears in subsequent edition.[152] Although the time of writing is uncertain, I agree with Chadwick that *De fide* was probably written first.[153] The Catholic faith is the basis of Boethius' arguments in other theological tractates. For example, at the beginning of *De trinitate*, he emphasizes this: "I think that the method of our inquiry must be borrowed from what is admittedly the surest source of all truth, namely, the fundamental doctrines of the Catholic faith." (*Utrum pater*, 3-5) In addition, if any argument is against the Catholic faith, Boethius will come to the conclusion that it is impious, "for the truth of the faith and the unwontedness of the miracle alike remain, for Catholics, unshaken." (*Contra Eutychen et Nestorium*, IV.61-62) Secondly, *De fide* is composed of a succinct survey of the Bible history and some great truths of Christianity, especially the doctrines of the Trinity and Christ's Nature and Person, which will be tackled in *De trinitate*, *Utrum pater*, and *Contra Eutychen et Nestorium*. And finally, *De fide* sets out the range of problems Boethius' other theological treatises will deal with, and leaves detailed explanations for later discussions. For instance, several heresies are discussed briefly in *De fide*, but Boethius leaves the refutation of these heresies for later discussions. Take Arius and the Manichaeans for example. Boethius introduces Arius and the Manichaeans in *De fide*. "Arius, for instance, who, while calling the Son God, declares Him to be vastly inferior

[151] Mair (1981), "The Text of the *Opuscula Sacra*," in Gibson (ed.), p. 212.
[152] Cf. Stewart (1974), p. 139. Also cf. Chadwick (1981b), pp. 175-180; Marenbon (2003), pp. 66-68.
[153] Cf. Chadwick (1981b), p. 180.

to the Father and of another substance." (*De fide*, 32-34) But the error of the views of the Arians is pointed out by Boethius in Chapter I of *De trinitate*. And in the case of the Manichaeans, they refuse the divine nature of Christ, and also refuse the Virgin birth of the Son. Boethius does not further consider these points here. He says that "the points will be presented in the proper place as the proper arrangement demands (*De fide*, 52-53)," which may imply the treatise against Eutyches and Nestorius.

Therefore, of the five theological treatises I will discuss *De fide* first. In this section, I focus on two important issues discussed in *De fide*, i.e. issue of the Trinity and Christology, and point out the relation between these two issues and Boethius' other theological treatises.

III.1.1. The Issue of the Trinity

The first important issue is on the Trinity[154], upon which Boethius says the Christian and Catholic religion is chiefly based (*De fide*, 7-9). This implies that issue of the Trinity is essential to discuss other Christian issues.

Boethius begins *De fide* with the sentence that "the Christian Faith is proclaimed by the authority of the New Testament and of the Old (*De fide*, 1-2)". In the Scripture, the word "trinity" is not found, but thoughts of the Trinity are deemed to emerge from Scripture by theologians. For example, the verse "Go ye therefore, and teach all nations, baptizing them in the name of the Father, and of the Son, and of the Holy Spirit" (Matthew 28:19) is taken to imply the faith that baptism is administered in God who is Father, Son, and Holy Spirit, but in his "name" in the singular (not "names"). That is the required unity-in-diversity. Although three names are provided, no explanation of the plurality is supplied. This is not a trinity (though trinity is not excluded) but a triad.[155] Phan believes that the understanding of Christian faith in the Trinity should not be based exclusively on explicitly triadic formulae such as the verse in Matthew 28:19.[156]

The word *trias* was first used in the East by Theophilus of Antioch in

[154] Cf. Davis [etc.] (1999/2002); Phan (2011); Wolfson (1956).
[155] Cf. Grant (1986), p. 151.
[156] Cf. Phan (2011), p. 3.

the second century. In the West, in the third century, the African theologian Tertullian[157] (160-about 225 A.D.) coined a Latin word *trinitas*, meaning "three persons in one substance".[158] At first the word "trinity" only implied the threefoldness of the Godhead, but later came to include the unity as well. The doctrine of the Trinitarian unity, says Grant, is not a product of the earliest Christian period. As a matter of fact, before the end of the second century, we could not find this doctrine expressed carefully.[159] Before Nicaea[160] the main problem was to derive the Trinity of persons from the unity of God; while after Nicaea the main issue was to achieve the unity of God from the Trinity of the persons.[161]

For the inquiry of the Trinitarian doctrine, Boethius writes two tractates, namely *De trinitate* and *Utrum pater.* There are some similar thoughts in these two tractates, which they have in common with *De fide*. For example, when Boethius introduces the Father, the Son, and the Holy Spirit, he says, "there has existed one divine substance of Father, Son, and Holy Spirit in such wise that we confess the Father God, the Son God, and the Holy Spirit God, and yet not three Gods but one God." (*De fide*, 9-14) And later in *De trinitate* and *Utrum pater*, the alike thoughts, words, and phrases relating to the Trinity are repeated.

> "I take together all three, Father, Son, and Holy Spirit, the result is not three substances but one substance. The one substance of the Three, then, cannot be separated or divided, nor is it made up of various parts, combined into one: it is simply one." (*Utrum pater*, 12-14)

[157] On the introduction on Tertullian, cf. Barnes (1985); Dunn (2004).
[158] Cf. Phan (2011), "Developments of the Doctrine of the Trinity," in Phan (ed.), p. 8.
[159] Cf. Grant (1986), p. 156.
[160] The First Council of Nicaea took place in 325 A.D.. According to the Nicene Creed, the belief "in one God, the Father; and in one Lord Jesus Christ, the Son of God" implies that there is unity between them. Cf. Ayres (2004).
[161] McKim (1988), p. 19. Barnes summarizes the form of Latin theologies of the Trinity from Tertullian to Augustine thus: "there is an explanation for how the Three are understood to be one"; "there is an explanation for how the Three are distinct from one another"; "there is a statement that the Three are eternally irreducible and unconvertible"; and "there is a word for what is three in God — that is, person (*persona*)". Cf. Barnes (2011), "Latin Trinitarian Theology," in Phan (ed.), p. 71.

> "For instance if we say 'the Father is God, the Son is God, and the Holy Spirit is God,' then Father, Son, and Holy Spirit are one God." (*Utrum pater*, 19-21)
>
> "The belief of this religion concerning the Unity of the Trinity is as follows: the Father is God, the Son is God, the Holy Spirit is God. Therefore, Father, Son, and Holy Spirit are one God, not three Gods." (*De trinitate*, I.6-9)

In *De fide*, Boethius states that Father, Son, and Holy Spirit have one divine substance, and they are not the same. However, he does not give the reason why they are not the same, which is the main topic of *De trinitate*.[162]

III.1.2. The Issue of the Nature and Person of Christ

In *De fide*, Boethius uses a long passage to state Christ's birth from the Virgin, his assumption of manhood, and Nestorius' and Eutyches' distortions of the orthodox teaching on the subject. The thought here is wholly concordant with which Boethius sets forth in his tractate against Eutyches and Nestorius. It should be said that *De fide* introduces the vital issue discussed in *Contra Eutychen et Nestorium*, which focuses on Christology, concerning the identity of Jesus Christ, or in other words, relating to Nature and Person of Christ.

In the first three centuries of the church, formulations of Jesus' identity were not explicit.[163] Jesus Christ stood in common with other humans in terms of his human characteristics, but he is also seen as one in whom God is realized and known in a special and unique way. Jesus Christ is considered both divine and human at the same time. Two important terms related to Jesus Christ, *ousia* and *hypostasis*, were first used as synonyms, and Basil of Caesarea (329 or 330-379 A.D.) was the first to attempt to distinguish explicitly between them: "*ousia* is related to *hypostasis* as the common to the proper; the Son is *homoousion* with the Father, but the Father, Son, and Holy Spirit must each be confessed in his own

[162] Cf. Section III.3.
[163] Cf. McKim (1988), p. 25.

hypostasis."[164] However, Basil was concerned with the words in a Trinitarian content rather than in a Christological context.

The terminology of Christ "two substances or natures in one person" was coined by Tertullian. He was the first theologian to deal with the issue of Christ's two natures, using the term "substance". He believes that both substances, divine and human, belong to the same person[165], excluding any division in God. In Christ, the divinity and humanity maintain their own distinctive qualities and activities.[166] Similar with Basil, Tertullian discusses the issue on "two natures, one person" by citing the Psalms and the apostle, so he did not have an explicit idea about nature and person of Christ from the Christological perspective. As a result, he did not give an exposition of what unity of Person in Christ means and how two natures united in one person. The point is mainly significant for him in the discussion on the two natures or the two substances. A new christological significance of discussions on one person finally appeared in the writings of Augustine in 411 A.D..[167]

Concerning Christology, there are two important heresies, which are refuted by Boethius in his fifth tractate, namely Nestorius and Eutyches. Nestorius was a Syrian monk who became patriarch of Constantinople from 428 A.D. and was deposed in 431 A.D.. He believes that the divine and human in Christ maintain their own properties. According to him, Nature and Person always exist side by side: Person could not exist without Nature, and when Nature exists, there must be Person. Therefore, for Nestorius, there are two persons with their respective natures in Christ.[168] In the year 444 A.D., in an attempt to stop a new outbreak of Nestorianism, Eutyches, an aged monk from Constantinople began teaching a subtle variation on the

[164] Davis [etc.] (1999/2002), pp. 103-107.
[165] Tertullian believes "there are two natures present in Christ Jesus, a divine and a human." Cf. Tertullian, *Against Praxeas*, 29; Souter (1920).
[166] Tertullian sums all up these ideas in a scriptural quotation: "Learn, therefore, with Nicodemus that 'what is born in flesh is flesh, and what is from spirit is spirit (John III.6).' Neither does flesh become spirit nor does spirit become flesh. However, they can, to be sure, be present in one. Of these Jesus consisted, as man, of flesh, as God, of spirit." Cf. Tertullian, *Against Praxeas*, 27; Souter (1920).
[167] Cf. Grillmeier (1975), pp. 124-125, p. 129, and p. 131.
[168] Cf. Wolfson (1956), p. 451-463.

traditional Christology. Eutyches denied that the human nature of Christ was consubstantial with ours, and held that there had been two natures before the Incarnation, but after the Incarnation, the human nature of Christ was completely absorbed by his divine nature. So this leaves Christ with a single, divine nature.[169] In the treatise *Contra Eutychen et Nestorium*, Boethius analyses the errors of Nestorius and Eutyches, and interprets orthodox Christology.

III.1.3. Conclusion

From the analysis of two important issues in *De fide*, it is clear that *De fide* is closely related to Boethius' other theological tractates. It is a summary of Christian doctrine, and it can also be regarded as the prologue of *De trinitate*, *Utrum pater*, and *Contra Eutychen et Nestorium*, introducing issues briefly and leaving detailed explanations for later discussions. From *De fide*, we can best understand the outlook of Boethius' theological works, so it may well be written first. Next, I will discuss how Boethius discusses the two issues of the Trinity and Christology in detail using his theory of basic logic and mathematics.

III.2. On Christology —*Contra Eutychen et Nestorium*

The motivation of the treatise *Contra Eutychen et Nestorium*[170] is the assembly described in the preface to Chapter III. The distinction between unions formed from two natures and unions that consist in two natures, in Boethius' perspective, had not been solved, and Boethius believed no one really touched the edge of it. In this situation, he decided to inquire the issue in order to solve it. The goal of this treatise, as Boethius says (*Contra Eutychen et Nestorium*, Pref. 54-58), is to clear away the extreme and self-contradictory errors of Nestorius and Eutyches, and then by God's help, to set forth the middle way of the Catholic faith, in response to the request

[169] Cf. Mair (1981), "The Text of the *Opuscula Sacra*," in Gibson (ed.), p. 207.
[170] Cf. Marenbon (2003), "Against Eutyches and Nestorius: The Argument," pp. 70-76.

of the Eastern bishops.[171]

While refuting Nestorius and Eutyches, Boethius employs basic logic, especially the theory of definition, to demonstrate where and how these two heresies are at fault. Boethius is not the first to use this method in this context, but by his use of the method he clarifies the issue of Christology considerably, and develops the application of logic to theology.

III.2.1. Interpreting Terms "Nature" and "Person"

The terms "nature" and "person" were first used as synonyms. Even though later the distinction between these two terms was discussed, most people are concerned with these terms in a Trinitarian context. For instance, Pope Leo I (440-461 A.D.) takes over the distinction between Person and Nature from the Latin tradition for the theology of the Trinity. His primary interest was not in terminology and its definition, but in the theological contents which he wanted to combine by means of it.[172] Unlike him, Boethius begins the interpretation of these two terms with their definitions, for in Boethius' logical works, definition is very important. The discussion of any problem begins from the definition of terms. Nature and Person are two important terms to understand the identity of Christ.[173] And the dispute in Nestorius and Eutyches also centers on these two terms. Thus, the correct interpretation of these two terms is indispensable. Boethius' method in opting for definitions is in line with the thoughts in his logical works.

III.2.1.1. Definition of Nature

The definition of Nature[174] had been discussed a lot before Boethius. In his works, Boethius does not give a new definition of this term, but he lists other people's opinions, analyzes them, and finally chooses the best one.

Boethius lists four definitions of Nature belonging to different types of definition, which are stated in Boethius' commentary on Cicero's *Topics*

[171] Cf. the preface to Chapter III above.
[172] Cf. Grillmeier (1987), pp. 159-160.
[173] Cf. Chadwick (1981b), "Nature and Person", pp. 190-202.
[174] Cf. Catholic Encyclopedia on the definition of nature;
http://www.catholic.org/encyclopedia/view.php?id=8348.

(*C.Topica*, 323/1096)[175]. The first kind of definition is that where accidents are gathered together into one thing and one thing is produced from them; it is a sort of enumeration of parts located not in substance but in a gathering together of accidents. This sort of definition is called a description. In this sense, Boethius lists the first definition of Nature, which is "Nature belongs to those things which, since they exist, can in some measure be apprehended by the mind." (*Contra Eutychen et Nestorium*, I.8-10) This definition describes all things, substantial or accidental.

The second kind of definition is at play if someone makes a definition by presenting species rather than members in the definition; this is called a definition from the division of species. In this sense, Boethius lists a definition of Nature that runs "Nature is either that which can act or that which can be acted upon." (*Contra Eutychen et Nestorium*, I.25-26) This definition refers to corporeal substances or incorporeal substances.

The third kind of definition is that when a definition is constructed of genus and differentiae, we unfold substantial parts. This is called definition in the strict sense of the term. In this sense, Boethius lists two definitions of Nature: one is "Nature is the principle of movement properly inherent in and not accidentally attached to bodies" (*Contra Eutychen et Nestorium*, I.41-42); the other is "Nature is the specific difference that gives form to anything" (*Contra Eutychen et Nestorium*, I.57-58).

Of these four definitions of Nature, it is the last one Boethius chooses to use in his argument.[176] In the last definition, Nature is the species of the genus "difference", and it differs from other kinds of difference with differentiae "specific" and "that gives form to anything". The same specific differences could not apply to both God and man, so Boethius assures that "both Catholics and Nestorians firmly hold that there are in Christ two natures of the kind laid down in our last definition" (*Contra Eutychen et Nestorium*, I.58-63).

Thanks to the last definition, it can be understood that Christ has two

[175] Cf. Section II.1.2.2.

[176] The context of the last definition is the system of genera and species of Aristotle's *Categories* which is put forward by Porphyry in his *Isagoge*. Cf. Marenbon (2003), p. 70.

natures. In other words, Christ has two specific differences: the specific difference for man and the specific difference for God. What exactly are the specific differences for God and man? This could be clear from the Porphyrian Tree of *Contra Eutychen et Nestorium* in Section III.2.1.2 (see Diagram III).

In the Porphyrian Tree of *Contra Eutychen et Nestorium*, man is corporeal, living, sensitive, and rational substance; God is incorporeal, immutable and impassible, and rational substance. There seems to be one specific difference in common, namely "rational substance" which both God and man possess. If so, it would be contradictory to what Boethius believes that "the same specific differences cannot apply to God and man". Actually, the contradiction does not exist, for the term "rational substance" is not used univocally. In this term, the two words — "rational" and "substance" — are both equivocal. In the case of "rational", God falls under the genus "incorporeal substance", while man falls under the genus "corporeal substance", thus rationality for God and for man does not have the same meaning. As for the word "substance", as Boethius says, substance is subject to accidents (*Contra Eutychen et Nestorium*, III.48-49). God is the origin of all things, so substance could not be predicated of Him. When Boethius applies Person to God, we must understand that God is not a substance in the same way as a man is a substance. In *De trinitate*, Boethius calls the substance of God "supersubstantial" (*De trinitate*, IV.14-16). Consequently, the specific differences applied to God and man are not same. In Christ, there are two kinds of specific differences, that is, there are two natures in Christ.

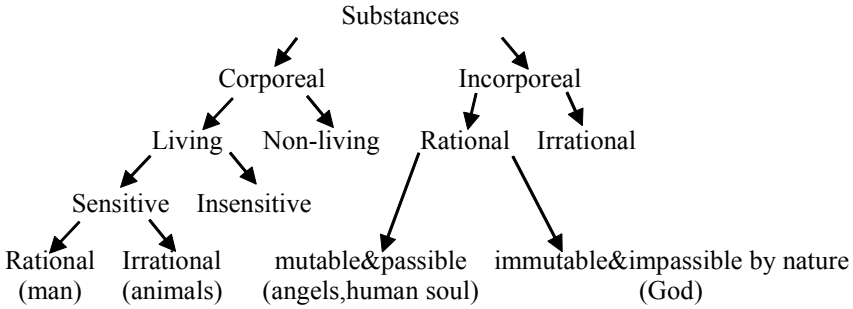
III.2.1.2. Definition of Person

The definition of Nature is given first, because "Person cannot be predicated apart from Nature." Here Boethius returns briefly to the second definition of Nature: Natures are either substances or accidents. However, a person cannot come into being among accidents, thus Person is identified as something predicated of Nature in the sense of substance, that is "Person is properly applied to substances." (*Contra Eutychen et Nestorium*, II.13-18)

Nevertheless, this is not the definition of Person,[177] because the term "substance" needs to be explained more clearly.

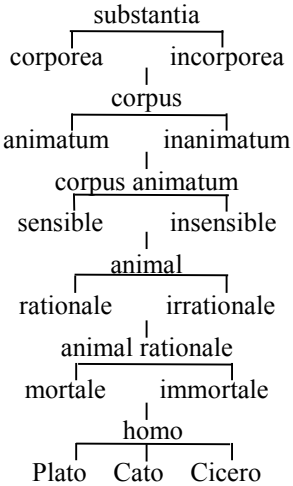
There are many types of substances, so to which substance is the term "person" properly applied? Boethius again uses the Porphyrian Tree to draw the distinction between substances (*Contra Eutychen et Nestorium*, II.18-28). Following Boethius' description, I illustrate the Porphyrian Tree as follows. (See Diagram III)

Diagram III: Porphyrian Tree of *Contra Eutychen et Nestorium*



The Porphyrian Tree here is similar to what Boethius describes in his commentaries on Porphyry's *Isagoge*. (*2InIsag.*, 103A-104D) (See Diagram IV)

Diagram IV: Porphyrian Tree of *2InIsag*.[178]



[177] The classic definition of person is that given by Boethius. Cf. Catholic Encyclopedia on definition of person; http://www.catholic.org/encyclopedia/view.php?id=9193.

[178] Diagram IV is from Boethius' *2InIsag*. by Brandt (1906), p. 209.

From this kind of division of substances both in Boethius' logical works and in his theological works, it is clearly shown that "Person cannot be affirmed of bodies which have no life, nor yet of living things which lack sense, nor finally of that which is bereft of mind and reason." (*Contra Eutychen et Nestorium*, II.28-35) Hence, Person only applies to man, God, and angels, who all belong to rational beings. Moreover, some substances are universal, and others are particular. From the Porphyrian tree, we can also find that individual persons, such as Plato, Cato, and Cicero, are located under *homo*, the particular. In any case, it is impossible that Person can be applied to universals, thus, Person is particular and individual (*Contra Eutychen et Nestorium*, 47-49).[179]

The idea can be found early in Boethius' commentary on Aristotle's *Categories*. In the third book of his commentary, Boethius makes a distinction between Socrates and Plato, saying that they "do not differ according to the species of humanity, but differ only by the quality of person of their own." (*In Categorias*, 241D) This distinction implies that the term "person" is individual rather than universal substance.

From the Porphyrian Tree, it is easy to point out that the term "person" is properly applied to an individual substance of a rational nature. However, it is not enough to understand this definition of Person. Boethius deems the term "person" needs a further exposition. In order to expound the term "person", he discusses the etymology of the word "person". Boethius observes that it is good to go back to the Greeks for a clear understanding of this conception, for it originated with them, while the Latins only know it through translation.

According to Boethius, the Greeks sometimes describe the same object using many synonyms for which the spellings are totally different. Additionally, they sometimes use the same word to refer to different objects, so the same word has different significations. Nevertheless, the Latins may not understand these, thus they are likely to make mistakes. The second situation is analogous to the division of a spoken sound into its significations, which Boethius discusses in *Divisio*.

[179] Cf. Koterski (2004) and Marshall (1950).

> "Division of a spoken sound into its proper significations occurs whenever a single spoken sound is revealed as signifying more than one thing and its plurality of signification is shown." (*Divisio*, 877d)

The Greek term *prosopon* has at least two meanings. The first one is "mask", and the Latin *persona* is derived from this. This may lead to the error of Sabellianism.[180] God was said to have three "faces" or "masks".[181] Modalists note that the only number ascribed to God in the Holy Bible is One and that there is no inherent threeness ascribed to God explicitly in the Scripture. Tertullian and Augustine both believed that one should guard against interpreting *persona* as mask, and Boethius follows them. Moreover, the Greeks also use the term *prosopon* and the term *hypostasis* as a synonym. As a matter of fact, the definition of Person should be corresponds to this word. "Greece with its richer vocabulary gives the name ὑπόστασις to the individual subsistence." (*Contra Eutychen et Nestorium*, III.23-25) Consequently, the term "person" should be defined as "the individual substance of a rational nature." (*Contra Eutychen et Nestorium*, III.4-5)

III.2.1.3. A Problem of Boethius' Definition of Person

When coining the terminology of Christ as "two substances or natures in one person", Tertullian used *substantia* and *persona* as equivalents of *ousia* and *prosopon* (or *hypostasis*) respectively.[182] Different from the translation of Tertullian, when tracing the etymology of terms "nature" and "person", Boethius equals the term "nature" to *ousia* (essence) and the term "person" to *hypostasis*. In the view of Boethius, the Greek word ὑπόστασις is translated to *substantia*, and πρόσωπον to *persona*.

> "But the Greeks called individual substances ὑποστάσεις because they underlie the rest and offer support and substrate to what are called accidents; and we in our term call them 'substances' as being substrate — ὑποστάσεις, and since they also term the same substances πρόσωπα, we too may call them 'persons'." (*Contra*

[180] Cf. the preface to this chapter.
[181] Cf. Lossky (1976), pp. 51-55.
[182] Cf. McManners (2001), pp. 49-50.

Eutychen et Nestorium, III.62-68)

By such translation, the word "substance" is identical to the word "person", which leads Boethius in certain troubles. At the end of Chapter III in *Contra Eutychen et Nestorium*, Boethius tries to apply the definition of Person he gives to the doctrine of the Trinity, saying that "And indeed, following this use, men have spoken of One essence, three substances and three persons of the Godhead." (*Contra Eutychen et Nestorium*, III.93-95) However, this application of substances and persons to God is not very successful. In *Contra Eutychen et Nestorium*, Boethius equals the word "substance" with the word "person", so there is no problem to say "three substances and three persons of the Godhead". However, later in *Utrum pater*, Boethius says, "I take together all three, Father, Son, and Holy Spirit, the result is not three substances but one substance. The one substance of the Three, then, cannot be separated or divided, nor is it made up of various parts, combined into one: it is simply one." (*Utrum pater*, 9-14) So the first problem is that Boethius' views in *Contra Eutychen et Nestorium* and in *Utrum pater* are contradictory. Another problem is that "One essence, three substances and three persons of the Godhead" is not quite in line with the teaching of the Church. Augustine also points out this problem in his *De trinitate* mentioning that "Our Greek friends have spoken of one essence, three substances; but the Latins of one essence or substance, three persons."[183]

Boethius realizes the difficulty, but he does not want to abandon his definition of Person. He insists: "For did not the language of the Church forbid us to say three substances in speaking of God, substance might seem a right term to apply to Him." (*Contra Eutychen et Nestorium*, III.98-101) Even though, later he has to say that "all I have said so far has been for the purpose of marking the difference between the terms 'nature' and 'person'. The exact terms that should be applied in each case must be left to the decision of ecclesiastical usage." (*Contra Eutychen et Nestorium*, IV.1-5) This confirms that Boethius writes his theological treatises as a philosopher but not as a theologian. Moreover, the connection between Boethius' own logical works and theological works is also shown, and will become much clearer in our later discussions.

[183] Augustine, *On the Holy Trinity*, VII.7; Schaff (1887).

Although there is a flaw in the definition of the term "person", the distinction between Nature and Person is sufficient to refute the heresies. Boethius goes on to refute Nestorius and Eutyches.

III.2.2. Refutations of Nestorius and Eutyches

The inability to distinguish between terms "nature" and "person", in Boethius' judgment, results in the errors of Nestorius and Eutyches. The forms of the reasoning of Nestorius and Eutyches are similar, which could be summaries as follows.

Nestorius may argue:

Person may be applied to every Nature.
There are in Christ two natures.
Therefore, there are likewise two persons.

Eutyches may argue:

Nature may be applied to every Person.
There is only one person in Christ.
Therefore, Nature in Christ should be regarded as single.

From these two statements above, we can see that the first proposition in each of the syllogisms has the same meaning, that is, Person and Nature are in the relationship of one-to-one correspondence. Accordingly, the errors of both Nestorius and Eutyches spring from the same source. Nestorius deems if Person were doubled, there could be a double Nature, so Nestorius confesses the double Nature in Christ. Just like Nestorius, Eutyches reckons that if Person was not double, Nature was not double, and since he did not confess a double Person, he thought it is a necessary consequence that Nature should be regarded as single. (*Contra Eutychen et Nestorium*, V.8-16)

The refutation of Nestorius is easier than the refutation of Eutyches, so Boethius first points out the errors of Nestorius and refutes him.

III.2.2.1. Refuting Nestorius

Boethius has confidence in definitions of Nature and Person he gives above. He believes these definitions are enough to prove Nestorius wrong (*Contra Eutychen et Nestorium*, IV.14-15). According to the definition of Person Boethius gives, a Person is an individual substance of a rational nature, which implies that the presence of a person entails the presence of a nature. The reverse of this statement is not true. It is not necessary that the presence of a nature involves the presence of a person. Hence, although Nestorius rightly holds Christ's Nature to be double, it is not inevitable to come to the conclusion that in Christ Person is twofold.

In Boethius' opinion, the name of one thing denotes its singular number, a unity. And because being and unity are convertible terms, what is not one cannot be, and whatever is one, can be. According to Nestorius, if there were two persons, it would mean that there were two individual substances, so Christ would not be one, that is to say, Christ would not exist. Or, from two Persons it could be argued that there are two Christs, which is "nothing else than madness of a distraught brain" (*Contra Eutychen et Nestorium*, IV.45-46). The same conclusion can be derived from the nature of definition, which Boethius discusses in his *In Ciceronis Topica*.

> "Only a definition, which is produced by a phrase, unfolds what a thing is; a genus and the rest, which are expressed for the most part by a single name, do not unfold what a thing is. ... A definition does not unfold what it defines with respect to quality or quantity or any of the other categories. Rather, the definition shows what the thing defined is; that is, it shows its substance." (*C.Topica*, 319/1091)

From the nature of definition, we could know that one definition of a thing shows that it has one substance. It is the same with the definition of Christ. Then the very name of Christ, which is defined, shows that Christ has one substance. Two different substances could not be covered by the same definition. Nestorius himself also admits that he could not apply the substance of one definition to both his Christs. Boethius argues that Nestorius wrongly uses the same name for two utterly different natures. "For if the substance of God is different from that of man, and the one name

of Christ applies to both, and the combination of different substances is not believed to have formed one person, the name of Christ is equivocal and cannot be comprised in one definition." (*Contra Eutychen et Nestorium*, IV.51-59) Moreover, in his *Divisio* Boethius defines a genus as that "which is predicated of a number of specifically different things in respect of what it is" (*Divisio*, 880a). Accordingly, two different things could be united in one genus, but there are some conditions. "Men indeed and oxen are united in one animal nature, for by genus they have a common substance and the same nature in the collection which forms the universal." (*Contra Eutychen et Nestorium*, IV.106-109) If the distinction of persons continues under a difference in nature, God and man will be fundamentally different at all points. Then the salvation has not been brought to us, and the human race has not been saved. In other words, "the birth of Christ has brought us no salvation, the writings of all the prophets have but beguiled the people that believed in them, contempt is poured upon the authority of the whole Old Testament which promised to the world salvation by the birth of Christ." (*Contra Eutychen et Nestorium*, IV.112-118) This is an impious conclusion.

In order to confute the thoughts of Nestorius, Boethius sets up some assumptions and raises some questions about Nestorian opinions, and then all of these lead to the impious conclusion which is against the Catholic faith. "For the truth of the faith and the unwontedness of the miracle alike remain, for Catholics, unshaken." (*Contra Eutychen et Nestorium*, IV.61-62)

Boethius admits that his method is only one of many strong weapons to wound and demolish the Nestorian view, but he is content with this. Then he proceeds to refute Eutyches.

III.2.2.2. Refuting Eutyches

Eutyches rightly believes the Person of Christ to be single, but impiously professes that the Nature of Christ is also single. As stated above, the definition of Nature is "Nature is the specific difference that gives form to anything." It is clear that divine nature is different from human nature. And Eutyches also seems to confess this, so he declares his belief to be "two Natures in Christ before the union and only one after the union" (*Contra Eutychen et Nestorium*, V.23-25). The refutation of Eutyches is more

complicated.

Due to the vagueness of what Eutyches declares, Boethius puts forward some suppositions and begins his arguments on the basis of these suppositions.

The union which Eutyches holds may take place:

(a) *either* at the moment of conception;
(b) *or* at the moment of resurrection — in one of the following two ways:
  (b1) Christ was conceived and did not assume a body from the Virgin Mary;
  (b2) Christ did assume a body from the Virgin Mary.

