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Pulses in singularly perturbed reaction-diffusion systems

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**Pulses in
singularly perturbed
reaction-diffusion systems**

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VOLGENS BESLUIT VAN HET COLLEGE VOOR PROMOTIES
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Jan Hanlo, *De Mus*

Table of contents

1	Introduction and Summary	1
1.1	How should I read this?	1
1.2	Concepts	3
1.2.1	Patterns	3
1.2.2	Dynamical systems	6
1.2.3	Patterns in reaction-diffusion systems	8
1.2.4	Perturbations	11
1.2.5	What is this thesis about?	17
1.3	Methods	17
1.3.1	Existence and construction	20
1.3.2	Stability	30
1.3.3	Dynamics	40
1.4	Thesis structure and main results	46
2	Pulses in a slowly nonlinear Gierer-Meinhardt equation	49
2.1	Introduction	49
2.2	Pulse construction	58
2.2.1	Geometric analysis	58
2.3	Pulse stability: analysis	64
2.3.1	The Evans function and its decomposition	65
2.3.2	The slow solution $\phi_{s,L}$ outside I_f	67
2.3.3	The fast components of $\phi_{s,L}$ inside I_f	70
2.3.4	The slow transmission function $t_{s,+}(\lambda, \varepsilon)$	73
2.4	Pulse stability: results	75
2.4.1	Immediate results: $\sigma < 0$ and $\gamma \downarrow 0$	76
2.4.2	Varying α and investigating the role of d	78
2.5	Numerical simulations	84

3	Pulses in a general reaction-diffusion system	89
3.1	Introduction	89
3.1.1	The model	94
3.2	The existence of pulses	98
3.3	Linearization and the reduced problems	108
3.3.1	The linear stability problem	108
3.3.2	The homogeneous fast reduced Sturm-Liouville problem	110
3.3.3	The inhomogeneous fast reduced Sturm-Liouville problem	115
3.3.4	The intermediate, slowly varying problem	117
3.4	The Evans function and the NLEP procedure	119
3.4.1	The construction of the Evans function	119
3.4.2	The NLEP procedure	121
3.5	Implications of Theorem 3.21: (in)stability results	124
3.5.1	The structure of $B_-(\lambda)$	125
3.5.2	The trivial eigenvalue $\lambda = 0$	130
3.5.3	Further instability results	133
3.6	Discussion	137
4	Hopf bifurcations for localised pulses	139
4.1	Introduction	139
4.2	Preliminaries	141
4.2.1	Existence	142
4.2.2	Linearisation and eigenfunctions	145
4.2.3	Translational symmetry and the trivial eigenvalue	148
4.3	The Hopf centre manifold	148
4.3.1	Choosing a function space	149
4.3.2	Foliation of the centre manifold along the translational eigenmode	151
4.4	Unfolding the Hopf bifurcation	155
4.4.1	Normal form reduction	156
4.4.2	Calculating the first Lyapunov coefficient to leading order	159
4.5	An alternative approach	164
4.5.1	Expanding the centre manifold	164
4.5.2	The coefficient c_1 (4.92)	168
4.6	Application: the slowly nonlinear Gierer-Meinhardt equation	171
4.6.1	Analytical preliminaries	172
4.6.2	Hopf bifurcations	173
4.7	Discussion	177

References	181
Samenvatting	187
Dankwoord	201
Curriculum vitae	203

