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## **Pulses in singularly perturbed reaction-diffusion systems**

Veerman, F.W.J.

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**Pulses in  
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**Jan Hanlo, *De Mus***



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