The arguments based on (a) and (b1) suggest the falsehood of the Godhead. In these arguments, no basic logic or mathematics is applied. Concerning the argument based on (b2), Boethius believes if Christ received the body from Mary, and later the human and divine nature united and did not continue, then this situation may have happened in one of three ways (*Contra Eutychen et Nestorium*, VI.3-8):

(b2.1) "Godhead was translated into manhood"

(b2.2) "manhood was translated into Godhead"

(b2.3) "both were so modified and mingled that neither substance kept its proper form"

Boethius demolishes these three ways by means of logic.

According to the Porphyrian Tree of *Contra Eutychen et Nestorium* in Section III.2.1.2 (See Diagram III), God falls under the genus "incorporeal", and of manhood, which consists of body and soul, human body falls under the genus "corporeal" and human soul falls under the genus "incorporeal". Therefore, the first two ways (b2.1-2) both concern transformations between the incorporeal and the corporeal. In the first commentary on Porphyry's *Isagoge*, Boethius pays emphatic attention to the relationship of the incorporeal and the corporeal. In Boethius' point of view, the corporeal could not be the genus of those things that are placed under the incorporeal

substances; and nothing corporeal in genus can fall under an incorporeal species, neither can a corporeal substance be changed into an incorporeal (*InIsag.*, 20A). There is a sharp disjunction between the corporeal and the incorporeal. Thus, Boethius says, "Only those things can be interchanged and transformed which possess the common substrate of the same matter, nor can all these so behave, but only those which can act upon and be acted on by each other." (*Contra Eutychen et Nestorium*, VI.24-27) But incorporeal things lack matter, so the corporeal and the incorporeal do not share any common underlying matter, therefore, they cannot be interchanged and transformed.

Accordingly, it is not possible for God, as a species of the incorporeal, and human body, as a species of the corporeal, to be changed into each other. Moreover, as a species of the incorporeal, God and human soul both lack matter, so they could not be translated into each other. In a word, it is irrational and also impious to say that God could be transformed into man; and it is impossible for man to be translated into God. "If neither body nor soul can be turned into Godhead, it could not possibly happen that manhood should be transformed into God." (*Contra Eutychen et Nestorium*, VI.75-77) Therefore, the first two ways are demolished.

As regards the third way (b2.3), Boethius takes an example of the mixture of honey and water.

> "When honey is mixed with water neither remains, but the one thing being spoilt by conjunction with the other produces a certain third thing, so that third thing which is produced by the combination of honey and water is said to consist of both, but not in both." (*Contra Eutychen et Nestorium*, 87-91)

This example is similar to the mixture of wine and water used by Aristotle for illustrating what a mixture is. In *On Generation and Corruption*, Aristotle maintains that "if mixture has taken place, the compound *must* be uniform — any part of such a compound being the same as the whole."[184] Mixture is a union of different constituents, and each constituent in this union, Aristotle says, changes out of its own nature towards a third property,

[184] Aristotle, *On Generation and Corruption*, 328a9-11; Barnes (1984a).

yet neither becomes the other, but both become that which is intermediate and common.[185] The third property is a compromise between the properties of the constituents. If Eutychians says that "Christ consists *of* two natures, but not *in* two natures" in this sense of union, a mixture, it means that God and man were mixed to become "a third property". According to Aristotle, such "third property" is different from the properties of the constituents, and in case of Eutychians, Christ will be different from the properties of both God and man. This conclusion is absolutely impossible. Consequently, the third way, "both God and man were so modified and mingled that neither substance kept its proper form" is demolished.

All in all, the upshot of Boethius' reasoning is that "no one of the three ways is a possibility" (*Contra Eutychen et Nestorium*, VI.108-109). In other words, Godhead could not be translated into manhood, nor could manhood be translated into Godhead, nor were both so modified and mingled that neither substance kept its proper form. Thus, the doctrine of the Eutychians is incorrect.

III.2.3. The Middle Way of the Catholic Faith

In the letter of the year 512, the Eastern bishops requested from Pope Symmachus a middle way between Nestorius and Eutyches. After clearing away the self-contradictory errors of Nestorius and Eutyches, Boethius sets forth the middle way of the Catholic faith.

From the refutations of Nestorius and Eutyches, it is clear that the terms "person" and "nature" are at the heart of the debate. Boethius lists four possible combinations of Nature and Person in Christ (*Contra Eutychen et Nestorium*, VII.80-93). In Christ, there are (1) two natures and two persons (as Nestorius says); (2) one nature and one person (as Eutyches says); (3) two natures and one person (as the Catholic faith holds); (4) one nature and two persons, which is manifestly absurd. The errors of (1) and (2) have

[185] Aristotle, *On Generation and Corruption*, 328a29-31; Barnes (1984a). Aristotle also describes a different situation in which "a great quantity (or a large bulk) of one of these materials be brought together with a liter (or with a small piece) of another." If so, "the effect produced is not mixture, but increase of the dominant": a small amount of wine in the ocean will turn into water.

already discussed by Boethius, and for (4), he only says that "there has never been anyone so mad as to believe that His Nature was single but His Person double." (*Contra Eutychen et Nestorium*, VII.90-91) Hence (3) is the only one left, and this is what the Catholic faith affirms, that is "Nature is double, but the Person is one."

The question arises *how*, in accordance with the affirmation of the Catholic faith, Christ consists as one person *in* and *of* both natures. Boethius confesses that the term "to consist *of* two natures" is equivocal. There are two ways to interpret this. One way is the way Eutyches adopts — which has been proven false. The other way is a union consisting *of* two natures in which the two natures continue to exist without changing into each other (*Contra Eutychen et Nestorium*, VII.12-15). This is the way to interpret the Catholic faith. Boethius illustrates this using an example of a crown that is composed *of* gold and gems. Just like a crown consisting of gold and gems, Christ consists *of* God and man. In a crown, the gold and the gems continue to exist without surrendering their proper forms (*Contra Eutychen et Nestorium*, VII.16-20). And it is the same with Christ: neither is God converted into man nor is man turned into God, but both natures continue to exist in Christ, and they both remain perfect, as the Catholic faith confesses. Therefore, that "Christ consists both *in* and *of* the two natures" is to be interpreted thus: "*in* the two because both continue, *of* the two because the One Person of Christ is formed by the union *of* the two continuing natures." (*Contra Eutychen et Nestorium*, VII.29-30)

The Eastern bishops requested the middle way, and their reason was that there was a middle way between Arius and Sabellius, so in imitation of them, there should be a middle way between Nestorius and Eutyches. Boethius regards the Catholic faith as the middle between two false extremes, because he says it is "just as virtues also hold a middle place" (*Contra Eutychen et Nestorium*, VII.74-76). The similar thought has already been stated early in Boethius' arithmetical works, and was introduced in Section II.2.1.3.a. When Boethius discusses the property of "perfect number", he makes an analogy. He says imperfect number and superabundant number are just like two unequal and intemperate elements, and between these two kinds of numbers, we can put a number "which holds

the middle place between the extremes like on who seeks virtue". And this number that maintains the place of the middle is called perfect number.[186] Similarly, as the virtues we seek hold a middle place, the Catholic faith also holds the middle place, and it is the right way.

In the treatise against Eutyches and Nestorius, Boethius makes an exposition of his orthodox thoughts of Christology, that is to say, Christ is one Person with two Natures. With the definitions of Nature and Person he establishes, Boethius confutes the errors of Eutyches and Nestorius, and he succinctly interprets the union of two natures in Christ. Boethius relies on his logical ability to solve the question of the Natures and Person of Christ.

The discussion of these topics is not new, but Boethius, unlike the Church Fathers before him, adopts the knowledge of basic logic to make these Christian issues much more articulate and lucid. In *Contra Eutychen et Nestorium* there are more connections to Boethius' own logical works, especially his theory of definition and his commentaries on Porphyry's *Isagoge*, and there are also some similar thoughts in *Contra Eutychen et Nestorium* and his work on arithmetic.

The solution to the question of Christology is clearly stated by Boethius. Christ is One God with two natures, divine nature and human nature. And Boethius believes that "quaternity is not added to the Trinity by the addition of human nature to perfect Godhead, but that one and the same Person completes the number of the Trinity." (*Contra Eutychen et Nestorium*, VII.51-55) Here Boethius just mentions the Trinitarian question, but later he makes a further elaboration about that. Thus, in the following section, I will go on to introduce applications of basic logic and mathematics in Boethius' works on the Trinity.

[186] *Arithmetica* I.19. See Section II.2.1.3.a. In that section, I also explain that "mean" in the notion "virtue is the mean" used by Aristotle is not a point exactly in the middle, but a stretch of the continuum around the middle. And here the meaning of "Catholic faith as the middle between two false extremes" is similar to what Aristotle says "the third holds the middle, which is the right, position." Cf. Aristotle, *Nicomachean Ethics*, 1116a7; Barnes (1984b).

III.3. On the Trinity — *De Trinitate* & *Utrum Pater et Filius et Spiritus Sanctus de Divinitate Substantialiter Praedicentur*

In *De fide* Boethius says that the Trinity became the orthodox teaching in the Church. In *Contra Eutychen et Nestorium*, he also touches on this topic, but he does not penetrate into this question. The main reason is that Boethius' research of theological issues is not due to systematic theological study, but arises out of the political purpose mentioned in the preface to Chapter III.

The early Church Fathers only restated basic principles of Christianity in the manner of faith, so they did not truly argue why the Trinity is one God, not three. They wanted to translate the doctrines to the common people in order to convince them. In this way, the early Church Fathers were unable to confute various heresies. For example, from the overview the Greek patristic theology of the Trinity one can see that there are different acts in this period, but the continuous development of these acts has its own basis in biblical premises. Specifically, these acts mediate through the Church's lived experience.[187] Unlike them, Boethius thinks people who cast their thoughts before the common herd "would seem to bring discredit on the study of divinity" (*De trinitate*, 12-16), so he writes only to Symmachus and himself. Boethius believes that logic and reason must be used to make a reinterpretation of the theological issues including issue of the Trinity.[188]

The interpretation of the Trinity, that is, God's identity, could be divided into two questions. One is the unity in God, and the other is the diversity in God. Boethius is also concerned with these two questions and examines the doctrine of the Trinity in *De trinitate*[189] and *Utrum pater*. Boethius admits that the truth about the Trinity, indeed, has already been established by

[187] Cf. McGruckin (2011), "The Trinity in the Greek Fathers," in Phan (ed.), p. 49.
[188] Cf. Chadwick (1981b), pp. 211-222; Marenbon (2003), pp. 79-87.
[189] Cf. Marenbon (1982) examines the discussion by three medieval scholars of a passage from Boethius' *De trinitate*.

Augustine,[190] as I will set out in this section. Especially, *Utrum pater* comes near to Augustine's *De trinitate*, but it is a short treatise and does not make a further argument on the Trinity, which is continued in Chapter V of Boethius' *De trinitate*. Boethius hopes to throw some further light on this argument by means of his logical training. He wants to set forth the problem about the One God in logical order and cast it into literary form (*De trinitate*, 1-5). In this section, I will introduce how Boethius applies basic logic and mathematics to issue of the Trinity in his two treatises. This section is divided into two parts: one deals with the unity in God, which is discussed in *De trinitate*; the other part deals with the diversity in God, which is discussed in both *De trinitate* and *Utrum pater*.

III.3.1. The Unity in God

Boethius starts his discussions with the Christian faith on the Trinity, which is "the Father is God, the Son is God, the Holy Spirit is God" (*De trinitate*, I.7-8). The statement of the Trinitarian orthodoxy in the formula of one *ousia* and three *hypostaseis* was finally fixed by the Cappadocian Settlement.[191] The Cappadocian Settlement was raised to refute Arian teaching. In agreement with the tradition, Boethius' route to examine the doctrine of the Trinity also takes it starting point from the refutation of Arian views.

Generally speaking, the pastoral task of clerics is to express the truths of faith in ways that their contemporaries could understand. The theologians who were busy with this task but laid great stress only on one aspect rather the whole of the Christian faith, are condemned as heresiarchs.[192] Arius is someone who advocates the oneness of God to the detriment of the divinity of the Son. The Arians ascribe grades to Persons, and so they break up the Unity of the Trinity and convert it into Plurality (*De trinitate*, 12-13). Boethius sets forth the error of Arianism on account of their lack of

[190] Bradshaw gives an explanation between Boethius' *De trinitate* and Augustine's writings; cf. Bradshaw (2009), "The *Opuscula Sacra*: Boethius and Theology," in Marenbon (ed.), pp. 109-115. And Marenbon discusses *Utrum pater* and Augustine's *On the Trinity*; cf. Marenbon (2003), pp. 77-79.
[191] Cf. Davis [etc.] (1999/2002), pp. 99-121; Prestige (1952), p. 233.
[192] Cf. Phan (2011), p. 5.

knowledge of difference, and demolishes Arianism on account of their theory of form. From these two respects, Boethius refutes Arianism, pointing out that the plurality in God is different from the plurality in men. If, however, there are three persons, how does the repetition of three persons in God *not* produce a plurality of number? Boethius tackles this question by means of a distinction between two kinds of number.

III.3.1.1. Arguing from Knowledge of Difference

The Arians covert the unity in God to plurality, which, Boethius informs us, is caused by difference or otherness, for "the essence of plurality is otherness" (*De trinitate*, I.13-14). Thus, Boethius begins his arguments from basic knowledge of sameness and otherness.

Sameness and otherness are the formal principles of the universe, which is elaborated by Nicomachus in *Introduction to Arithmetic*.[193] Nicomachus points out that when these two principles enter into the composition of things, sameness assures the things to preserve their identity, and to continue being in the same manner, and otherness causes things to "change from their original forms and assume others."[194] Although in the *Arithmetica* Boethius omits the discussion of this principle, he has the same conviction, as I mentioned in Section II.2.1.3.a. Boethius believes that otherness inheres in the number two and the whole even series, and that otherness is the essence of plurality. Therefore, it is impossible to understand plurality apart from otherness (*De trinitate*, I.14-15).

Concerning sameness, Boethius adopts most of Aristotle's point of view. Aristotle states in his *Topics*,

> "Sameness would be generally regarded as falling, roughly speaking, into three divisions. We generally apply the term numerically or specifically or generically — numerically in cases where there is more than one name but only one thing; specifically, where there is more than one thing, but they present no differences in respect of their species. Similarly, too, those things are called

[193] Cf. Nicomachus, *Introduction to Arithmetic*, II.17.1; 18.1 and 4; 19.1 and 20.2; D'Oog (1938).
[194] D'Ooge (1938), p. 99.

generically the same which fall under the same genus."[195]

Since "otherness", or in other word "difference", is the necessary correlative of "sameness", "difference" is similarly expressed in these three ways: by genus, species, and number (*De trinitate*, I.23-24).

However, there is a little difference between Boethius and Aristotle on "sameness". In the second commentary on Porphyry's *Isagoge*, Boethius takes an example of human property to explain sameness in number. He affirms that Socrates, Plato, and other people are plural in number, and that the sameness of them as human beings, the unique property, is arising out of accidents (*2InIsag.*, 114D). Unlike Aristotle, Boethius reckons that sameness in number is sameness in accidents. So the difference in number is also difference in accidents. In other words, "numerical difference is caused by variety of accidents" (*De trinitate*, I.24-25); hence, three men do not differ by genus or species, but only by accidents. If all other accidents are absent, there will always remain the place, which is also an accident (*De trinitate*, I.24-30). The point that the place is an accident is made by Boethius early in his commentary on Aristotle's *Categories*. In that commentary, Boethius takes an example of a man departing a theatre. A man can depart a theatre without destroying the substantial notion of his humanity, which implies that the theatre as a place is an accident. "Wherefore it is because men are plural by their accidents that they are plural in number." (*De trinitate*, I.31-32) This is possibly the reason of the Arian error, for they consider God the same as men, so God is also plural. As a matter of fact, the plurality in God is different from that in men, which will be clear from the theory of form.

III.3.1.2. Arguing from Theory of Form

In God, Father, Son, and Holy Spirit are three persons, so why they does this not result in a plurality of number? The reason which will be explained by Boethius in the following is that the three persons are in God without accidents. The subject of the Divine Substance belongs to Theology, thus Boethius gives a brief description of the division of speculative science first.

[195] Aristotle, *Topics*, 103a7-14; Barnes (1984a).

In the second commentary on Porphyry's *Isagoge*, Boethius points out that one branch of philosophy is speculative philosophy, which concerns the nature of things or elaborates the knowledge of things (*2InIsag.*, 73D). Further division of speculative science is discussed in Chapter II of *De trinitate*.[196] "Speculative Science may be divided into three kinds: Physics, Mathematics, and Theology." (*De trinitate*, II.5-16) As one branch of speculative science, theology does not deal with motion and is abstract and separable using intellectual concepts, that is to say, it deals with the Divine Substance which is without either matter or motion. Boethius alarms us that we should not "be diverted to play with imaginations" in Theology. Everything owes its being not to matter, but to the form which is imprinted on it. What we should do is to "simply apprehend that Form which is pure form and no image, which is being itself and the source of Being" (*De trinitate*, II.17-21).

Boethius distinguishes two kinds of forms. The first kind consists of Plato's Forms, in which being is separate from matter. The second kind of forms is found in an Aristotelian framework, where universals solely have being in so far as they are instantiated in the particulars that compose them.[197] As Boethius says: "Form which is without matter cannot be a substrate, and cannot have its essence in matter, else it would not be form but a reflexion" (*De trinitate*, II.48-51). Boethius believes that the entities that reside in bodies are misnamed by us, for "they are mere images; they only resemble those forms which are not incorporate in matter." (*De trinitate*, II.55-56) But the Divine Substance is pure Form without matter (*De trinitate*, II.29-31). The Divine Substance is what it is and its own essence, which "does not consist of This and That, but is only This". Therefore, the Divine Substance is truly One, in which there is no number, and in which except its own essence nothing is present (*De trinitate*, II.37-42). This thought is similar to what Augustine expresses in his *De trinitate*.

"He is, however, without doubt, a substance, or, if it be better so to

[196] Kijewska gives a discussion on the division of philosophy in Boethius' *De trinitate* and his commentary on Porphyry's *Isagoge*. Cf. Kijewska (2003), pp. 628-631.
[197] Cf. Chadwick (1981b), p. 215.

> call it, an essence, which the Greeks call οὐσία. ... But other things that are called essences or substances admit of accidents, whereby a change, whether great or small, is produced in them. But there can be no accident of this kind in respect to God; and therefore He who is God is the only unchangeable substance or essence, to whom certainly being itself, whence comes the name of essence, most especially and most truly belongs."[198]

Unlike men who are plural by accidents, God, in whom there is no accident, is not plural in number, and is absolutely simple. Therefore, in God, there is no difference, and then there is no plurality arising out of difference, and no multiplicity arising out of accidents, and accordingly no number (*De trinitate*, II.56-58). The error of the Arians is refuted now. By accidents men are plural in number, but unlike men, in God there is no accident and no difference, therefore, God is not plural. Father, Son, and Holy Spirit is one God, but not three Gods.

Nevertheless, we indeed call God "Father", "Son", and "Holy Spirit". Why does the repetition of three persons not produce a plurality of number? This can be explained from a distinction between two kinds of number.

III.3.1.3. Arguing from Two Kinds of Number

The question why the repetition of three persons does not produce a plurality of number can be further explained by dividing the type of number into abstract number and concrete number.[199] The number with which we count is the abstract number; and the number inherent in the things counted is the concrete number. "In the case of abstract number a repetition of single items does produce plurality; but in the case of concrete number the repetition and plural use of single items does not by any means produce numerical difference in the objects counted." (*De trinitate*, III.10-14) Therefore, when a repetition of unities is a question of abstract number, it produces plurality, but when it is a question of concrete number, it does not produce plurality.

[198] Augustine, *On the Holy Trinity*, V.3; Schaff (1887).
[199] Cf. Kijewska (2003), pp. 634-637.

This formula is derived from Aristotle, who states that if three synonyms are used for the same object, then the object is one, not three. Boethius states the same point of view in his second commentary on *On Interpretation*.

> "And when in the same language many terms are applied to one thing, it is demonstrated that the thing in question is not named naturally but by applying names; for if each thing were called by a natural name, we would signify one thing by just one name." (*2InInter.*, 56.4-9)

For example, in saying "one sword, one brand, one blade", synonymous terms are used, for each of these names denotes one and the same thing, a sword. So the repetition of single terms here does not result in plural number. This idea can be applied to God. "Similarly, if God be predicated thrice of Father, Son, and Holy Spirit, the threefold predication does not result in plural number." (*De trinitate*, III.29-31) God is One.

Thus far, from three respects, i.e. the knowledge of difference, the theory of forms, and the distinction between two kinds of number, Boethius explains the unity of God. However, at the end of the discussion of the two kinds of number, one question seems to remain. While "sword", "brand", and "blade" are synonymous, "Father", "Son", and "Holy Spirit" are not synonymous terms. Though these three terms denote the same thing, they are not identical. "There is not, therefore, complete indifference between Them; and so number does come in — number which we explained was the result of diversity of substrates." (*De trinitate*, III.50-53) Boethius discusses this view by examining how the ten categories can be applied to God. Hence we arrive at the second question about the Trinity, which concerns the diversity in God.

III.3.2. The Diversity in God

The ten categories can be universally predicated of things, viz. substance, quality, quantity, relation, place, time, condition, situation, activity, passivity. They are divided into two kinds, Boethius says, viz. substantial categories and accidents. The categories in the first kind "denote

the reality of a thing", declaring "that a thing is something"; and categories in the second kind "denote a thing's accidental circumstances", saying "nothing about its being anything, but simply attach to it, so to speak, something external." (*De trinitate*, IV.100-104)

How are the categories applied to God? Boethius tries to apply every category to God to examine whether it is possible. The same use of categories in relation to God is found in Augustine's *De trinitate*.

> "We may understand God, if we are able, and as much as we are able, as good without quality, great without quantity, a creator though He lack nothing, ruling but from no position, sustaining all things without "having" them, in His wholeness everywhere, yet without place, eternal without time, making things that are changeable, without change of Himself, and without passion."[200]

When applied to God, the meanings of these categories are changed entirely. In reference to God, it is only possible to employ the category of substance. God is substance not in the common sense, and God is supersubstantial. That, which is an accidental quality in man, is an essence in God, or rather a supersubstantial quality. It is not possible to apply the remaining categories (time, place, condition, activity, situation, and passivity) to God. The category of relation is considered last.

In *Utrum pater*, Boethius gives a concise account of the question "whether Father, Son, and Holy Spirit may be predicated of the Divinity substantially or otherwise." *Utrum pater* comes near to Augustine's *De trinitate*.

> "But because the Father is not called the Father except in that He has a Son, and the Son is not called Son except in that He has a Father, these things are not said according to substance; because each of them is not so called in relation to Himself, but the terms are used reciprocally and in relation each to the other; nor yet according to accident, because both the being called the Father, and the being called the Son, is eternal and unchangeable to them.

[200] Augustine, *On the Holy Trinity*, V.2; Schaff (1887).

> Wherefore, although to be the Father and to be the Son is different, yet their substance is not different; because they are so called, not according to substance, but according to relation, which relation, however, is not accident, because it is not changeable."[201]

Following Augustine, Boethius ends *Utrum pater* with the conclusion that "The Trinity is effected by diversity of Persons, wherefore Trinity does not belong to Substance. Hence neither Father, nor Son, nor Holy Spirit, nor Trinity can be substantially predicated of God, but only relatively." (*Utrum pater*, 62-68) Similarly to Augustine, *Utrum pater* does not take the argument that far. However, Boethius continues this argument in Chapter V of *De trinitate*.

The theory used in Chapter V of *De trinitate* is Aristotelian theory of relation. In the *Categories*, Aristotle begins his statement of relation with it definition: "We call relatives all such things as are said to be just what they are, *of* or *than* other things, or in some other way *in relation* to something else."[202] In God, Father is said to be the Father of the Son, for Father begets Son; and Holy Spirit is said to be in some other way in relation to Father and Son, for Holy Spirit is begotten by Father and Son. Hence, Father, Son, and Holy Spirit are called relatives.

"The category of relation", Boethius says, "has nothing to do with the essence of the subject; it simply denotes a condition of relativity, and that not necessarily to something else, but sometimes to the subject itself." (*De trinitate*, V.19-22) So when the category of relation is applied to God, the essential nature of God cannot be altered, changed, or disturbed in any way. The Father is God, the Son is God, and the Holy Spirit is God. There is only one essence of the Three. Father, Son, and Holy Spirit are predicated of the Divinity relatively.

The diversity in God is expounded through the category of relation. "The relation of Father to Son, and of both to Holy Spirit is a relation of identicals." (*De trinitate*, VI.20-22)

---

[201] Augustine, *On the Holy Trinity*, V.6; Schaff (1887).
[202] Aristotle, *Categories*, 6a.36-38; Ackrill (1963).

Therefore, in God, there are three Persons, Father, Son, and Holy Spirit. And They are not separated by any difference.

> "But where there are no differences there is no plurality; where is no plurality there is Unity. Again, nothing but God can be begotten of God, and lastly, in concrete enumerations the repetition of units does not produce plurality. Thus the Unity of the Three is suitably established." (*De trinitate*, V.52-57)

III.3.3. Conclusion

From the analysis above, we may surmise that the seeds for reconciling unity and plurality in God are sown in Boethius' mind, as he acknowledges, by Augustine's writings (*De trinitate*, 31-33), especially Augustine's *On the Holy Trinity*. Boethius discusses the Trinitarian doctrine along the same lines as Augustine. In his work on the Trinity, Augustine employs Aristotelian categories when addressing the issue of Trinitarian relations. At the same time, he always appeals to the Scripture for evidence. For example, Augustine says,

> "But position, and condition, and places, and times, are not said to be in God properly, but metaphorically and through similitudes. For He is both said to dwell between the cherubims, which is spoken in respect to position; and to be covered with the deep as with a garment, which is said in respect to condition; and 'Thy years shall have no end,' which is said in respect of time; and, 'If I ascend up into heaven, Thou art there,' which is said in respect to place. And as respects action (or making), perhaps it may be said most truly of God alone, for God alone makes and Himself is not made. Nor is He liable to passions as far as belongs to that substance whereby He is God. So the Father is omnipotent, the Son omnipotent, and the Holy Spirit is omnipotent; yet not three omnipotents, but one omnipotent: 'For of Him are all things, and through Him are all things, and in Him are all things; to whom be glory.' "[203]

[203] Augustine, *On the Holy Trinity*, V.9; Schaff (1887).

In this quotation, Augustine employs a combination of exegesis, argument, and prayerful meditation. Unlike Augustine, in the tractates on the Trinity, *De trinitate* and *Utrum pater*, Boethius does not refer to verses in the Scripture, but he attempts to demonstrate the coherence of Trinitarian doctrine on purely philosophical grounds, relying on arithmetic and basic logic, such as his commentaries on Porphyry's *Isagoge* and Aristotle's *On Interpretation*.

By examining how categories can be applied to God, Boethius explains the diversity of God. He secures the diversity of the Trinity by means of the category of relation, thus removing simple unity. The unity which is preserved by the category of substance consists in simplicity of substance, and is proven by the absence of all diversity in the Divine nature.

As demonstrated in the discussions on the Trinity and Christology above, Boethius investigated two important issues in Christian doctrine. These issues are the main topics in his theological tractates. There is only one so-called theological treatise left, but it is more like a philosophical essay. The *Quomodo substantiae* is close to Boethius' last famous work, the *Consolatio*, and not only similar in content but also similar in method, as I will introduce in Section III.4 and then give a detailed explanation in Chapter IV. In *Quomodo substantiae* we also find applications of basic logic and mathematics.

III.4. On the Treatise *Quomodo Substantiae in eo Quod Sint Bonae Sint cum non Sint Substantialia Bona*

The four treatises discussed above are all about the Christian doctrines, but unlike them, *Quomodo substantiae*[204] appears to have little direct relation to Christian doctrines, and it has nothing to do with the theological controversy at that time either. Thus it looks odd to rank *Quomodo substantiae* among Boethius' theological tractates. As a matter of fact, the content of *Quomodo substantiae* is in accordance with Christian theology.

[204] Cf. Chadwick (1981b), pp. 203-211.

Quomodo substantiae is concerned with the demonstration of the Prime Good, and Its relation to the created existences. "The Prime Good", Boethius admits, "can be deduced from the religious beliefs of savage races". (*Quomodo substantiae*, 92-95) As Marenbon discusses, the area of problems *Quomodo substantiae* is concerned with, is the same as that of *De trinitate* and *Utrum pater*, but the particular difficulty *Quomodo substantiae* deals with does not involve a specifically Christian doctrine, thus its language is neutral.[205] Hence, there is no doubt that *Quomodo substantiae* is one of the theological tractates.

The motivation to write *Quomodo substantiae,* which is given at the beginning of this treatise, is to reply to a request of John the Deacon. As Boethius says, John asked him to "state and explain somewhat more clearly that obscure question in my *Hebdomads* concerning the manner in which substances are good in virtue of existence without being substantial goods" (*Quomodo substantiae*, 1-2). In the Middle Ages the title of this tractate (as accepted by Thomas Aquinas) was derived from these opening lines: it was called *De Hebdomadibus* (*On the Hebdomads*).[206] Most scholars nowadays regard this title as mistaken. In some manuscripts, this tractate has no title; however, in most manuscripts it is given a long title, viz. *Quomodo substantiae in eo quod sint bonae sint cum non sint substantialia bona*.[207]

The *Quomodo substantiae* has a conspicuous character. John the Deacon urged Boethius to write an essay because "the method of this kind of treatise is not clear to all" (*Quomodo substantiae*, 6-7). Boethius decides to follow "the example of the mathematical and cognate sciences" (*Quomodo substantiae*, 15). And later in his last composition, *Consolatio*, Boethius gives an exposition of this mathematical method again. The mathematical method is as follows:

> "As the geometricians are wont, out of their propositions which they have demonstrated, to infer something which they call *porismata* (deductions) so will I give thee as it were a *corollarium*." (*Consolatio*, III.pX.80-83)

[205] Cf. Marenbon (2003), p. 87.
[206] Cf. Casey (1987); Crouse (1982); Solère (2003).
[207] Cf. Chadwick (1981b), pp. 203-204.

In this method, there is a key term, *porismata* (porism) which is from the Greek word πόρισμα. Proclus distinguishes two senses in which this word is used.

> "The first is that of *corollary* where something appears as an incidental result of a proposition, obtained without trouble or special seeking, a sort of bonus which the investigation has presented us with. The other sense is that of Euclid's *Porisms*. In this sense *porism* is the name given to things which are sought, but need some finding and are neither pure bringing into existence nor simple theoretic argument."[208]

Here in *Quomodo substantiae* and also in *Consolatio*, Boethius uses the word porism in the first sense, which is equal to *corollary*. And the method Boethius uses here is similar to what Euclid calls "analysis" and "synthesis".

> "Analysis is the assumption of that which is sought as if it were admitted <and the arrival> by means of its consequences at something admitted to be true."
>
> "Synthesis is an assumption of that which is sought as if it were admitted <and the arrival> by means of its consequences at something admitted to be true."[209]

Boethius first uses the method of synthesis to the attainment of what is sought, and then uses problematical analysis[210] — one kind of the method analysis — to come to the conclusion. Section III.4 will be divided into three parts: the first one concerns the premises for the demonstration (Section III.4.1), the second one introduces the mode of mathematical example (Section III.4.2), and the last one is the solution to the problem

[208] Heath (1956), p. 13.
[209] Euclid, *Elements*, XIII.1; Heath (1956).
[210] Cf. Heath (1956), pp. 138-139: "In the problematical kind we assume that which is propounded as if it were known, after which we pass through its successive consequences, taking them as true, up to something admitted: if then (a) what is admitted is possible and obtainable, that is, what mathematicians call given, what was originally proposed will also be possible, and the proof will again correspond in reverse order to the analysis, but if (b) we come upon something admittedly impossible, the problem will also be impossible."

(Section III.4.3).

III.4.1. The Premises for the Demonstration

Boethius first lays down "bounds and rules" according to which he develops nine rules for the demonstration.[211]

> Rule I: A common conception is a statement generally accepted as soon as it is made. (*Quomodo substantiae*, 18-19)
>
> Rule II: Being and a concrete thing are different. Simple Being awaits manifestation, but a thing is and exists as soon as it has received the form which gives it Being. (*Quomodo substantiae*, 28-30)
>
> Rule III: A concrete thing can participate in something else; but absolute Being can in no wise participate in anything. (*Quomodo substantiae*, 31-32)
>
> Rule IV: That which exists can possess something in addition to itself. But absolute Being has no admixture of anything besides Itself. (*Quomodo substantiae*, 35-37)
>
> Rule V: Merely to be something, and to be something absolutely are different; the former signifies an accident; the latter substance. (*Quomodo substantiae*, 38-40)
>
> Rule VI: Everything that is participates in absolute Being in order to exist. (*Quomodo substantiae*, 41-42)
>
> Rule VII: Every simple thing possesses as a unity its existence and its particular being. (*Quomodo substantiae*, 45-46)
>
> Rule VIII: In every composite thing existence and it particular being are different. (*Quomodo substantiae*, 47-48)
>
> Rule IX: Diversity repels; likeness attracts. (*Quomodo substantiae*, 49-50)

[211] Cf. Marenbon (2003), pp. 87-90.

The first rule concerns the common conception. One kind of it is universally intelligible, which could be considered as maximal proposition in Boethius' logical works. In his two works on topics, *In Ciceronis Topica* and *De Topicis Differentiis*, Boethius sets forth the maximal proposition.

> "We call highest and maximal propositions those propositions that are universal and known and manifest to such an extent that they need no proof but rather themselves provide proof for things that are in doubt, for those proposition for those propositions that are uncertain." (*C.Topica*, 280/1051)
>
> "Those for which where is no proof are called maximal and principal, because it is necessary that these prove those which do not deny that they can be demonstrated." (*TopicisD.*, 1176C.21-24)

After the two statements about the maximal proposition above, and also after the statement of the universally intelligible common conception in the first premise, Boethius uses the same example from mathematical theory, that is, "if equals be taken from equals the remainders are equal." Rule II and Rule VIII are all regarding the difference between being and a concrete thing. Being is absolute, and it can "in no wise participate in anything" and "have no admixture of anything besides Itself" (*Quomodo substantiae*, 31-32; 36-37). The points here are in accord with the analysis of pure form in *De trinitate*, which I have already discussed in Section II.3.1.2 on the Trinity. The last rule is about diversity and likeness.

After laying down these nine preliminary rules, Boethius takes up his argument in the way he proposed in the context of mathematical lines.

III.4.2. Demonstrations of a Problem[212]

Armed with nine rules, Boethius first employs the method of synthesis to get what is sought.

> Suppose "Every existing thing tends to good." (This is in line with Rule I. The second kind of common conception is intelligible only

[212] Cf. Marenbon (2003), pp. 90-94.

to the learned, and it is just agreed by all learned.)

According to Rule IX, everything tends to its like.

Therefore, things are themselves good.[213]

According to Rule I, things are good in one of the following two ways: either by participation or by substance.

Here we arrive at what is sought, and this proposition is what Boethius plans to show, which results in a problem. Then Boethius employs the method of analysis to demonstrate two parts of this problem in turn. The first part of the problem: things are good by participation.

Suppose the problem solved.

Then according to the definition of "participation", the thing itself does not have what it participates in as part of its essence. (This is in line with Rule I.)

Therefore, things do not possess goodness in themselves.

But according to Rule IX, what does not own goodness in itself cannot be good in itself.

Therefore, these things are not good, which is opposite to the supposition.

Therefore, the supposition is impossible, that is, things cannot be good by participation.

Accordingly, things are good in the other way: things are good by substance.

Suppose the problem solved.

Then according to Rule V, to be something and to be something absolutely are different.

If something's substance is good, its particular being is good.

According to Rule VIII, since the absolute and particular being of

[213] Cf. MacDonald (1988).

all things is the same, they are substantial goods like the First Good.

Therefore, all things are like the First Good, and are God, which is an impious assertion.

Therefore, the supposition is impossible, that is, things cannot be good by substance.

From this analysis, the problem follows that things are in no way good, for they are good neither by participation nor by substance. Boethius solves this problem in the following way.

III.4.3. The Solution to the Problem

At the beginning of the solution, Boethius offers a process of mathematics related to mental abstraction.

> "There are many things which can be separated by a mental process, though they cannot be separated in fact." (*Quomodo substantiae*, 87-88)

In order to make this process clear, Boethius gives an example from mathematics:

> "No one can actually separate a triangle or other mathematical figures from the underlying matter; but mentally one can consider a triangle and its properties apart from matter." (*Quomodo substantiae*, 88-91)

By imitating this mathematical process, Boethius removes the presence of the Prime Good from the mind for a moment. So the proposition that needs be proved is: All things are good, which is the repetition of the first proposition, and then inquires how they could possibly be good if they did not derive from the Prime Good.

The imitation of this mathematical process suggests that "the Goodness of all things and their existence are two different things." (*Quomodo substantiae*, 98-100) Hence, the being of all things would be not identical

with their goodness. Simple goodness is the distinguishing feature of the one sole Good, thus if the things were nothing else but good substances, "they (or rather it) would seem to be not things but the principle of things" (*Quomodo substantiae*, 113-115). But this conclusion is out of the question. For things derive their existence from the Will of the Good, so they are neither independent nor simple.

> "For the Prime Good is essentially good in virtue of Being; the secondary good is in its turn good because it derives from the good whose absolute Being is good." (*Quomodo substantiae*, 121-124)

In the case of the Prime Good, Being and Goodness are identical, and It is good due to Its existence, irrespective of all conditions. Unlike the Prime Good, absolute Being of all things is not good under all circumstances, though they be good in virtue of their existence. Unless the true good had produced them, these things could not actually exist. In this way the problem is solved.

Augustine also discussed the topic of goodness and existence. For example, "A nature with no good in it cannot exist."[214] Augustine neither states the internal relationship between the created existence and its goodness, nor does he explain how the existence of a substance makes it good. However, unlike Augustine, Boethius adopts the mathematical method and the demonstrative method to address these issues. Boethius employs the mathematical method, and regards his demonstration as an example of mathematical demonstration. Boethius also employs a similar mathematical method for the arguments in his famous work *Consolatio*. Lady Philosophy begins her argument by an imitation of this geometrical method, which I will set out in Chapter IV.

The issue of goodness discussed in *Quomodo substantiae* will also continue in *Consolatio*. In Book III, the ninth prose, of *Consolatio*, when Lady Philosophy argues for the proposition that unity and the good are identical, she draws the conclusion that "All things then desire goodness, which you may define thus: goodness is that which is desired of all things." (*Consolatio*, III.pXI.110-112) In absolute terms, the good is the final cause

[214] Augustine, *City of God*, XIX.13; Schaff (1890).

of all things, and the good is identical with happiness. People also seek the good in order to be happy. However, most of the time, they are not happy, for the good they seek is not the true good. So where is the true good? How do people to get to it? These will be discussed in the following chapter on the *Consolatio*.

III.5. Conclusion

Boethius' five theological treatises are brief but greatly contribute to the intellectual tradition of the West, not only in content but also importantly in method.[215]

The content, as Boethius himself admits, follows Augustine' thoughts, but the terminology and the method in these treatises are Boethius' own, independent from Augustine.

Sutherland speaks very highly of Boethius' contribution:

> "In *On the Trinity*, he employed the term *theology* for the first time as a technical Christian term denoting the philosophical inquiry into the nature of God. Methodologically, his contribution lies in the use of formal Aristotelian demonstrative logic for the first time in the service of Christian theology."[216]

This estimate is not very accurate, for before Boethius, there are other people who used Aristotelian logic to explain Christian issues. However, Boethius may have been the first to systematically apply formal Aristotelian demonstrative logic to solve Christian issues. In doing so, Boethius conjoins philosophy and faith, which is the aim stated in his second treatise, viz. to "reconcile faith and reason" (*Utrum pater*, 71).

From the chapters on logic and mathematics, it is obvious that Boethius

[215] Cf. Erismann, (2009), "The Medieval Fortunes of the *Opuscula Sacra*," in Marenbon (ed.), pp. 155-177; Gibson (1981), "The *Opuscula Sacra* in the Middle Ages," in Gibson (ed.), pp. 214-235; Nash-Marshall (2012), "Boethius's Influence on Theology and Metaphysics to c.1500," in Kaylor and Phillips (eds.), pp. 163-192; Marenbon (2003), "The Method and Character of the *Opuscula*," pp. 94-95.
[216] Eliade (2005), p. 277.

is a philosopher, which is confirmed again by his theological tractates. Boethius is also labeled as a theologian by some scholars, such as Sutherland, who called Boethius "Roman philosopher, theologian, and statesman".[217] The label of "theologian" is not accurate in Boethius' case. Indeed, Boethius wrote five theological tractates. However, this is not enough to prove that he is a "theologian". The term *theologia* was used by Boethius to denote a subdivision of philosophy. In Boethius' view, theology as an academic discipline deals with motionless and incorporeal reality, which is different from the theology of the theologians. Boethius did not write the theological treatise as a theologian in the ordinary sense of the word and his audiences are the few people who are actually capable of tackling properly theological disputation.[218]

From the motivations for writing the theological treatises, it has been shown that Boethius' active interest in Christian disputes is of the greatest political importance. The political reason aside, Boethius' interest on these Christian issues is also scientific and speculative. As Boethius admits, in his works on the Trinity, no new Christian issues are discussed, and the seeds in his mind are sown by Augustine's writings (*De trinitate*, 31-32). However, he decides to try his best to make these issues clear. For theology, like all Liberal Arts, has a limit: the inquiry about God is beset by the weakness of man's wit and reason. In spite of this, Boethius is confident that philosophy could be a little useful. He purposely uses brevity and wraps up the ideas he draws from the deep questionings of philosophy in new and unaccustomed words (*De trinitate*, 16-18). That is to say, the theological tractates of Boethius are written from the perspective of philosophy.[219] This is the reason why it is not accurate to call Boethius a theologian. The accurate label for Boethius is Philosopher, which is confirmed again by his last

[217] Eliade (2005), p. 276.

[218] Chadwick points out that Boethius is "addressing himself only indirectly to a pastoral or 'political' situation in the Church, as a logician who thought there was some tidying up to be done in the ecclesiastical garden." Cf. Chadwick (1981a), "Introduction," in Gibson (ed.), p. 1. The audiences of Boethius' theological tractates imply that "one must have both piety and knowledge, and be able to assess clearly the proper relationship between faith and reason." Cf. Reiss (1982), "The Question of Audience," pp. 66-70.

[219] This implies an answer to the question why Boethius chooses philosophy to console him when facing death. Cf. Section IV.4.

composition, *Consolatio*, to which we now turn.

IV. Applications of Basic Logic and Mathematics in *Consolatio Philosophiae*

IV.1. Introduction to *Consolatio Philosophiae*

At the end of previous chapter, I gave only one label to Boethius, i.e. Philosopher, which will be confirmed again by his last but famous work, *Consolatio*. At the end of his life, Boethius lost his power, wealth, and fame, and he was suddenly condemned as a traitor, stripped of his goods, imprisoned, and sentenced to death.[220] When facing the death, Boethius appeals to the solace from philosophy, which is implied by the title of his last work. If Boethius is a Christian, then it looks odd, because it is good for those who trust in Christ to appeal to Christ's redeeming work or to the religious consolation of St. Paul. However, in *Consolatio*, it seems that there is no direct reference to Christian doctrine and no citation of Christian scripture.[221] It is philosophy that supplies him with consolation. At the end of this chapter I shall discuss the reason why he makes this choice.[222]

There are numerous discussions of the sources of the *Consolatio* among

[220] Cf. Magee (2005); Reiss (1981); Relihan (2007); Shanzer (1984).

[221] Courcelle (1969, pp. 318-322) says that the problem of Boethius' Christianity has produced a spate of discussion from the ninth century, and he lists some different opinions in Note 1 of p. 318. Astell (1994) compares Boethius' *Consolatio* with books of Job, and she tries to give a new approach to the well-established problem of the ascendancy of romance over epic. Chadwick (1981b, p. 249) claims that *Consolatio* "contains no sentence that looks like a confession of faith either in the gods of paganism or in Christian redemption." In Shanzer's point of view, it is more likely that Boethius is neutral. It appears that Boethius consciously avoided any specific attitude. Cf. Shanzer (2009), "Interpreting *The Consolation*," in Marenbon (ed.), pp. 241-242. In that article Shanzer lists some proofs to show why Boethius remained neutral. Sharples (1991, pp. 46-48) suggests different kinds of answer to this problem of the *Consolatio* and Christianity. On Boethius' Christianity and Philosophy, also cf. Bernardine (1949); Chadwick (1981b), pp. 247-253; De Vogel (1973); Lewis (1967), pp. 76-79; Magee (1988); Marenbon (2003), pp. 154-159; Olmsted (1989); Starnes (1981); Shanzer (2009), "Interpreting *The Consolation*," in Marenbon (ed.), pp. 241-254; Varvis (1991), p. 1.

[222] Cf. Section IV.4.

Boethius' predecessors, including Plato, Aristotle, the Stoics, Cicero, Platonists and Neoplatonists.[223] Stewart points out that, in the various themes of *Consolatio*, the recollection of earlier studies and modes of thought is palpable.[224] And Chadwick regards *Consolatio* as the work of a refined humanist scholar with a richly stocked memory.[225] In these scholars' works on Boethius' *Consolatio*, they sum up the sources and possible sources of *Consolatio* from earlier scholars.[226] There are very few scholars who focus on the relation between Boethius' own earlier works and his last work.[227] Sometimes authors do discuss this relation, but without explaining how Boethius' earlier ideas function in his consolation. For example, Courcelle mentions that in *Consolatio* Boethius refers quite often to his earlier scientific or logical works,[228] and Reiss regards *Consolatio* as representing "the memorializing of work Boethius had been involved in for some time,"[229] but they do not give detailed expositions on the connection between *Consolatio* and Boethius' earlier works. Also Sharples signals that Boethius draws on his own earlier studies and goes beyond them in a striking way in *Consolatio*, but he limits himself to Book IV.5-7 and Book V of *Consolatio*. Sharples, too, points out only a few connections between

[223] Cf. Silk (1939) discusses Boethius' sources from Augustine.
[224] Cf. Stewart (1974), p. 56.
[225] Cf. Chadwick (1981b), p. 223.
[226] On sources of *Consolatio* in general, cf. Chadwick (1981b), pp. 228-234; Courcelle (1969), pp. 295-318; Marenbon (2003), pp. 97-99; Rand (1904); Reiss (1982), pp. 147-153; Sulowski (1961). Also cf. Sharples (1991, pp. 41-46) discusses the sources of IV.5-7 and V in *Consolatio*. And Shanzer discusses the classical sources of *Consolatio*; cf. Shanzer (2009), "Interpreting *The Consolation*," in Marenbon (ed.), p. 230.
[227] Chamberlain (1970, p. 80) points out that "*Consolatio* is a major instance of the close relations among philosophy, music, and literature in late classical and mediaeval culture, but an instance that has been overlooked by scholars in all three disciplines." Chamberlain does connect Boethius' work on music and *Consolatio*, but what he focuses on is the literature of these two works. Neither Sharples nor Chamberlain give full discussions on the function of Boethius' earlier ideas in his whole consolation. Uhlfelder gives an outline of the role of Liberal Arts in Boethius *Consolatio*. She points out two uses of arithmetic. One is connected with the number five, which she considers may be the reason of Boethius' dividing of *Consolatio* into five books. The other use of arithmetic Uhlfelder points out is metaphorical adaptations of proportions. On these two uses of arithmetic and the use of astronomy, cf. Uhlfelder (1981), "The Role of the Liberal Arts in Boethius' *Consolatio*," in Masi (ed.), pp. 24-25.
[228] Cf. Courcelle (1969), p. 319.
[229] Cf. Reiss (1982), p. 99.

Boethius' early works and *Consolatio,* apart from more numerous connections between Boethius' sources and *Consolatio*.[230]

It is clear, however, that Boethius' early works on mathematics, logic, and theology are all applied in his last work. In *Consolatio*, he employs mathematics most, and then logic, and just some methods from the theology. In this chapter, I will focus on applications of basic logic[231] and mathematics in *Consolatio* and identify applications of method in these cases.

The consolation, in Boethius' point of view, is like the therapy for a patient.[232] He employs the style of dialogue to console himself.[233] One person in the dialogue is, of course, Boethius himself, who has become a sick man;[234] the other person is his physician, the personification of philosophy, called Lady Philosophy[235], in whose house Boethius had

[230] Cf. Sharples (1991), p. 41.

[231] Cf. Section II.1.2.

[232] Cf. Duclow (1979) regards Boethius' *Consolatio* as one of the most remarkable works in the therapeutic tradition, and he examines *Consolatio* as an exercise in psychotherapy, the healing of the soul.

[233] In later antiquity, the genre of consolation literature flourished. This genre was both philosophical and rhetorical and embraced a variety of forms, including letters, treatises, and poetry. And as its title indicates, *Consolatio* stands within this genre, and it is written in alternating meters of poetry and prose. In order to distinguish poetry and prose in citations, I use "p" for prose and "m" for meters of poetry. I will not focus on the literary method of *Consolatio* (except the relation between meters and music); on this type of discussion, cf. Cooper and Santo (2009), "Boethius, Gregory the Great and the Christian 'Afterlife' of Classical Dialogue," in Goldhill (ed.), pp. 173-190; Crabbe (1981a), "Literary Designs in the *De Consolatione Philosophiae*," in Gibson (ed.), pp. 237-277; Crabbe (1981b); Curley (1986 and 1987); Dronke (1994), pp. 41-46; Dwyer (1976); Lerer (1985); Lewis (1967), pp. 79-90; Marenbon (2003), "Prose and Verse: *The Consolation* as Menippean Satire," pp. 159-163; Payne (1981), Chapter 3: "*The Consolation of Philosophy* as Menippean Satire," pp. 55-85; Shanzer (2009), "Interpreting *The Consolation*," in Marenbon (ed.), pp. 228-254; Walz (2011).

[234] In the following discussions, in order to distinguish Boethius as the author of *Consolatio* and as the interlocutor in the dialogue of this work, I will call the person in the dialogue "the patient". There is a question on the difference between the author and the patient Boethius. It is very difficult to determine where the patient stops and the historical Boethius begins, which is a problem I shall not discuss. On this question, cf. Chadwick (1981b), p. 247; Albrecht [etc.] (1997), pp. 1708-1710.

[235] Cf. Chadwick (1981b), pp. 225-228; Courcelle (1981); Gualtieri (1971); Marenbon (2003), pp. 153-154; Shanzer (2009), "Interpreting *The Consolation*," in Marenbon (ed.), pp. 231-232.

remained from his youth (*Consolatio*, I.pIII.3-6). The nature of Lady Philosophy can be defined by the interplay of Boethius' own experience and the image of Lady Philosophy.

Lady Philosophy comes from heaven (*Consolatio*, I.pIII.6-8), but she is not a god, as she says herself quoting Homer (Iliad XII.176) (*Consolatio*, IV.pVI.196). Boethius also admits that Lady Philosophy did instil into his ears and thoughts daily that saying of Pythagoras, "Follow God." (*Consolatio*, I.pIV.140-141) In the ninth meter of the third book, Lady Philosophy and her patient pray to God together to ask Him for help.

> "Thou and God Himself who had inserted thee in the minds of the wise, are my witnesses that nothing but the common desire of all good men brought me to be a magistrate." (*Consolatio*, I.pIV.28-31)

The role of Lady Philosophy[236] could be considered that of being an interlocutor between God and man. She derives from God and was put into the human mind by God. She personifies the tradition of philosophical thinking, which nourished Boethius' mind. This could be confirmed by Boethius' own words. "I am endued with thy knowledge, and adorned with thy virtues." (*Consolatio*, I.pIV.151-152) In Boethius' work on arithmetic, he defines philosophy, following Pythagoras, as "the love of wisdom" (*Arithmetica*, I.1), and in his first commentary on Porphyry's *Isagoge*, he calls philosophy "the love and pursuit of wisdom and in some way the friendship with it" (*1InIsag.*, 10D). For Boethius, wisdom belongs to God, so "the love and pursuit of wisdom" means the love and pursuit of God. This interpretation suggests that philosophical thought is the means to reach God. However, I must here point out that God in this context does not refer to the Christian God, but to divine wisdom.[237] The task of Lady Philosophy is to try and lead her student back to this God and shape him into such

[236] There are various discussions on the different roles of Lady Philosophy, such as: (1) as the text says, Lady Philosophy is a healer; (2) Lady Philosophy is the philosophical tradition; cf. Marenbon (2003), p. 154; (3) Lady Philosophy is an educator in eloquence: from silence to speech; cf. Lerer (1985), pp. 96-99.

[237] This corresponds with the claim that *Consolatio* is written from the perspective of philosophy, and not Christian in character. Cf. Section IV.4.

excellence as to make him like to God (*Consolatio*, I.pIV.145). This means that Lady Philosophy does her best to make her student get the highest knowledge, the divine wisdom.

On Lady Philosophy's arrival, Boethius gives a detailed description of her appearance.[238] Lady Philosophy is both young and ancient; in her left hand she carries a book and in her right hand she has a scepter[239]; the height of Lady Philosophy is uncertain — both normal and enormous height.[240] The reason for her uncertain height may be that since she can provide practical knowledge, she seems normal; since she is also able to provide the speculative philosophy, she looks enormous. This uncertain height could be connected with the symbolic meaning of the images in her dress, which is the more important part of her appearance.

> "In the lower part of them was placed the Greek letter Π, and in the upper Θ, and betwixt the two letters, in the manner of stairs, there were certain degrees made, by which there was a passage from the lower to the higher letter: this her garment had been cut by the violence of some, who had taken away such pieces as they could get. In her right hand she had certain books, and in her left hand she held a sceptre." (*Consolatio*, I.p1.18-22)

The symbol Π and Θ represent practical and theoretical (or speculative) philosophy respectively.[241] This division of philosophy can also be found in Boethius' early logical works.

In the second commentary on Porphyry's *Isagoge*, when analyzing what logic itself should be, a part of philosophy or an instrument of philosophy,

[238] Cf. *Consolatio*, I.pI.1-25.

[239] The book Lady Philosophy carries in her left hand may be "her text, the writings of the philosophers, or the symbol of study", and the scepter she has in her right hand is "a symbol of her majesty, authority, and rule." Cf. Varvis (1991), p. 32.

[240] The uncertain height of Lady Philosophy indicates, as Marenbon shows, "how the study of philosophy progresses from the easily graspable towards an intimation of what is beyond comprehension." Cf. Marenbon (2003), p. 153.

[241] Chadwick (1980b) says that there is no doubt that these two symbols stand for the two main divisions of philosophy, practical and theoretical. He also argues that the theta on the dress of Lady Philosophy represents the mark of a condemned man, and thus mirrors the letter on Boethius' own clothing.

Boethius first gives two kinds of opinions on the division of philosophy.[242] One opinion is that philosophy should consist in three parts, that is, speculative, practical, and rational. Of course, rational philosophy refers to logic, so logic is considered a part of philosophy. The other opinion is that logic is not a part but an instrument of philosophy, so that philosophy should be divided only two parts, naming speculative and practical. As for the role of logic in philosophy, Boethius reckons that logic serves at the same time the function of part and instrument (*InIsag.*, 73D-74D). In both divisions of philosophy we find practical and speculative (or theoretical) philosophy, which correspond to the letters Π and Θ in the dress of Lady Philosophy.

Practical philosophy concerns morals, and speculative philosophy works out the knowledge of the nature of things. The ladder between the symbol Π and Θ in the dress of Lady Philosophy represents the hierarchy of philosophy, ascending through practical philosophy (the inferior disciplines) up to speculative philosophy, which is divided into physics, mathematics, and theology in Boethius' treatise on the Trinity.[243]

As a teacher, Lady Philosophy is faithful to her students, so when one of her students is ill, she arrives as a physician to heal him. Generally, in order to cure Boethius' illness completely, the diagnosis of the illness is followed by two treatments, which I shall call Treatment 1 and Treatment 2. In *Consolatio*, the diagnosis and Treatment 1 are closely related in the first three books, and the three diagnostic reasons of the disease[244] as figured out by Lady Philosophy, are elucidated in the first three books, so that it seems that the whole work should come to the end at Book III. As a matter of fact, apart from the three diagnostic reasons, there is still a root cause that is not solved. As a result, the treatment should continue with Treatment 2, a further treatment, which covers Books IV and V.

Accordingly, my discussion of the whole work is divided into two

[242] On the detailed discussion of this, cf. Section I.2.3.
[243] Varvis (1991, p. 32) claims, "the ladder between represents the movement of the human mind from the lower to the higher realms of understanding." Also Chadwick (1980b, p. 176) says that "The steps connecting the two are then natural enough: they represent the road from morality and political science to more contemplative and abstract mental enquiries."
[244] Cf. Section IV.2.

sections:[245] the first section covers Book I-III, the second section deals with Book IV-V. There is a further reason for this division. As I introduced above, *Consolatio* is Boethius' last work. In it, Boethius uses his knowledge, which was presented in his early works, to console himself. In addition, he adds some new thoughts the seed of which was sown in his early works. So in *Consolatio* we find both Boethius' old knowledge and his more recent insights, which are included in the first and second sections respectively. Thus the division of sections in this chapter rests on these two reasons.

In both sections, basic logic and mathematics are applied as part of the remedy, which Lady Philosophy administers to console her patient.[246] Mathematics, especially arithmetic and music, are often used in the first section, while basic logic and a geometrical method are used in both sections. In this chapter, I investigate applications of basic logic and mathematics in *Consolatio*. In Section IV.2, I will discuss applications of basic logic and mathematics in Book I-III. The application of mathematics in the second section consists of only one geometrical method which will be introduced in the first section. Hence, in Section IV.3, I shall only focus on applications of basic logic in Book IV-V.

[245] Magee points out that according to a certain formal consideration, *Consolatio* should be divided into three parts: Book I both begins and ends with poetry; Book II-IV begins with prose and ends with poetry; Book V both begins and ends with prose. Magee further divides Book II-IV into two parts according to the tasks to accomplish. Cf. Magee (2009), "The Good and Morality: *Consolatio* 2-4," in Marenbon (ed.), p. 181 and p. 184. Courcelle (1969, p. 297) divides *Consolatio* into four parts according to its design. He says that the design is based on a dual conversion in three stages: self-knowledge (Book II); knowledge of the purpose of things (Books III and IV to paragraph 5); knowledge of the laws that govern the universe (end of Book IV and Book V). For the division of *Consolatio*, also cf. Lerer (1985), pp. 96-99.

[246] Uhlfelder claims that the belief Boethius associates with his study of Liberal Arts is one of the reasons that lead to the existence of *Consolatio*; logic and mathematics "provide a mean between earth and the Creator, reveals the physical universe as a mirror of the divine plan, and so makes it possible for man to reach the third level." Cf. Uhlfelder (1981), "The Role of the Liberal Arts in Boethius' *Consolatio*," in Masi (ed.), p. 23 and p. 29.

IV.2. Applications of Basic Logic and Mathematics in Book I-III

The first scene in *Consolatio* is that the patient is sorrow, and his physician Lady Philosophy is standing over his head. Facing her silent and even mute and dumb patient, Lady Philosophy encourages him to open his mouth. After the patient's complaint and the outburst against the iniquity of Fortune, Lady Philosophy responds to him with the criticism that he has exiled himself from his country (*Consolatio*, I.pV.4-6). Lady Philosophy tells her patient that no one could exile him from his country except himself.[247] The patient's country refers to the kingdom of God, which refers to the divine wisdom, not to the Christian God.[248]

Continuing with a few questions, Lady Philosophy touches and tries the state of her patient's mind (*Consolatio*, I.pVI.1-3). With the reflection and answers of her patient, Lady Philosophy makes a diagnosis of his illness and finds out the means of his recovery.

> "For the confusion which thou art in, by the forgetfulness of thyself, is the cause why thou art so much grieved at thy exile and the loss of thy goods. And because thou art ignorant what is the end of things, thou thinkest that lewd and wicked men be powerful and happy; likewise, because thou hast forgotten by what means the world is governed, thou imaginest that these alternations of fortune do fall out without any guide, sufficient causes not only of sickness, but also of death itself." (*Consolatio*, I.pVI.42-49)[249]

Lady Philosophy elicits three reasons of her patient's sickness. Terms that Lady Philosophy uses to describe the attitude of her patient are

[247] The exile here means the patient's spiritual exile which is mirrored by his physical exile. Cf. Chadwick (1981b), p. 227.
[248] Cf. Section IV.1.
[249] Magee analyses the questions in the diagnosis and assesses the patient's condition by these questions. Cf. Magee (2009), "The Good and Morality: *Consolatio* 2-4," in Marenbon (ed.), p. 193.

"forgetful"[250], "ignorant", and "forgotten". These words imply that the so-called three reasons for his illness were familiar to the patient at an earlier time. However, the distress has clouded the recollection of the patient.

The first reason is that he has no idea of what true happiness, and the final cause of all things are. In *Consolatio*, happiness is equal to goodness; the topic of the good was touched upon in Boethius' theological tractate called *Quomodo substantiae*.[251] That tractate proves that everything is good. Based on this conclusion, in *Consolatio*, Boethius raises some issues related to the Good and attempts to solve them.[252]

The second reason is that though he does not doubt that the world is governed by reason, he is at a loss by what methods the world is governed. He does not remember the nature and origin of the order of the world. Actually, the subject of how the world is governed was discussed in Boethius' mathematical works, especially on arithmetic and music.[253]

The third reason is that he holds a wrong definition of man. He forgets what man, or the nature of man, is. The discussion of human nature was involved in Boethius' commentary on Porphyry's *Isagoge*.[254] By her treatment, Lady Philosophy helps her patient to solve his three problems and lead him to the right way to his country with Treatment 1, and at last bring him back to the kingdom of God.

With these three reasons, the remedy for the illness begins. In Treatment 1 Boethius employs basic logic (especially the Porphyrian Tree and the theory of definition) and mathematics (especially music and arithmetic).

---

[250] Cf. Donato (2013b) emphasizes the significance of forgetfulness in Boethius' *Consolatio*.

[251] Cf. Section III.4.

[252] Cf. the preface to Section IV.3.1.

[253] In his *Arithmetica*, Boethius assures his readers that God binds the world's elements together by number; cf. Section II.2.1 and Section IV.2.2.2. In his *Musica*, Boethius gives a short introduction to cosmic music which means the cosmic harmony holding together four elements of earth, air, fire, and water, or the cycle of the four seasons in consonance and equilibrium. Cf. Section II.2.2.3 and Section IV.2.2.1.

[254] Cf. *InIsag.*, 10C-13D and 17C-D.

IV.2.1. Applications of Basic Logic in Book I-III

In order to explain the diagnostic reasons of the patient's illness, Lady Philosophy adopts an order similar to the one Boethius used in *Contra Eutychen et Nestorium* in which he first refuted the faith of heresies, and then by reflecting of the contrary, he gave the Catholic faith.[255] In *Contra Eutychen et Nestorium*, Boethius writes in the preface about the procedure of that treatise, saying that, "But since the pen is now to take the place of the living voice, let me first clear away the extreme and self-contradictory errors of Nestorius and Eutyches. After that, by God's help, I will temperately set forth the middle way of the Christian Faith." (*Contra Eutychen et Nestorium*, 54-58) Similar to this order, in *Consolatio*, Boethius uses a poem to show how to discern the first diagnostic reason, viz. what is the true happiness.

> "So thou, whom seeming goods do feed, first shake off yokes which so thee press, that truth may then thy mind possess." (*Consolatio*, III.mI.11-13)

To insure a fruitful harvest, one must clear the ground before sowing the seed, which can be deemed as reproducing the principle of the procedure in *Contra Eutychen et Nestorium*. That is to say, in order to find what true happiness is, the first thing is the analysis of false happiness. Then the contrary of false happiness will be true happiness.[256] After recognizing false goods, the mind should abandon those false goods to make way for the true good, so as to make the mind more receptive to true one. There is another reason to begin with false happiness. Because the patient is turmoiled with the multitude of affections, and his mind is so blinded and impeded by his grief and anger, he cannot understand reasonable arguments immediately. In this case, Lady Philosophy decides to cure her patient with the easier remedies first, but not to use more forcible remedies, so that the patient's affections, "which are, as it were, hardened and swollen with perturbations, may by gentle handling be mollified and disposed to receive the force of

[255] Cf. Section III.2.

[256] De Vogel gives another reason for this order, viz. that Lady Philosophy "does precisely according to the rules laid down for such a case by the philosopher Plotinus in that well-known first treatise of the fifth *Ennead*." Cf. De Vogel (1973), pp. 357-358.

sharper medicines." (*Consolatio*, I.pV.41-44) Later, the forcible medicine will be applied to him. Lady Philosophy attempts to declare in words and to give shape to that which is better known to her patient, so that her patient could understand it thoroughly (*Consolatio*, III.pI.22-24). False happiness is familiar to all human beings, including the patient here, which is the beginning of an initial cure of a gentle kind.

Boethius complains that though he once owned riches, dignities, kingdoms, glory, and pleasure, which are the forms of human felicity, these things have now abandoned him.[257] People seek these material goods separately in different ways. Some people seek one of them, and some people seek some of them. Some people believe that possessing these material goods will make them happy. Are they right? The truth could be discovered through the Porphyrian Tree. As the Porphyrian Tree of *Contra Eutychen et Nestorium* in Section III.2.1.2 shows,[258] human beings consist of soul and body. Human soul falls under the genus of incorporeal substance, so "the souls of men are in no wise mortal." Unlike human soul, human body falls under the genus of corporeal substance, which suggests that the body will die. If the human body would disappear, the casual felicity, or material goods people sought would end. It follows that, without doubt, these material goods could not bring human beings to happiness (*Consolatio*, II.pIV.91-101). Lady Philosophy makes the exposition of institutions when chasing these common goods.

In the case of riches, money cannot keep itself from being taken from those who possess it (*Consolatio*, III.pIII.30-33). Human beings are greedy, and they want more and more money. However, money is limited, so if a person wants to earn more money, he has to take it from other people. Thus, everyone with money will need a bodyguard to protect him from losing his

[257] The discussions on these forms of human felicity are split between two parallel treatments, and the distribution of themes is as follows:

Riches	II.p5, m5	III.p3, m3
Dignities	II.p6, m6	III.p4, m4
Kingdoms	II.p6, m6	III.p5, m5
Glory	II.p7, m7	III.p6, m6
Pleasure	—	III.p7, m7

[258] Cf. Diagram III.

money. The rich man will thus not be free. In other words, the riches require additional protection, and the greed of human beings is insatiable. Therefore, "riches cannot make a man wanting nothing nor sufficient of himself, and this was that they seemed to promise." (*Consolatio*, III.pIII.28-30) In the case of dignity, the person who wants dignities has to beg for them from those who can bestow them. Therefore he may become vile by crouching to the giver. "The desire of bodily pleasures is full of anxiety, and the enjoying of them breeds repentance." (*Consolatio*, V.pVII.1-3) Other forms of happiness[259] people seek after will not bring people to the happiness either.

> "These goods, which can neither perform that they promise, nor are perfect by having all that is good, do neither, as so many paths, lead men to happiness, nor make men happy of themselves." (*Consolatio*, III.pVIII.31-35)

By reviewing these material goods, to which men aspire, Lady Philosophy concludes that happiness cannot be found in these forms of human happiness, which are false goods. The searches for material goods are only uncertain by-ways to happiness, "which can never bring any man thither whither they promise to lead him" (*Consolatio*, III.pVIII.1-3). So the search for them is the wrong avenue, which fails to lead to happiness.

These material goods are controlled by Fortune.[260] Then Lady Philosophy exploits rhetoric[261] as one method of treatment.

> "Wherefore let us use the sweetness of Rhetoric's persuasions, which then only is well employed when it forsaketh not our ordinances." (*Consolatio*, II.pI.21-22)

Rhetoric was regarded by Aristotle as a close ally of philosophy. In his monograph on topics, Boethius devotes the fourth book to the discussion of rhetoric.

---

[259] Cf. *Consolatio*, II.p6-II.m7, and III.p3-m7.

[260] For the discussion on Fortune in *Consolatio*, cf. Magee (1987).

[261] Rhetoric here and Music later are described by Boethius as slaves belonging to Lady Philosophy which represents the power of philosophy. On the rhetoric in *Consolatio*, cf. Uhlfelder (1981), "The Role of the Liberal Arts in Boethius' *Consolatio*," in Masi (ed.), pp. 19-20.

> "The rhetorical [discipline] investigates and discusses hypotheses, that is, questions hedged in by a multitude of circumstances. Circumstances are who, what, where, when, why, how, by what means." (*TopicisD.*, 1205D.15-19)

The use of the rhetoric by the speaker functions as the appropriate persuasion. There are three species of rhetoric: judicial, epideictic, and deliberative. In *Consolatio*, what Lady Philosophy adopts is epideictic rhetoric, since "every one that has to do with praise or censure is put under the epideictic genus." (*TopicisD.*, 1207B.10-11) The instrument of rhetoric is discourse. So Fortune uses her own speech to censure the patient for his complaint. She believes that riches, dignities, and the rest of that sort belong to her. So if those things are taken from the patient, he should not have lost them. In other words, the search for riches, dignities, kingdoms, glory, and pleasure cannot lead human beings to true happiness.

What, then, is true happiness?[262] Lady Philosophy provides a definition. She defines happiness (or blessedness) as "a state replenished with all that is good" (*Consolatio*, III.pII.10-12). The good grants happiness, because attainment of the good leaves no room for further desires (*Consolatio*, III.pII.2-7). This means that this good must be the chief of all goods and contains whatsoever is good within it. Were it to lack anything of whatsoever is good, there would still be something left to desire (*Consolatio*, III.pII.7-10).

Lady Philosophy declares that her patient must abandon those false goods, and by reflecting of the contrary, he may discover the true goods. Lady Philosophy gives the nature of the true good with a dialectic technique.[263] As Boethius states in *De Topicis Differentiis*, "The dialectical discipline examines the thesis only; a thesis is a question not involved in circumstance." (*TopicisD.*, 1205C.14-15)

As part of the gentle remedy Lady Philosophy employs rhetoric, whereas in the strong remedy she chooses the technique of dialectic. Similar

[262] Cf. Marenbon (2003), pp. 102-108.
[263] On the dialectic in *Consolatio*, cf. Uhlfelder (1981), "The Role of the Liberal Arts in Boethius' *Consolatio*," in Masi (ed.), pp20-22.

to rhetoric, dialectic uses discourse as its instrument. Using dialectic, Lady Philosophy encourages her patient to cooperate with her in the search for the true good. Besides their definitions, rhetoric and dialectic are also different in some respects, which are discussed in *De Topicis Differentiis*, IV.1205C-1206C.

> (1) "Dialectic is restricted to question and answer. Rhetoric, on the other hand, goes through the subject proposed in unbroken discourse. Similarly, dialectic uses complete syllogisms. Rhetoric is content with the brevity of enthymemes."
>
> (2) "The rhetorician has as judge someone other than his opponent, someone who decides between them. But for the dialectician, the one who is the opponent also gives the decision because a reply [which is], as it were, a decision is elicited from the opponent by the cunning of the questioning."

According to two differences above, rhetoric seems softer and sweeter than dialectic.[264] With the technique of rhetoric, Fortune gives an unbroken discourse and the patient does not to respond to her, so the technique of rhetoric is fit for the patient who is dumb in the soft remedy. After the patient has become ready for the forcible medicine, the technique of dialectic is used, and Lady Philosophy and her patient ask and answer the questions in an interrupted discourse.

In theory of definition, there is one important standard, which is that the definition of something must equal the thing itself. Otherwise, the definition is not complete (*Divisio*, 886a). According to this definition, the definition of man that the patient first gives, which is "Man is a rational and mortal animal", is not a good one. Indeed, "rational" and "mortal" are two differences of the species "man", but when they are joined to the genus "animal", the definition is not equal to "man". Thus, Lady Philosophy claims that her patient forgets himself because he is so much grieved at his exile and the loss of his goods (*Consolatio*, I.pVI.42-46).

[264] According to Lerer's analysis, the rhetorical self-defence follows the patient's initial silence, and then debate and questioning takes the place. Cf. Lerer (1985).

With the Porphyrian Tree, theory of definition, and rhetoric and dialectic, the wrong way to happiness is cleared away, the general picture of true happiness is given, and the incomplete definition of man is explained. However, the detailed discussion of true happiness and the definition of man need the help of mathematics, which will be discussed in the Section IV.2.2.

Before the discussion of applications of mathematics, some methods that the arguments will use should be introduced first. They can be found in Boethius' theological treatises.

In his theological treatises, one kind of argument is often used by Boethius. Boethius gets into an argument from one respect, and then reaches an impious conclusion, so that argument is refuted. The examples of this method are as follows.

> "Hence all things that are, are God — an impious assertion." (*Quomodo substantiae*, 79-80)
>
> "Wherefore man's nature has not been saved by the birth of Christ — an impious conclusion." (*Contra Eutychen et Nestorium*, IV.123-125)

Similar to the method of arguments above, in *Consolatio*, Boethius also draws some impious conclusions.

> "Which it were impious to think of God, than whom, we know certainly, nothing is better." (*Consolatio*, III.pX.54-55)
>
> "For if He judgeth that those things shall happen inevitably, which it is possible shall not happen, He is deceived, which is not only impious to think, but also to speak." (*Consolatio*, V.pIII.67-69)
>
> "And — than which there can be nothing invented more impious — since that all order of things proceedeth from Providence, and human counsels can do nothing, it followeth that our vices also shall be referred to the author of goodness." (*Consolatio*, V.pIII.94-97)

Other important methods are mathematical methods, which are used in

Quomodo substantiae. These methods are analysis and synthesis.[265] In the following discussions I will show how these methods are used in Boethius' arguments. Now I move on to applications of mathematics in Book I-III.

IV.2.2. Applications of Mathematics in Book I-III

IV.2.2.1. Applications of Music in Book I-III

In Treatment 1 music is an important part of the therapy. [266] Music is used both in the diagnosis and in the remedies (gentle and strong).

In his work on music, Boethius divides music into three kinds: instrumental music, human music, and cosmic music.[267] Instrumental music, the lowest level of music, is often used in the process of consolation in the form of meters of poetry and the singing voice. In the soft remedy, except instrumental music, human music is also applied. Cosmic music as the highest level of music is applied only as part of the strong remedy. In this section, I will introduce how these three kinds of music are used and what functions they fulfil to the consolation.

Among three kinds of music, the only audible one is instrumental music, which resides in various instruments. It is governed either by tension, or by breath, or by a certain percussion. In this case, the singing voice in songs is also included in instrumental music.

First of all, instrumental music is used in the form of this composition. The whole work of *Consolatio* is written in alternating poetry[268] and prose. One of the basic elements of poetic language is rhythm. When meters of poetry are read, the rhythm sounds like music. For rhythm and meter are

[265] Cf. Section III.4. "Analysis is the assumption of that which is sought as if it were admitted (and the arrival) by means of its consequences at something admitted to be true. Synthesis is an assumption of that which is sought as if it were admitted (and the arrival) by means of its consequences at something admitted to be true."

[266] Cf. Chamberlain (1970); Uhlfelder (1981), "The Role of the Liberal Arts in Boethius' *Consolatio*," in Masi (ed.), pp. 25-26.

[267] Cf. Section II.2.2.3.

[268] Harpur (2007) translates the poems in *Consolatio*, and he argues for the reappraised of Boethius as poet. However, in my dissertation, I do not focus on the poetry of *Consolatio*. About discussions on the poetry of *Consolatio*, see Barnish (1990); Blumenthal (1986); Marenbon (2003), pp. 146-153; Newman (2003); O'Daly (1991); Sweeney (2006).

essential to song, and the meters, sometimes, are just sung by Lady Philosophy.[269] For example, "Philosophy had sung these verses with a soft and sweet voice." (*Consolatio*, IV.pI.1-2) When one hears rhythms and modes which reach the intellect through the ears, it is doubtless that the mind is affected and reshaped according to the particular characters of those rhythms and modes (*Musica*, I.1.181). This may be one of the reasons that Boethius uses the style of meters. The meters in *Consolatio* play a vital role in changing the patient's psychological disposition. The meters have the power to soothe the patient (*Consolatio*, I.pV.38-44), to refresh wearied minds (*Consolatio*, III.pI.4-6), and to delight the patient (*Consolatio*, IV.pVI.17-20).[270] These roles of meters are all related to instrumental music.

Boethius insists that "music is associated not only with speculation but with morality as well" and it forms a part of us through nature,[271] "for nothing is more characteristic of human nature than to be soothed by pleasant modes or disturbed by their opposites." (*Musica*, I.1.179) This function of music refers directly to instrumental music but the fulfilment of this function needs the cooperation with human music. Music has effects on mood and moral growth and it can ennoble or debase character. That is to say, different styles of instrumental music have different effects upon human nature. A harsh tune can interrupt the mind. This is the reason why at the arrival of Lady Philosophy, she cries out to and shoos away the "Sirens", the poetical Muses, who suggest words to the patient and make him burst into tears using sad music.[272] In contrast, a sweet song can have positive effects on the human mind. Accordingly, human behavior can be modified by hearing pleasant music. In his writing on music, Boethius lists some examples of persons with a serious illness who were saved or cured by the

[269] Cf. *Consolatio*, III.pIX.105, IV.pVI.17-18 and 206-209.
[270] Cf. Scarry (1980), p. 102.
[271] Cf. Levin (2009).
[272] Cf. *Consolatio*, I.pI.28-34: Lady Philosophy says with angry look that, "Who hath permitted these tragic harlots to have access to this sick man, which will not only not comfort his grief with wholesome remedies, but also nourish them with sugared poison? For these be they which with the fruitless thorns of affections do kill the fruitful crop of reason, and do accustom men's minds to sickness, instead of curing them."

assistance of pleasant songs.[273] Accordingly, Lady Philosophy has her own Muses, who, she assures, can cure and heal her patient. In other words, Lady Philosophy refers to the help from Music.[274] From the beginning to the end of the treatment, Lady Philosophy sings verses or songs,[275] which gives her patient the feeling of comfort.

The functions of instrumental music cannot be fulfilled without the help of human music. Human music, in its turn, is the interpretation of the cosmic harmony. Whoever penetrates into himself perceives human music. This music is a certain harmony uniting the incorporeal nature of reason with the body. Human music unites the parts of the soul, which, according to Aristotle, is composed of the rational and the irrational. It intermingles elements of the body or holds together the parts of the body in an established order (*Musica,* I.2.188-189). The whole structure of our soul and body has been joined by means of musical coalescence. When a human being hears an instrumental music, his soul responds to this because of human music. Therefore the patient feels comfort when listening to the songs. At the beginning of Prose I of Book III, though Lady Philosophy ended her verse, the sweetness of the song made her patient remain astonished, attentive, and desirous to hear her longer. Human music can also unite the rational with the irrational soul. When the patient is listening to instrumental music, not only is his soul affected by this music, but his irrational soul is also united with his rational soul. In this manner, with a song, Lady Philosophy also makes her patient come to himself and recover his judgment (*Consolatio*, I.pIII.1-2).

In the soft remedy, Lady Philosophy sings a lot of beautiful songs. The sweetness of the songs arouses her patient's interest in the stronger medicine. Boethius writes,

[273] For example, "Terpander and Arion of Methymna saved the citizens of Lesbos and Ionia from very serious illness through the assistance of song. Moreover, by means of modes, Ismenias the Theban is said to have driven away all the distresses of many Boeotians suffering the torments of sciatica." (*Musica*, I.1.185)

[274] Cf. *Consolatio*, II.pI.22-25: Lady Philosophy says that, "Let Music, a little slave belonging to our house, chant sometime lighter and sometime sadder notes."

[275] Although Lady Philosophy is not a human being, as an interlocutor between God and men, when she talks with her patient, she uses human style. So her singing voice, like human singing voice, belongs to instrumental music.

> "O most effectual refreshment of wearied minds, how have I been comforted with thy weighty sentences and pleasing music! ... Wherefore, I am not now afraid, but rather earnestly desire to know those remedies, which before thou toldest me were too sharp." (*Consolatio*, III.pI.1-9)

After the help of instrumental music and human music in the soft remedy, the patient can enter into discussions with his physician Lady Philosophy, and he is now well enough to engage in deeper reasoning. The gentle and pleasant things in the soft remedy prepare the patient for stronger potions (*Consolatio*, II.pI.18-21). Accordingly, a much stronger medicine can be applied to him now, and cosmic music, the highest level of music, is applied.

In the diagnosis of his illness, the patient admits that God governs the world. However, the patient could not tell by what means the world is governed. In the ninth poem of Book III,[276] Lady Philosophy sings a beautiful song to show how the world is governed.[277]

> "And, standing still Thyself, yet fram'st all moving laws,
> Who to Thy work wert moved by no external cause:
> But by a sweet desire, where envy hath no place,
> Thy goodness moving Thee to give each thing his grace,
> Thou dost all creatures' forms highest patterns take,
> From Thy fair mind the world fair like Thyself doth make.
> Thus Thou perfect the whole perfect each part dost frame.
> Thou temp'rest elements, making cold mixed with flame
> And dry things join with moist, lest fire away should fly,
> Or earth, opprest with weight, buried too low should lie."
> (*Consolatio*, III.m9.3-12)

The world is teeming with many contrary parts or elements, such as cold and flame, but those different things are bound together and achieve a balance to make up the world in harmony. There are disagreeing natures,

[276] This poem is among the most famous ones in *Consolatio*, and many authors have discussed it. Cf. Chadwick (1981b), "O qui perpetua," pp. 234-235; Wiltshire (1971).
[277] Cf. Marenbon (2003), pp. 112-114.

which could separate the concord, but they, too, are held together and united by the One. This One is called God by all men. (*Consolatio*, III.pXII.15-26)

This is also a theme expounded in Boethius' work on music. The harmony of the universe, both motion and rest, is studied by music. The term "harmony" was originally used for the whole field of music. Through music people could get to the principle of nature, which is harmony.

> "For when we hear what is properly and harmoniously united in sound in conjunction with that which is harmoniously coupled and joined together within us and are attracted to it, then we recognize that we ourselves are put together in its likeness." (*Musica,* I.1.180)

Boethius believes that people can go beyond instrumental music to achieve our own music, human music, and then proceed to the harmonious proportion of human music in general. People could even arrive at cosmic music. In cosmic music, one can understand the pure concept of proportion or harmony itself, and then get to the state of happiness. Through cosmic music, the patient understands that God governs the world in a harmonious way, but the question "by what specific method does God govern the world" is not solved. The musical therapy is not enough to cure the patient from this illness, so another one needs to be added in the strong remedy, which is arithmetic. In Section IV.2.2.2, I will discuss applications of arithmetic in the strong remedy.

IV.2.2.2. Applications of Arithmetic in Book I-III

With applications of basic logic and music, the three diagnostic reasons of the patient's illness are touched upon to some extent but not solved completely. With applications of basic logic, Lady Philosophy refutes false happiness, and she provides a definition of happiness without explanation of what true happiness looks like. The answer to "what is the end of everything" is not given. With the theory of definition, Lady Philosophy points out that the definition her patient provides in reply at first is incomplete. She claims that man is not only a rational and mortal animal, but she has not given the full definition either. By means of the musical therapy, the patient recalls that God governs the world in a harmonious way, but Lady Philosophy does not explain how this works. All unsolved

questions will be solved by applications of arithmetic.

Just like "They [*i.e.* the souls] stream, like fire returning, back to Thee, their God" (*Consolatio*, III.mIX.21), the patient should also return to God. The kingdom of God is just the patient's own country. Before the treatment of his illness, his physician Lady Philosophy gave the diagnosis that he had exiled himself from his country (*Consolatio*, I.pV.4-6). Here she determines to lead her patient to "return to thy country in safety" (*Consolatio*, III.pXII.28-29). As the remedy for the illness continues, applications of arithmetic are needed.

God binds the world's elements together on mathematical principles by number; the exemplar of His thought is arithmetic. Arithmetic studies number itself, and it introduces people to the numerical expression of quantity.

> "From the beginning, all things whatever which have been created may be seen by the nature of things to be formed by reason of numbers. Number was the principal exemplar in the mind of the creator." (*Arithmetica*, I.2)

Through numbers of an assigned order, all things exhibit the logic of their maker, God. "From Thy fair mind the world fair like Thyself doth make" (*Consolatio*, III.m9.8). Consequently, all things found harmony. The geometrical figures follow the laws of harmony, and the movements of the celestial bodies of astronomy follow the harmonious interval.

God is the beginning of all things, which is known by the patient at first. However, he forgets what the end of all things is. Then the method of synthesis[278] is used.

> Suppose a living creature given.
>
> Since unity is the substance and principle of any constant quantity, (*Arithmetica*, II.1)
>
> Then this living creature is a unity.

[278] On the explanation of this method, cf. Section III.4.and the end of Section IV.2.1.

> According to the Porphyrian Tree, this living creature consists of body and soul. (Diagram III)
>
> Therefore, body and soul are united as a unity.
>
> According to the nature of unity, when a unity is dissolved by their separation, it is manifest that it perishes.
>
> Thus, the body or the soul of a living creature is not representative of this living creature. If one of them is separated and sundered, there is no longer a living creature.
>
> Therefore, we get what is sought after, that is, in order to keep the identity of a living creature, it must not lose its unity. In other words, all things desire unity.

Furthermore, according to Rule I of *Quomodo substantiae,*[279] it is a common conception that complete and absolute goodness is only found when all is united as it were into one form. When things differ, they are not good, and when they are one, they become good, so such things are made good by obtaining unity (*Consolatio*, III.pXI.18-21). The good and unity of course do not have diverse effects, so they have the same substance (*Consolatio*, III.pXI.24-26). And it is clear that all that is good is good by partaking of goodness (*Consolatio*, III.pXI.22-23). Therefore, goodness and unity are identical.

Now we arrive at a proposition from the synthesis (i.e. All things desire unity) and an equation (i.e. goodness=unity). We arrived at the conclusion that all things desire goodness. Everything, both animate and inanimate, seeks goodness. Thus, goodness is that which is desired by all things, that is to say, "we must confess that goodness is the end of all things." (*Consolatio*, III.pXI.121-123) With the idea of unity (*unitas*) or, as she more often says, the one (*unum*), Lady Philosophy explains the final goal of all things.

In musical therapy, the patient recalls that God governs the world in a harmonious way. Harmony is the sufficient state, neither more nor less, which is perfect. From Chapter 19-20 of Book I in *Arithmetica*, we know

[279] Cf. Section III.4.1.

that the perfect numbers does not extend in a superfluous progression nor are they reduced in a contracted reduction, "the sum of its parts is not more than the total nor does it suffer from a lack in comparison with the total." Such as 6, its parts are 1, 2, and 3, and the sum of its part just equals to 6. For "Thou perfect the whole perfect each part dost frame" (*Consolatio*, III.m9.9), God commands its perfect parts to form a perfect world. And "within the first ten numbers there is only one perfect number, 6; within the first hundred, there is 28; within a thousand, 496; within ten thousand, 8128." This means that within the first ten thousand numbers, there are only four perfect numbers, that is, 6, 28, 496, 8128, thus perfect numbers are rare, and like this goodness is rare.

The definition of happiness is "a state replenished with all that is good" (*Consolatio*, III.pII.10-12), and by means of this definition we can produce a second equation, that is, goodness=happiness. Since goodness is rare, happiness is rare. The way in which human beings can achieve such rare happiness can be explained by two deductions:[280] one is concerned with the question whether the perfect happiness exists, and the other asks, if true happiness does exist, where we can find it. Here the method of synthesis is used again.

> Suppose equality given.
>
> Then according to theory of equality,[281] the world begins with equality.
>
> Equality entails being entire and absolute, and inequality implies being defective and incomplete.
>
> Hence the world began with perfections, from which it lapsed into the present diminished and exhausted state.
>
> For the less perfect derives from what is more perfect and never vice versa.
>
> Thus "if in any kind we find something imperfect, there must need

[280] Deduction is translated from the word *porismata*, which I give an explanation in Section III.4. Lady Philosophy here regards it as a *corollarium.*

[281] Cf. Section II.2.1.3.a.

> be something perfect also in the same kind." (*Consolatio*, III.pX.11-13)
>
> But we indeed find imperfect happiness, or in other words false happiness, which is explained in Section IV.2.1.
>
> Then imperfect happiness must derive from perfect happiness.
>
> Therefore, perfect happiness exists.

Now we get what we are seeking: perfect happiness does exist. However, what's the difference between perfect happiness and false happiness? Before answering this question, I need go back to a distinction between form and matter put forward by Boethius in his earlier work on the Trinity.

> "Form which is without matter cannot be a substrate, and cannot have its essence in matter, else it would not be form but a reflexion (*imago*). ... We misname the entities that reside in bodies when we call them forms; they are mere images (*imagines*); they only resemble those forms which are not incorporate in matter." (*De trinitate*, II.48-51; 53-56)

If an analogy is drawn between human beings and the goods, then the goods could be regarded as the combination of the goods of body and the goods of soul. The goods of body are a substrate with matter, thus they are just the images of their form, "the images (*imagines*) of the true good, or certain imperfect goods, but they cannot give them the true and perfect good itself." (*Consolatio*, III.pIX.91-94) Unlike the goods of the body, the goods of the soul are without matter, since they are form. In fact, the material goods, including riches, dignities, kingdoms, glory, and pleasure are just the goods of the body, and what they counterfeit are the goods of the soul or the form of goods, which are the true goods, including sufficiency, power, respect, renown, and joy. The goods of the soul equal perfect happiness, and the goods of the body equal false happiness. Therefore, perfect happiness and false happiness are just like form and matter.

The goods of the soul are simple and undivided. The division is unlike the division of a whole into parts. If the goods of soul could be divided into

parts, such as sufficiency, power, respect, renown, and joy, then, as Boethius describes in his *Divisio*, each part would differ from another. However, this is not the truth. "Every one of these things (the goods of soul) is the same with the rest, whosoever seeketh for any of them without the rest obtaineth not that which he desireth." (*Consolatio*, III.pIX.64-67) Goods of the soul are one and the same thing. When human beings are seeking happiness, they wrongly divide it into different things, which cannot be owned separately, but only as one substance. As a result, the goods are translated from true and perfect to false and imperfect. The goods of the soul have different names, but do not differ in substance (*Consolatio*, III.pIX.41-44).

In his treatise against Eutyches and Nestorius, Boethius chooses one definition of Nature from the four he listed, that is, "Nature is the specific difference that gives form to anything."[282] From this definition of Nature, we could see that the goods of the soul, i.e. sufficiency, power, respect, renown, and joy, are one and simple by nature. However, man wrongly divides it and tries to obtain parts of the goods of that soul that have no parts, so the result is that "he neither getteth a part, which is none, nor the whole, which he seeketh not after." (*Consolatio*, III.pIX.45-49)

True and false happiness are different; they are like form and matter, so that "neither sufficiency by riches, nor power by kingdoms, nor respect by dignities, nor renown by glory, nor joy can be gotten by pleasures." (*Consolatio*, III.pIX.4-6)

So, if true happiness does indeed exist, where can it be found? The method of analysis[283] is applied first.

> Suppose God is not the source of all things.
>
> Then there must be something that is the source of God, say that is α.
>
> According to a common concept, i.e. if one thing is source of another thing, then this thing must be prior to that thing, α is prior to God.

[282] Cf. Section III.2.1.
[283] On the explanation of this method, cf. Section III.4 and the end of Section IV.2.1.

But God is given to prevent an infinite series, and He is the beginning of all things.

Then nothing is prior to God.

Therefore, the supposition is wrong.

From the method of analysis, we know that God is the source of all things. There is no doubt that "the nature of nothing can be better than the beginning of it" (*Consolatio*, III.pX.56-59), which is just what Boethius calls a common concept in *Quomodo substantiae*. Thus there is nothing better than God, which means that God is the perfect good. Alternatively, this can be expressed in an equation: God=perfect good. And from the definition of happiness given by Lady Philosophy, viz. happiness is "a state replenished with all that is good" (*Consolatio*, III.pII.10-12), it can be inferred that true happiness=perfect good. So according to the nature of the equation, true happiness=God, which means that happiness resides in God.[284]

With the help of mathematical methods and knowledge, we know where true happiness is, but how can human beings get it? The answer to this question needs to explain the last diagnostic reason of the patient's illness, concerning what man is.[285]

The question what man is asks for the nature of man. In his arguments to this question, Boethius combines the ideas of Plato and Aristotle.[286] When Lady Philosophy asks, "Well then, canst thou explicate what man is?" the answer of the patient is as follows, "Dost thou ask me if I know that I am a reasonable and mortal living creature? I know and confess myself to be

[284] On the arguments on "God, the Good, and Happiness", cf. Marenbon (2003), pp. 108-112.

[285] Cf. Dougherty (2004).

[286] Boethius made a great plan to harmonize Plato and Aristotle, but he did not finish his plan. Cf. Section I.2.1. In *Consolatio*, it seems that in content he prefers Plato. For example, on the definition of man, Lady Philosophy asserts that Aristotle's definition is not enough, and the complete definition of man should include man's higher self which is learnt from Plato. And also when defining chance, Lady Philosophy is not satisfied with Aristotle's definition, and the difference from Aristotle's concept of chance is characteristic of the Platonic tradition, cf. Section IV.3.2.1.b. In method, however, Boethius prefers Aristotle, as the use of Aristotle's logic shows.

so." (*Consolatio*, I.pVII.35-37) In Section IV.2.1, with reference to the theory of definition, Lady Philosophy claims that this definition is incomplete. "Man is a rational and mortal animal"[287] is the Aristotelian definition, which is not enough to explain the nature of man. Besides this, man could be something else, which is man's higher self or abstract being, which is learnt from Plato. Here Boethius harmonizes the definition of man from Aristotle and Plato, for the Aristotelian definition is closer to this world, whereas the Platonic definition sets men free from this world and makes it possible for the patient to return to God.

According to what have already discussed, true happiness or blessedness and God are the same, which is equal to the perfect good. From this it follows that "blessedness is equal to divinity". As we all know, "All men can be made blessed by obtaining of blessedness." Thus, "all men can be made blessed by the obtaining of divinity." (*Consolatio*, III.pX.83-86) The divinity is a nature of God. If men can obtain this nature of the divinity, then they who obtain divinity must become gods. Therefore, man's higher self is a god by participation, for there is only one God by nature (*Consolatio*, III.pX.86-90).

Hitherto, all of three diagnostic reasons of the patient's illness have been expounded by Boethius employing knowledge and methods of basic logic and mathematics.

IV.2.3. How to Get to Happiness

After the exposition of the three diagnostic reasons, it is clear that the goods of the body human beings use to seek after are not true and perfect goods. These false avenues do not lead to real happiness. These false goods should now be gathered into a unity, or, in other words, be harmonized.

God rules the world in a harmonious way. As human music is human expression of natural harmony, human mind can imitate the nature or God and liberate himself from this world. God disposes all things by Himself, for "Who to Thy work wert moved by no external cause" (*Consolatio*, III.m9.4).

[287] This definition of man appears in Boethius' first commentary on Porphyry's *Isagoge*; cf. *InIsag.*, 12D-13D and 17C-D.

And it has already been proved that God is goodness itself. Therefore, God, Whom we have argued to be Goodness, governs the world in a harmonious way by Himself. As I discussed above, man could become a god by obtaining divinity which is equal to blessedness (or happiness). And man is comprised of soul and body which are joined by means of musical coalescence. Imitating God, man needs to find the way within himself.

> "O mortal men, why seek you for your felicity aboard, which is placed within yourselves? ... Wherefore, if thou enjoyest thyself, thou shalt possess that which neither thou wilt ever wish to lose nor fortune can take away. And that thou mayst acknowledge that blessedness cannot consist in these casual things, gather it thus." (*Consolatio*, II.pIV.72-79)

Nothing is more precious to us human beings than our selves. Therefore, if man wants to get to the end, to goodness and happiness, he can do so in the same manner as God: by himself. When we learn this, we should not be sad about the things outside us.

All discussions set out above are not new to Boethius: "what thou hast uttered, though the force of grief had made me forget it of late, yet heretofore I was not altogether ignorant of it." (*Consolatio*, IV.pI.7-9) This has already been proven from Boethius' earlier works on mathematics and logic.[288]

Three diagnostic reasons of the patient's illness have been explained, so the course of treatment prescribed by Lady Philosophy in the first book of *Consolatio* seems to be completed, and the dialogue seems to be brought to a close.

However, as I announced in the preface of this chapter, the patient's illness has not really been cured. Treatment 1 in the first three books is not enough. Treatment 2 should be provided as a further treatment and will comprise Books IV and V. Unlike Treatment 1, in Treatment 2, the patient will take the initiative. Boethius will again apply basic logic in Treatment 2, but most of the ideas expressed will be new to him. In Section IV.3, I will

[288] Cf. Section IV.2.1 and Section IV.2.2.

focus on applications of basic logic in Book IV and Book V.

IV.3. Applications of Basic Logic in Book IV-V

During Treatment 1 in Book I-III, the patient at first was mute and dumb, and Lady Philosophy asked him some questions to help him. At the end of Treatment 1, the patient recalls something and can reason by himself now. However, apart from the three diagnostic reasons of the patient's illness, there is another vital question in his mind, which is related to evil.[289] Lady Philosophy claims that "there is nothing that God who is almighty cannot do," and "God cannot do evil," therefore, "evil is nothing." (*Consolatio*, III.pXII.74-78) However, evil does exist in a world that is universally governed by the God! This is the root cause of the patient's illness. The patient makes the diagnosis himself,

> "O thou who bringest us to see true light, those things which hitherto thou hast treated of have manifestly appeared both to be divine when contemplated apart, and invincible when supported by thy reasons, and what thou hast uttered, though the force of grief had made me forget it of late, yet heretofore I was not altogether ignorant of it. But this is the chiefest cause of my sorrow, that since the governor of all things is so good, there can either be any evil at all, or that it pass unpunished. Which alone I beseech thee consider, how much admiration it deserveth. But there is another greater than this; for wickedness bearing rule and sway, virtue is not only without reward, but lieth also trodden under the wicked's feet, and is punished instead of vice. That which things should be done in the kingdom of God, who knoweth all things, can do all things, but will do only that which is good, no man can sufficiently admire nor complain." (*Consolatio*, IV.pI.5-19)

[289] Cf. Chadwick (1981b), pp. 239-241; Marenbon (2003), pp. 114-117; Sharples (1991), pp. 31-34. On the discussions on evil and punishment also cf. Evans (1982a), about Augustine on evil; Mackenzie (1985), about Plato on punishment; Sorabji (2004a), pp. 95-108, about the texts of the commentators from 200-600AD on evil; Steel (1998), about Proclus on evil.

The patient's diagnosis contains two main questions. One is that evil seems powerful than the good; and the other one is that it appears that the good is not rewarded while evil is not punished. Both of these seem abnormal in a world governed by an almighty and good God. Therefore, the patient needs further treatment.

Treatment 2 is different from Treatment 1. The questions discussed in Treatment 1 used to be known by Boethius, but were temporarily forgotten, while the issues argued in Treatment 2 are not discussed nor solved by Boethius in his earlier works. The necessary continuation applies the metaphysics of the Good to moral considerations.[290] The questions in the patient's diagnosis derive from Plato's *Gorgias*.[291] In *Gorgias*, especially 466b-481b, Socrates discusses with Polus what rhetors or tyrants do, which deals with concrete examples.[292] Unlike Plato, in *Consolatio* Boethius discusses the good, and evil men who are abstract and general.

In the following two sections, I will first introduce the analysis of the questions in Treatment 2, viz. the power of good and evil; and the reward of good and the punishment of evil. Then I will give the explanation Lady Philosophy provides for these questions, which relates to Providence. In the discussion of these two sections, Boethius adopts basic logic.

IV.3.1. Analysis of Questions in Treatment 2

The questions in Treatment 2 are related to good and evil, which are contraries. Contrary is one of four classes of opposites.[293] In Chapter 11 of *Categories*, Aristotle gives a further discussion of contraries with special mention of good and evil. Good and evil are contraries, so the power of the good and the evil can be proved as follows. If a potent feature of the good is obvious, then it will be acknowledged that the evil is frail; and on the contrary, if a weakness of evil is discerned, then it will be manifest that the

[290] Cf. Magee (2009), "The Good and Morality: *Consolatio* 2-4," in Marenbon (ed.), pp. 198-200.
[291] Cf. Marenbon (2009), p. 198.
[292] Cf. Plato, *Gorgias*, 466b-481b; Nichols (1998).
[293] Opposites can be divided into correlatives, contraries, positives and privatives, affirmation and negation.

good is strong. (*Consolatio*, IV.pII.6-8)

Before the discussion, one theorem is given first, that is, "There be two things by which all human actions are effected, will and power, of which if either be wanting, there can nothing be performed." (*Consolatio*, IV.pII.12-14) This theorem is developed from Plato's *Gorgias*.

> "SOC.: By preparing what, then, would a human being help himself, so as to have both of these benefits — that of not doing injustice and that of not suffering injustice? Is it power or wish? This is how I mean it: if he does not wish to suffer injustice, will he not suffer injustice; or if he has prepared a power of not suffering injustice, will he not suffer injustice?"[294]

Unlike Socrates, Boethius immediately divides the whole of human actions into two elements, will and power. This is the use of the logical theory of division, here the division of a whole into parts.[295] There are many ways in which "whole" is used, and among these ways, one is that what consists of certain powers is called a whole (*Divisio*, 888c). The division of human actions is the division of a whole consisting of powers. One characteristic of this kind of division is that "if a part of the whole perishes then that of which one part has been destroyed will not be *whole*" (*Divisio*, 879c). Accordingly, if one of these two parts, will or power, is absent, the whole of human actions cannot be accomplished.

> "For if there want will, no man taketh anything in hand against his will, and if there be not power, the will is in vain. So that, if thou seest any willing to obtain that which he doth not obtain, thou canst not doubt but that he wanted power to obtain what he would." (*Consolatio*, IV.pII.14-19)

Based on this theorem, Lady Philosophy begins her argument, again, in the way of the *Quomodo substantiae*, i.e. by means of synthesis.

> Suppose two kinds of men, the good and the evil.

[294] Cf. Plato, *Gorgias*, 509d; Nichols (1998).
[295] Cf. Section II.1.2.2.

> Then according to the theorem, their actions depend on their will and power.
>
> All men both good and evil without difference of intentions endeavour to obtain goodness. (This is proved in Section IV.2.2)
>
> Hence, the good and the evil have the same will, that is to say, obtaining goodness.
>
> Thus, their actions depend on how much power they have.
>
> But every man is mighty in that which he can do, and weak in that which he cannot do. (*Consolatio*, IV.pII.22-24)
>
> Then the more powerful men are, the easier men can get their will.
>
> The good men are so-called because they achieve goodness. If evil men could obtain goodness, they would not be called evil men.
>
> Thus, the good men get what they will, but the evil men do not get that they will.
>
> Therefore, the good men are more powerful than the evil men.

With the method of synthesis, the first question on the power of the good and the evil is explained. In order to explain the second question on the good and the evil, the argument continues.

> Since goodness is identical with happiness. (This is proved in Section IV.2.2)
>
> Then the good who get their will, i.e. goodness, are happy.
>
> Since happy men are gods. (This is proved in Section IV.2.2)
>
> Therefore, the good men become gods, which is the reward for the good.

Contrary to the reward of the good, the evil could not get their will, i.e. goodness, so they are unhappy. However, if the wicked men get to the end of evil, they will have gone too far away from goodness; therefore, they should be happier in failing their ends than in attaining them. As we know,

justice is a subordinate species of the genus good, which is a common concept. So by the reason of justice, punishment is good. This means that the wicked who are punished have some good annexed. If wicked men accept their punishment, which is good, they will be happy, for goodness is identical with happiness. Otherwise, if wicked men escape the hands of justice, they will not get any good, and then they are less happy than when they are punished (*Consolatio*, IV.pIV.42-44). Deductively, the wicked who are punished justly will be far happier if than those who escape punishment unjustly (*Consolatio*, IV.pIV.71-73).

From arguments of the reward of the good and the punishment of evil, it is manifest that good men are happy and get their reward by becoming gods, whereas evil men are unhappy. However, there is a far worse outcome of evil. Because of their all-embracing wickedness, evil men have lost the nature of men (*Consolatio*, IV.pIII.50-51). This can be shown from the Porphyrian Tree. As the Porphyrian Tree of *Contra Eutychen et Nestorium* in Section III.2.1.2[296] shows, men are composed of body and soul, which is also a division of a whole into parts. When either of these two parts is destroyed, the whole "man" will disappear. "For as faintness is a disease of the body, so is vice a sickness of the mind." (*Consolatio*, IV.pIV.149-151) As for the wicked, though they keep the outward shape of human body, their souls are destroyed; as a result, in this case, the inward states of the wicked's mind are changed into brute beasts. If the human soul, as one part of the whole human being, is damaged, then the whole human being of evil men will disappear, and they could not be called men.

> "So that he who, leaving virtue, ceaseth to be a man, since he cannot be partaker of the divine condition, is turned into a beast." (*Consolatio*, IV.pIII.66-69)

With the method of synthesis in mathematics, knowledge of the category of contrary and the Porphyrian Tree in logic, Lady Philosophy expounds the patient's questions about the good and evil. However, the reason why these questions on the good and the evil are misunderstood is not solved, and Treatment 2 continues.

[296] Cf. Diagram III.

IV.3.2. A Reason Related to Providence

After the explanation of the questions, the patient points out a confusion. Specifically, the patient wonders why, if the world is governed by God, a good ruler, there still exists apparent injustice and irrationality.[297]

> "Wherefore I much marvel why these things are thus turned upside down, and the punishments of wickedness oppress the good, while evil men obtain the rewards of the good. ... Now God being the Governor, my astonishment is increased." (*Consolatio*, IV.pV.13-17; 18-19)

At the request of her patient, Lady Philosophy offers a reason, which concerns providence. The issue of Providence is entangled with many other questions, including questions "of the simplicity of Providence; of the course of Fate; of sudden chances; of God's knowledge and predestination, and of free will" (*Consolatio*, IV.pVI.11-13).[298] Though these questions had had a long history of discussion, from Boethius' point of view they had not been solved completely. Boethius discusses them with his own methods to make them much clearer, especially by applying division and definition. In the following discussions, some ideas or examples in his second commentary on Aristotle's *On Interpretation* 9 are resumed and advanced by Boethius here.

IV.3.2.1. Distinction between Providence and two Terms, Fate and Chance

In order to understand the argument about providence, we first need to distinguish providence from fate and chance.

IV.3.2.1.a. Providence and Fate

The first term related to providence is "fate".[299] The Stoics, as

[297] Throughout Chapter VI of Book IV in *Consolatio*, the emphasis is laid on the limitations of human understanding; cf. Lerer (1985), pp. 204-213.
[298] Cf. Dubs (1981); Sharples (2009), "Fate, Prescience and Free Will," in Marenbon (ed.), pp. 207-227.
[299] Sorabji (2004a, pp. 89-92) lists texts of the commentators from 200-600AD on the relation of providence to fate. Also cf. Chadwick (1981b), pp. 242-244; Marenbon (2003),

Courcelle indicates, did not know the subordination of fate to providence, for to the Stoics, it is meaningless except in a philosophy that establishes a hierarchy among beings. It is the Neoplatonists that state this relationship between the two conspicuously.[300] But it should be said that the development of the precise nature of Providence has its origin in Boethius.[301] Boethius begins the discussion of the distinction between providence and fate from their definitions. It is the Neoplatonists that "link providence with the timeless unity of intellect, and fate with the nature of the physical universe extended in time and governed by soul."[302] This link affects Boethius' differentiation.

From the four types of definition in his commentary on Cicero's *Topics*,[303] Boethius here employs the definition in the strict sense of the term. This type of definition is constructed of genus and differentiae. Boethius gives two definitions of each concept.

In the first definition, providence and fate both fall into the genus called the "way" in which all things behave. The differentiae are that this way is either "considered in the purity of God's understanding", or "referred to those things which He moves and disposes". The former is named Providence, and the latter is called Fate by the ancients (*Consolatio*, IV.pVI.27-30). The differentiae determine their different forces. Providence arranges all things and embraces all things together, however diverse and infinite; while Fate is a disposition inherent in changeable things that arranging the motion of every particular thing as it is being distributed in place, form, and time.

In the second definition providence and fate fall into another genus, called "unity". Therefore, when the unfolding of temporal order is a unity within the foresight of God's mind, it is Providence, but when that unity is unfolded in time, it is called Fate.

Regardless of the differences between them, providence and fate are

pp. 117-121; Sharples (1991), pp. 29-31.
[300] Cf. Courcelle (1969), p. 305.
[301] Cf. Craig (1988b) and Patch (1929).
[302] Sharples (1991), p. 31.
[303] Cf. Section II.1.2.2.

also related to each other. Fatal order proceeds from the simplicity of Providence, and Fate depends on Providence.[304] So fate obeys to providence and all that is under fate is also subject to providence. However, it is not the same contrariwise. There are other things except fate being placed under providence, which "above the course of fate and nigh to the first Divinity, being stable and fixed, exceed the order of fatal mobility" (*Consolatio*, IV.pVI.60-65).

To illustrate this relation, Lady Philosophy uses an analogy of geometrical images.[305] She describes how orbs turn about the same centre.

> "For as of orbs which turn about the same centre, the inmost draweth nigh to the simplicity of the midst, and is as it were the hinge of the rest, which are placed without it, about which they are turned, and the outmost, wheeled with a greater compass, by how much it departeth from the middle indivisibility of the centre, is so much the more extended into larger spaces, but that which is joined and coupled to that middle approacheth to simplicity, and ceaseth to spread and flow abroad." (*Consolatio*, IV.pVI.65-73)

In a similar way, Providence is the center, and what distances itself farthest from this mind is involved more deeply in the chains of Fate. The closer things are to the sovereign mind, the more they are free from motion, and surpass the necessity of Fate. Therefore, the way a circle moves about the centre represents the course of movable Fate in relation to the stable simplicity of Providence.

Now that Providence and Fate have been defined, the nature of providence has become clear. As the author of all nature, God by His Providence directs and disposes all things to goodness. "While He endeavoureth to retain in His own likeness those things which He hath produced, He banisheth all evil from the bounds of His commonwealth, by the course of fatal necessity." (*Consolatio*, IV.pVI.201-204) The nature of

[304] Boethius stresses that fate is subordinate to providence. On this point, Sharples compares Boethius' opinion with those of his sources, cf. Sharples (1991), pp. 204-205.
[305] On the sources of this analogy, cf. Courcelle (1969), pp. 305-306; Patch (1929); Sharples (1991), p. 205.

Providence is that it is "the very Divine reason itself, seated in the highest Prince" (*Consolatio*, IV.pVI.32-33). If so, it will be perceived that there is no place left for the so-called evil in the world. Accordingly, it is clear that all manner of fortune is good.[306] The reason is that when fortune is directed to rewarding the good, it is just, which is good; and when fortune is directed to punish the wicked, it is profitable, which is also good.

IV.3.2.1.b. Providence and Chance

Another term related to providence is chance.[307] On the basis of Aristotle's account of chance, Lady Philosophy explains the nature of chance and gives a definition of chance. Aristotle's definition of chance in *Physics*[308] has already been offered by Boethius in his second commentary on Aristotle's *On Interpretation* 9.

> "They (the Peripatetics) affirm that there is chance in physical things: whenever something is done and the outcome is not that for which the thing which was done was initiated, then the outcome must be considered to have occurred by chance. Thus chance does not come about without any action, but as long as something unexpected happens through the action which is put into effect, the traditional view of the Peripatetics is that it has occurred by chance." (*2InInter.*, 194.2-9)

Later in his commentary on Cicero's *Topica*, Boethius distinguishes the Ciceronian definition of chance from the Aristotelian definition of chance, and he sums up that of these two definitions have in common that "what is subject to fortune is always included among uncertain occurrences" (*C.Topica*, 374/1153).

In *Consolatio*, the Aristotelian definition of chance is again given. However, this time, Boethius does not just state Aristotle's definition, but also points out its imperfection. Lady Philosophy says that Aristotle's account of chance is "near the truth" (*Consolatio*, V.pI.35-37), which means

306 Cf. Cioffari (1935), p.89.
307 Cf. Marenbon (2003), pp. 121-124. And Sorabji (2004a, pp. 92-95) lists some texts of the commentators from 200-600AD on the relation of providence to chance.
308 On Aristotle's discussion of chance, cf. Aristotle, *Physics*, II.4-6; Ross (1995).

it is not exactly the truth. The definition of chance given by Boethius is as follows.

> "Wherefore, we may define chance thus: That it is an unexpected event of concurring causes in those things which are done to some end and purpose. Now the cause why causes so concur and meet so together, is that order proceeding with inevitable connexion, which, descending from the fountain of Providence, disposeth all things in their places and times." (*Consolatio*, V.pI.53-58)

Between Boethius' definition of chance and Aristotle's, there are differences. From Aristotle's perspective, "by chance" means that the result of the action is not intended, for this action was done with some other primary purpose. In other words, there is no explanation of the connection between the action and the result attaching to it, apart from happening "by chance". Like Aristotle, Boethius admits that the action and the result are coincident and separate. However, unlike Aristotle, Boethius argues that chance is also the working out of Providence. The difference between the definitions of chance given by Boethius and Aristotle can also be seen from the examples of chance. Both in the commentary on Aristotle's *On Interpretation* and *Consolatio*, Boethius takes man's digging, which appears first in abbreviated form in Aristotle's *Metaphysics*, 1025A, as an example to explain chance,[309] but what he states about the example in the two works is not the same. In the commentary on Aristotle's *On Interpretation*, Boethius states that,

> "For if someone while digging the ground or making a ditch for agricultural purposes finds treasure, the treasure has been found by chance though not without some action (for the ground had been dug when the treasure was found), but it was not the agent's intention that treasure be found. Thus though the man was doing something, a different result happened to him while he was doing something else." (*InInter*., 194.9-15)

Boethius stops the example at the point of stating that this man's action and what he gets are coincident. However, in *Consolatio*, Boethius advances this

[309] Cf. Aristotle, *Metaphysics*, 1025A; Ross (1995).

example.

> "... One digging his ground with intention to till it, findeth a hidden treasure. This is thought to have fallen thus out by fortune, but it is not of nothing, for it hath peculiar causes whose unexpected and not foreseen concourse seemeth to have brought forth a chance. For unless the husbandman had digged up his ground, and unless the other had hidden his money in that place, the treasure had not been found. There are therefore the causes of this fortunate accident, which proceedeth from the meeting and concourse of causes, and not from the intention of the doer." (*Consolatio*, V.pI.41-52)

Here Boethius admits the coincident of the man's action and his result, but he also claims that "it is not of nothing", without any cause. There is also the other person who had hidden the treasure in the place where the man was digging, which is the working out of Providence.

From the exposition of definitions above, fate and chance both depend on Providence, and all things are arranged and embraced by Providence, so it seems that there is no free will. Nevertheless, Lady Philosophy confirms that free will does exist.

> "We have [free will] for there can be no reasonable nature, unless it be endued with free-will. ... Now every one seeketh for that which he thinketh is to be desired, and escheweth that which in his judgment is to be avoided. Wherefore, they which have reason in themselves have freedom to will and nill." (*Consolatio*, V.pII.5-6; 9-11)

As a result of the proper definition of providence, an important issue comes out, namely how it is possible to reconcile free will with providence. The solution to this is the solution to the patient's root cause of his illness.

IV.3.2.2. The Solution to the Issue of Providence and Free Will

The patient complains of this issue of Providence and free will[310] and

[310] Cf. Bobzien (1998 and 2000); Brennan (2001); Chadwick (1981b), pp. 244-247; Kenny (1969); LaChance (2004).

concludes that there are three possible scenarios for God to foreknow uncertain things:[311]

(a) God judges that things will happen inevitably, but what He foreknows does not happen;
(b) God knows uncertain things for what they are: they can either happen or not happen;
(c) God foresees all things without error, and nothing can be uncertain.

(*Consolatio*, V.pIII.67-84)

In the first situation (a), the conclusion is that God is mistaken, "which is not only impious to think, but also to speak" (*Consolatio*, V.pIII.68-69). In the second case (b), God could not exceed human opinion, which is also impious.[312] The last situation (c) suggests that there is no free will. The consequences of the last situation will be far worse. All rewards and punishments will be arbitrary and in vain, because the good and the wicked are compelled by the certain necessity proceeding from Providence, and their own wills lead them to neither. What is worse is that human vices will also be referred to the author of goodness. Furthermore, prayer to God will be futile, "since an unflexible course connecteth all things that can be desired." (*Consolatio*, V.pIII.99-100) From this situtation the core of the patient's complaint has become clear.

After the patient's long complaint,[313] Lady Philosophy claims that the issue of the compromise between Providence and free will has a long history, and that early thinkers already presented the solution. They advanced that what providence foresees depends on what actually comes to pass, and not vice versa. In other words, "they suppose that foreknowledge is not the cause of any necessity in things to come." (*Consolatio*, V.pIV.13-15) And Boethius himself also searched this solution in his commentary on

[311] In his commentary on Aristotle's *On Interpretation*, Ammonius Hermiae contends that divine foreknowledge makes the contingent void; Boethius has a different opinion. Cf. Ammonius, *On Aristotle's On interpretation 9*; Blank (1998).
[312] The method of arguments (i.e. coming to an impious conclusion) is similar with one Boethius used in his theological treatises. Cf. Section IV.2.1.
[313] Cf. *Consolatio*, V.pIII.

Aristotle's *On Interpretation*, 225.9ff.[314] However, now Lady Philosophy is not satisfied with those solutions and refutes them. She says that,

> "This is an ancient complaint of providence, vehemently pursued by Marcus Tullius in his *Distribution of Divination*, and a thing which thou thyself hast made great and long search after. But hitherto none of you have used sufficient diligence and vigour in the explication thereof." (*Consolatio*, V.pIV.1-6)

Lady Philosophy indicates that her solution is a novel one. Actually, in *Consolatio*, that Boethius lets Lady Philosophy give a solution to the issue on Providence and free will, which is different from Cicero and Augustine. Cicero regards this issue as the pagan problem of divination and what he is concerned with are the implications of divination and prophecy for human freedom.[315] Augustine considers this issue as the Christian problem of free will and grace.[316] Unlike Cicero and Augustine, Boethius treats this issue as a philosophical problem, for which Lady Philosophy gives a solution in *Consolatio*. There are two key terms on which this solution hinges, viz. God's knowledge and God's eternity, which will be explained as follows.

IV.3.2.2.a. God's knowledge

As we have seen, the patient analyses three scenarios of God foreknowing uncertainties.[317] In his argument, two conclusions are impious, and the last one is that foreknowledge and necessity should be bound together, for "if future things be foreseen, there followeth necessity, if there be no necessity, that they that are not foreknown, and that nothing can be perfectly known unless it be certain." (*Consolatio*, V.pIV.65-68)

Lady Philosophy points out the reason of her patient's error. This error

[314] Cf. *2InInter.*, 225.9ff.
[315] Cicero, *On Fate*; Sharples (1991).
[316] Augustine, *The Confessions*; Schaff (1886).
[317] For three situations of God foreknowing the uncertainties, cf. the preface of Section IV.3.2.2. Sharples (1991, pp. 25-29) gives a short introduction to the development of the discussion of divine foreknowledge from Cicero to Boethius; Sorabji (2004a, pp. 69-77) lists the texts of the commentators from 200-600AD on divine knowledge. And for the discussions on divine foreknowledge, also cf. Craig (1988a); Davies (1983); Evans (2004); Knuuttila (2010), pp. 77-80; Obertello (1981).

is caused by the misapprehension of knowledge. "For all that is known is not comprehended according to the force which it hath in itself, but rather according to the faculty of them who know it" (*Consolatio*, V.pIV.75-77). Knowledge depends on the force of the knower who comprehends things, rather than on the nature of the things known.[318] Since the knowers are different, it is without doubt that their knowledge of things is not the same.

The first move to correct her patient's error is to expound the modes of knowing, which is divided into four levels. From the inferior to the superior, they are sense, imagination, reason, and understanding.

> "For sense looketh upon his form as it is placed in matter or subject, the imagination discerneth it alone without matter, reason passeth beyond this also and considereth universally the species or kind which is in particulars. The eye of the understanding is higher yet. For surpassing the compass of the whole world it beholdeth with the clear eye of the mind that simple form in itself." (*Consolatio*, V.pIV.84-91)

The rudiment of this division could be found in Boethius' earlier works. In his mathematical work on music, he shows a hierarchical relation between sense and reason. He reminds people not to grant all judgment to the senses.

> "Yet the sense of hearing holds the origin in a particular way, and, as it were, serves as an exhortation; the ultimate perfection and the faculty of recognition consists of reason, which, holding itself to fixed rules, does not falter by any error." (*Musica*, I.9.195)

And later in his second commentaries on Porphyry's *Isagoge*, Boethius discusses the triple power of the soul, without offering a neat correspondence with the four modes of knowing.

> "Of these (viz. the triple power of the soul), the first one supports the life for the body, that it may arise by birth and subsist by nourishment; another lends judgment to perception; the third is the

[318] This derives from Iamblichus, who claimed that knowledge is intermediate between the knower and the known, since it is the activity of the knower concerning the known. Cf. Inglis (2002), p. 128.

> foundation for the strength of the mind and for reason." (*2InIsag.*, 71B)

There is no counterpart of the first power in the modes of knowing. The second power of soul equals sense. The third power "is constituted completely in reason, and it is occupied in the very firm conception of present things, or in the understanding of absent things, or in the investigation of unknown things" (*2InIsag.*, 71C-72A). Even though Boethius touched the topic of the modes of knowledge in his earlier works, he did not give a complete division. Here in his last work, he gives a detailed exposition of four modes of knowledge.[319]

Just as the triple power of the soul could be found in different animated bodies, the four modes of knowing also belong to different levels of knower, who only use their own force and faculty. Among them, reason (*ratio*) belongs only to human beings, and understanding (*intellegentia*) belongs to things divine. Since all judgment is the act of the knower, "it is necessary that everyone should perfect his operation by his own power and not by the force of any other" (*Consolatio*, V.pIV.118-120). This implies that divine knowledge and human knowledge are not on the same level. Among the triple powers of the soul, the first one could not exercise the judgment of the latter two powers, but the highest carries with it the lower powers and uses them as slaves and servants. In the same way, among the four modes of knowing, "the superior force of comprehending embraceth the inferior; but the inferior can by no means attain to the superior" (*Consolatio*, V.pIV.92-94). As a result, the divine understanding knows everything that human reason knows, but not vice versa. As for future things, human reason does not think in the same way in which the divine understanding knows. In fact, that we cannot know in advance the result of uncertain things does not mean that the divine understanding cannot do so.

> "And that is, how a certain and definite foreknowledge seeth even those things which have no certain issue, and that this is no opinion, but rather the simplicity of the highest knowledge enclosed within no bounds." (*Consolatio*, V.pV.53-56)

[319] Cf. Marenbon (2003), pp. 130-135.

The nature of knowledge suggests that what the divine understanding knows on the issue of future things and necessity is different from what human reason apprehends. Nevertheless, one might ask what the relation is between future things and necessity in the divine understanding. The nature of knowledge is a necessary preparation to the explanation, but it alone does not offer an adequate explanation. What is more, the issue on how to reconcile Providence with free will is not yet solved. The last term that is crucial to the solution is the eternity of God.

IV.3.2.2.b. Eternity of God

Eternity is usually compared with temporal things.[320] As Sharples claims, the contrast between eternity and temporal things goes back "arguably to Parmenides and certainly to Plato, *Timaeus* 37d-38b"[321]. Plato and Aristotle[322] both agree that this world had neither beginning of time nor would ever come to an end. Boethius goes along with them, which verifies his attempt to harmonize Plato and Aristotle as he planned to do in his earlier life.[323] The world, in Aristotle's thought[324], never "began nor were ever to end, and its life did endure with infinite time" (*Consolatio*, V.pVI.18-22). However, it should not be called everlasting. Following Plato,[325] Boethius confirms that "God is everlasting and the world perpetual" (*Consolatio*, V.pVI.57-59).[326]

Boethius distinguishes eternity (*aeternitas*) from perpetuity (*perpetuitas*). The definition of eternity given by Boethius is as follows. This definition is not the type of definition in the strict sense, but it is a

[320] Cf. Ford (1968); Marenbon (2003), pp. 135-138; Obertello (2003). And Sorabji (2004a, pp. 221-225) lists some texts of the commentators from 200-600AD on eternity.
[321] Sharples (1991), p. 228.
[322] The names of Plato and Aristotle appear in the last chapter of Book V in *Consolatio*. Lerer (1985, p. 230) claims that it may be no accident that Boethius had intended in his works to present their doctrines as a harmonized whole. Actually, it is surely not an accident, for Boethius announced to reconcile the ideas of Plato and Aristotle in his second commentary on Aristotle's *On Interpretation*, 58; cf. Section I.2.1.
[323] Cf. Section I.2.1.
[324] Cf. Aristotle. *On the Heavens*, I.12; Ross (1995).
[325] Cf. Sharples (1991), p. 228.
[326] Cf. Bubacz (1981) discusses Boethius and Augustine on knowledge of the physical world.

description.[327]

> "That then which comprehendeth and possesseth the whole fulness of an endless life together, to which neither any part to come is absent, nor of that which is past hath escaped, is worthy to be accounted everlasting (*aeternitas*), and this is necessary, that being no possession in itself, it may always be present to itself, and have an infinity of movable time present to it." (*Consolatio*, V.pVI.25-31)

Consequently, the eternity of God is like a perpetual present. He always has an everlasting and present state. This notion has already been stated in Boethius' earlier theological treatise on the Trinity. In Chapter IV of *De trinitate*, while examining the change of categories when applied to God, Boethius applies the category of time to God.

> "But the expression 'God is ever (*semper*)' denotes a single Present, summing up His continual presence in all the past, in all the present — however that term be used — and in all the future. ... He is ever, because 'ever (*semper*)' is with Him a term of present time, and there is this great difference between 'now,' which is our present, and the divine present. Our present connotes changing time and sempiternity; God's present, abiding, unmoved, and immoveable, connotes eternity (*aeternitas*)." (*De trinitate*, IV.64-66; 69-74)

In his theological works, Boethius does not explain God's eternity, but in *Consolatio*, he gives an exposition.

From the definition of eternity, it could be inferred that God sees even future events as present, for His knowledge remains in the simplicity of His presence, and He considers all things in His simple knowledge as if they were happening now.

In the criticism of earlier thinker's solution, the patient declared that future things and necessity are bound together, that is to say, if there be necessity, then things must be foreseen; and vice versa, if future things be foreknown, then there must follow necessity; if there be no necessity, then

[327] Cf. Section II.1.2.2.

nothing can be known. And here Lady Philosophy gives the correct exposition of the issue. The necessities are divided into two: simple and conditional.[328]

> "The same thing is necessary when it is referred to the Divine knowledge; but when it is weighed in its own nature that it seemeth altogether free and absolute. ... Likewise those things which God hath present, will have doubtless a being, but some of them proceed from the necessity of things, other from the power of the doers." (*Consolatio*, V.pVI.100-103; 131-134)

Such future things become necessary when they are related to God by condition of divine knowledge, but when they are being considered in themselves, they still have the total freedom of their own nature.

In his theological treatise against Eutyches and Nestorius, Boethius resorted to the method of etymology to trace the source of the word "person",[329] so that the meaning of it could be more stated more accurately. Here in *Consolatio*, Boethius briefly uses this method, too. It is not exact to call God's foresight a foreknowledge of the future. The foresight by which God discerns all things should be regarded as "the knowledge of a never fading instant" (*Consolatio*, V.pVI.68-69). For this reason, the so-called foreknowledge of God should rather be termed *providentia* ("looking forward spatially") rather than *praevidentia* ("looking forward in time"), because it is "placed far from inferior things, it overlooketh all things, as it were, from the highest top of things" (*Consolatio*, V.pVI.69-72).

Although the issues in Treatment 2 were discussed by Boethius and others before, here in *Consolatio* Boethius explains the issue in unparalleled clarity. It is striking to see how Boethius' resolution of the thorny issue of Providence and free will is solved by wielding knowledge of basic logic, especially the theory of definition and division.

[328] Cf. Marenbon (2003), pp. 138-143; Patch (1935a).
[329] Cf. Section III.2.1.2.

IV.4. Back to the Fatherland

Through Treatment 1 and Treatment 2, the patient in *Consolatio* is cured and back to his Fatherland.[330] This Fatherland is the kingdom of God, which does not refer to Christian God but the divine wisdom, the understanding of the true nature of all things.

Boethius, though a Christian,[331] and though he wrote theological treatises, does not resort to the Christian faith and Scripture when he faces death. In his last work, *Consolatio*, as the title implies, Boethius is in quest of consolation from philosophy not from Christ.[332] One reason Boethius chose to write his last work in such philosophical language is in line with his earlier proposals. In his theological treatises, Boethius tries to "reconcile faith and reason" (*Utrum pater*, 70-71), so he employs basic logic and mathematics to investigate theological issues. As a Christian, Boethius writes his theological treatises triggered by political circumstances, and not from the perspective of a Christian but from the perspective of philosophy, which shows his preference.

Now in his last work, when he faces death, the first thing he remembers must be philosophy. He displays the power and significance of the elementary disciplines of philosophy (i.e. mathematics and logic, and especially music) with which he is so familiar, and the study of which he sought to promote. Even the most complex issues become clear by applying

[330] Cf. Sharples (1991, p. 213) compares this notion with the Stoics and Plato.

[331] It is sure that Boethius is a Christian. Cf. Courcelle (1969), p. 318; Sharples (1991), p. 46.

[332] Steward and Rand believe it is easy to answer the question "why *The Consolation of Philosophy* contains no conscious or direct reference to the doctrines which are traced in the Tractates with so sure a hand, and is, at most, not out of harmony with Christianity." This answer is that "in *Consolation* he is writing philosophy; in the *Tractates* he is writing theology." Cf. Steward and Rand (1926), p.XI. Similar to the answer given by Steward and Rand, Lewis (1964, pp. 76-79) gives the answer imitating Boethius, saying "If we had asked Boethius why his book contained philosophical rather than religious consolations, I do not doubt that he would have answered, 'But did you not read my title? I wrote philosophically, not religiously, because I had chosen the consolation of philosophy, not those of religion, as my subject. You might as well ask why a book on arithmetic does not use geometrical methods."

simple methods, and simple distinctions.

A further reason for his choice may be related the situation he describes. At the beginning of *Consolatio*, he is confused and even complains about Fortune. If he had relied on faith from the start, he would not plausibly have fallen into such a lamentable condition. As it is, the best way to console him is not by faith but by reason. As a result, Boethius has Lady Philosophy lead him out of his state of confusion.

The task of Lady Philosophy in *Consolatio* is indeed to lead her patient back to his Fatherland.

> "Thou in consenting parts fitly disposed hast
> Th' all-moving soul in midst of threefold nature placed,
> Which, cut in several parts that run a different race,
> Into itself returns, and circling doth embrace
> The highest mind, and heaven with like proportion drives.
> Thou with like cause dost make the souls and lesser lives,
> Fix them in chariots swift, and widely scatterest
> O'er heaven and earth; then at Thy fatherly behest
> They stream, like fire returning, back to Thee, their God."
> (*Consolatio*, III.m9.13-21)

The whole work of *Consolatio* is Boethius' course of return to his fatherland, guided by Lady Philosophy. From the beginning onward, Lady Philosophy repeatedly promises to bring her student home:

> "I think with little labour thou mayest be capable of felicity, and return to thy country in safety." (*Consolatio*, III.pXII.28-29)

> "I will show thee the way which will carry thee home. And I will also fasten wings upon thy mind, with which she may rouse herself, that, all perturbation being driven away, thou mayest return safely into thy country by my direction, by my path, and with my wings." (*Consolatio*, IV.pI.35-38)

> "I make haste to perform my promise, and to show thee the way by which thou mayest return to thy country." (*Consolatio*, V.pI.7-9)

In my arguments above, I have divided the process of the consolation into two parts. The first part is called Treatment 1. In this part, the questions discussed used to be known by Boethius, as could be shown from his earlier works, so Lady Philosophy helps her patient to recall them. By Treatment 1, the patient moves from numbness, sadness, and passivity to consciousness and initiative. He is back to the peace of his mind and realizes that true happiness is in one's heart, which no person can take away. The patient then diagnoses himself and he awaits further treatment from Lady Philosophy, which is the second part. The questions dealt with in the second part are not discussed or solved by Boethius, although he touched upon some of these questions in his earlier years.

In both Treatment 1 and Treatment 2, applications of basic logic and mathematics, and even theology could be found. Mathematics, especially arithmetic and music, are mainly applied in Treatment 1; basic logic is employed in both Treatment 1 and 2. These applications help render the organization of *Consolatio* extraordinary. Though Boethius' predecessors are the source of many doctrine set forth in this work, *Consolatio* can safely be said to be novel owing to its organization along the lines of basic mathematics and logic.[333]

[333] On the structure of *Consolatio*, cf. Rand (1904) and Ranneft (1983). On the influence of *Consolatio*, cf. Kneepkens (2003); Cropp (2012), "Boethius in Medieval France: Translations of the *De consolatione philosophiae* and Literary Influence," and Hehle (2012), "Boethius's Influence on German Literature to c.1500," and Love (2012), "The Latin Commentaries on Boethius's *De consolatione philosophiae* from the 9th to the 11th Centuries," and Szarmach (2012), "Boethius's Influence in Anglo-Saxon England: The Vernacular and the *De consolatione philosophiae*," in Kaylor and Phillips (eds.), pp. 319-356, and pp. 255-318, and pp. 75-134, and pp. 221-254; Elliott (2012); Lewis (1967), "Boethius," pp. 75-91; Magee (2003); Minnis (1987); Nauta (1997, 2003, and 2009); Wetherbee (2009), "*The Consolation* and Medieval Literature," in Marenbon (ed.), pp. 279-302; Sharples (1991), pp. 48-49.

V. Conclusion

The study of the oeuvre of Boethius from the perspective of his application of basic logic and mathematics has provided an answer to the question whether Boethius is an original thinker. Boethius is indeed an original thinker, not because the topics he touches upon are original or new, but because the way he discusses these topics is original. He displays originality in deciding whose ideas to follow and which logical or mathematical tools to use in order to obtain the clearest results in the theological or philosophical matter at hand. Even in a basic work as the *De institutione arithmetica,* his rendering differs from his sources by employing logic.[334]

Influenced by his father-in-law, Boethius adopted the mission to pass on Greek culture and thoughts to Rome. In his time, most of the body of Greek thought was preserved in the seven liberal arts (Section I.1). Grammar and rhetoric aside, Roman teaching in the liberal arts was rather weak, so Boethius dedicated himself to the remaining five disciplines, viz. the four mathematical disciplines (arithmetic, music, geometry and astronomy) and logic (Section I.2.1). He regarded mathematics and logic as the foundations of philosophy. Mathematics should be the beginning of the study of philosophy, for Boethius considered it an indispensable preliminary to the investigation of the physical world, and to the purification of the human soul that is needed to comprehend the divine (Section I.2.2). In addition, Boethius was convinced that logic plays a double role in philosophy, both as an instrument and as a part of philosophy (Section I.2.3).

Boethius' logical works include his commentaries on the logical works of Aristotle, Porphyry, and Cicero, and his monographs on division, syllogism and topics (Section II.1.1). In these works we find the theory of categories, division and definition, and also techniques for constructing arguments. Boethius uses all of these theories and techniques in his discussions. In this dissertation, I have focused on basic logic, including

[334] Cf. Section II.2.1.1.

theories of categories, division and definition (Section II.1.2), because these are used most often in Boethius' works. From Boethius' mathematical works, only the discussions of two of the four mathematical arts are (partly) extant, viz. arithmetic and music. Hence, my discussion of mathematics has been limited to these two arts. Again, because not every detail of basic arithmetical and musical knowledge is employed by Boethius, I have first introduced the basic ideas of arithmetic, such as the theory of equality, unity, number and divinity (Section II.2.1.3). Equality is like the matrix taking the force of a root, so it is prior to inequality from which all the other things are derived. This means that equality is the beginning of the world. The theory of equality is related to "sameness" and "otherness" which are the principles of the universe, and it is also related to virtue and vice. Although in his *Arithmetica* Boethius omits discussion of these topics, he has them in mind, for in his other works he does employ them. The idea of unity and the relationship between number and divinity are the basis of his consolation. From the *De institutione musica* I have taken two important ideas. The first is the division of people in relation to music. Boethius distinguishes musicians from performers and composers (Section II.2.2.2.b). In his point of view, performers act as slaves, and composers just take advantage of a certain natural ability but not so much of reason, so neither could comprehend musical knowledge. Unlike them, musicians in the right sense can make full use of reason in forming judgments in musical speculation. The second important idea taken from Boethius' *Musica* is his division of the kinds of music, viz. instrumental, human, and cosmic music. By the comparison between Boethius' extant work on music and that of Ptolemy, it is revealed that Boethius intended to follow Ptolemy's first book in both basic content and detail. In the introduction of the first book, Boethius promises to return to human and cosmic music which we cannot find in his extant work, so it is probable that in the original plan of *Musica* the first four books were based on Nicomachus and the last three on Ptolemy. Therefore, it is a good guess that if Boethius had finished the work on music, he was likely to follow Ptolemy and then he would have returned to human and cosmic music. Since in Boethius' extant *Musica* discussions on human music and cosmic music are few, I have introduced Ptolemy's thoughts on human music and cosmic music in order to grasp the applications of music

in Boethius' other works (Section II.2.2.3).

As two elementary disciplines, mathematics and logic may seem independent, but I have nevertheless explored their relationship in Boethius' works. Most applications of mathematics in logical works are examples to clarify logical notions, which renders the logic much easier to grasp (Section II.1); the applications of basic logic to the mathematical works render Boethius' *Arithmetica* and *Musica* even more tightly argued and well-organized than their sources (Section II.2.1.2 and Section II.2.2.2).

My research has shown that Boethius considered mathematics and logic so essential to philosophy that he has applied them throughout his theological and philosophical treatises. Boethius has written five theological tractates, among which *De fide* was probably written first. It can be regarded as the prologue of *De trinitate*, *Utrum pater*, and *Contra Eutychen et Nestorium* (Section III.1). The fundamental doctrines of the Catholic faith that it describes, concerning the Trinity (Section III.1.1) and the nature and person of Christ (Section III.1.2), serve as the foundation of Boethius' subsequent inquiries. There is no application of mathematics and basic logic in *De fide*, but in the subsequent four treatises, applications of mathematics and basic logic can be found. In order to preserve the spectrum of Boethius' arguments as a whole, I have followed the order of his discussions in each theological tractate and pointed out applications of mathematics and basic logic where they are used.

The chronological order of Boethius' theological treatises is related to his interest in political affairs. In the year 512, a letter from a number of Eastern bishops to Pope Symmachus set Boethius thinking about Christology. In Boethius' opinion, no one had solved or even touched upon the true distinction between a union formed *from* two natures and a union which consists *in* two natures. Hence he decided to inquire into this issue, and he wrote the treatise against Eutyches and Nestorius. Boethius begins his refutations with definitions of Nature and Person, for the inability to distinguish between these two terms has resulted in the errors of Nestorius and Eutyches (Section III.2.1). After clearing away the errors of Nestorius and Eutyches (Section III.2.2.1 and Section III.2.2.2), Boethius sets out the middle way of the Catholic Faith (Section III.2.3), just as the Eastern

bishops requested from Pope Symmachus. In the whole treatise, there is no application of mathematics, apart from the notion of virtues holding a middle place, which is also found in *Arithmetica*. There are some applications of basic logic, most notably the highly significant theory of definition, and notions taken from his commentaries on Porphyry's *Isagoge*, not least Porphyry's Tree.

In the year 519, Scythian monks from the East had offered the slogan, "one of the Trinity suffered in the flesh", which ignited questions about the Trinity. Hence Boethius had occasion to discuss doctrines of the Trinity in *De trinitate* and *Utrum pater*. The issue of the Trinity focuses on two questions. The first one regards the unity in God, and is discussed in *De trinitate* (Section III.3.1). Boethius builds his argument on three pillars: knowledge of otherness or difference; the theory of form; and the two kinds of number. The first and the last topic are closely related to arithmetic (Section III.3.1.1 and Section III.3.1.3); and discussions of all three pillars have notions in common with Boethius' commentary on Porphyry's *Isagoge* (Section III.3.1.2) and his commentary on Aristotle's *On Interpretation* (Section III.3.1.3). Applying arithmetic and logic, then, Boethius concludes that men are plural in number by the accidents. Accident and difference are essential to plurality, but in God there is no accident and no difference, so God is not plural. Father, Son, and Holy Spirit constitute one God and not three Gods. Father, Son, and Holy Spirit do not result in a plural number, because we are dealing with a repetition of unities in a question of concrete number; this does not produce plurality.

However, Father, Son, and Holy Spirit are the same but not identical, and here the second question concerning the Trinity arises. Boethius' discussions on the diversity in God make use of the category of relation (Section III.3.2). He explains that when relation is applied to God, the essential nature of God is not disturbed in any way, and in God, the Father is said to be the Father of the Son, for the Father begets the Son; and the Holy Spirit is said to be related to Father and Son in another way, for the Holy Spirit is begotten by the Father and the Son. Accordingly, Father, Son, and Holy Spirit are predicated of the Divinity relatively.

I have discussed Boethius' treatise *Quomodo substantiae* last, because

of its peculiar nature and its close relation to his last work, *Consolatio*. In *Quomodo substantiae*, in order to discuss how substances can be good in virtue of their existence without being absolute goods, Boethius employs geometrical methods. First he lays down nine rules as premises (Section III.4.1); then he uses the methods of synthesis and analysis to demonstrate the question. By means of synthesis, Boethius reaches the conclusion that things are good either by participation or by substance. However, by means of analysis, he rules out that things are good by participation (which means they are not good themselves) or by substance (which would make them absolute goods, a blasphemy) (Section III.4.2). Therefore, he arrives at the problem that things are in no way good, for they are good neither by participation nor by substance. In order to solve this problem, Boethius offers a process of mathematics similar to mental abstraction (Section III.4.3). He removes the presence of the Prime Good from the mind for a moment, and then the proposition which need be proved is: all things are good. But how could they possibly be good if they did not derive from the Prime Good? With the help of synthesis and analysis borrowed from geometry, and in imitation of the mathematical process of abstraction, Boethius comes to the conclusion that in the case of the Prime Good, Being and Goodness are identical, and It is good due to Its existence, irrespective of all conditions. Unlike the Prime Good, the absolute being of all things is not good under all circumstances, though they are good in virtue of their existence. For unless the True Good had produced them, these things could not actually exist.

From Boethius' discussions on theological issues, we can see that Boethius does not proceed as a theologian, as Augustine did. He does not concern himself with the common understanding of the Christian doctrines; moreover, Boethius does not use the term *theologia* in the ordinary sense of the word. He regards theology as a speculative discipline within philosophy. Therefore he uses a lot of logic and a bit of mathematical knowledge in his theological treatises. Boethius shows himself to be a philosopher, which is also confirmed in his *Consolatio*.

When facing death, Boethius seeks consolation from Philosophy. I have divided the *Consolatio Philosophiae* into two parts (Section IV.1). The first

three books are closely related, and constitute the first part of the medical treatment of the patient Boethius, which I labelled Treatment 1; the last two books concern the second part of the medical treatment, labelled Treatment 2. What is more, the discussions in the first three books consist of material that was already familiar to Boethius. The explanations for the three diagnostic reasons of the patient's illness can be found in his earlier works but due to the difficulties he is in, he has merely forgotten them. The discussions in the last two books are related to misconceptions concerning Providence, which are the root cause of his illness. The solution to these questions is as new to Boethius as it was to his contemporaries. In the *Consolatio*, Lady Philosophy says that the ancient problem of providence was pursued by Cicero, and that the patient in the *Consolatio* also inquired into this question for a long time, but neither of them solved this question — until now.

Thus, throughout his *Consolatio*, Boethius makes full use of the basic logic and mathematics he had explored in earlier works. In the first three books of the *Consolatio*, he abandons the pursuit of false happiness with the help of basic logic, although he merely provides the definition of happiness without explanation. While applying the theory of definition, Boethius argues that the definition of man as a rational and mortal animal is still incomplete (Section IV.2.1). Using musical therapy, he makes clear that God governs the world in a harmonious way, but he does not explain in what kind of way (Section IV.2.2.1). To answer the unsolved questions in the first three books Boethius applies arithmetic (Section IV.2.2.2). We have established that Boethius applies basic logic, music and arithmetic to complete Treatment 1 (Section IV.2.3). By means of these tools the patient of the *Consolatio* comes to himself and recovers from the numb state he was in at first.

In Treatment 2, the patient has sufficiently recovered to take the initiative in the discussion, and himself digs up the root cause of his illness, which is related to the issue of Providence (Section IV.3.1). By means of basic logic and geometrical methods of analysis and synthesis, Boethius explores difficult questions on providence and fate (Section IV.3.2.1.a), providence and chance (Section IV.3.2.1.b), and providence and free will

(Section IV.3.2.2) and finally solves them. The tools of philosophy empower the patient of the *Consolatio* to go back to his fatherland (Section IV.4), which is divine wisdom.

From the beginning of the Middle Ages, Boethius is considered as one of the leading thinkers. This is mainly due to his mission of preserving classical culture and thoughts for the Roman world. He carried out his mission by exploring basic ancient logic and mathematics and by showing how these tools can be decisive for theology, philosophy and personal happiness alike.[335]

[335] I have not discussed the influence of Boethius in my dissertation. On the influence of Boethius, see Black and Pomaro (2000); Bolton (1977); Daley (1984); D'Onofrio (1986); Evans (1978 and 1980); Gibson (1982); Hehle (2012), "Boethius's Influence on German Literature to c.1500," and Nash-Marshall (2012), "Boethius's Influence on Theology and Metaphysics to c.1500," and Szarmach (2012), "Boethius's Influence in Anglo-Saxon England: The Vernacular and the *De consolatione philosophiae*," and Troncarelli (2012), "Afterword: Boethius in Late Antiquity and the Early Middle Ages," in Kaylor and Phillips (eds.), pp. 255-318, pp. 163-192, pp. 221-254, and pp. 519-550; Humphrey (2010); Liebeschütz (1970); Patch (1935b); Woodcock (1943).

Bibliography

I. Boethius' Works

Mathematical Works

-(1867). *Anicii Manlii Torquati Severini Boetii: De Institutione Arithmetica (libri duo); De institutione musica (libri quinque)*. Friedlein, G. (ed.). Lipsiae: in aedibus B.G. Teubner.

-(1983). *Boethian Number Theory: A Translation of the De Institutione Arithmetica*. Masi, M. (transl.). Amsterdam: Rodopi.

-(1989). *Fundamentals of Music*. Bower, C.M. (transl.). New Haven [etc.]: Yale University Press.

-(1999). *Anicii Manlii Severini Boethii De Arithmetica*. Oosthout, H. and Schilling, J. (eds.). Turnhout: Brepols.

Logical Works

-*An. Manl. Sev. Boetii In Porphyrium Dialogi A Victorino Translati*. Partrologia Latina (Migne), Vol. 64 (LXIV): 9A-70D.

-*An. Manl. Sev. Boetii Commetaria In Porphyrium A Se Translatum*. Partrologia Latina (Migne), Vol. 64 (LXIV): 71A-158D.

-*An. Manl. Sev. Boethii In Categorias Aristotelis Libri Quatuor*. Partrologia Latina (Migne), Vol. 64 (LXIV): 159A-294C.

-*An. Manl. Sev. Boetii In Librum Aristotelis De Interpretatione Libri Duo: Editio Prima, Seu Minora Commentaria*. Partrologia Latina (Migne), Vol. 64 (LXIV): 293D-640A.

-*An. Manl. Sev. Boetii De Syllogismo Hypothetico Libri Duo*. Partrologia Latina (Migne), Vol. 64 (LXIV): 831B-876C.

-(1906). *Anicii Manlii Severini Boethii In Isagogen Porphyrii Commenta*. Brandt S. *Corpus Scriptorum Ecclesiasticorum Latinorum*, Vol. XXXXVIII.

Vindobonae [etc.]: Tempsky [etc.].

-(1957-1958). “The Second Edition of the Commentaries on the Isagoge of Porphyry, (Book I),” in McKeon, R. (ed.) *Selections From Medieval Philosophers (I): Augustine to Albert the Great*. New York: Charles Scribner’s Sons, pp. 65-99.

-(1978). *Boethius's ‘De Topicis Differentiis’*. Stump, E. (transl.). Ithaca; London: Cornell University Press.

-(1988). *Boethius's ‘In Ciceronis Topica’*. Stump, E. (transl.). Ithaca; London: Cornell University Press.

-(1988). “Boethius: On Division,” in Kretzmann, N. and Stump, E. (eds.). *The Cambridge Translations of Medieval Philosophical Texts (Vol. 1): Logic and the Philosophy of Language*. Cambridge: Cambridge University Press, pp. 11-38.

-(1994). “Boethius, From His *Second Commentary on Porphyry's Isagoge*,” in Spade, P.V. (transl. and ed.). *Five Texts on the Mediaeval Problem of Universals: Porphyry, Boethius, Abelard, Duns Scotus, Ockham*. Indianapolis, Ind. [etc.]: Hackett, pp. 20-25.

-(1998). *Anicii Manlii Severini Boethii De Divisione Liber*. Magee, J. (transl.). Leiden [etc.]: Brill.

-(1998). *Boethius On Aristotle's On Interpretation 9*. Kretzmann, N. (transl.). Ithaca, N.Y: Cornell University Press.

-(2008a). *Anicii Manlii Severini Boethii De Syllogismo Categorico*. Thörnqvist, C.T. (transl.). Gothenburg: University of Gothenburg.

-(2008b). *Anicii Manlii Severini Boethii Introductio ad Syllogismos Categoricos*. Thörnqvist, C.T. (transl.). Gothenburg: University of Gothenburg.

-(2010). *Boethius On Aristotle On Interpretation 1-3*. Smith, A. (transl.) London: Duckworth.

-(2011). *Boethius On Aristotle On Interpretation 4-6*. Smith, A. (transl.). London: Bristol Classical Press.

Theological and Philosophical Works

-(1926). *The Theological Tractates; The Consolation of Philosophy*. Stewart, H.F. and Rand, E.K. (transls.). London [etc.]: Heinemann [etc.].

-(1957). *Anicii Manlii Severini Boethii Philosophiae Consolatio*. Bieler, L. Turnholti: Brepols.

-(1999). *The Consolation of Philosophy*. Walsh, P.G. (transl.). Oxford: Clarendon Press.

-(2007). *Fortune's Prisoner: The Poems of Boethius's The Consolation of Philosophy*. Harpur, J. (transl.). London: Anvil Press Poetry.

II. Secondary Literature

-Albrecht, M. Von [etc.] (1997). *A History of Roman Literature: From Livius Andronicus to Boethius*. Leiden · New York · Köln: E.J. Brill.

-Alexander of Aphrodisias. (1991). *On Aristotle Prior Analytics 1.1-7*. Barnes, J.; Bobzien, S.; Flannery, K.; Ierodiakonou, K. (tansls.). London: Duckworth.

-Ammonius Hermiae (1996). *On Aristotle On Interpretation 1-8*. Blank, D. (transl.). London: Duckworth.

-Ammonius Hermiae (1998). *On Aristotle's On Interpretation 9*. Blank, D. (transl.). Ithaca, N.Y: Cornell University Press.

-Anatolios, K. (2011). *Retrieving Nicaea: The Development and Meaning of Trinitarian Doctrine*. Grand Rapids: Baker Academic.

-Aquinas, St. T. (1953). *The Division and Methods of the Sciences: Questions V and VI of His Commentary on the 'De Trinitate' of Boethius*. Maurer, A. (transl.). Toronto: Pontifical Institute of Mediaeval Studies.

-Aquinas, St. T. (1987). *Faith, Reason and Theology: Questions I-IV of His Commentary on the 'De Trinitate' of Boethius*. Maurer, A. (transl.). Toronto: Pontifical Institute of Mediaeval Studies.

-Aquinas, St. T. (2001). *An Exposition of the 'On the Hebdomads' of Boethius*. Schultz, J.L. and Synan, E.A. (transls.). Washington: CUA Press.

-Arber, A. (1942). "Spinoza and Boethius." *Isis* 34, pp. 399-403.

-Aristotle (1938). *Aristotle: 'Posterior Analytics' and 'Topica'*. Tredennick, H. and Forster, E.S. (transls.). Cambridge, Mass.: Harvard University Press; London: William Heinemann LTD.

-Aristotle (1963). *Aristotle's 'Categories' and 'De Interpretatione'*. Ackrill, J.L. (transl.). Oxford: Oxford University Press.

-Aristotle (1984a). *The Complete Works of Aristotle (Volume One)*. Barnes, J. (ed.). Princeton: Princeton University Press.

-Aristotle (1984b). *The Complete Works of Aristotle (Volume Two)*. Barnes, J.

(ed.). Princeton: Princeton University Press.

-Aristotle (1995). *Aristotle*. Ross, W.D. (transl.). London: Routledge.

-Aristotle (2009). *The Nicomachean Ethics*. Ross, D. (transl.). Oxford: Oxford University Press.

-Armstrong, D.M. (1978). *Universals and Scientific Realism I. Nominalism and Realism*. Cambridge: Cambridge University Press.

-Armstrong, D.M. (1989). *Universals: An Opinionated Introduction*. Boulder, CO: Westview Press.

-Astell, A.W. (1994). *Job, Boethius, and Epic Truth*. Ithaca and London: Cornell University Press.

-Asztalos, M. (1993). "Boethius as a Transmitter of Greek Logic to the Latin West: The *Categories*." *Harvard Studies in Classical Philology* 95, pp. 367-407.

-Asztalos, M. (2003). "Boethius on the *Categories*," in Galonnier, A. (ed.). *Boèce Ou La Chaîne Des Savoirs*. Louvain [etc.]: Peeters, pp. 195-205.

-Augustine (1886). *Nicene and Post-Nicene Fathers of the Christian Church — (Vol. I) The Confessions and Letters of St. Augustine, With a Sketch of His Life and Work*. Schaff, P. (ed.). New York: The Christian Literature Publishing Co..

-Augustine (1887). *Nicene and Post-Nicene Fathers of the Christian Church — (Vol. III) St. Augustine: On the Holy Trinity; Doctrinal Treatises; Moral Treatises*. Schaff, P. (ed.). New York: The Christian Literature Publishing Co..

-Augustine (1890). *Nicene and Post-Nicene Fathers of the Christian Church — (Vol. II) St. Augustine's City of God and Christian Doctrine*. Schaff, P. (ed.). New York: The Christian Literature Publishing Co..

-Ayres, L. (2004). *Nicaea and Its Legacy: An Approach to Fourth-Century Trinitarian Theology*. Oxford [etc.]: Oxford University Press.

-Baker, D. (1973). *Relations Between East and West in the Middle Ages*.

Edinburgh: Edinburgh University Press.

-Bark, W. (1944). "Theodoric vs. Boethius: Vindication and Apology." *The American Historical Review* 49(3), pp. 410-426.

-Bark, W. (1946). "Boethius' Fourth Tractate, the So-Called *De Fide Catholica*." *The Harvard Theological Review* 39(1), pp. 55-69.

-Barnes, J. (1981). "Boethius and the Study of Logic," in Gibson, M. (ed.), pp. 73-89.

-Barnes, M.R. (2011). "Latin Trinitarian Theology," in Phan, P.C. (ed.), pp. 70-84.

-Barnes, T.D. (1985). *Tertullian: A Historical and Literary Study*. Oxford: Clarendon Press.

-Barnish, S.J.B. (1990). "Maximian, Cassiodorus, Boethius, Theodahad: Literature, Philosophy, and Politics in Ostrogothic Italy." *Nottingham Medieval Studies* 34, pp. 16-32.

-Barrett, H.M. (1940). *Boethius: Some Aspects of His Times and Work*. Cambridge [etc.]: University Press.

-Behr, J. (2004). *The Nicene Faith* (2 vols.). Crestwood, NY: St. Vladimir's Seminary Press.

-Bernardine, M. (1949). "Boethius, A Christian Socrates." *The Classical Bulletin* 26, pp. 28-29.

-Betsey, A. (1991). "Boethius and *The Consolation of Philosophy*, or, How to be a Good Philosopher." *Ratio* 4(1), pp. 1-15.

-Black, R. and Pomaro, G. (2000). *Boethius' 'Consolation of Philosophy' in Italian Medieval and Renaissance Education: Schoolbooks and Their Glosses in Florentine Manuscripts*. Florence: SISMEL.

-Blumenthal, A. (1986). "New Muses: Poetry in Boethius's *Consolatio*." *Pacific Coast Philology* 21(1/2), pp. 25-29.

-Bobzien, S. (1998). "The Inadvertent Conception and Late Birth of the Free-Will Problem." *Phronesis* 43, pp. 133-175.

-Bobzien, S. (2000). "Did Epicurus Discover the Free Will Problem?" *Oxford Studies in Ancient Philosophy* 19, pp. 287-337.

-Bobzien, S. (2002). "A Greek Parallel to Boethius' *De Hypotheticis Syllogismis*." *Mnemosyne* 55, pp. 285-300.

-Bolton, D. (1977). "The Study of *The Consolation of Philosophy* in Anglo-Saxon England." *Archives D'Histoire Doctrinale et Littéraire du Moyen Age* 44, pp. 33-78.

-Bower, C.M. (1978). "Boethius and Nicomachus: An Essay Concerning the Sources of *De institutione musica*." *Vivarium* 16, pp. 1-45.

-Bower, C.M. (1981). "The Role of Boethius' *De Institutione Musica* in the Speculative Tradition of Western Musical Thought," in Masi, M. (ed.), pp. 157-174.

-Bower, C.M. (1984). "The Modes of Boethius." *The Journal of Musicology* 3(3), pp. 252-263.

-Bradshaw, D. (2004). *Aristotle East and West: Metaphysics and the Division of Christendom*. Cambridge: Cambridge University Press.

-Bradshaw, D. (2009). "The *Opuscula Sacra*: Boethius and Theology," in Marenbon, J. (ed.), pp. 105-128.

-Brancato, D. (2012). "Readers and Interpreters of the *Consolatio* in Italy, 1300-1500," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 357-412.

-Brennan, T. (2001). "Fate and Free Will in Stoicism." *Oxford Studies in Ancient Philosophy* 21, pp. 259-286.

-Brennan, T. (2005). *The Stoic Life*. Oxford: Clarendon Press.

-Bubacz, B.S. (1981). "Boethius and Augustine on Knowledge of the Physical World," in Obertello, L. (ed.). *Congresso Internazionale di Studi Boeziani*. Roma: Herder, pp. 287-296.

-Bury, J.B. (1958). *History of the Later Roman Empire From the Death of Theodosius I. to the Death of Justinian (Volume 1)*. New York: Dover Publications.

-Caldwell, J. (1981). "The *De Institutione Arithmetica* and the *De Institutione Musica*," in Gibson, M. (ed.), pp. 135-154.

-Cally, P. and Nauta, L. (1996). "Platonic and Cartesian Philosophy in the Commentary on Boethius's *Consolatio Philosophiae*." *British Journal for the History of Philosophy* 4(1), pp. 79-100.

-Cameron, M. (2009). "Boethius on Utterances, Understanding and Reality," in Marenbon, J. (ed.), pp. 85-102.

-Casey, G. (1987). "An Explication of the *De Hebdomadibus* of Boethius." *The Thomist* 51(3), pp. 419-434.

-Casey, J.P. (2012). "Boethius's Works on Logic in the Middle Ages," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 193-220.

-Cassiodorus (1946). *An Introduction to Divine and Human Readings*. Jones, L.W. (transl.). New York: Columbia University Press.

-Cassiodorus (1992/2006). *Variae*. Barnish, S.J.B. (transl.). Liverpool: Liverpool University Press.

-Chadwick, H. (1980a). "The Authenticity of Boethius' Fourth Tractate *De Fide Catholica*." *Journal of Theological Studies NS* 31, pp. 551-556.

-Chadwick, H. (1980b). "*Theta* on Philosophy's Dress in Boethius." *Medium Aevum* 49, pp. 175-179.

-Chadwick, H. (1981a). "Introduction," in Gibson, M. (ed.), pp. 1-12.

-Chadwick, H. (1981b). *Boethius: The Consolations of Music, Logic, Theology, and Philosophy*. Oxford: Clarendon Press.

-Chamberlain, D.S. (1970). "Philosophy of Music in *The Consolatio* of Boethius." *Speculum* 45(1), pp. 80-97.

-Charanis, P. (1974). *Church and State in the Later Roman Empire: The Religious Policy of Anastasius the First, 491-518*. Thessalonikē: Kentron Buzantinōn Ereunōn.

-Cioffari, V. (1935). *Fortune and Fate From Democritus to St. Thomas Aquinas*. New York: Columbria University.

-Clarke, M.L. (1971). *Higher Education in the Ancient World*. London: Routledge & Kegan Paul.

-Coolidge, J.S. (1963). "Boethius and 'That Last Infirmity of Noble Mind'." *The Philological Quarterly* 42(2), pp. 176-188.

-Cooper, K. and Santo, M.D. (2009). "Boethius, Gregory the Great and the Christian 'Afterlife' of Classical Dialogue," in Goldhill, S. (ed.). *The End of Dialogue in Antiquity*. Cambridge: University of Cambridge, pp. 173-190.

-Cooper, L. (1928). *A Concordance of Boethius: The Five Theological Tractates and 'The Consolation of Philosophy'*. Cambridge and Massachusetts: The Medieval Academy of America.

-Correia, M. (2009). "The Syllogistic Theory of Boethius." *Ancient Philosophy* 29(2), pp. 391-407.

-Coster, C.H. and Patch, H.R. (1948). "Procopius and Boethius." *Speculum* 23(02), pp. 284-287.

-Coster, C.H. (1968). "The Fall of Boethius: His Character," in Coster, C.H. *Later Roman Studies*. Cambridge, Mass.: Harvard University Press, pp. 54-103.

-Courcelle, P. (1969). *Late Latin Writers and Their Greek Sources*. Wedeck, H.E. (transl.). Cambridge and Massachusetts: Harvard University Press.

-Courcelle, P. (1981). "Boethius, Lady Philosophy, and the Representations of the Muses," in Masi, M. (ed.), pp. 211-218.

-Cousins, A.D. (2002). "Augustine, Boethius and the Fortune Verses of Thomas More." *Moreana* 39(149), pp. 17-40.

-Crabbe, A. (1981a). "Literary Designs in the *De Consolatione Philosophiae*," in Gibson, M. (ed.), pp. 237-277.

-Crabbe, A. (1981b). "Anamnesis and Mythology in the *De Consolatione Philosophiae*," in Obertello, L. (ed.). *Congresso Internazionale di Studi Boeziani*. Roma: Herder, pp. 311-325.

-Craig, W.L. (1988a). *The Problem of Divine Foreknowledge and Future*

Contingents From Aristotle to Suarez. Leiden [etc.]: E.J. Brill.

-Craig, W.L. (1988b). "Boethius on Theological Fatalism." *Ephemerides Theologicae Lovanienses* 64, pp. 324-347.

-Croke, B. and Harries, J. (1982). *Religious Conflict in Fourth-Century Rome: A Documentary Study*. Australia: Sydney University Press.

-Cropp, G.M. (2012). "Boethius in Medieval France: Translations of the *De consolatione philosophiae* and Literary Influence," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 319-356.

-Cross, R. (2012). "Form and Universal in Boethius." *British Journal for the History of Philosophy* 20(3), pp. 439-458.

-Crouse, R. (1982). "The Doctrine of Creation in Boethius: The *De Hebdomadibus* and *The Consolatio*," in Livingstone, E.A. (ed.). *Studia Patristica (Vol. XVII)*. Oxford [etc.]: Pergamon Press, Part One, pp. 417-421.

-Curley, T.F. (1986). "How to Read *The Consolation of Philosophy*." *Interpretation* 14(2&3), pp. 211-264.

-Curley, T.F. (1987). "*The Consolation of Philosophy* as a Work of Literature." *The American Journal of Philology* 108(2), pp. 343-367.

-Daley, B.E. (1984). "Boethius' Theological Tracts and Early Byzantine Scholasticism." *Mediaeval Studies* 46, pp. 158-191.

-Davies, M. (1983). "Boethius and Others on Divine Foreknowledge." *Pacific Philosophical Quarterly* 64(4), pp. 313.

-Davis, R. (1989). *The Book of Pontiffs (Liber Pontificalis)*. Liverpool: Liverpool University Press.

-Davis, S.; Kendall, D.; O'Collins, G. (eds.) (1999/2002). *The Trinity: An Interdisciplinary Symposium on the Trinity*. Oxford [etc.]: Oxford University Press.

-Demacopoulos, G. and Papanikolaou, A. (eds.) (2008). *Orthodox Readings of Augustine*. Grand Rapids: Eerdmans.

-De Rijk, L.M. (1964a). "On the Chronology of Boethius' Works on Logic I." *Vivarium* 2, pp. 1-49.

-De Rijk, L.M. (1964b). "On the Chronology of Boethius' Works on Logic II." *Vivarium* 2, pp. 122-162.

-De Rijk, L.M. (1988). "On Boethius's Notion of Being," in Kretzmann, N. (ed.). *Meaning and Inference in Medieval Philosophy: Studies in Memory of Jan Pinborg.* Dordrecht: Kluwer, pp. 1-29.

-De Rijk, L.M. (2003). "Boethius on *De Interpretatione* (Chapter 3): Is He a Reliable Guide?" in Galonnier, A. (ed.). *Boèce Ou La Chaîne Des Savoirs*. Louvain [etc.]: Peeters, pp. 207-227.

-De Vogel, C.J. (1973). "The Problem of Philosophy and Christian Faith in Boethius' *Consolatio*." in den Boer, W.; van der Nat, P.G.; Sicking, C.M.J.; van Winden, J.C.M. (eds.). *Romanitas et Christianitas*. Amsterdam and London: North-Holland Publishing Company, pp. 357-370.

-Donato, A. (2013a). *Boethius' Consolation of Philosophy as a Product of Late Antiquity*. London [etc.]: Bloomsbury Academic.

-Donato, A. (2013b). "Forgetfulness and Misology in Boethius's *Consolation of Philosophy*." *British Journal for the History of Philosophy* 21(3), pp. 463-485.

-D'Onofrio, G. (1986). "Dialectic and Theology: Boethius' *Opuscula Sacra* and Their Early Medieval Readers." *Studi Medievali* 27, pp. 45-67.

-Dougherty, M.V. (2004). "The Problem of *Humana Natura* in *The Consolatio Philosophiae* of Boethius." *American Catholic Philosophical Quarterly* 78(2), pp. 273-292.

-Dronke, P. (1994). *Verse With Prose From Petronius to Dante: The Art and Scope of the Mixed Form*. Cambridge, MS. [etc.]: Harvard University Press.

-Dubs, K.E. (1981). "Providence, Fate, and Chance: Boethian Philosophy in The Lord of the Rings." *Twentieth Century Literature* 27(1), pp. 34-42.

-Duclow, D.F. (1979). "Perspective and Therapy in Boethius's *Consolation of Philosophy*." *The Journal of Medicine and Philosophy* 4(3), pp. 334-343.

-Dunn, G.D. (2004). *Tertullian*. London [etc.]: Routledge.

-Dürr, K. (1951). *The Propositional Logic of Boethius*. Amsterdam: North-Holland Publishing Company.

-Dvornik, F. (1951). "Emperors, Popes, and General Councils." *Dumbarton Oaks Papers* 6, pp. 1-23.

-Dwyer, R.D. (1976). *Boethian Fictions: Narratives in the Medieval French Versions of 'The Consolatio Philosophiae'*. Cambridge, Mass.: Medieval Academy of America.

-Eastwood, B. (2002). *The Revival of Planetary Astronomy in Carolingian and Post-Carolingian Europe*. Aldershot: Ashgate.

-Ebbesen, S. (1990). "Boethius as an Aristotelian Commentator," in Sorabji, R. (ed.). *Aristotle Transformed*. London: Duckworth, pp. 373-392.

-Ebbesen, S. (2003). "Boethius on the Metaphysics of Words," in Galonnier, A. (ed.). *Boèce Ou La Chaîne Des Savoirs*. Louvain [etc.]: Peeter, pp. 257-275.

-Ebbesen, S. (2009). "The Aristotelian Commentator," in Marenbon, J. (ed.), pp. 34-55.

-Eliade, M. (2005). *The Encyclopedia of Religion (Volume 2)*. New York: Macmillan Publishing Company.

-Elliott, E. (2012). *Remembering Boethius: Writing Aristocratic Identity in Late Medieval French and English Literatures*. Farnham [etc.]: Ashgate.

-Erismann, C. (2009). "The Medieval Fortunes of the *Opuscula Sacra*," in Marenbon, J. (ed.), pp. 155-177.

-Evangeliou, C. (1985). "Aristotle's Doctrine of Predicables and Porphyry's *Isagoge*." *Journal of the History of Philosophy* 23(1), pp. 15-34.

-Evangeliou. C. (1988). *Aristotle's Categories and Porphyry*. Leiden: E. J. Brill.

-Evans, G.R. (1975). "The Influence of Quadrivium Studies in the Eleventh- and Twelfth-Century Schools." *Journal of Medieval History* 1(2), pp.

151-164.

-Evans, G.R. (1978). “Introductions to Boethius’s *Arithmetica* of the Tenth to the Fourteenth Century.” *History of Science* 16(1), pp. 22-41.

-Evans, G.R. (1980). “Boethian and Euclidean Axiomatic Method in the Theology of the Later Twelfth Century.” *Archives Internationales D' Histoire des Sciences* 30, pp. 36-52.

-Evans, G.R. (1981). “Boethius’ Geometry and the Four Ways.” *Centaurus* 25(2), pp. 161-165.

-Evans, G.R. (1982a). *Augustine on Evil.* Cambridge: Cambridge University Press.

-Evans, G.R. (1982b). “Thierry of Chartres and the Unity of Boethius’ Thought,” in Livingstone, E.A. (ed.). *Studia Patristica (Vol. XVII)*. Oxford [etc.]: Pergamon Press, Part One, pp. 440-445.

-Evans, J. (2004). “Boethius on Modality and Future Contingents.” *American Catholic Philosophical Quarterly* 78(2), pp. 247-272.

-Fletcher, R. (1998). *The Barbarian Conversion: From Paganism to Christianity*. New York: H. Holt and Co..

-Ford, L.S. (1968). “Boethius and Whitehead on Time and Eternity.” *International Philosophical Quarterly* 8(1), pp. 38-67.

-Fouracre, P. (ed.) (2005). *The New Cambridge Medieval History I. c.500-700*. Cambridge: Cambridge University Press.

-Fuhrmann, M. and Gruber, J. (eds.) (1984). *Boethius*. Darmstadt: Wissenschaftliche Buchgesellschaft.

-Gerson, L.P. (ed.) (2010). *The Cambridge History of Philosophy in Late Antiquity*. Cambridge [etc.]: Cambridge University Press.

-Gibson, M. (ed.) (1981). *Boethius: His Life, Thought and Influence*. Oxford: Blackwell.

-Gibson, M. (1982). “Boethius in the Carolingian Schools.” *Transactions of the Royal Historical Society* 5(32), pp. 43-56.

-Gillett, A. (2003). *Envoys and Political Communication in the Late Antique West, 411-533*. Cambridge: Cambridge University Press.

-Gracia, J.J.E. (1981). "Boethius and the Problem of Individuation in the Commentaries on the *Isagoge*," in Obertello, L. (ed.). *Congresso Internazionale di Studi Boeziani*. Roma: Herder, pp. 169-182.

-Grant, R.M. (1986). *Gods and the One God*. Philadelphia: The Westminster Press.

-Gray, P.T.R. (1979). *The Defense of Chalcedon in the East (451-553)*. Leiden: Brill.

-Green-Pedersen, N.J. (1984). *The Tradition of the Topics in the Middle Ages*. Munich: Philosophia Verlag.

-Grillmeier, A. (1975). *Christ in Christian Tradition (Volume One)*. Bowden, J. (transl.). London [etc.]: Mowbray.

-Grillmeier, A. (1987). *Christ in Christian Tradition (Volume Two: Part One)*. Allen, P. and Cawte, J. (transls.). London [etc.]: Mowbray.

-Gualtieri, A. (1971) "Lady Philosophy in Boethius and Dante." *Comparative Literature* 23, pp. 141-150.

-Guillaumin, J.Y. (2012). "Boethius's *De institutione arithmetica* and Its Influence on Posterity," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 135-162.

-Hall, D.C. (1992). *The Trinity: An Analysis of St. Thomas Aquinas' 'Expositio' of the 'De Trinitate' of Boethius*. Leiden: Brill.

-Hankey, W.J. (1981). "The *De Trinitate* of St. Boethius and the Structure of the *Summa Theologiae* of St. Thomas Aquinas," in Obertello, L. (ed.). *Congresso Internazionale di Studi Boeziani*. Roma: Herder, pp. 367-375.

-Hanson, R.P.C. (1988). *The Search for the Christian Doctrine of God: The Arian Controversy (318-381)*. Edinburgh: T. & T. Clark.

-Häring, N.M. (ed.) (1966). *The Commentaries on Boethius by Gilbert of Poitiers*. Toronto: Pontifical Institute of Mediaeval Studies.

-Häring, N.M. (1971). *Commentaries on Boethius by Thierry of Chartres and His School*. Toronto: Pontifical Institute of Mediaeval Studies.

-Heath, T.L. (1956). *The Thirteen Books of Euclid's Elements (Vol.1)* (Second Edition). New York: Dover Publications, INC.

-Hehle, C. (2012). "Boethius's Influence on German Literature to c.1500," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 255-318.

-Holloway, J.B. (1981). " 'The Asse to the Harpe': Boethian Music in Chaucer," in Masi, M. (ed.), pp. 175-186.

-Huffman, C.A. (2005). *Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King*. Cambridge [etc.]: Cambridge University Press.

-Humphrey, I. (2010). *Boethius — His Influence on the European Unity of Culture: From Alcuin of York (804) to Thierry of Chartres (1154)*. Nordhausen: Bautz.

-Ierodiakonou, K. (1997). "Aristotle's Logic: An Instrument, Not a Part of Philosophy," in Peonidis, F. (eds.). *Aristotle on Logic, Language and Science*. Thessaloniki: Sakkoulas Publications, 33-53.

-Inglis, J. (2002). *Medieval Philosophy and the Classical Tradition: In Islam, Judaism and Christianity*. London [etc.]: Routledge Curzon.

-Isocrates (1929). *Isocrates (Vol. II)*. Norlin, G. (transl.). London: Willian Heinemann Ltd; New Yordk: G.P. Putnam's Sons.

-Jefferson, B.L. (1917). *Chaucer and 'The Consolation of Philosophy' of Boethius*. Princeton: Princeton University Press.

-Johnson, I. (2012). "Making the *Consolatio* in Middle English," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 413-446.

-Kahn, C.H. (2001). *Pythagoras and the Pythagoreans: A Brief History*. Indianapolis/Cambridge: Hackett Publishing Company, Inc.

-Kaylor, N.H. (1992). *The Medieval 'Consolation of Philosophy' An Annotated Bibliography*. New York: Garland.

-Kaylor, N.H. and Philips, P.E. (2007). *New Directions in Boethian Studies*.

Kalamazoo, Michigan: Medieval Institute Publications, Western Michigan University.

-Kaylor, N.H. and Phillips, P.E. (eds.) (2012). *A Companion to Boethius in the Middle Ages*. Leiden [etc.]: Brill.

-Kelly, J.N.D. (1977). *Early Christian Doctrines* (Fifth Revised Edition). London: Adam & Charles Black.

-Kenny, A. (1969). "Divine Foreknowledge and Human Freedom," in Kenny, A. (ed.). *Aquinas: A Collection of Critical Essays*. London: Macmillan, pp. 255-270.

-Kibre, P. (1981). "The Boethian *De Institutione Arithmetica* and the *Quadrivium* in the Thirteen Century University Milieu et Paris," in Masi, M. (ed.), pp. 67-80.

-Kijewska, A. (2003). "Mathematics as a Preparation for Theology: Boethius, Eriugena, Thierry of Chartres," in Galonnier, A. (ed.). *Boèce Ou La Chaîne Des Savoirs*. Louvain [etc.]: Peeters, pp. 625-647.

-Kirkby, H. (1981). "The Scholar and His Public," in Gibson (ed.), pp. 44-69.

-Kline, M. (1972). *Mathematical Thought From Ancient to Modern Times*. New York: Oxford University Press.

-Kneale, M. And W. (1984). *The Development of Logic*. Oxford: Clarendon Press.

-Kneepkens, C.H. (2003). "The Reception of Boethius' *De Consolatione* in the Later Middle Ages: The Wolfenbüttel Quaestiones and Buridan's *Quaestiones* on the Ethics," in Galonnier, A. (ed.). *Boèce Ou La Chaîne Des Savoirs*. Louvain [etc.]: Peeters, pp. 713-739.

-Knuuttila, S. (2010) "Medieval Commentators on Future Contingents in *De Interpretatione 9*." *Vivarium* 48, pp. 75-95.

-Koterski, J.W. (2004). "Boethius and the Theological Origins of the Concept of Person." *American Catholic Philosophical Quarterly* 78(2), pp. 203-224.

-Kretzmann, N. and Stump, E. (eds.). (1988). *The Cambridge Translations of Medieval Philosophical Texts (Vol. 1): Logic and the Philosophy of Language*. Cambridge: Cambridge University Press.

-LaChance, P.J. (2004). "Boethius on Human Freedom." *American Catholic Philosophical Quarterly* 78(2), pp. 309-328.

-Lerer, S. (1985). *Boethius and Dialogue: Literary Method in 'The Consolation of Philosophy'*. Princeton University Press.

-Levin, F.R. (2009). *Greek Reflections on the Nature of Music*. Cambridge [etc.]: Cambridge University Press.

-Lewis, C.S. (1967). *The Discarded Image: An Introduction to Medieval and Renaissance Literature*. Cambridge [etc.]: Cambridge University Press.

-Lewry, O. (1981). "Boethian Logic in the Medieval West," in Gibson, M. (ed.), pp. 90-134.

-Liebeschütz, H. (1970), "Boethius and the Legacy of Antiquity," in Armstrong, A.H. (ed.). *The Cambridge History of Later Greek and Early Medieval Philosophy*. Cambridge: Cambridge University Press, pp. 538-564.

-Long, A.A. and Sedley, D.N. (1987). *The Hellenistic Philosophers*. Cambridge: Cambridge University Press.

-Lossky, V. (1976). *The Mystical Theology of the Eastern Church*. Crestwood: St Vladimir's Seminary Press.

-Love, R.C. (2012). "The Latin Commentaries on Boethius's *De consolatione philosophiae* From the 9th to the 11th Centuries," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 75-134.

-MacDonald, S. (1988). "Boethius's Claim That all Substances are Good." *Archiv für Geschichte der Philosophie* 3, pp. 245-279.

-MacDonald, S. (ed.). (1991). *Being and Goodness: The Concept of the Good in Metaphysics and Philosophical Theology*. Ithaca, NY: Cornell University Press.

-Mackenzie, M.M. (1985). *Plato on Punishment*. Berkely/Los Angeles/London: University of California Press.

-Magee, J. (1987). "The Boethian Wheels of Fortune and Fate." *Mediaeval Studies* 49, pp. 524-533.

-Magee, J. (1988). "Note on Boethius, *Consolatio* I.1,5; 3,7: A New Biblical Parallel." *Vigiliae Christianae* 42(1), pp. 79-82.

-Magee, J. (1989). *Boethius on Signification and Mind*. Leiden and New York: Brill.

-Magee, J. (2003). "Boethius' Anapestic Dimeters (Acatalectic), with Regard to the Structure and Argument of *The Consolatio*," in Galonnier, A. (ed.). *Boèce Ou La Chaîne Des Savoirs*. Louvain [etc.]: Peeter, pp. 147-169.

-Magee, J. (2005). "Boethius' *Consolatio* and the Theme of Roman Liberty." *Phoenix* 59(3/4), pp. 348-364.

-Magee, J. (2009). "The Good and Morality: *Consolatio* 2-4," in Marenbon, J. (ed.), pp. 181-206.

-Magee, J. (2010). "On the Composition and Sources of Boethius' Second *Peri Hermeneias* Commentary." *Vivarium* 48(1), pp. 7-55.

-Mansi, G.D. (1901-1927). *Sacrorum conciliorum nova et amplissima collectio, cujus Johannes Dominicus Mansi et post ipsius mortem Florentius et Venetianus editores ab anno 1758 ad annum 1798 priores triginta unum tomos ediderunt, nunc autem continuatat et absoluta: tomus 0, 1-53 (V.8)*. Parisiis: expensis H. Welter.

-Marenbon, J. (1982). "Making Sense of the *De Trinitate*: Boethius and Some of His Medieval Interpreters," in Livingstone, E.A. (ed.). *Studia Patristica (Vol. XVII)*. Oxford [etc.]: Pergamon Press, Part One, pp. 446-452.

-Marenbon, J. (2003). *Boethius*. Oxford: Oxford University Press.

-Marenbon, J. (ed.) (2009). *The Cambridge Companion to Boethius*. Cambridge: Cambridge University Press.

-Mair, J. (1981). "The Text of the *Opuscula Sacra*," in Gibson, M. (ed.), pp. 206-213.

-Marshall, M.H. (1950). "Boethius' Definition of *Persona* and Mediaeval Understanding of the Roman Theater." *Speculumn* 25(4), pp. 471-482.

-Martin, C.J. (2009). "The Logical Textbooks and Their Influence," in Marenbon, J. (ed.), pp. 56-84.

-Masi, M. (1979). "The Liberal Arts and Gerardus Ruffus' Commentary on the Boethian *De Arithmetica*." *The Sixteenth Century Journal* X(2), pp. 23-41.

-Masi, M. (ed.) (1981a). *Boethius and the Liberal Arts: A Collection of Essays*. Berne·Francfort/M·Las Vegas: Peter Lang.

-Masi, M. (1981b). "Boethius' *De Institutione Arithmetica* in the Context of Medieval Mathematics," in Obertello, L. (ed.). *Congresso Internazionale di Studi Boeziani*. Roma: Herder, pp. 263-272.

-Matthews, J. (1981). "Anicius Manlius Severinus Boethius," in Gibson, M. (ed.), pp. 15-43.

-McCluskey, S.C. (2012). "Boethius's Astronomy and Cosmology," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 47-74.

-McGeachy, J.A. (1942). *Quintus Aurelius Symmachus and the Senatorial Aristocracy of the West*. Chicago: The University of Chicago Libraries.

-McGuckin, J.A. (2011). "The Trinity in the Greek Fathers," in Phan, P.C. (ed.), pp. 49-69.

-McKeon, R. (ed.) (1957-1958). *Selections From Medieval Philosophers (I): Augustine to Albert the Great*. New York: Charles Scribner's Sons.

-McInerny, R. (1990). *Boethius and Aquinas*. Washington, DC: Catholic University of America Press.

-McInerny, R. and Bourke, V.J. (1992). "Boethius and Aquinas." *Speculum* 67(2), pp. 452-454.

-McKim, D.K. (1988). *Theological Turning Points: Major Issues in*

Christian Thought. Louisville, Kentucky: John Knox Press.

-McKinlay, A.P. (1907). "Stylistic Tests and the Chronology of the Works of Boethius." *Harvard Studies in Classical Philology* 18, pp. 123-156.

-McManners, J. (2001). *Oxford Illustrated History of Christianity*. Oxford: Oxford University Press.

-Merlan, P. (1968). "Ammonius Hermiae, Zacharias Scholasticus and Boethius." *Greek, Roman and Byzantine Studies* 9(2), pp. 193-203.

-Meyendorff, J. (1975). *Christ in Eastern Christian Thought*. Crestwood, NY: St. Vladimir's Seminary Press.

-Meyendorff, J. (1983). *Byzantine Theology: Historical Trends and Doctrinal Themes* (2nd ed.). New York: Fordham University Press.

-Minnis, A.J. (ed.) (1987). *The Medieval Boethius: Studies in the Vernacular Translations of 'De Consolatione Philosophiae'*. Cambridge: D. S. Brewer.

-Moorhead, J. (1992). *Theoderic in Italy*. Oxford: Oxford University Press.

-Moorhead, J. (2009). "Boethius' Life and the World of Late Antique Philosophy," in Marenbon, J. (ed.), pp. 13-33.

-Morton, C. (1981). "Boethius in Pavia: The Tradition and the Scholars," in Obertello, L. (ed.). *Congresso Internazionale di Studi Boeziani*. Roma: Herder, pp. 49-58.

-Moss, C.B. (1943). *The Christian Faith: An Introduction to Dogmatic Theology*. New York: Morehouse-Gorham.

-Moyer, A.E. (2012). "The Quadrivium and the Decline of Boethian Influence," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 479-518.

-Mueller, I. (1969). "Stoic and Peripatetic Logic." *Archiv für Geschichte der Philosophie*, 51(2), pp. 173-187.

-Nash-Marshall, S. (2000). *Participation and the Good: A Study in Boethian Metaphysics*. New York: Crossroads Publishing.

-Nash-Marshall, S. (2012). "Boethius's Influence on Theology and

Metaphysics to c.1500," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 163-192.

-Nauta, L. (1996). "Platonic and Cartesian Philosophy in the Commentary on Boethius' *Consolatio Philosophiae* by Pierre Cally." *British Journal for the History of Philosophy* 4(1), pp. 79-100.

-Nauta, L. and Hoenen, M.J.F.M. (ed.) (1997). *Boethius in the Middle Ages: Latin and Vernacular Traditions of 'The Consolatio Philosophiae'*. Leiden · New York · Köln: Brill.

-Nauta, L. (2003). "Some Aspects of Boethius' *Consolatio Philosophiae* in the Renaissance," in Galonnier, A. (ed.). *Boèce Ou La Chaîne Des Savoirs*. Louvain [etc.]: Peeters, pp. 767-778.

-Nauta, L. (2009). "*The Consolation*: The Latin Commentary," in Marenbon, J. (ed.), pp. 255-278.

-Newman, B. (2003). *God and the Goddesses: Vision, Poetry, and Belief in the Middle Ages*. Philadelphia: University of Pennsylvania Press.

-Nicomachus of Gerasa (1938). *Introduction to Arithmetic*. D'Ooge, M.L. (transl.). New York: University of Michigan Press.

-Nicomachus of Gerasa (1994). *The Manual of Harmonics of Nicomachus the Pythagorean*. Levin, F. (transl.). Michigan: Phanes Press.

-Obertello, L. (1981). "Proclus, Ammonius and Boethius on Divine Foreknowledge." *Dionysius* 5, 127-164.

-Obertello, L. (2003). "Ammonius of Hermias, Zacharias Scholasticus and Boethius: Eternity of God and/or Time?" in Galonnier, A. (ed.). *Boèce Ou La Chaîne Des Savoirs*. Louvain [etc.]: Peeters, pp. 465-479.

-O'Daly, G.J.P. (1991). *The Poetry of Boethius*. London: Duckworth.

-Olmsted, W.R. (1989). "Philosophical Inquiry and Religious Transformation in Boethius's *The Consolation of Philosophy* and Augustine's *Confessions*." *The Journal of Religion* 69(1), pp. 14-35.

-O'Meara, D.J. (1989). *Pythagoras Revived: Mathematics and Philosophy*

in Late Antiquity. Oxford: Clarendon Press.

-O'Meara, D.J. (2003). *Platonopolis: Platonic Political Philosophy in Late Antiquity*. Oxford: Clarendon Press.

-Patch, H.R. (1929). "Fate in Boethius and the Neoplatonists." *Speculum* 4(1), pp. 62-72.

-Patch, H.R. (1935a). "Necessity in Boethius and the Neoplatonists." *Speculum* 10(4), pp. 393-404.

-Patch, H.R. (1935b). *The Tradition of Boethius: A Study of His Importance in Medieval Culture*. New York: Oxford University Press.

-Patch, H.R. (1947). "The Beginnings of the Legend of Boethius." *Speculum* 22(03), pp. 443-445.

-Payne, F.A. (1981). *Chaucer and Menippean Satire*. Madison: University of Wisconsin Press.

-Pelikan, J. (1975). *The Christian Tradition: A History of the Development of Doctrine. 1: The Emergence of the Catholic Tradition (100-600)*. Chicago [etc.]: University of Chicago Press.

-Phan, P.C. (ed.) (2011). *The Cambridge Companion to the Trinity*. Cambridge: Cambridge University Press.

-Phillips, P.E. (2012). "Anicius Manlius Severinus Boethius: A Chronology and Selected Annotated Bibliography," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 551-590.

-Pingree, D. (1981). "Boethius' Geometry and Astronomy," in Gibson, M. (ed.), pp. 155-161.

-Pizzani, U. (1981). "The Influence of the *De Institutione Musica* of Boethius up to Gerbert D'Aurillac: A Tentative Contribution," in Masi, M. (ed.), pp. 97-156.

-Plato (1993). *Republic*. Waterfield, R. (transl.). Oxford and New York: Oxford University Press.

-Plato (1997). *Plato: Complete Works*. Cooper, J.M. (ed.). Cambridge:

Hackett Publishing Company.

-Plato (1998). *Gorgias and Phaedrus*. Nichols, J.H.(Jr) (transl.). Impressum Ithaca, NY [etc.]: Cornell University Press.

-Plotinus (1966). *Plotinus. I: Porphyry on the Life of Plotinus and the Order of His Books: Enneads I. 1-9*. Armstrong, A. H. (transl.). London: Heinemann; Cambridge, Mass.: Harvard University Press.

-Plotinus (1991). *The Enneads*. MacKenna, S. (transl.). London [etc.]: Penguin Books.

-Porphyry (1992). *Porphyry on Aristotle 'Categories'*. Strange, S.K. (transl.). London: Duckworth.

-Porphyry (2003). *Introduction*. Barnes, J. (transl.). Oxford: Clarendon Press.

-Prestige, G.L. (1952). *God in Patristic Thought* (2nd ed.). London: SPCK.

-Prior, A.N. (1953). "The Logic of Negative Terms in Boethius." *Franciscan Studies* 13(1), pp. 1-16.

-Proclus (1970). *A Commentary on the First Book of Euclid's Elements*. Morrow, G.R. (transl.). Princeton, N.J.: Princeton University Press.

- Ptolemy (2000). *Harmonics*. Solomon, J. (transl.). Leiden · Boston · Köln: Brill.

-Rand, E.K. (1904). "On the Composition of Boethius' *Consolatio Philosophiae*." *Harvard Studies in Classical Philology* 15, pp. 1-28.

-Rand, E.K. (1928). *Founders of the Middle Ages*. Harvard: Harvard University Press.

-Rand, E.K. (1934). "The Supposed Commentary of John the Scot on the *Opuscula Sacra* of Boethius." *Revue Néoscolastique de Philosophie* 36, pp. 67-77.

-Ranneft, D.M. (1983). *Structures in Boethius 'Consolation of Philosophy'*. Ann Arbor, Mich.: University Microfilms International.

-Reiss, E. (1981). "The Fall of Boethius and the Fiction of *The Consolatio Philosophiae*." *The Classical Journal* 77(1), pp. 37-47.

-Reiss, E. (1982). *Boethius*. Boston: Twayne Publishers, cop.

-Relihan, J.C. (2007). *The Prisoner's Philosophy: Life and Death in Boethius's 'Consolation'*. Notre Dame, IN: University of Notre Dame Press.

-Richards, J. (1979). *The Popes and the Papacy in the Early Middle Ages, 476-752*. London [etc.]: Routledge and Kegan Paul.

-Rimple, M.T. (2012). "The Enduring Legacy of Boethian Harmony," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 447-478.

-Rist, J.M. (1967). *Plotinus: The Road to Reality*. Cambridge: Cambridge University Press.

-Rosheger, J.P. (2001). "Boethius and the Paradoxical Mode of Theological Discourse." *American Catholic Philosophical Quarterly* 75(3), pp. 323-344.

-Rudavsky, T. (ed.). (1985). *Divine Omniscience and Omnipotence in Medieval Philosophy.* Dordrecht: Reidel.

-Scarry, E. (1980). "The Well-Rounded Sphere: The Metaphysical Structure of *The Consolation of Philosophy*," in Eckhardt, C.D. (ed.). *Essays in the Numerical Criticism of Medieval Literature*. Lewisburg, Pa: Bucknell University Press, pp. 91-140.

-Schrade, L. (1947). "Music in the Philosophy of Boethius." *The Musical Quarterly* 33(2), pp. 188-200.

-Scott, A.B. (1993). "Extracts From Trevet's Commentary on Boethius: Texts and Translation," in Minnis, A.J. (ed.). *Chaucer's Boece and the Medieval Tradition of Boethius*. Cambridge: D. S. Brewer, pp. 35-81.

-Shanzer, D.R. (1984). "The Death of Boethius and *The Consolation of Philosophy*." *Hermes* 112, pp. 352-366.

-Shanzer, D.R. (1986). *A Philosophical and Literary Commentary on Martianus Capella's De Nuptiis Philologiae et Mercurii Liber 1*. Berkeley: University of California Press.

-Shanzer, D.R. (2009). “Interpreting *The Consolation*,” in Marenbon, J. (ed.), pp. 241-242.

-Sharples, R.W. (1991). *Cicero: 'On Fate' & Boethius: 'The Consolation of Philosophy' IV.5-7, V*. Warminster, England: Aris &Philips Ltd.

-Sharples, R.W. (2009). “Fate, Prescience and Free Will,” in Marenbon, J. (ed.), pp. 207-227.

-Shiel, J. (1957). “Boethius and Andronicus of Rhodes.” *Vigiliae Christianae* 11(3), pp. 179-185.

-Shiel, J. (1958). “Boethius’ Commentaries on Aristotle.” *Medieval and Renaissance Studies* 4, pp. 217-244.

-Shiel, J. (1964). “Boethius the Hellenist.” *History Today* 14(10), pp. 678-686.

-Shiel, J. (1974). “Brighton, Sussex, Boethius and Eudemus.” *Vivarium* 12, pp. 14-17.

-Shiel, J. (1982). “A Recent Discovery: Boethius’ Notes on the *Prior Analytics*.” *Vivarium* 20, pp. 128-141.

-Silk, E.T. (1939). “Boethius’ *Consolatio Philosophiae* as a Sequel to Augustine’s *Dialogues* and *Soliloquia*.” *Harvard Theological Review* 32, pp. 19-39.

-Solère, J.L. (2003). “Bien, Cercles et Hebdomades: Formes et Raisonnement Chez Boèce et Proclus,” in Galonnier, A. (ed.). *Boèce Ou La Chaîne Des Savoirs*. Louvain [etc.]: Peeters, pp. 55-110.

-Solmsen, F. (1944). “Boethius and the History of the *Organon*.” *The American Journal of Philology* 65(1), pp. 69-74.

-Sorabji, R. (ed.). (1990). *Aristotle Transformed: The Ancient Commentators and Their Influence*. London: Duckworth.

-Sorabji, R. (2004a). *The Philosophy of the Commentators 200-600 AD — A Sourcebook Volume 2: Physics*. London: Duckworth.

-Sorabji, R. (2004b). *The Philosophy of the Commentators 200-600 AD — A*

Sourcebook Volume 3: Logic and Metaphysics. London: Duckworth.

-Spade, P.V. (transl. and ed.) (1994). *Five Texts on the Mediaeval Problem of Universals: Porphyry, Boethius, Abelard, Duns Scotus, Ockham*. Indianapolis, Ind,. [etc.]: Hackett.

-Speca, A. (2001). *Hypothetical Syllogistic and Stoic Logic*. Leiden: Brill.

-Stahl, W.H. (1962). *Roman Science: Origins, Development, and Influence to the Later Middle Ages*. Madison: The University of Wisconsin Press.

-Stahl, W.H.; Johnson, R. and Burge, E. (1971). *The Quadrivium of Martianus Capella: Latin Traditions in the Mathematical Sciences, 50 B.C.-A.D. 1250*. New York: Columbia University Press.

-Stahl, W.H.; Johnson, R. and Burge, E. (1971-1977). *Martianus Capella and the Seven Liberal Arts* (2 vols.). New York: Columbia University Press.

-Starnes, C.J. (1981). "Boethius and the Development of Christian Humanism: The Theology of *The Consolatio*," in Obertello, L. (ed.). *Congresso Internazionale di Studi Boeziani*. Roma: Herder, pp. 27-38.

-Stead, C. (1994). *Philosophy in Christian Antiquity*. Cambridge: Cambridge University Press.

-Steel, C. (1998). "Proclus on the Existence of Evil." *Proceedings of the Boston Area Colloquium of Ancient Philosophy* 14(1), pp. 83-102.

-Stewart, H.F. (1974). *Boethius: An Essay*. New York: Burt Franklin.

-Stump, E. (1974). "Boethius's Works on the Topics." *Vivarium* 12, pp. 77-93.

-Stump, E. (1981a). "Boethius and Peter of Spain on the Topics," in Masi, M. (ed.), pp. 35-50.

-Stump, E. (1981b). "Boethius' Theory of Topics and Its Place in Early Scholastic Logic," in Obertello, L. (ed.). *Congresso Internazionale di Studi Boeziani*. Roma: Herder, pp. 249-262.

-Stump, E. (1983). "Hamartia in Christian Belief: Boethius on the Trinity", in Stump, D. V. et al. (ed.). *Hamartia: The Concept of Error in the Western*

Tradition. New York and Toronto: Edwin Mellen Press.

-Stump, E. (1987). "Boethius's *In Ciceronis Topica* and Stoic Logic," in Wippel, J. (ed.). *Studies in Medieval Philosophy*. Washington: Catholic University of America, pp. 1-22.

-Sulowski, F.J. (1961). "The Sources of Boethius' *De Consolatione Philosophiae*." *Sophia* 29, pp. 67-94.

-Suto, T. (2012). *Boethius on Mind, Grammar and Logic: A Study of Boethius' Commentaries on 'Peri Hermeneias'*. Leiden·Boston: Brill.

-Sweeney, E.C. (2006). *Logic, Theology, and Poetry in Boethius, Abelard, and Alan of Lille: Words in the Absence of Things*. Basingstoke: Palgrave Macmillan.

-Szarmach, P.E. (2012). "Boethius's Influence in Anglo-Saxon England: The Vernacular and the *De consolatione philosophiae*," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 221-254.

-Tarán, L. (1969). "Asclepius of Tralles Commentary of Nicomachus' *Introduction to Arithmetic*." *Transactions of the American Philosophical Society* 59(4), pp5-81.

-Tertullian (1920). *Against Praxeas*. Souter, A. (transl.). London: Society for Promoting Christian Knowledge; New York: The Macmillan Company.

-Theon of Smyrna (1979). *Mathematics Useful for Understanding Plato*. Lawlor, R. and D. (transl.). San Diego: Wizards Bookshelf.

-Townsend, W.T. (1937). "Councils Held Under Pope Symmachus." *Church History* 6(3), pp. 233-259.

-Troncarelli, F. (2012). "Afterword: Boethius in Late Antiquity and the Early Middle Ages," in Kaylor, N.H. and Phillips, P.E. (eds.), pp. 519-550.

-Uhlfelder, M.L. (1981). "The Role of the Liberal Arts in Boethius' *Consolatio*," in Masi, M. (ed.), pp. 17-34.

-Varvis, S. (1991) '*The Consolation' of Boethius: An Analytical Inquiry into His Intellectual Processes and Goals*. San Francisco, California: Mellen

Research University Press.

-Walz, M.D. (2011). "Stoicism as Anesthesia: Philosophy's 'Gentler Remedies' in Boethius's *Consolation*." International Philosophical Quarterly 51(4), pp. 501-519.

-Weinbrot, H. (2005). *Menippean Satire Reconsidered: From Antiquity to the Eighteenth Century*. Baltimore: Johns Hopkins University Press.

-Wetherbee, W. (2009). "*The Consolation* and Medieval Literature," in Marenbon, J. (ed.), pp. 279-302.

-White, A. (1981). "Boethius in the Medieval Quadrivium," in Gibson, M. (ed.), pp. 162-205.

-Williams, R. (2001). *Arius: Heresy and Tradition* (2nd ed.). London: SCM Press.

-Wiltshire, S.F. (1971). "Boethius and the *Summum Bonum*." *The Classical Journal* 67(3), pp. 216-220.

-Wolfson, H.A. (1956). *The Philosophy of the Church Fathers: Faith, Trinity, Incarnation (Vol. 1)*. Cambridge, Massachusetts: Harvard University press.

-Woodcock, E. (1943). "The Influence of Boethius on Musical Thought." *Bulletin of the American Musicological Society* 7, pp. 30-32.

-Zhmud, L. (2012). *Pythagoras and the Early Pythagoreans*. Windle, K. and Ireland, R. (transls.). Oxford: Oxford University Press.

Index of Name

Nederlandse Samenvatting

Dit proefschrift is een studie van het werk van Boethius (480-525 n.Chr.) vanuit het gezichtspunt van basale logica en wiskunde en hun toepassingen. Vanuit dit gezichtspunt verschijnt Boethius als de auteur van een samenhangend oeuvre, niet als de logicus, de theoloog, de literator, of de wegbereider voor de Middeleeuwen zoals zijn werk in het verleden vaak is opgesplitst vanuit verschillende academische disciplines. Een eerste resultaat van deze benadering is een antwoord op de vraag naar de originaliteit van Boethius. Veel inhoudelijke aspecten van zijn werk zijn niet nieuw, maar de organisatie en de heldere articulatie van begrippen en argumenten zijn dat wel. Juist daarop berust zijn betekenis voor zijn tijd, en voor de eeuwen na hem.

Boethius nam de taak op zich om de Griekse cultuur en Grieks gedachtengoed door te geven aan de Romeinse cultuur, in het Latijn. Het Griekse cultuurgoed was in zijn tijd vooral voorhanden in de vorm van de 7 vrije kunsten: logica, grammatica, rhetorica (later *trivium* genoemd); getallenleer, muziek, meetkunde, en astronomie (door Boethius *quadrivium* gedoopt). Met name de logica en de vakken van het *quadrivium* waren ondervertegenwoordigd in het Romeinse curriculum, terwijl Boethius juist deze vakken als fundamenteel beschouwde voor de filosofie. Wiskunde is voor Boethius noodzakelijk voor de reiniging van de ziel, en het begin van de studie van de fysische werkelijkheid. Logica is zowel instrument van de filosofie als een integraal onderdeel ervan. Vandaar het plan van Boethius om juist in deze vakken Latijnse inleidingen en commentaren te bieden.

Van de wiskundige werken van Boethius zijn alleen zijn inleidingen op de getallenleer en, gedeeltelijk, de muziek overgeleverd. Tot deze teksten is mijn discussie van het *quadrivium* dan ook beperkt. Binnen deze terreinen worden de theorie van gelijkheid, éénheid, getal en goddelijkheid het meest gebruikt in andere werken, alsmede het onderscheid tussen uitvoerende musici en componisten, en de verdeling van muziek in instrumenteel, menselijk (harmonie van ziel en lichaam), en kosmisch (harmonie van de

kosmos). Helaas zijn de op Ptolemaeus gebaseerde laatste delen van Boethius' *De musica*, waarin de menselijke en kosmische muziek uitgebreid zouden moeten zijn behandeld, niet overgeleverd. Om toepassingen van deze theorie in de *Consolatio* van een passende context te voorzien heb ik direct geput uit het werk van Ptolemaeus.

De logische werken van Boethius omvatten zijn commentaren op de logica van Aristoteles, Porphyrius en Cicero, en monografieën over classificatie, syllogisme en 'gemeenplaatsen' (*topoi*). Met name de theorie van categorieën, classificatie en definitie, en delen van de argumentatieleer spelen in alle geschriften van Boethius een rol. In dit proefschrift heb ik mij geconcentreerd op die aspecten van de basale logica, die de meeste toepassingen vinden.

Een vergelijking van de inleidende werken op de getallenleer en muziek met hun bronnen (Nicomachus van Gerasa en de genoemde Ptolemaeus) toont Boethius' terughoudendheid ten aanzien van de meer speculatieve aspecten van de Platoonse getallenleer, ten gunste van de meer wiskundige aspecten ervan. Wel heeft Boethius een korte analogie tussen getalsverhoudingen en politieke verhoudingen ingevoegd: een indicatie van zijn politieke werk in Rome. Boethius gebruikt logische terminologie om zijn bronnenmateriaal strakker te ordenen dan in het origineel het geval was. In de inleidende logische werken van Boethius vinden we een lichte neiging om logische argumenten vooral met wiskundige voorbeelden te illustreren, of om wiskundige voorbeelden die elders in Aristoteles of Porphyrius zijn te vinden op meer plaatsen in te zetten.

Meer uitgebreide toepassingen van basale logica en wiskunde vinden we in de theologische tractaten, die de filosoof Boethius schreef vanwege zijn bemoeienis met de politieke verwikkelingen tussen Rome en de Oosterse kerk, die voortkwamen uit onenigheid over de Christologie. In de meest waarschijnlijke chronologische volgorde, die in ieder geval de ideale leesvolgorde is, worden *De fide*, *Contra Eutychen et Nestorium, De trinitate,* en *Utrum pater* behandeld. *De fide* beschrijft de christelijke orthodoxie ten tijde van Boethius, zonder vermelding van logica of wiskunde. De *Contra Eutychen* slaagt erin door heldere definities van de omstreden termen 'natuur' en 'persoon' de extreme posities van Eutyches resp. Nestorius te

weerleggen, en de positie van de Kerk als gulden middenweg te schetsen, precies zoals Oosterse kerkleiders in hun brief aan Paus Symmachus hadden gevraagd. Hier is het vooral de basale logica die het werk doet.

De trinitate en *Utrum pater* behandelen de vraag hoe de éénheid van God samengaat met de diversiteit van de Drieëenheid. Met het gelijknamige werk van Augustinus in het achterhoofd, benut Boethius hier op eigen wijze de getallenleer voor het onderscheid van 'verschil' in termen van genus, species of getal; en het onderscheid tussen abstract en concreet getal. Uit de logische geschriften benut hij de Boom van Porphyrius uit diens *Isagoge*, en argumenten uit de commentaren op Aristoteles ´ *De interpretatione*. Het resultaat is dat, volgens Boethius, God niet veelvuldig is omdat Hij geen accidenten of differentiae kent, en omdat de herhaling van éénheden in het onderscheid tussen Vader, Zoon, en Heilige Geest in de context van concrete getallen per definitie geen veelvoud oplevert. Uit *Utrum pater* blijkt dat de diversiteit van de Drieëenheid berust op een onderscheid tussen de onderlinge *relaties* tussen Vader, Zoon en Geest, zodat de dienovereenkomstige relatieve predicatie de enkelvoudige essentie van God niet kan bedreigen.

De *Quomodo substantiae* is een meer filosofische beschouwing van de vraag hoe substanties goed kunnen zijn op grond van hun bestaan zonder zelf absoluut goed te zijn. Naar toon en onderwerp vormt dit werk een goede overgang naar Boethius' laatste geschrift, de *Consolatio.* In *Quomodo substantiae* gebruikt Boethius argumentatievormen uit de geometrie, vooral een specifieke vorm van analyse en synthese, om het probleem te verhelderen. Middels synthese concludeert Boethius dat iets ofwel door participatie ofwel van substantie goed moet zijn. Middels analyse laat hij zien dat beide opties onaanvaardbaar zijn. De oplossing wordt geboden door een gedachtenexperiment dat Boethius vergelijkt met de mentale abstractie die in de wiskunde gebruikelijk is. Het resultaat is dat Zijn en Goedheid alleen in het hoogste Goede samenvallen. Al het andere is goed op grond van hun bestaan dat immers gegrond is in het hoogste Goede, zonder dat hun eigen zijn in alle opzichten en omstandigheden goed is.

Boethius' discussie van theologische kwestie toont hem niet als theoloog voor wie het begrip van de Christelijke leer het doel is, maar als

filosoof die de theologie ziet als onderdeel van de theoretische filosofie. Deze visie legitimeert het gebruik van filosofische, i.c. wiskundige en logische, middelen.

In de beroemde *Consolatio Philosophiae* tenslotte figureert de auteur als patiënt die door het verlies van zijn politieke positie en naderende executie de wanhoop nabij is. Vrouwe Philosophia biedt hem de filosofie aan als therapie, en wel in twee achtereenvolgende behandelingen die verschillend van aard zijn. In het eerste deel (boek I-III) neemt Vrouwe Philosophia het voortouw en laat haar hulpeloze patiënt zijn vroegere inzichten in de basale logica weer inzien. Dat helpt hem om een onjuiste opvatting van geluk te verwerpen, zij het nog zonder een adequate definitie van geluk te kunnen geven. De patiënt gaat ook inzien dat de standaard definitie van de mens als rationeel sterfelijk wezen incompleet is. Via muziektherapie wordt de patiënt duidelijk gemaakt dat God de wereld op harmonieuze wijze bestuurt, zij het zonder precies uit te werken hoe dit in zijn werk gaat. Zo passeren thema's uit Boethius' voorafgaande werk opnieuw de revue, en wordt de grote betekenis van basale logica en wiskunde voor het welbevinden van de mens opnieuw bevestigd.

In het tweede deel van de *Consolatio* (boek IV-V) is de patiënt dankzij deze filosofische middelen reeds zover genezen dat hij zelf het initiatief kan nemen. De vragen die nog open staan worden door de patiënt herleid tot de fundamentele vraag naar de aard en betekenis van de voorzienigheid. Opnieuw worden analyse en synthese, en de theorie van definitie, gebruikt om voorzienigheid te onderscheiden van noodlot resp. toeval, en om de relatie tussen voorzienigheid en vrije wil te bepalen. Als zo ook deze fundamentele vraag naar de voorzienigheid een bevredigende oplossing heeft gevonden wordt duidelijk hoe de filosofie, middels de instrumenten van de basale logica en de wiskunde, erin slaagt om de aanvankelijk wanhopige patiënt Boethius terug te leiden naar zijn vaderland, de goddelijke wijsheid.

Het succes van Boethius' oeuvre in de Middeleeuwen is de vervulling geworden van zijn missie om de Griekse cultuur en het Griekse gedachtengoed door te geven aan de Romeinse wereld. Dit proefschrift heeft laten zien dat dit oeuvre duidelijk het stempel draagt van Boethius in de

consequente en overtuigende toepassing van basale logica en wiskunde in filosofische vragen van allerlei aard: theologisch, kosmologisch, en ethisch. Juist hierdoor werd het oeuvre van de filosoof Boethius een goudmijn voor de Middeleeuwse cultuur.

Curriculum Vitae

Qi Wang was born on April 21st 1982, in Huludao (China). After six years at the elementary school and seven years at the high school, she went to Dongbei University of Finance and Economics from 2002 to 2006. There she majored in English and received her Bachelor's Degree. In 2007, she began to study philosophy at Beijing Normal University. During three-year master study, she organized and participated in several conferences. In addition, she gained twice the first prize of "National Science and Technology of Philosophy and Cross-disciplinary Forum for Graduate Students" (2008 and 2010), in 2009 she received the Beijing Normal University Comprehensive Scholarship of Academic Excellence 2008-2009, and in 2010 she gained the second prize of Chongxue Award of College of Philosophy & Sociology, Beijing Normal University. In 2010, she received her Master's Degree of Philosophy with a thesis on Boethius' quadrivium and at the same time she got Academic Excellence Award of the Outstanding Graduates. After finishing her master study, in 2010 she started her PhD research at the Institute for Philosophy at Leiden University, funded by a China Research Council scholarship. During her years as a PhD candidate, she presented papers at international conferences in Belgium, Netherlands, China, and Greece.